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# Different approaches for topology optimization of building structures subjected to thermo-mechanical loads due to fire

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**Abstract**

Building structures are designed to withstand a lot of different actions. Apart from mechanical loading (service loads, self-weight, etc.), the structural behavior can be affected by thermal loading as well, such as seasonal temperature fluctuations or accidental fire loads. As a consequence of the large temperature gradient imposed on the structure, fire loading can cause severe structural damage if the structure is not designed accordingly. Typically, structural optimization procedures for building components do not include thermal loading due to fire, such that a post-optimization analysis is required to check the fire performance. Therefore, it is more efficient to include fire loading in the optimization procedure directly and evaluate how this affects the optimized topology. In this paper, a topology optimization procedure is developed for building components subjected to thermo-mechanical loads, considering transient heat conduction as the main contribution for the thermal loading. A back-calculation approach for the sensitivity analysis of the transient temperature field is described. The proposed optimization process is applied to a case study to show the influence of different approaches for the thermal analysis. The difference between a steady-state and transient thermal analysis is investigated and their influence on the optimized topologies is discussed. Overall, the results indicate that fire loading strongly affects the result and the importance of considering a transient thermal analysis in the optimization process is confirmed.

**Keywords:** *Topology optimization, compliance, transient heat conduction, thermo-mechanical loads, fire exposure*

## 1. Introduction

Building structures are not only subjected to mechanical loads, but are affected by a large variety of multi-physical influences. One of such influences is the thermal loading, either due to environmental temperature changes or, in a more extreme case, due to a large thermal gradient imposed by fire exposure. As the occurrence of a fire is plausible during the service life of a building, performance-based optimization of structural components, that also takes into account the influence of elevated temperatures among other related phenomena, can be beneficial. However, since fire loading can be described as a highly transient thermal phenomenon, the resulting temperature distribution is nonlinear and has a complex stress field due to internally restrained thermal expansion, as illustrated in Fig. 1. Furthermore, thermal loads are considered design-dependent, as the associated temperature field is dependent on the material layout in each optimization cycle [2]. Consequently, the design- and time-dependent behavior of transient thermal loads complicate the calculation of the sensitivities required for gradient-based optimization.

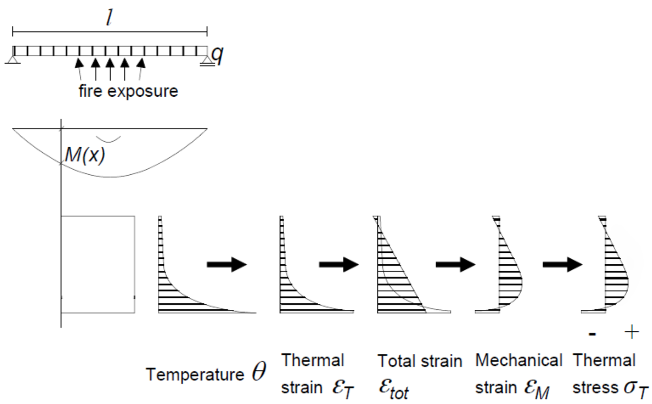


Figure 1. Transient thermal stress for a linear elastic problem (adapted from Fellinger [1]).

Since the seminal paper from Rodrigues and Fernandes [3] regarding topology optimization of thermo-elastic structures using the homogenization method for compliance-based problems, thermo-mechanical loading for structural optimization procedures have been frequently developed and discussed in literature [4-12]. However, very few research papers have been published on the topic of topology optimization with fire loading. Diaz and Benard [13] discussed topology optimization with thermal loads due to fire exposure for heat-resistant structures. More recently, Madsen et al. [14] explicitly proposed a topology optimization strategy for simplified structural fire safety. Herein, they used a constraint based on a time-dependent and heat-controlled structural degradation model driven by convective boundary conditions.

In this paper, a topology optimization procedure is discussed, taking into account the influence of elevated temperatures through transient heat conduction as a first step towards the optimization of structural building components exposed to fire. First, the governing equations for the thermo-mechanical model for both steady-state and transient problems are briefly described, followed by the topology optimization formulation. In this section, an iterative back-calculation approach to calculate the transient sensitivities is proposed, based on the work of Zhu et al. [15], who provided a clear description for the sensitivity of the thermal load vector for steady-state thermo-mechanical problems. Furthermore, the effect of a transient thermal analysis is investigated, in comparison to the assumption of steady heat transfer. Finally, a case study of a Messerschmitt-Bölkow-Blohm (MBB) beam subjected to thermo-mechanical loads is presented and discussed in detail.

## 2. Thermo-mechanical analysis

The thermo-mechanical action on the structure due to fire exposure is approximated by a sequentially coupled finite element analysis (FEA) where the calculated temperature field is used to apply a thermal load in the mechanical analysis governed by thermal expansion, in contrast to Madsen et al. [14]. The material distribution within a continuum structure subjected to elevated temperatures is interpreted as a bi-material (solid/void) layout assuming heat transfer through conduction only. Convection and radiation are neglected in the internal voids created during the optimization process, because an insulating material is considered to be inserted, which has low stiffness and conductivity. The governing equations for conductive heat transfer are given by Fourier's law of conduction in 2D for steady-state and transient thermal problems in Eq. (1) and Eq. (2) respectively, assuming isotropic and constant thermal properties.

$$-k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=q \tag{1}$$

$$\rho c_p \frac{\partial T}{\partial t}-k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=q \tag{2}$$

where $k$ is the isotropic coefficient of thermal conductivity, $\rho$ is the material density, $c_p$ is the specific heat capacity at constant pressure, $q$ the internally generated heat and $T$ is the temperature field.

The thermal FEA allows for the calculation of a nonuniform temperature field $T$, which is determined by solving the linear system of equations for the steady-state or transient thermal problem, given by Eq. (3) and Eq. (4) respectively.

$$K_T T = Q \tag{3}$$

$$C\dot{T} + K_T T = Q \tag{4}$$

where $K_T$ is the conductivity matrix, $C$ is the capacity matrix, $\dot{T}$ the time-dependent temperature field and $Q$ is the heat flux vector.

Because the thermal properties are temperature-independent, the conductivity and capacity matrices are independent of the temperature field, resulting in a linear heat conduction problem. In the case of transient heat transfer, a time-discretization scheme is used to approximate the time-dependent temperature field. In this paper, an implicit backward-Euler scheme (based on the Crank-Nicholson method) is implemented to calculate the temperature field given by Eq. (5). The benefit of using a fully implicit scheme is that it is unconditionally stable, allowing for a larger time step compared to explicit discretization schemes. This is very useful for computationally expensive optimization procedures, as the linear system for the thermal analysis is solved in each iteration. However, one needs to be careful in choosing an appropriate time step as it also influences the accuracy and convergence of the solution.

$$T^n = \left(\frac{1}{\Delta t}C + K_T\right)^{-1}\left(\frac{1}{\Delta t}CT^{n-1} + Q\right) \tag{5}$$

where $T^n$ is the temperature field after $n$ time steps $\Delta t$, which is equal to the time period (e.g. 1 hour) for the time-dependent thermal analysis.
Based on the temperature field $T^n$, the global thermal load vector can be determined from Eq. (6).

$$F_{th} = \sum_{e=1}^{N} \int_{\Omega_e} B_e^T D_e \varepsilon_{th} t d\Omega \tag{6}$$

where $t$ is the thickness, $D_e$ is the element elasticity matrix (for 2D plane stress), $B_e^T$ is the element strain-displacement matrix, $\varepsilon_{th} = \alpha\left(T_e - T_0\right)\phi$ is the element thermal strain tensor, for which $\alpha$ is the coefficient of thermal expansion, $T_e$ is the element temperature (calculated as the average of the nodal temperatures), $T_0$ is the initial reference temperature and $\phi$ is $\begin{pmatrix}1 & 1 & 0\end{pmatrix}^T$ for isotropic thermal expansion in 2D.
Finally, the mechanical FEA is expressed by the governing equation for a static analysis in Eq. (7), assuming linear elastic material behavior.

$$KU = F = F_m + F_{th} \tag{7}$$

where $K$ is the stiffness matrix, $F$ is the global load vector, $F_m$ is the mechanical load vector, $F_{th}$ is the thermal load vector and $U$ is the displacement field.

**3. Topology optimization formulation**

In this paper a density-based, FEM-based topology optimization procedure is carried out for the minimization of the elastic compliance as the objective [16], subjected to the weakly-coupled thermo-mechanical FEA and a volume constraint, with the element pseudo-densities as design variables. For the case of steady-state heat transfer and a detailed derivation of the sensitivities, the reader is referred to the work of Zhu et al. [15].
In this section, the proposed strategy for transient heat transfer and the corresponding sensitivity analysis are briefly described. The compliance-based topology optimization problem considering transient heat transfer is formulated in Eq. (8).

$$\begin{aligned}
\min_{x} \quad & c = F^T U = U^T K U = \sum_{e=1}^{N} u_e^T k_e u_e \\
\text{s.t.} \quad & V = \sum_{e=1}^{N} x_e v_e \leq f V_0 \\
& C\dot{T} + K_T T = Q \\
& KU = F = F_m + F_{th} \\
& 0 \leq x_e \leq 1 \qquad \text{with} \quad x_e \in x
\end{aligned} \tag{8}$$

where $c$ is the elastic compliance, $V$ is the volume (calculated as the sum of the element densities multiplied with the element volume $v_e$), $f$ the desired fraction of the initial volume of the design domain $V_0$, $k_e$ is the element stiffness matrix, $u_e$ the nodal displacement vector for element $e$ and $x$ the vector of the design variables $x_e$ (i.e. the relative element densities).
The general expression for the derivative of the elastic compliance with respect to the element densities is well known [16]. In case of design-dependent thermal loads, the adapted expression is given in Eq. (9).

$$\frac{\partial c}{\partial x_e} = 2U^T \frac{\partial F_{th}}{\partial x_e} + U^T \frac{\partial K}{\partial x_e} U \tag{9}$$

From Zhu et al. [15], the sensitivity of the thermal load vector $F_{th}$ is determined in Eq. (10).

$$\frac{\partial F_{th}}{\partial x_e} = \frac{\beta}{(1-\nu)} I_{th}^T \left( \frac{\partial f(x_e)}{\partial x_e} \Delta T(x_e) + \frac{f(x_e)}{4} \frac{\partial \left( t_1^e(x_e) + t_2^e(x_e) + t_3^e(x_e) + t_4^e(x_e) \right)}{\partial x_e} \right) \tag{10}$$

where $I_{th}^T = [-b \quad -a \quad b \quad -a \quad b \quad a \quad -b \quad a]$ for an element with sides $2a$ and $2b$, $f(x_e)$ represents an expression for the material interpolation, e.g. Solid Isotropic Material with Penalization (SIMP) [16], for the thermal stress coefficient $\beta = E\alpha$.

The derivative of the nodal temperatures with respect to the element density is determined based on the work of Zhu et al. [15] by taking into account the governing equation for transient thermal analysis as stated in Eq. (5) and is given by Eq. (11).

$$\frac{\partial t_i}{\partial x_e} = \frac{1}{\Delta t} \left( -T_i^T \frac{\partial C}{\partial x_e} T^n - \Delta t \left( T_i^T \frac{\partial K_T}{\partial x_e} T^n \right) + T_i^T \frac{\partial C}{\partial x_e} T^{n-1} + T_i^T C \frac{\partial T^{n-1}}{\partial x_e} \right) \tag{11}$$

where $T_i$ is the temperature field that corresponds to a unit transient pseudo-load $e_i$ at the $i$th degree of freedom in an additional, equivalent thermal FEA, which is similar to the concept of virtual work. Herein, $T_i$ is defined in Eq. (12).

$$T_i = \left( \frac{1}{\Delta t} C + K_T \right)^{-1} e_i \tag{12}$$

where the unit transient pseudo-load $e_i$ is defined in Eq. (13).

$$e_i = \frac{1}{\Delta t} C T^{n-1} + Q \tag{13}$$

In Eq. (11), $T^n$ and $T^{n-1}$ are consecutive temperature fields after $n$ and $n-1$ time steps in the transient thermal FEA, which requires back-calculation to fully determine the sensitivity of the nodal temperature $t_i$.

**4. Applications**

In this section the proposed optimization procedure, using transient thermal FEA and corresponding sensitivities, is compared to its steady-state equivalent. This mainly investigates the importance of using a fully transient approach for both the thermal analysis and sensitivities. A case study of a simply supported, prismatic beam (see Fig. 2) with dimensions 4000 × 400 × 10 mm (L × H × t) is considered. The full beam is modelled as a half-MBB beam with 200 × 40 square elements. The results shown in the following sections are mirrored to illustrate the full beam topology.

The beam is subjected to a mechanical point load, which varies between 0 kN (absent) and 200 kN, in a 3-point bending setup. Additionally, a large thermal gradient by an imposed Dirichlet boundary condition, 20°C (blue) and 800°C (red) at the top and bottom edge respectively, is applied. The sides of the beam are assumed to be adiabatic. The elastic compliance is minimized with a volume constraint of 30%.

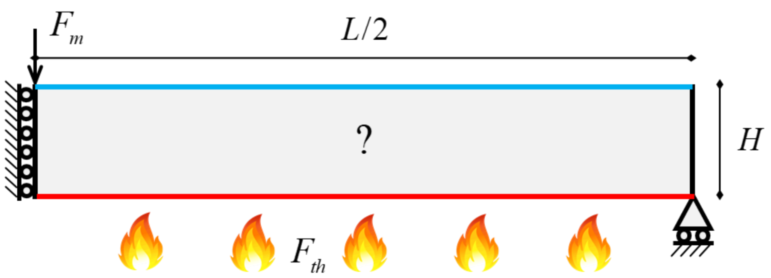


Figure 2. Design optimization problem of the MBB beam subject to thermo-mechanical loads.

A linear elastic material is assumed, in this case resembling concrete for its elastic and thermal properties. The lower limits for the material properties correspond to a fictitious insulating material in order to assume heat transfer through conduction only, neglecting convection and radiation in air-filled voids otherwise. The material properties are summarized in Table 1.

Table 1. Material properties.

| Material | $E$ (MPa) | $\nu$ (-) | $\alpha$ (m/m°C) | $k$ (W/mK) | $\rho$ (kg/m³) | $c_p$ (J/kgK) |
|---|---|---|---|---|---|---|
| Solid | 30000 | 0.3 | 12e-6 | 1 | 2400 | 900 |
| Void (insulation) | 1e-3 | 0.3 | 12e-6 | 0.025 | 23 | 900 |

Furthermore, the SIMP approach for the material interpolation is used with a penalization parameter of 3 for $E$, $k$ and $\beta$. No penalization is used for the volumetric specific heat $c_p\rho$, as it linearly changes with the density. The practical implementation of the optimization procedure using thermo-mechanical loading is based on the 88-line MATLAB code from Andreassen et al. [17], modified in order to use the MMA from Svanberg [18] as the optimization algorithm. The reader is referred to the work of Andreassen et al. for the definition of the used convergence criterion and applied density filtering technique to ensure mesh-independent solutions and to avoid checkerboard patterns in the optimized results.

4.1. Steady-state

In this section the results for the steady-state case are discussed. Overall, the results with dominant thermal loading struggle to converge. This can be attributed to many parameters in the optimization, such as the material interpolation, material properties and the ratio between their respective solid/void values. The outcome of the results was found to be affected more by the ratio of the material parameters than by the material interpolation scheme used.

The first result for the optimized beam subjected to a linear thermal gradient in absence of a mechanical load (0 kN), shows no realistic structural layout (see Fig. 3). Instead a functionally graded material layout from intermediate densities is the outcome with a final volume much lower than the imposed 0.3 volume constraint. Furthermore, the ratio of the thermal conductivity of the solid/void material influences the resulting material distribution. By increasing the point load, structural members of solid material are introduced into the final structure, approximating the intended 0-1 (void/solid) material layout. In this particular case, from the 100 kN point load onwards, the expected truss system becomes visible. Further increasing the point load will eventually lead to an optimized solution where only a mechanical load would be considered.

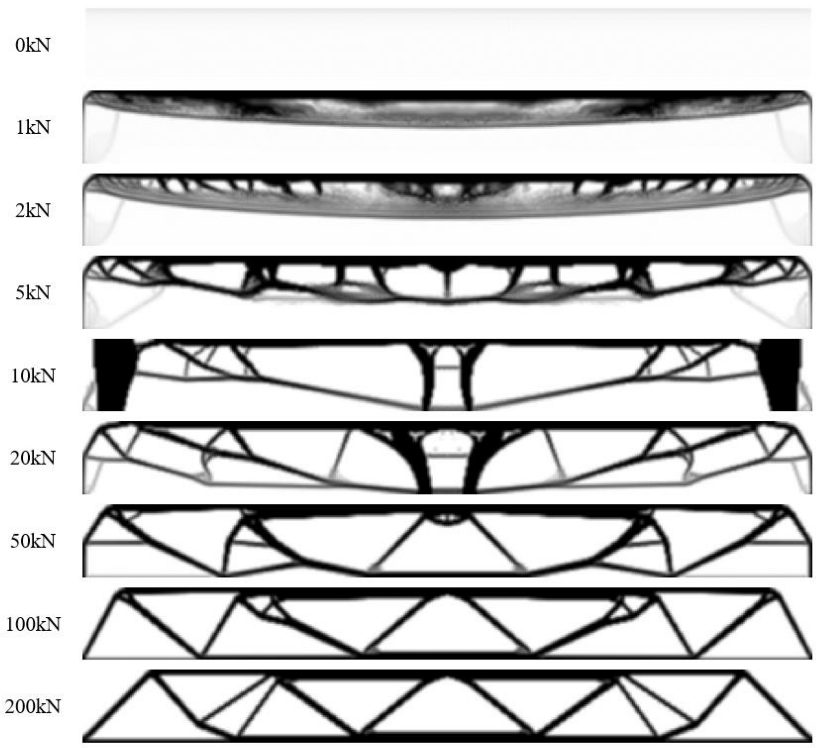


Figure 3. Results for different ratios of mechanical/thermal loading (steady-state).

4.2. Transient for different time periods

Similar to the case of the steady thermal analysis, the ratio between the thermal load and the mechanical load largely influences the optimized result. In the thermally dominant load cases with a point load between 0 kN and 20 kN, the results show a thin layer of insulating material between the main structure and the imposed boundary condition (see Fig. 4), similar to a results from [11]. After the transition into the mechanically dominant load cases from 50 kN onwards, the insulating layer disappears as it does not provide sufficient stiffness to the structure for the given load ratio. Further increasing the mechanical load will eventually approximate the optimized result for a purely mechanical load case, similar to its steady-state equivalent. The transient thermal analysis for a shorter time period (1 hour) allows for faster convergence compared to a longer time period (4 hours) or a steady-state equivalent. This is mainly due to the limited region of elements that is affected by the thermal gradient. Additionally, the insulation layer increased slightly for a time period of 4 hours (see Fig. 5) compared to the results after 1 hour in order to protect the structure more from the thermal loading. As the time period progresses further, the insulation layer thickens until it reaches the result from the steady-state case.

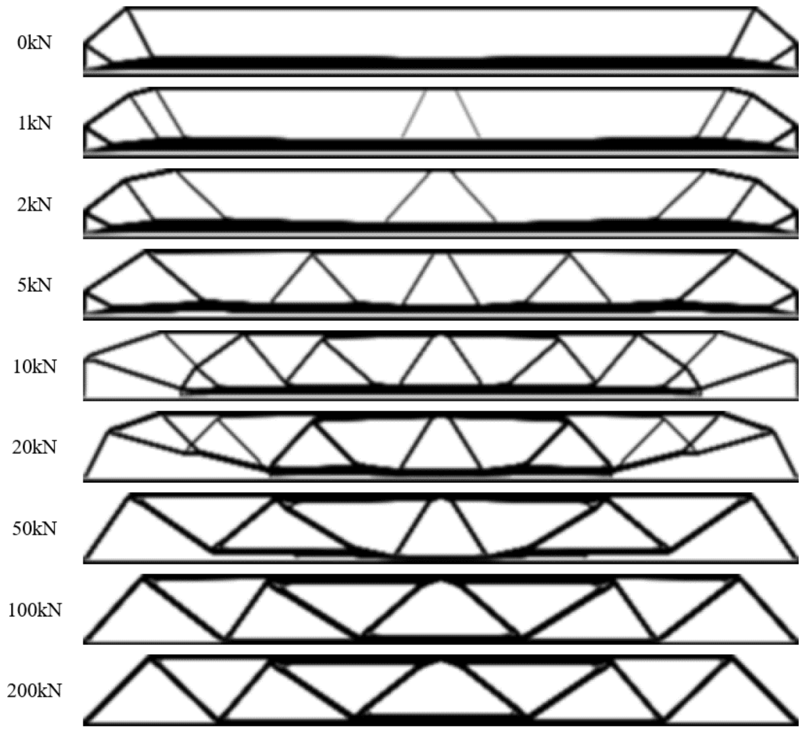

Figure 4. Results for different ratios of mechanical/thermal loading (transient – 1 hour).

In conclusion, a preliminary parameter study confirmed the influence of the ratio between the mechanical and thermal load on the optimized topology. Furthermore, the ratio of the material property values for solid/void regions largely influences the convergence rate of the optimization. For a transient problem, the result is less affected because of the smaller thermally affected region in the design domain.

**5. Conclusion**

A topology optimization procedure considering thermo-mechanical loads and transient heat conduction was proposed. A simplified approach for the sensitivity analysis of the nodal temperatures was suggested using back-calculation, facilitated by a fully implicit time-discretization scheme for the transient thermal problem. Several concluding remarks can be drawn from the preliminary results:

- The results for the optimization procedure considering thermal loads for a steady-state and transient problem differ significantly for thermally dominant load cases.
- The ratio of the mechanical load to the thermal load has a large influence on the resulting topologies.
- A steady-state thermal load case appears to be more extreme than its transient equivalent. However, steady thermal loading is unrealistic for optimization of building structures subjected to large thermal gradients, given that, during the course of a fire, the steady-state temperature distribution is never reached in concrete structures.
- The optimized topology changes depending on the specified time period for the transient thermal analysis.

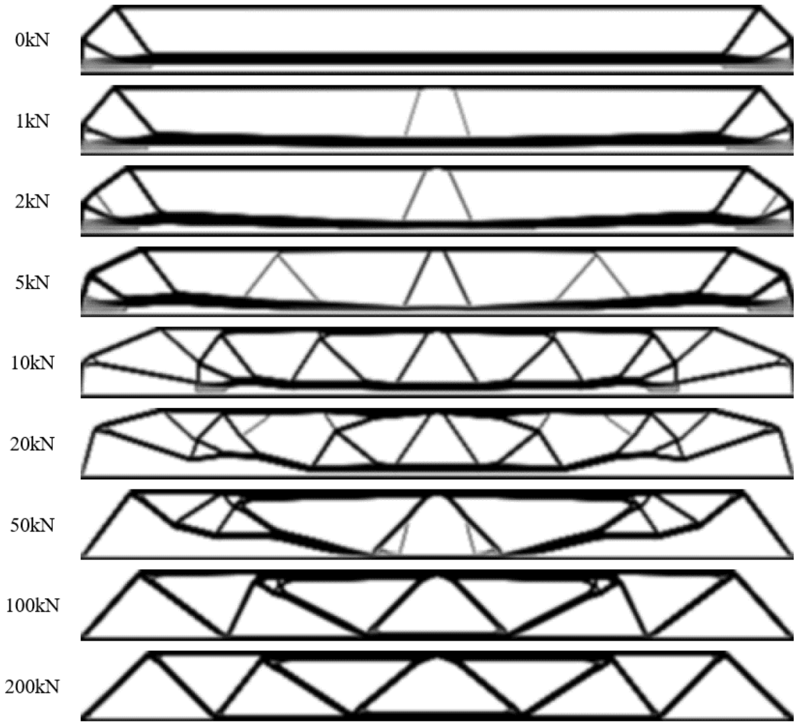

Figure 5. Results for different ratios of mechanical/thermal loading (transient – 4 hours).

In conclusion, it is relevant to continue investigations into the transient thermal analysis and corresponding sensitivities in the optimization procedure for building components subjected to accidental fire exposure. Further research is directed towards the implementation of multiple loads and varying thermal conditions during optimization, nonlinear material behavior for (reinforced) concrete and specific objectives related to structural fire safety.

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