

Design optimization-under-uncertainty of a forward swept wing unmanned aerial vehicle using SAMURAI

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Abstract

In this paper the design optimization-under-uncertainty of a forward swept wing (FSW) blended wing body (BWB) unmanned aerial vehicle (UAV) is examined. Conventional BWBs are often tailless, which leads to a backward swept wing to ensure longitudinal static stability. This in turn can induce flow separation at the tip, leading to a loss of lift, controllability and the appearance of a nose-up pitching moment. A possible solution to this problem is a conceptual redesign by introducing a forward swept wing, which is inherently free of tip-stall, but needs a careful design in order to be controllable. However, fixed wing UAVs are often produced by means of direct injection expanded foam moulding, which is characterized by not negligible production tolerances. This lead to a reliability-based robust design optimization problem, for which a novel framework is employed: SAMURAI. Firstly, the method accounts for computational cost by means of surrogate modelling, an analytical treatment of the problem and an asynchronous updating scheme that balances design space exploration and objective exploitation. Secondly, the method treats the problem as a multi-objective problem, which leads to a Pareto front of robust and reliable designs. The result is a novel series of UAV designs that are inherently free of tip stall, perform robustly and meet the stability requirements with the target reliability obtained with a computationally feasible budget.

Keywords

Optimization-under-uncertainty, Bayesian optimization, unmanned aerial vehicles, forward swept wing

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Introduction

The widespread use of *unmanned aerial vehicles* (UAV) has become hard to overlook, amongst other thanks to its versatility in applicability. This has led to an enormous research boost in that field.^{6,3,12,10,4,1,8,7,5,11,9,2} In order to improve endurance, range, efficiency and the payload capacities of the drones, the ability to optimize these complex structures that are often subjected to variations in geometry due to production tolerances, changes in operating conditions and complex aerodynamic behaviour is fundamental. This in turn may lead to the development of novel optimization schemes and new aerial structures with a decreased drag force, increased lift force, delayed stall angle, reduced noise and vibrations, further extending their capabilities.^{17,16,14,15,13}

A common returning design choice for fixed-wing UAVs is the *blended wing body* (BWB): a more often than not tailless configuration with the fuselage, the aircraft's main body, integrated in the wing.^{21,23,24,18,22,19,25–27,20} The absence of a horizontal tailplane combined with profiles

that have positive camber enforces the sweeping and twisting of the wing to obtain longitudinal static stability, the intrinsic desire of the airplane to correct minor changes in its *angle of attack* (α).^{28,16,14,29} When swept backward, the sweeping of the wings introduces a pressure gradient on the wing normal to the free stream, decreasing from the root to the tip. This leads to a more rapid growth of the thickness of the boundary layer towards the tip and results in a higher likelihood of flow separation at that location. Furthermore, the vortex-shedding caused by the inboard sections results in a decreased downwash and consequently a higher loading at the tip. In addition, tapering of

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the wing, often present in order to reduce the structural load at the root and improve the effectiveness of the control surfaces, locally introduces a higher lift coefficient at the tip, also leading to a local increased likelihood of separation. Finally, the BWB design is typically characterized by wing tip devices that take up the job of providing lateral and directional stability.³⁰ These winglets or end-plates introduce secondary flows at the tip due to the interaction of the boundary layers on them and on the wing, also resulting in a flow more prone to separation at the tip. This all leads to a high probability of the occurrence of a special case of stall at higher angles of attack: tip-stall.^{31,32} As its name indicates, the tips of the wings start stalling first. Since the tips are generally located behind the *center of gravity* (CoG), tip stalling will result in a pitch-up moment, thus pulling the plane further into stall. Even more dangerous is the asymmetric initiation of separation at the tips, possibly leading to spin.

Even though a number of solutions were introduced to tackle the problem of tip stalling, only a handful can be used on the blended wing body aerial vehicle after its manufacturing (typically addressed as aerodynamic afterthoughts).³³ An alternative approach is a conceptual redesign (typically addressed as an aerodynamic aforethought) of the aircraft to make it inherently free of the occurrence of tip-stall. This can for example be obtained by means of sweeping the wings forward.^{34,35} From an aerodynamic point of view, the forward swept wing introduces a pressure gradient on the wing pointing in the opposite direction of backward swept wings, resulting in a higher likelihood of flow separation at the root. Since the root of the wing is now typically found behind the CoG, it will still introduce a nose-up pitching moment. However, the control surfaces are often found near the tips of the wing or are not part of the wing, for example when they are part of the conventional T-tail or through the use of the more exotic V-tail,^{37,36} as will be discussed further on. Overall, this leads to smaller tip-stall and spin tendencies and thus a safer aircraft. Secondly, the vortex-shedding results in a lift distribution that more closely resembles the idealized elliptical lift distribution, characterized by a minimal induced drag and thus a highly efficient planform. Thirdly, regarding stability, longitudinal static stability is now conventionally obtained by twisting the tips of the wing nose-up, as opposed to the conventional backward swept wing, which leads to a higher lift coefficient of the aircraft as a whole and must not be compensated.^{38,39} As such, the overall efficiency of the wing is higher. However, the sweeping of the wings in a forward direction leads to directional instability: in the event of side slip, a higher lift and higher drag is generated on the side opposite of the side slip direction, since its chord is more directly aligned with the flow, which introduces an aggravating yawing moment. Winglets or end-plates can no longer be placed at the wing tips to mitigate this issue, since they

are typically found in front of the CoG. Therefore additional stabilizers must be introduced on the aircraft. Secondly, the sweeping of the wings in a forward direction can lead to aeroelastic divergence, which has been actively been examined in recent years.^{41,40,42,43,46,48,47,44,49,45}

The production of a small (span $\approx 1m$) unmanned aerial vehicle on an industrial scale is typically performed through direct injection expanded foam moulding using *expanded polypropylene* (EPP). This process has the tendency to shrink the wing 2% on average compared to the mould shape. This might lead to a varying performance, which must be accounted for during the design optimization. Since the publication of the 2002 NASA report on the needs and opportunities for uncertainty-based multidisciplinary design methods,⁵⁰ a large body of research has been conducted to account for its current shortcomings. Broadly speaking *optimization-under-uncertainty* (OUU) can be divided into two classes: *robust design optimization* (RDO), which treats objective variability, and *reliability-based design optimization* (RBDO), which treats constraint variability.^{52,51} In this paper the merger of the two classes, *reliability-based robust design optimization* (RBRDO) is applied, which can be defined as a problem that “*seeks a design that is relatively insensitive to small changes in the uncertain quantities*” and “*that has a probability of failure that is less than some acceptable (invariably small) value*”.^{50,52} The importance of OUU has been displayed by a number of review papers that have been written in the last decade.^{55,54,57,56,53}

However, the classic approaches to OUU remain severely limited by the computational cost that is related to the uncertainty propagation.⁵⁰ OUU is therefore prone to, as⁵⁸ puts it: “*violating a fundamental tenet of numerical optimization - that one should avoid doing too much work until one nears a solution*”. In an attempt to account for the shortcomings of conventional approaches, a number of meta-model assisted methods have appeared that may lead to a significant time reduction.^{53,56} Among the different surrogate-modelling techniques that exist, particular attention has been given to Gaussian process interpolation, also called Kriging.⁵⁹ On the one hand this can be explained by the model’s flexibility, being a parameter-free method, and on the other hand this can be attributed to the model’s ability to quantify the (epistemic) uncertainty related to a prediction. The latter has inspired to formulation of different acquisition functions that are tailored to simultaneously pursue design space exploration and objective exploitation, leading to so-called sequential or Bayesian optimization mythologies. The most well known of these acquisition functions is the *expected improvement* (EI), which led to the *efficient global optimization* (EGO) framework.^{61,60} A recent *surrogate-assisted* (SA) OUU method is SAMURAI, combining a sequential multi-objective approach of the RDO problem with a sequential single-loop formulation of the RBDO.⁶² The multi-objective treatment

presents the designer with a Pareto front of robust and reliable designs with varying degrees of robustness. A novel acquisition function was formulated through the assessment of the predictive error of robustness and reliability measures. Furthermore, the efficiency was enhanced by formulating the acquisition function in such a way that it permits the selection of a multitude of infill points at once, while a multitude of infill points is still being evaluated. In doing so full use is made of parallelized computational power.

Consequently, this work goes beyond the state-of-the-art by designing a series of forward swept wing blended wing body unmanned aerial vehicles with varying degrees of robustness that are inherently free of tip-stall and robust and reliable in regards to the production tolerances and operating conditions by means of SAMURAI. In *Problem Description* the problem is formalized and the parameterization of the planform is presented. Subsequently, in *Methodology* the methodological tools to solve the problem at hand are presented. Finally, in *Results and Discussion* the resulting designs are discussed.

Problem description

Conceptual aerodynamic design

The design of a *forward swept wing (FSW) blended wing body (BWB) unmanned aerial vehicle (UAV)* is presented as a possible conceptual solution to the occurrence of tip-stall. From the idea that a FSW designs are inherently insensitive to tip-stall,^{34,35} the optimization can occur at cruise conditions. Optimal range can be obtained by maximizing the lift-to-drag ratio C_L/C_D while enforcing the pitching moment equilibrium $C_{M,pitch} = 0$.

In this paper the outperformance of a conventional fixed wing blended wing body UAV used for aerial photography (span $s \approx 1m$ and *mean aerodynamic chord* (MAC) based Reynolds number $Re_{MAC} \approx 5e5$) with the same level of complexity is pursued, so a maximum of two actuators is used to steer the aircraft.³³ This limits the number of possible tail configurations. Here the use of the V-tail configuration is proposed, which can be seen as two surfaces set in a V-shaped configuration when viewed from the front, to which the control surfaces are hinged. The control surfaces are subsequently denoted as ruddervators, a portmanteau of rudder and elevator, which in a conventional aircraft tail configuration respectively provide yaw and pitch control. Even though mechanically the control system is simplified, from an avionics point of view control becomes somewhat more tedious: pitch, roll and yaw are now coupled. Nonetheless, the reduced interference drag that comes with the configuration makes it interesting to be used from an aerodynamic point of view.³⁷

Beyond the requirement of longitudinal static stability $\partial C_{M,pitch}/\partial \alpha < 0$, the natural tendency of the aircraft to correct minor changes in angle of attack α , directional

static stability $\partial C_{M,yaw}/\partial \beta > 0$ is enforced, the natural tendency of the aircraft to correct minor changes in the side-slip angle β . Furthermore, slip-roll neutrality $\partial C_{M,roll}/\partial \beta = 0$ is enforced, which implies that a side slip component will not lead to rolling tendency. This requirement comes forth from the application for which the UAV is designed: aerial photography. Rolling motions, much more than yawing motions, might lead to blurred images.

A final deterministic constraint that is enforced is that the ruddervator deflection δ is within 5° of its neutral position. While the effect of the deflection on C_L and C_D is taken into account by the model (see further) during the optimization, it is important that the ruddervators can be deflected from their neutral position during cruise flight to ensure the initiation of manoeuvres.

To account for the variability in range and stability constraints due to production tolerances and varying operating conditions, the design problem is formulated as a reliability-based robust design optimization problem. This implies that the maximization of the mean of the range is pursued, while simultaneously minimizing the variability on the range. Furthermore, it is intended to meet the stability constraints with a pre-specified probability, as to ensure that production variability does not lead to undesirable flight behaviour. This can be formally formulated as follows

$$\begin{aligned} \mathbf{x} = \arg \min_{\mathbf{x}} & \left\{ -\mathbb{E}_{\{\mathbf{x}, \mathbf{u}\}} \left[\frac{C_L}{C_D}(\mathbf{x}, \mathbf{u}) \right], \mathbb{V}_{\{\mathbf{x}, \mathbf{u}\}} \left[\frac{C_L}{C_D}(\mathbf{x}, \mathbf{u}) \right] \right\} \\ \text{s.t.} & \begin{cases} \mathbb{P}_{\{\mathbf{x}, \mathbf{u}\}} [\partial C_{M,pitch}(\mathbf{x}, \mathbf{u})/\alpha \leq 0] > 1 - p_f \\ \mathbb{P}_{\{\mathbf{x}, \mathbf{u}\}} [\partial C_{M,yaw}(\mathbf{x}, \mathbf{u})/\beta \geq 0] > 1 - p_f \\ \mathbb{P}_{\{\mathbf{x}, \mathbf{u}\}} [-\epsilon \leq \partial C_{M,roll}(\mathbf{x}, \mathbf{u})/\partial \beta \leq \epsilon] > 1 - p_f \\ |\delta| < 5^\circ \end{cases} \end{aligned} \quad (1)$$

with $\mathbf{x}$ a stochastic design variable (*Type II*⁶³), $\mathbf{u}$ a stochastic parameter (*Type I*⁶³), $\mathbb{E}$, $\mathbb{V}$ and $\mathbb{P}$ respectively the expected value, variance and probability operators and $1 - p_f$ the target reliability, in this work taken equal to 0.99. A stochastic variable is presented in this work as a deterministic mean $\mathbf{x}$ corrupted by some noise $\epsilon_{\mathbf{x}} \sim \mathcal{N}(\mathbf{0}, \Psi_{\mathbf{x}})$, assumed to be normally distributed (similar for parameters $\mathbf{u}$) and added to the stochastic parameter set. Non-normal distributions can be included by means of iso-probabilistic transformations, but are not examined here.

Parameterization

The conventional planform and control surface parameterization accustomed to aerodynamic conceptual design is incorporated here.³⁵ First the axes system is defined, to simplify our definitions. The x -axis is defined along the chord of the aircraft at the symmetry-plane with its origin at the leading edge. Along this axis x_{CoG} is defined as the position of the centre of gravity. The z -axis is defined perpendicular

to the x -axis in the upward direction in the symmetry plane. Finally, the y -axis is defined perpendicular to the aforementioned two to form a right-handed base. The chord length of the wing and control surfaces at the fuselage are respectively denoted by $c_{W,root}$ and $c_{C,root}$. The fuselage is represented as a lifting body with width w_F and the total spanwidth is w_W fixed to $1m$. The taper ratio of the wing λ_W and control surfaces λ_C respectively correspond to the ratio of the chord at the tip to the chord at the root of the respective surfaces: $\lambda_W = c_{W,tip}/c_{W,root}$ and $\lambda_C = c_{C,tip}/c_{C,root}$. The sweep angle of the wing Λ_W is defined as the angle between the leading edge projected on the xy -plane and the y -axis, with a negative value indicating a sweeping of the wing in forward direction. In a similar manner, the sweep angle of the control surfaces Λ_C is defined as the angle between the leading edge projected on the symmetry plane and the negative z -axis. h_C is defined as the height of the control surface when projected on the symmetry plane. The remaining 3D design variables are the rotation of the tip in reference to the root of the lifting surfaces, respectively denoted as twist of the wing γ_W and twist of the control surfaces γ_C , and the

dihedral angles Γ_W and Γ_C , respectively corresponding to the angle between the horizontal plane and the plane through the leading-edge and the chord at the root and the angle between symmetry plane and the plane through the leading-edge and the chord at the root. Finally, the fraction of the control surface that can be tilted to initiate manoeuvres is denoted as f_C . In summary, all design variables are listed below:

- $c_{W,root} \in [0.4m, 1.0m]$:
Chord length wing at the fuselage
- $c_{C,root} \in [0.2m, c_{W,root}/2]$:
Chord length control surface at fuselage
- $w_F \in [0.10m, 0.25m]$:
Width fuselage
- $\lambda_W \in [0.5, 1.0]$:
Taper ratio wing
- $\lambda_C \in [0.5, 1.0]$:
Taper ratio control surface
- $\Lambda_W \in [-10^\circ, -45^\circ]$:
Sweep angle wing (negative = forward sweep)
- $\Lambda_C \in [0^\circ, 45^\circ]$:
Sweep angle control surface
- $\gamma_W \in [-5^\circ, 5^\circ]$:
Twist angle wing
- $\gamma_C \in [-5^\circ, 5^\circ]$:
Twist angle control surface
- $\Gamma_W \in [-5^\circ, 5^\circ]$:
Dihedral angle wing
- $\Gamma_C \in [30^\circ, 90^\circ]$:
Dihedral angle control surface
- $h_C \in [0m, 0.5m]$:
Height control surface
- $f_C \in [0.0, 1.0]$:
Fraction control surface
- $x_{CoG} \in [0.0m, c_{W,root}]$:
x-position of the centre of gravity
- $V \in [15m/s, 25m/s]$:
Velocity

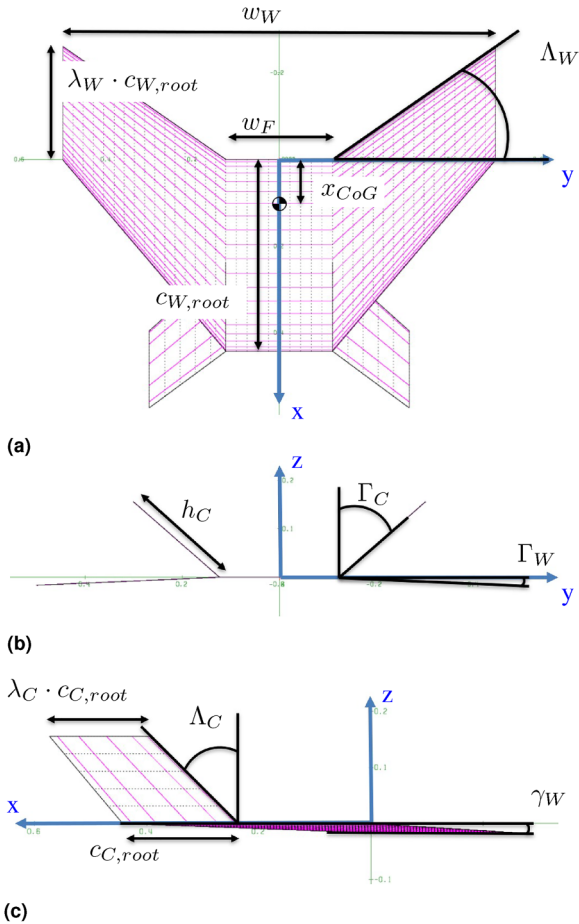


Figure 1. Parameterization of the C2-series UAV. (a) Bottom view; (b) Front view and (c) Side view.

The velocity is treated as a stochastic parameter with a *Coefficient of Variation* (CoV), corresponding to the standard deviation over the mean, equal to 0.1. This leads to a total of 15 deterministic design variables (D_x) and 1 stochastic parameter (D_u). The parameterization is visualized in Figure 1 (γ_C and f_C are not visualized).

Methodology

In this section the *Surrogate-assisted Asynchronous Multi-objective optimization-under-Uncertainty framework for Robust and reliable solutions to Applications in Industry* (SAMURAI) is presented.⁶² The SAMURAI technique is motivated by the author's belief that there is a region of overlap between the regions of applicability of

robust design optimization and reliability based design optimization based on frequency and impact of events⁶⁴ to which is referred as *reliability-based robust design optimization* (RBRDO). In SAMURAI the RBRDO problem is presented as a constrained multi-objective problem. This presents the designer with a Pareto front of reliable solutions with varying degrees of robustness instead of only one design with an arbitrary level of robustness as is often the case in literature. Secondly, SAMURAI answers the 2002 NASA report on the needs and opportunities for uncertainty-based multidisciplinary design methods regarding and computational cost by means of the introduction of surrogate-modeling techniques, namely Kriging, in the design framework.⁵⁰

Kriging is a well-established surrogate-modeling technique, extending on the concepts of Gaussian processes. Jones' *efficient global optimization* (EGO) framework⁶⁰ forms a natural transition to optimization by introducing the *expected improvement* (EI) acquisition function, which, when optimized, indicates which design contributes most strongly to the surrogate when added to the training data in pursuit of the optimal point. The optimization of the acquisition function is a computationally cheap problem, since it occurs on the surrogate itself. The methodological cornerstones of SAMURAI: GAMO,⁶⁵ ERGO⁶⁶ and ESLA⁶⁷ formulate novel acquisition functions within the EGO framework to respectively enable asynchronous multi-objective optimization with exploration and exploitation control, RDO by means of an analytical uncertainty propagation through the surrogate and a multi-objective treatment of the problem and RBRDO by means of an asymptotic reliability analysis in a single-loop formulation using the KKT-conditions. Combining all the aforementioned, a surrogate-assisted design optimization-under-uncertainty methodology can be created (Figure 2).

In the SAMURAI framework, Kriging models are constructed for the objective and each of the constraint functions following a *design of experiments* (DoE). Training of the surrogate model corresponds to maximizing the likelihood that the aforementioned surrogate can reproduce the evaluated data. This boils down to determining the parameters of the covariance function, referred to as hyperparameters. In practice, the concentrated log-likelihood function is maximized. The Matérn covariance function, an often used class in the aerospace community, is used. The construction of the Kriging model is performed using an open-source toolbox ooDACE (*object-orientated Design and Analysis of Computer Experiments*)⁶⁸ using a multi-start SQP methodology. For the DoE a *Latin Hypercube Sampling* (LHS) approach is used⁶⁹ and Morris & Mitchell's maximin-criterion to impose the space-filling property by maximizing in ascending order the distance between pairs of points and simultaneously minimizing the number of corresponding pairs.⁷⁰ Conform Jones et al.'s suggestion, a DoE-size of $11 \cdot (D_x + D_u) - 5$

samples is selected.⁶⁰ From the constructed surrogates, the expected value and variance of the objective, along with the probability of failure of the constraints of each design can be assessed. Furthermore, an internal clock is present, which regularly checks the number of available (λ) and computing (μ) nodes. Based on this information λ new infill points to be evaluated can be selected by optimizing SAMURAI's acquisition function (more details in the appendix). This can either be the *generalized multi-points reliability-based robust expected improvement* (GRBREI) to improve upon the current prediction of the Pareto front or the *probability of single-loop reliability* ($P_{r,sl}$) in pursuit of feasible designs. This process is repeated until the stopping criteria are met. The stopping criteria used are a maximum of 200 function evaluations after the DoE or if the normalized expected improvement drops below 0.1%.

To evaluate a UAV design, Drela's *Athena Vortex Lattice* (AVL) method is used.⁷¹ AVL requires $C_D(C_L)$ data for every section of the plane, for which Youngren & Drela's open source XFOIL (version 6.97) software is used.⁷² AVL provides the lift coefficient C_L , drag coefficient C_D , ruddervator δ and angle of attack α settings for which steady flight is obtained. Furthermore, stability derivatives $\partial C_{M,pitch}/\partial \alpha$, $\partial C_{M,yaw}/\partial \beta$ and $\partial C_{M,roll}/\partial \beta$ are directly obtained. If AVL is unable to converge during the evaluation of the DoE, this design is omitted. If AVL is unable to converge during an optimization step, the design is treated as *missing at random* (MAR) and filled equal to the surrogate's prediction plus the standard deviation on the prediction: $Y(\mathbf{x}) + s(\mathbf{x})$. In order to assess the interaction between of fuselage and wing on the one hand and assess the pitch up tendencies of stall on the other hand computational fluid dynamics (CFD) simulations would need to be performed. While this has already been employed to optimize wing fences in the context of tip stall avoidance of backward swept wings of UAVS,³³ this was performed deterministic and not in a stochastic context as is examined here and is left as future research.

Results and discussion

The result of the aerodynamic optimization-under-uncertainty is a front of Pareto optimal solutions (Figure 3). Furthermore, the robust and reliable Pareto front predicted by means of *Monte-Carlo Sampling* (MCS) of the surrogates and optimized by means of NSGA-II is added along with the validation of the aforementioned fronts using $1e^3$ exact function evaluations. Overall it can be said that the prediction of the mean values is fairly accurate, however, there is a strong offset in regards to the prediction of the variance (cf. predicted and validated Pareto). The variance is most strongly influenced by the curvature of the surrogates, which in turn is most strongly influenced by the covariance function, which

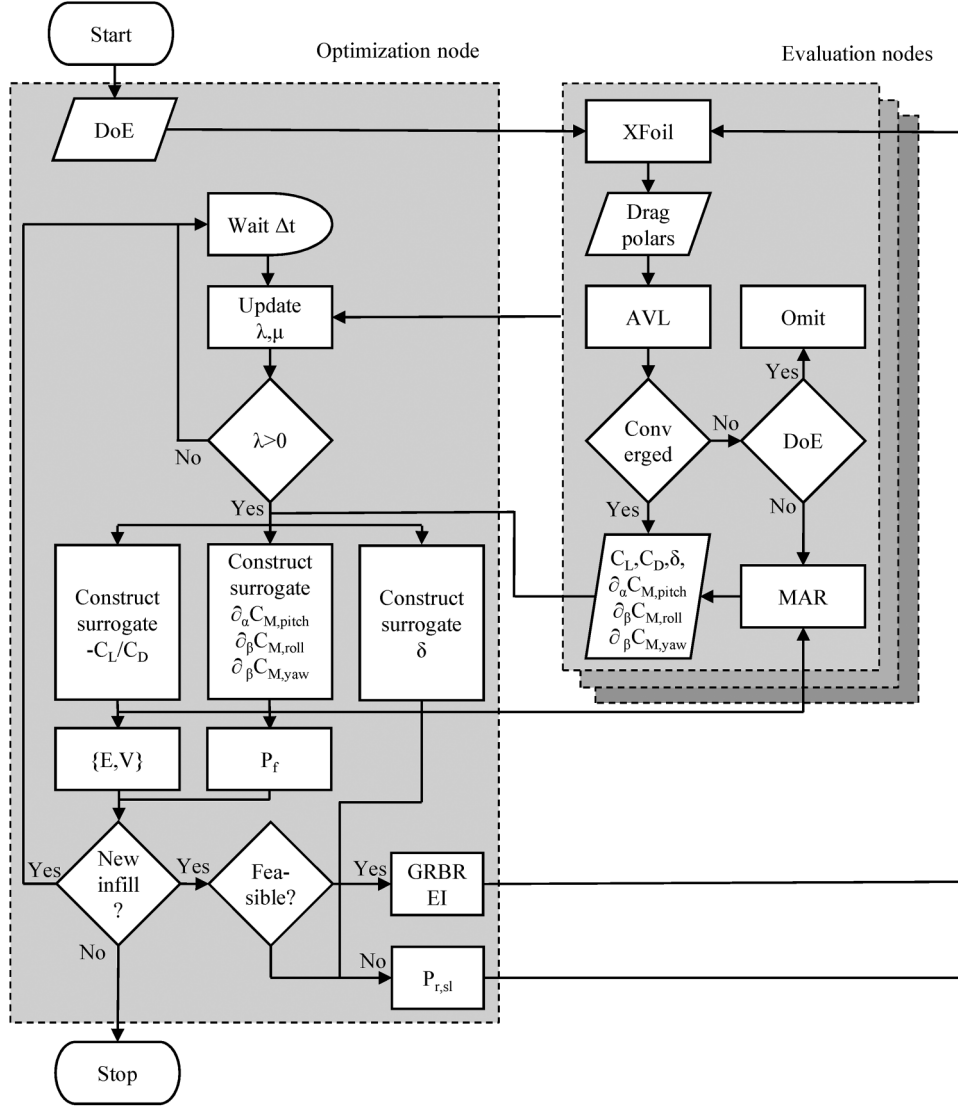


Figure 2. Flowchart of the SAMURAI optimization framework applied to the FSW design.

might still be too strongly continuous for the problem at hand and needs future research.

Seven designs are considered, denoted by the CoV of the objective. The two extremes of the Pareto front are considered, along with five intermediate designs. The design with the lowest mean (and highest variance on the Pareto front), thus the deterministically best performing UAV has a CoV equal to 0.0040. In the same manner, the UAV with the lowest variance (and the highest mean on the Pareto front), thus the most robust UAV has a CoV equal to 0.0011. The five intermediate designs, going from lowest mean to lowest variance have CoVs respectively equal to 0.0038, 0.0035, 0.0030, 0.0026 and 0.0023. Their parameter settings are presented in table 1 and the two designs at the extremes of the front are visualized in Figure 4.

To illustrate the importance of a multi-objective approach, the objective values of the designs for a single-

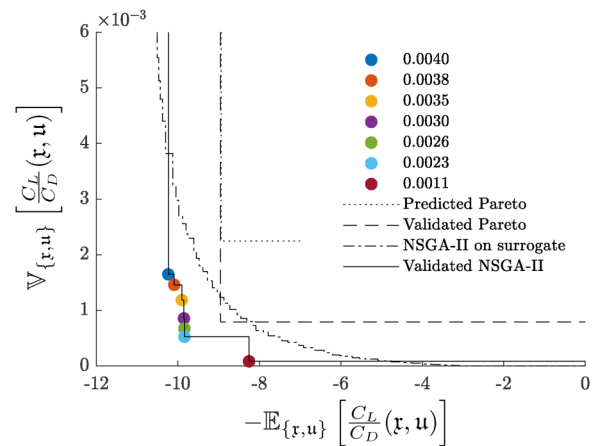


Figure 3. Robust and reliable Pareto front of mean versus variance of lift over drag for the C2-series designs.

CoV	$c_{W,root}$ [m]	$c_{C,root}$ [m]	w_F [m]	λ_W [—]
0.0040	0.4427	0.2055	0.1232	0.5923
0.0038	0.4365	0.2080	0.1169	0.7489
0.0035	0.4346	0.2091	0.1092	0.8266
0.0030	0.4296	0.2091	0.1058	0.8638
0.0026	0.4361	0.2128	0.1086	0.8685
0.0023	0.4386	0.2138	0.1151	0.8916
0.0011	0.4299	0.2136	0.1120	0.9469

CoV	Λ_W [°]	Λ_C [°]	f_C [—]	x_{CoG} [m]
0.0040	−27.5378	33.8450	0.7388	0.0184
0.0038	−27.2270	30.8115	0.8137	0.0187
0.0035	−27.2515	24.4215	0.8800	0.0169
0.0030	−29.2325	24.0480	0.9124	0.0163
0.0026	−26.9575	24.0705	0.9086	0.0160
0.0023	−26.4395	24.3585	0.9292	0.0165
0.0011	−31.0068	17.9027	0.9653	0.0149

CoV	k=0	k=10	k=50	k=100	k=200
0.0040	-10.3	-9.8	-8.3	-6.2	-2.1
0.0038	-10.2	-9.7	-8.3	-6.3	-2.5
0.0035	-10.0	-9.6	-8.3	-6.5	-3.1
0.0030	-9.9	-9.6	-8.5	-7.1	-4.1
0.0026	-9.9	-9.6	-8.7	-7.4	-4.8
0.0023	-9.9	-9.6	-8.8	-7.7	-5.4
0.0011	-8.2	-8.2	-8.2	-8.2	-8.2

angle of the control surfaces are noted to change most significantly. However, a global sensitivity study should be performed to make a definite claim regarding the importance of the design variables, which is left as future research. Nonetheless, lumping part of this information together, moving towards a lesser robust design is characterized by a more pronounced tail: larger sweep and taper ratio of the control surfaces and a smaller taper ratio of the main wing. On the other hand, the twist of the tail of more robust designs is more strongly positive, which increases its stabilizing contribution at the cost of an increased drag. Furthermore noteworthy, for all designs the twist angle of the wing is slightly negative, as one would

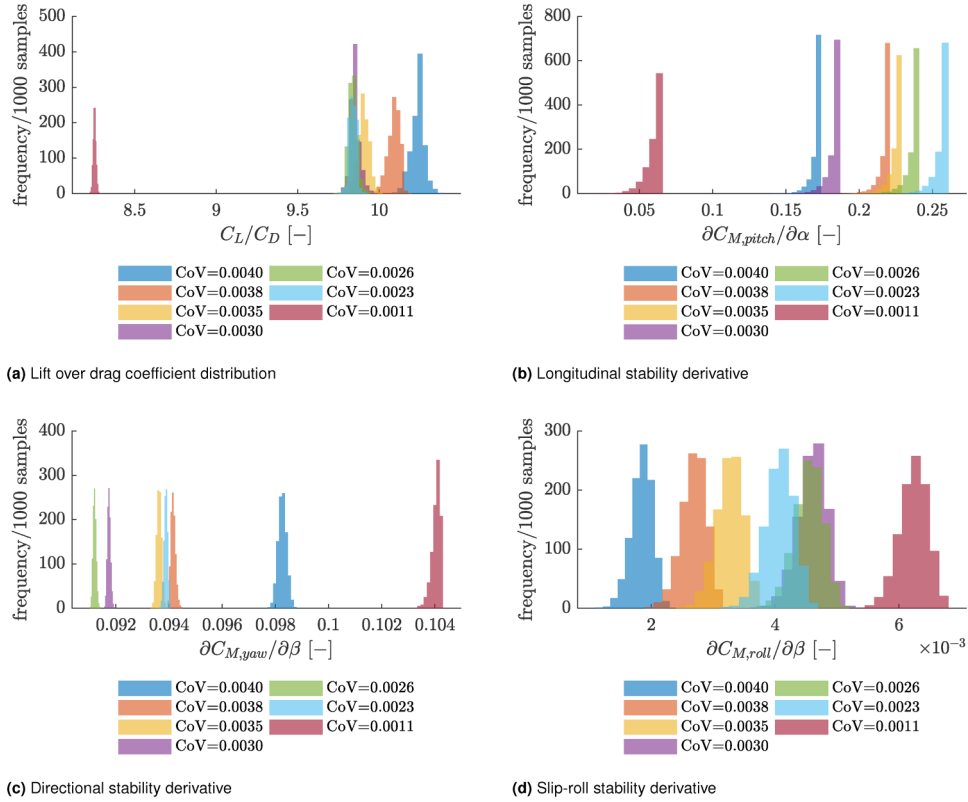


Figure 6. Distributions of objective and constraints of the C2-series FSW-BWB-UAVs.

expect from a backward swept design. This particularity can be attributed to the presence of the tail.

For all designs the position of the centre of gravity x_{CoG} is found close near the leading edge. This particularity might enforce the use of a puller propeller system to meet the weight balance requirements. Alternatively, part of the carried load can be placed in the wings, which, since the wings are swept forward, appears to be a feasible solution. From a structural point of view, weight balancing is a challenging endeavour. Therefore, it can be argued that in the future, position of the CoG of the payload should be treated as a stochastic design variable.

The distributions of the objective and constraints of the forward swept designs after $1e3$ exact evaluations is presented in Figure 6. It can be observed that the deterministic best but least robust performing designs have wider and overlapping objective distributions found most strongly to the right, once more emphasizing the importance of a multi-objective approach. When considering the distributions of the stability derivatives no particular order or consistency in variance is to be observed along varying CoVs, besides the slip-roll derivative where the order is somewhat contained. Furthermore, while the target reliability levels are met for all constraints, the spread of distributions indicates that the prediction of the constraints by the surrogates is not entirely accurate, since one would expect the solutions to the problem to lie on the boundary of the constraints and

as such observe an overlap of distributions of at least one of the constraints.

It can be noticed on the more ‘conventional’ forward swept wing aircraft such as the *Grumman X-29*, *Sukhoi Su-47* and the *Bugatti Model 100* that the nose of the aircraft is positioned in front of the tips of the wings, as opposed to the obtained designs. The corresponds to a more forward position of the CoG on the aircraft such that the trailing edge of the wing is positioned behind the CoG. Consequently, the control surfaces can be positioned on the main wing, which might simplify the tail design.

Conclusion and future work

In this paper the conceptual aerodynamic design optimization-under-uncertainty of a fixed wing unmanned aerial vehicle (UAV) with span $1m$ was performed. The use of a blended wing body (BWB) configuration with backward swept wings for UAVs is common, but characterized by the increased likelihood of tip stall, leading to a nose up pitching moment and loss of control. A possible solution is a conceptual redesign by sweeping the wings forward. Secondly, the unoverlookable production tolerances that often come with foam moulding are taken into account. With the objective of maximizing the mean and minimizing the variance on the range, the planform and tail of the

FSW-BWB-UAV were optimized. In addition to the objectives, stability constraints were enforced to be met with a target reliability.

A framework was built around the SAMURAI technique, which introduces surrogate modelling to the reliability-based robust design optimization problem, which reformulated as a constrained multi-objective optimization problem. By building surrogates for both the objectives and constraints, a trustworthy, but relatively inexpensive optimization can be obtained. The result is a Pareto front of robust and reliable UAV designs with varying degrees of robustness that are inherently free of tip stall through the design choices made and furthermore are relatively insensitive to production tolerances and changes in operating conditions, leading to a consistent flight behaviour.

Future research is pointed towards a more detailed optimization using high-fidelity tools, such as computational fluid dynamics simulations, that permit modelling the interaction of the fuselage with the wings and assessing the pitch-up tendencies in stall-regime. During this phase the design space can be narrowed to the optimal region found during the design optimization presented in this work. As such, this forms a natural follow-up step to the current work.

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Acronyms

<i>BLUP</i>	Best Linear Unbiased Prediction
<i>BWB</i>	Blended Wing Body
<i>DoE</i>	Design of Experiments
<i>EGO</i>	Efficient Global Optimization
<i>(C/G/MO)</i>	(Constraint/Generalized/Multi-Objective)
<i>EI</i>	Expected Improvement
<i>ERGO</i>	Efficient Robust Global Optimization
<i>ESLA</i>	Efficient Single-Loop Approach
<i>FSW</i>	Forward Swept Wing
<i>GAMO</i>	Generalized Asynchronous Multi-objective Optimization
<i>K&D</i>	Kaiser & Dickman
<i>KKT</i>	Karush-Kuhn-Tucker
<i>MC(I/S)</i>	Monte Carlo (Integration/Sampling)
<i>MPFP</i>	Most Probable Failure Point
<i>MSE</i>	Mean Square Error
<i>OUU</i>	Optimization-Under-Uncertainty
<i>(SA)RA</i>	(Surrogate-Assisted) Reliability Analysis

(RB/R)DO (Reliability-Based/Robust) Design Optimization
 UAV Unmanned Aerial Vehicle

Greek symbols

α Angle of attack
 $\boldsymbol{\alpha}$ BLUP coefficients
 β Sideslip angle
 $\boldsymbol{\beta}$ Trend function coefficients
 δ Control surface deflection
 γ Twist angle
 Γ Dihedral angle
 λ Taper ratio
 Λ Sweep angle
 Ψ Covariance matrix
 σ^2 Process variance

Roman symbols

c Chord length
 C_D Drag coefficient
 C_L Lift coefficient
 C_M Moment coefficient
 f_C Fraction control surface
 $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), \dots, f_f(\mathbf{x})]$ trend function
 $\mathbf{F}$ Model matrix
 $\mathbb{F}$ Failure domain
 $\mathcal{F}$ Limit-state surface
 $g(\mathbf{x})$ Constraint function
 $\mathbf{g}$ Vector of constraint function evaluations
 $\mathcal{G}$ Surrogate of the constraint function
 $G(\mathbf{x})$ MSPE of the constraint function
 h_C Height control surface
 $L_{\{\cdot\}}$ Cholesky decomposition
 $\mathbf{M}$ Multi-variate sampling
 N_{sim} Sample size
 p_f Target probability of failure
 $s^2(\mathbf{x})$ MSPE of the stochastic process
 $\mathbb{S} = \{\mathbb{X}, y(\mathbb{X})\}$ Sampling set
 $\mathbf{u} = [\mathbf{u}^{(1)}, \dots, \mathbf{u}^{(D_u)}]$ Parameters sample
 $\mathbf{u} = \mathbf{u} + \epsilon_u$ Stochastic parameters sample
 $\mathbb{U} = [\mathbf{u}_1, \dots, \mathbf{u}_n]$ Parameters set
 V Free-stream velocity
 x Chordwise position
 $\mathbf{x} = [\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(D_x)}]$ Design variables sample
 $\mathbf{x} = \mathbf{x} + \epsilon_x$ Stochastic design variables sample
 $\mathbb{X} = [\mathbf{x}_1, \dots, \mathbf{x}_n]$ Variables set
 $y(\mathbf{x})$ Objective function
 $\mathbf{y}$ Vector of objective function evaluations
 $\mathcal{Y}$ Surrogate of the objective function
 $\mathbf{Y}(\mathbf{x})$ BLUP of the objective function

$\mathcal{Y}$ Mixed aleatory & epistemic stochastic process
 w_F Width fuselage

Operators

$\psi(\cdot, \cdot)$ Covariance function
 $\mathbb{E}[\cdot]$ Expected value operator
 $\mathcal{Z}(\cdot, \cdot)$ Gaussian Process
 $\mathcal{H}(\cdot)$ Heaviside function
 $\mathcal{N}(\cdot, \cdot)$ Normal distribution
 $\mathbb{P}[\cdot]$ Aleatory probability operator
 $P[\cdot]$ Epistemic probability operator
 $P_f(\cdot)$ Probability of failure prediction
 $\mathbb{V}[\cdot]$ Variance operator

Superscript

$*$ Pareto/MPFP optimal sample location
 λ Available computing nodes
 μ Evaluating computing nodes

Subscript

C Control surface
 r Reliability
 sl Single-loop
 W Wing

Appendix: SAMURAI

The SAMURAI framework solves RBRDO problems from an EGO perspective. To accomplish this, it introduces a novel acquisition function by building forth on the GAMO,⁶⁵ ERGO⁶⁶ and ESLA⁶⁷ methodologies. This novel acquisition function, the *Generalized multi-points Reliability-Based Robust Expected Improvement (GRBREI^(g,μ,λ))*, corresponds to the product of the *Robust Expected Improvement (REI)*, which pursues the Pareto front of robust solutions, and the *Probability of Single-Loop Reliability (P_{r,sl})*, which ensures that the Pareto front is reliable. Furthermore, the acquisition function is formulated on the one hand in an asynchronous manner, such that multiple points (λ) can be selected while multiple points (μ) are being evaluated, and on the other hand in a generalized fashion (g), such that design space exploration and objective exploitation can be controlled.

EGO relies on *Kriging*, which in turn builds forth on the concepts of Gaussian processes and the theorem of Bayes. Kriging can be seen a Gaussian process with mean trend function: $\mathcal{Y}(\mathbf{x}) = \mathbf{f}(\mathbf{x})^T \boldsymbol{\beta} + \mathcal{Z}(\mathbf{x})$ with $\mathbf{f}(\mathbf{x}) = [f_i(\mathbf{x}), i = 1, \dots, N_b]$ the vector of basis functions, $\boldsymbol{\beta}$ the vector of coefficients and $\mathcal{Z}(\mathbf{x})$ a Gaussian process $\mathcal{GP}(0, \psi(\mathbf{x}_i, \mathbf{x}_j))$, with zero mean and fully described by the covariance function $\psi(\cdot, \cdot)$. Where $\mathbf{x}_i$ refers to the i^{th} sample ($i = 1, \dots, N_{\mathbb{S}}$) and $\mathbf{x}^{(i)}$ refers to the i^{th} component ($i = 1, \dots, D_x$). The trend function is typically the solution of a regression

problem and the Gaussian process captures the variation on this trend to exactly interpolate the evaluated data.

Solving the *maximum likelihood estimation* (MLE) problem, the *Best Linear Unbiased Prediction* (BLUP) can be defined and the *Mean Square Predictive Error* (MSPE), which allows the prediction of unsampled locations $\mathbf{x}$ with respectively the predicted mean and predicted variance:

$$\mathbb{E}[\mathcal{Y}(\mathbf{x})] \equiv Y(\mathbf{x}) = \mathbf{f}(\mathbf{x})^\top \boldsymbol{\beta} + \boldsymbol{\psi}(\mathbf{x})^\top \boldsymbol{\alpha} \quad (\text{A.1})$$

$$\mathbb{V}[\mathcal{Y}(\mathbf{x})] \equiv s^2(\mathbf{x}) = \sigma^2 \{1 - h(\mathbf{x}) + \mathbf{g}(\mathbf{x})^\top \boldsymbol{\Gamma} \mathbf{g}(\mathbf{x})\} \quad (\text{A.2})$$

with $\boldsymbol{\alpha} = \boldsymbol{\Psi}^{-1}(\mathbf{y} - \mathbf{F}\boldsymbol{\beta})$, $\boldsymbol{\beta} = \boldsymbol{\Gamma}^{-1}\boldsymbol{\Pi}\mathbf{y}$, $\boldsymbol{\Pi} = \mathbf{F}^\top \boldsymbol{\Psi}^{-1}$, $\boldsymbol{\Gamma} = (\boldsymbol{\Pi}\mathbf{F})^{-1}$, $h(\mathbf{x}) = \boldsymbol{\psi}(\mathbf{x})^\top \boldsymbol{\Psi}^{-1} \boldsymbol{\psi}(\mathbf{x})$, $\mathbf{g}(\mathbf{x}) = \boldsymbol{\Pi}\boldsymbol{\psi}(\mathbf{x}) - \mathbf{f}(\mathbf{x})$, σ the process variance and $\mathbf{F}$ the model matrix: $\mathbf{F}^{(i,j)} = f_i(\mathbf{x}_j)$ and $\boldsymbol{\Psi}$ the correlation matrix: $\boldsymbol{\Psi}^{(i,j)} = \psi(\mathbf{x}_i, \mathbf{x}_j)$, $\boldsymbol{\psi}(\mathbf{x}) = [\psi(\mathbf{x}_1, \mathbf{x}), \dots, \psi(\mathbf{x}_N, \mathbf{x})]$ and the MLE of the coefficient vector and the process variance defined by: $\sigma^2 = \frac{1}{N}(\mathbf{y} - \mathbf{F}\boldsymbol{\beta})^\top \boldsymbol{\Psi}^{-1}(\mathbf{y} - \mathbf{F}\boldsymbol{\beta})$. Furthermore, the analytical formulation of BLUP and MSPE permits the formulation of the Jacobian ($\nabla Y(\mathbf{x})$) and Hessian ($\Delta Y(\mathbf{x})$) and their respective MSPEs ($s_{\nabla Y}^2(\mathbf{x})$ and $s_{\Delta Y}^2(\mathbf{x})$). For more details I refer to.⁶⁶

In this paper implicitly a categorization of uncertainties is introduced: changing environmental and operating conditions (*Type I*), production tolerances and actuator imprecisions (*Type II*).⁶³ Furthermore an epistemological categorization is introduced: aleatory uncertainties, “the inherent variation associated with the physical system or environment being considered”, which is denoted with the *Fraktur* font ($\mathfrak{Y}$). And the epistemic uncertainty, “the cause of non-deterministic behaviour from some level of ignorance or lack of knowledge about the system or the environment”, which is denoted with the *calligraphic* font ($\mathcal{Y}$).⁷³ The *Spencerian* font is used to denote mixed aleatory and epistemic uncertainties ($\mathscr{Y}$).

The first part of SAMURAI’s acquisition function relies on the formulation of the generalized asynchronous/multi-points formulation of the robust expected improvement as introduced in⁶⁶ and evaluated through Monte Carlo integration:

$$GRE_{MC}^{g,\mu,\lambda}(\mathbb{X}_{\lambda+\mu}) = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} RI_{sim}^{(g,\mu,\lambda)}(i) \quad (\text{A.3})$$

$$RI_{sim}^{(g,\mu,\lambda)}(i) = |\text{Pareto}(\{\mathbb{E}[\mathscr{Y}(\mathbb{X})], \mathbb{V}[\mathscr{Y}(\mathbb{X})]\})|,$$

$$\{\mathbf{M}_{\mathbb{E},\mu}(i), \mathbf{M}_{\mathbb{V},\mu}(i)\}, \{\mathbf{M}_{\mathbb{E},\lambda}(i), \mathbf{M}_{\mathbb{V},\lambda}(i)\}^g|_{min}^+$$

Here $|\text{Pareto}(\mathbf{Y}(\mathbb{X})), \mathbf{Y}(\mathbf{x})|^g$ is defined as the distance measured from the Pareto front of evaluated infills to $\mathbf{Y}(\mathbf{x})$ taken to a power g and $[\cdot]_{min}^+$ is defined as the smallest value evaluated outside of the Pareto front. The sampling of the $(\lambda + \mu)$ -variates of the surrogate’s prediction (epistemic mean) of the aleatoric mean $\mathbf{M}_{\mathbb{E},\mu+\lambda}(i)$ and variance $\mathbf{M}_{\mathbb{V},\mu+\lambda}(i)$ when μ computing nodes are available and λ

designs are being evaluated is obtained using *Kaiser & Dickman’s* (K&D) method (Algorithm 1).

The surrogate’s prediction of the mean $\mathbb{E}_{\mathfrak{r}}[\mathscr{Y}(\mathfrak{r})]$ and variance $\mathbb{V}_{\mathfrak{r}}[\mathscr{Y}(\mathfrak{r})]$ due to aleatoric variability $\boldsymbol{\Psi}_{\mathfrak{r}}$ cannot be evaluated analytically without approximations. The analytical propagation method used in SAMURAI corresponds to computing the mean and variance of the new (non-Gaussian) predictive distribution by assuming that the output distribution is Gaussian: $f_{\mathscr{Y}}(\mathscr{Y} | \mathbf{x}, \mathbb{S}, \boldsymbol{\Psi}_{\mathfrak{r}}) \approx \mathcal{N}(\mathbf{Y}_{\mathfrak{r}}(\mathbf{x}, \boldsymbol{\Psi}_{\mathfrak{r}}), s_{\mathfrak{r}}(\mathbf{x}, \boldsymbol{\Psi}_{\mathfrak{r}}))$. Mean $\mathbb{E}_{\mathfrak{r}}[\mathscr{Y}(\mathfrak{r})] \equiv Y_{\mathfrak{r}}(\mathbf{x}, \boldsymbol{\Psi}_{\mathfrak{r}})$ and variance $\mathbb{V}_{\mathfrak{r}}[\mathscr{Y}(\mathfrak{r})] \equiv s_{\mathfrak{r}}^2(\mathbf{x}, \boldsymbol{\Psi}_{\mathfrak{r}})$ are respectively given by:

$$\mathbb{E}_{\mathfrak{r}}[\mathscr{Y}(\mathfrak{r})] = \mathbb{E}_{\mathfrak{r}}[Y(\mathfrak{r})] \quad (\text{A.5})$$

$$\mathbb{V}_{\mathfrak{r}}[\mathscr{Y}(\mathfrak{r})] = \mathbb{E}_{\mathfrak{r}}[s^2(\mathfrak{r})] + \mathbb{E}_{\mathfrak{r}}[Y(\mathfrak{r})^2] - \mathbb{E}_{\mathfrak{r}}[Y(\mathfrak{r})]^2 \quad (\text{A.6})$$

Filling in the equations of noise-free predictive mean and variance (equations A.1 and A.2) in equations A.5 and A.6, simplifying the equations by introducing $\boldsymbol{\alpha} = \boldsymbol{\Psi}^{-1}(\mathbf{y} - \mathbf{F}\boldsymbol{\beta})$ and making use of the multinomial theorem, the following formulas can be obtained for respectively the expected value of the prediction, variance on the prediction and prediction squared:

$$\mathbb{E}_{\mathfrak{r}}[Y(\mathfrak{r})] = \mathbb{E}_{\mathfrak{r}}[\boldsymbol{\psi}(\mathfrak{r})]^\top \boldsymbol{\alpha}$$

$$\mathbb{E}_{\mathfrak{r}}[s^2(\mathfrak{r})] = \sigma^2 \{1 - \boldsymbol{\Psi}^{-1} : \mathbb{E}_{\mathfrak{r}}[\boldsymbol{\psi}(\mathfrak{r})\boldsymbol{\psi}(\mathfrak{r})^\top] + \boldsymbol{\Gamma}\boldsymbol{\Pi} : \boldsymbol{\Pi}\mathbb{E}_{\mathfrak{r}}[\boldsymbol{\psi}(\mathfrak{r})\boldsymbol{\psi}(\mathfrak{r})^\top]\} \quad (\text{A.7})$$

$$\mathbb{E}_{\mathfrak{r}}[Y(\mathfrak{r})^2] = \boldsymbol{\alpha}\boldsymbol{\alpha}^\top : \mathbb{E}_{\mathfrak{r}}[\boldsymbol{\psi}(\mathfrak{r})\boldsymbol{\psi}(\mathfrak{r})^\top] \quad (\text{A.8})$$

The expected values of the covariance function and the product with itself in the equations above are the only unknowns remaining, which can be obtained by using a Taylor approximation of the covariance function. By means of Isserlis’ theorem, these terms can be evaluated. Details are found in.⁶⁶

In order to evaluate the covariance matrix of the mean ($\boldsymbol{\Psi}_{\mathbb{E}}$) and variance ($\boldsymbol{\Psi}_{\mathbb{V}}$), the MSPE of mean and variance must be known. This is obtained by means of a first order Taylor series uncertainty propagation neglecting the covariance contribution of the second-order Taylor

Algorithm 1 Generalized Asynchronous Multi-Points Robust Expected Improvement $GRE_{MC}^{g,\mu,\lambda}$

- 1: $\mathbf{L}_{\mathcal{F}} = \text{chol}(\boldsymbol{\Psi}_{\mathcal{F}}[\{\mathbb{X}_{\mu+\lambda}, \mathbb{U}_{\mu+\lambda}\} | Y(\mathbb{X}) \equiv y])$ with $\mathcal{F} \in \{\mathbb{E}, \mathbb{V}\}$ $\triangleright$ Cholesky decomposition
 - 2: **for** $i \leftarrow 1, N_{sim}$ **do** $\triangleright$ Loop over simulation size
 - 3: $\mathbf{N}(i) \sim \mathcal{N}(\mathbf{0}_{\mu+\lambda}, \mathbf{I}_{\mu+\lambda})$ $\triangleright$ Sampling
 - 4: $\mathbf{M}_{\mathcal{F},\mu+\lambda}(i) = \mathcal{F}[\mathbb{X}_{\mu+\lambda}] + \mathbf{L}_{\mathcal{F}} \cdot \mathbf{N}(i)$ with $\mathcal{F} \in \{\mathbb{E}, \mathbb{V}\}$ $\triangleright$ K&D’s method
 - 5: $RI_{sim}^{(g,\mu,\lambda)}(i) = |\text{Pareto}(\{\mathbb{E}(\mathbb{X}), \mathbb{V}(\mathbb{X})\}, \{\mathbf{M}_{\mathbb{E},\mu}(i), \mathbf{M}_{\mathbb{V},\mu}(i)\}, \{\mathbf{M}_{\mathbb{E},\lambda}(i), \mathbf{M}_{\mathbb{V},\lambda}(i)\})^g|_{min}$
 - 6: $GRE_{MC}^{g,\mu,\lambda}(\mathbb{X}_{\lambda}) = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} RI_{sim}^{(g,\mu,\lambda)}(i)$
-

approximation of the surrogate which is considered exact (*Kriging Believer*). This results in the following:

$$s_{\mathbb{E}[\mathcal{Y}]}^2(\mathbf{x}) = s_Y^2(\mathbf{x}) + \frac{1}{2} s_{\Delta Y}^2(\mathbf{x}) : \Psi_{\mathbf{x}}^2 \quad (\text{A.9})$$

$$s_{\mathbb{V}[\mathcal{Y}]}^2(\mathbf{x}) = s_{\nabla Y}^2(\mathbf{x}) \nabla Y(\mathbf{x})^{\top \circ 2} : \Psi_{\mathbf{x}}^2 + 2s_{\Delta Y}^2(\mathbf{x}) \Psi_{\mathbf{x}}^{\circ 2} : \Psi_{\mathbf{x}}^{\circ 2} \Delta Y^{\circ 2} \quad (\text{A.10})$$

The second part of SAMURAI's acquisition function relies on the formulation of the multi-points single-loop reliability R_{sl} of the constraint functions ($\mathbf{g}(\mathbf{x})$) as the product of the feasibility of the prediction of the probability of failure constraint, F_{P_r} , and feasibility of the prediction of the *Karush-Kuhn-Tucker* (KKT) conditions of the underlying *most probable failure point* (MPFP) optimization problem, F_{KKT_1} and F_{KKT_2} . As such, the following is obtained by means of Monte Carlo integration:

$$R_{sl, sim}(i) = F_{P_r}(i) \cdot F_{KKT_1}(i) \cdot F_{KKT_2}(i)$$

$$F_{\mathcal{F}}(i) = \prod_{l=1}^{\mu+\lambda} \mathcal{H}(\{\mathbf{M}_{\mathcal{F}, \mu+\lambda}\}_l(i)) \quad (\text{A.11})$$

with $\mathcal{F} \in \{P_r, |KKT_1| - \epsilon, |KKT_2| - \epsilon\}$

$$P_{r, sl, MC}^{(g, \mu, \lambda)}(\mathbb{X}_{\mu+\lambda}) = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} R_{sl, sim}(i) \quad (\text{A.12})$$

The multivariate sampling of the probability of reliability $M_{P_r, \mu+\lambda}(i)$, the first *Karush-Kuhn-Tucker* condition

Algorithm 2 Asynchronous Multi-Points Probability of Single-Loop Reliability $P_{r, sl, MC}^{(g, \mu, \lambda)}$

```

1:  $\mathbf{L}_{\mathcal{F}} = \text{chol}(\Psi_{\mathcal{F}}[\{\mathbb{X}_{\mu+\lambda}, \mathbb{U}_{\mu+\lambda}\} | G(\mathbb{X}) \equiv \mathbf{g}])$  with
    $\mathcal{F} \in \{P_r, |KKT_1| - \epsilon, |KKT_2| - \epsilon\}$ 
2: for  $i \leftarrow 1, N_{sim}$  do  $\triangleright$  Loop over simulation size
3:    $\mathbf{N}(i) \sim \mathcal{N}(\mathbf{0}_{\mu+\lambda}, \mathbf{I}_{\mu+\lambda})$   $\triangleright$  Sampling
4:    $\mathbf{M}_{\mathcal{F}, \mu+\lambda}(i) = \mathcal{F}(\mathbb{X}_{\mu+\lambda}) + \mathbf{L}_{\mathcal{F}} \cdot \mathbf{N}(i)$  with
      $\mathcal{F} \in \{P_r, |KKT_1| - \epsilon, |KKT_2| - \epsilon\}$ 
5:    $F_{\mathcal{F}}(i) = \prod_{l=1}^{\mu+\lambda} \mathcal{H}(\{\mathbf{M}_{\mathcal{F}, \mu+\lambda}\}_l(i))$  with
      $\mathcal{F} \in \{P_r, |KKT_1| - \epsilon, |KKT_2| - \epsilon\}$ 
6:    $R_{sl, sim}(i) = F_{P_r}(i) \cdot F_{KKT_1}(i) \cdot F_{KKT_2}(i)$ 
7:  $P_{r, sl, MC}^{(g, \mu, \lambda)}(\mathbb{X}_{\mu+\lambda}) = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} R_{sl, sim}(i)$ 

```

$M_{KKT_1, \mu+\lambda}(i)$ and the second *Karush-Kuhn-Tucker* condition $M_{KKT_2, \mu+\lambda}(i)$ is once more obtained using Kaiser & Dickman's method (Algorithm 2). The inclusion of the probability of feasibility of the constraint(s) in a sampling-based evaluation corresponds to the numerical integration (through sampling) of the heaviside function evaluation $\mathcal{H}$ of these samples drawn from the multivariate distribution of the constraints.

Reliability analysis, the process of determining the probability of failure P_f can be understood as calculating the

area under the (joint) probability distribution $f_{\mathbf{u}}(\mathbf{u})$ cut-off by the *limit-state surface* $\mathcal{F} = \{\mathbf{u} \in \mathbf{U} : g(\mathbf{u}) = 0\}$, the boundary determining failure, so over the *failure domain* $\mathbb{F} = \{\mathbf{u} \in \mathbf{U} : g(\mathbf{u}) \leq 0\}$. In SAMURAI the asymptotic reliability analysis is applied which approximates this integral in the following way when evaluated in the standard normal space and evaluated in a single point, the MPFP:

$$P_f \equiv \mathbb{P}[g(\mathbf{u}) \leq 0] = \int_{\mathbb{F}} f_{\mathbf{u}}(\mathbf{u}) d\mathbf{u} \quad (\text{A.13})$$

$$\approx \Phi(-\beta(\mathbf{u}^*)) |\mathbf{X}(\mathbf{u}^*)|^{-1/2},$$

$$\mathbf{u}^* = \arg \min_{\mathbf{u}} \phi(\mathbf{u}) \quad \text{s.t.} \quad g(\mathbf{u}) \leq 0. \quad (\text{A.14})$$

with Φ the *standard normal cumulative density function* (CDF) and with

$$\mathbf{X}(\mathbf{u}) = \mathbf{I}_{D_{\mathbf{u}}} + \|\mathbf{u}\|_2 \cdot \|\nabla g(\mathbf{u})\|_2^{-1} \cdot \Delta g(\mathbf{u}) \quad (\text{A.15})$$

$$\beta(\mathbf{u}) = \|\nabla \phi(\mathbf{u})\|_2 \cdot \|\nabla g(\mathbf{u})\|_2^{-1} \cdot \sigma(\mathbf{u}) \quad (\text{A.16})$$

$$\sigma(\mathbf{u})^2 = \mathbf{X}(\mathbf{u})^{-1} : \nabla g(\mathbf{u}) \nabla g(\mathbf{u})^{\top} \quad (\text{A.17})$$

where $\|\cdot\|_2$ refers to the L_2 -norm. To fit this nested optimization problem in a single-loop approach, the KKT

Algorithm 3 Surrogate-assisted Asynchronous Multi-objective optimization-under-Uncertainty framework for Robust and reliable solutions to Applications in Industry (SAMURAI)

Require: Evaluated sampling plan $(\{\mathbb{X}, \mathbb{U}\}, \gamma(\{\mathbb{X}, \mathbb{U}\}))$ and $(\{\mathbb{X}, \mathbb{U}\}, \mathbf{g}(\{\mathbb{X}, \mathbb{U}\}))$

Ensure: The robust and reliable Pareto front

```

1: while stopping criteria are not met do
2:   Wait predefined time
3:   Determine available  $\lambda$  and evaluating  $\mu$  nodes
4:   Update  $\{\mathbb{X}, \mathbb{U}\}$  and  $\{\mathbb{X}_{\mu}, \mathbb{U}_{\mu}\}$ 
5:   if computation nodes available:  $\lambda > 0$  then
6:     Estimate hyperparameters Kriging models
7:      $\{\mathbb{E}_{\mathbf{x}}[\mathcal{Y}(\mathbf{x}^*)], \mathbb{V}_{\mathbf{x}}[\mathcal{Y}(\mathbf{x}^*)]\}$ 
        $\leftarrow$  Pareto( $\{\mathbb{E}_{\mathbf{x}}[\mathcal{Y}(\mathbb{X})], \mathbb{V}_{\mathbf{x}}[\mathcal{Y}(\mathbb{X})]\}$ )
       s.t.  $P_f(\mathbf{x}, \mathbf{u}) \leq p_f$ 
8:     if feasible points exist then
9:        $\{\mathbb{X}_{\lambda}, \mathbb{U}_{\lambda}\} = \arg \max_{(\mathbf{x}, \mathbf{u})} P_{r, sl, i}^{(g, \mu, \lambda)}(\mathbf{x}, \mathbf{0}) \prod_i P_{r, sl, i}^{(g, \mu, \lambda)}(\mathbf{x}, \mathbf{u})$ 
10:    else
11:       $\{\mathbb{X}_{\lambda}, \mathbb{U}_{\lambda}\} = \arg \max_{(\mathbf{x}, \mathbf{u})} \prod_i P_{r, sl, i}^{(g, \mu, \lambda)}(\mathbf{x}, \mathbf{u})$ 
12:      Start evaluating  $\{\mathbb{X}_{\lambda}, \mathbb{U}_{\lambda}\}$ 
13:       $\{\mathbb{X}_{\mu}, \mathbb{U}_{\mu}\} \leftarrow \{\mathbb{X}_{\lambda}, \mathbb{U}_{\lambda}\} \cup \{\mathbb{X}_{\mu}, \mathbb{U}_{\mu}\}$ 
14:      Update iteration counter  $l \leftarrow l + 1$ 
    else
16:      Return to line 2
17: Estimate hyperparameters Kriging models
18:  $\{\mathbb{E}_{\mathbf{x}}[\mathcal{Y}(\mathbf{x}^*)], \mathbb{V}_{\mathbf{x}}[\mathcal{Y}(\mathbf{x}^*)]\} \leftarrow$  Pareto( $\{\mathbb{E}_{\mathbf{x}}[\mathbb{X}], \mathbb{V}_{\mathbf{x}}[\mathbb{X}]\}$ )
    s.t.  $P_f(\mathbf{x}, \mathbf{u}) \leq p_f$ 
19:  $\mathbb{X}^* \leftarrow \arg \text{Pareto}_{\mathbf{x}}\{\mathbb{E}_{\mathbf{x}}[\mathcal{Y}(\mathbb{X})], \mathbb{V}_{\mathbf{x}}[\mathcal{Y}(\mathbb{X})]\}$ 
    s.t.  $P_f(\mathbf{x}, \mathbf{u}) \leq p_f$ 

```

conditions of the nested problem are formulated and added as constraints to the original RBDO.

$$KKT_1 \equiv g(\mathbf{x}, \mathbf{u}) = 0 \quad (A.18)$$

$$KKT_2 \equiv \mathbf{u}^T \nabla_{\mathbf{u}} g(\mathbf{x}, \mathbf{u}) + |\mathbf{u}| \cdot |\nabla_{\mathbf{u}} g(\mathbf{x}, \mathbf{u})| = 0 \quad (A.19)$$

To evaluate the covariance matrix of the probability of failure (Ψ_{P_f}) and KKT conditions (Ψ_{KKT_1} and Ψ_{KKT_2}), the predictive errors of the probability of failure (equation A.13) $s_{P_f}^2$ and the KKT conditions $s_{KKT_1}^2$ and $s_{KKT_2}^2$ must be known. The expected feasibility of the first KKT conditions can be directly evaluated from the surrogate of the constraint. The MSPE of the second KKT condition can once more be approximated through a Taylor series based error propagation in which the covariance between the components is omitted. This leads to the following expression

$$s_{KKT_2}^2(\mathbf{x}, \mathbf{u}) \approx \mathbf{u}^{\circ 2} s_{\nabla_{\mathbf{u}} g}^2(\mathbf{x}, \mathbf{u})^T + \|\mathbf{u}\|_2^2 \cdot \|s_{\nabla_{\mathbf{u}} g}(\mathbf{x}, \mathbf{u})\|_2^2 \quad (A.20)$$

The predictive error of the probability of failure is given by

$$s_{P_f}^2(\mathbf{x}, \mathbf{u}) \approx s_{\Phi}^2(\mathbf{x}, \mathbf{u}) \cdot |\mathbf{X}(\mathbf{x}, \mathbf{u})|^{-1} + s_{|\mathbf{X}|^{-1/2}}^2(\mathbf{x}, \mathbf{u}) \cdot \Phi(\beta(\mathbf{x}, \mathbf{u}))^2. \quad (A.21)$$

In ESLA an approximation of the CDF is propagated, which is adopted in SAMURAI. For details, I refer to.⁶⁷ Now, the Generalized multi-points Reliability-Based Robust Expected Improvement can be introduced as:

$$GRBREI_{MC}^{(g, \mu, \lambda)} = \frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} R_{sim}^{(g, \mu, \lambda)}(i) \cdot \prod_{k=1}^{N_{com}} \left(\frac{1}{N_{sim}} \sum_{i=1}^{N_{sim}} R_{sl, sim}^{(k)}(i) \right) \quad (A.22)$$

Incorporated in the GAMO-framework⁶⁵ the SAMURAI framework is obtained (Algorithm 3).⁷⁴