

1 **Title:** Theta oscillations shift towards optimal frequency for cognitive control

2

3 **Author list:** Mehdi Senoussi^{1*}, Pieter Verbeke¹, Kobe Desender^{1,2,3}, Esther De Loof¹, Durk Talsma¹,

4 Tom Verguts¹

5

6 **Affiliations:**

7 1. Department of Experimental Psychology, Ghent University, Belgium

8 2. Department of Neurophysiology and Pathophysiology, University Medical Center Hamburg-

9 Eppendorf, Germany

10 3. Brain and Cognition, KU Leuven, Belgium

11 * corresponding author

12

13 **Corresponding author:**

14 Dr. Mehdi Senoussi

15 Department of Experimental Psychology, Ghent University

16 Henri Dunantlaan 2,

17 9000, Ghent, Belgium

18 E-mail: mehdi.senoussi@ugent.be

19 **Abstract:**

20 Cognitive control allows to flexibly guide behaviour in a complex and ever-changing environment. It is
21 supported by theta band (4-7Hz) neural oscillations that coordinate distant neural populations. However,
22 little is known about the precise neural mechanisms permitting such flexible control. Most research has
23 focused on theta amplitude, showing that it increases when control is needed, but a second essential
24 aspect of theta oscillations, their peak frequency, has mostly been overlooked. Here, using computational
25 modelling, behavioural and electrophysiological recordings, in three independent datasets, we show that
26 theta oscillations adaptively shift towards optimal frequency depending on task demands. We provide
27 evidence that theta frequency balances reliable set up of task representation and gating of task-relevant
28 sensory and motor information and that this frequency shift predicts behavioural performance. Our study
29 presents a mechanism supporting flexible control and calls for a re-evaluation of the mechanistic role of
30 theta oscillations in adaptive behaviour.

31 Introduction

32 Cognitive control permits adapting behaviour to task demands, crucial in an ever-changing
33 environment. The flexibility of such a fundamental cognitive ability is at the core of intelligent behaviour.
34 Cognitive control is supported by neural oscillations in the theta band (4-7Hz)^{1,2} (also called frontal midline
35 theta) that coordinate distant neural populations to create task-relevant functional networks through
36 synchronization³⁻⁷. Medial frontal cortex (MFC) generates theta oscillations when cognitive control is
37 needed, i.e. during conflict or in preparation of a difficult task¹. Task rules and goals dictating behaviour are
38 instead encoded in lateral frontal cortex (LFC)⁸⁻¹⁰. The coordination of these two areas through theta-
39 rhythmic processes, has been shown to support successful task performance¹¹. It has been proposed that
40 task-relevant functional networks are established through top-down gating from these frontal areas, by
41 synchronizing distant neural populations allowing efficient communication, i.e. communication-through-
42 coherence^{3,12}.

43 Theta oscillations thus play a critical role in the implementation of cognitive control, but to be
44 adaptive, theta oscillation characteristics must change with task demands. However, the exact neural
45 mechanisms that support flexible control remain largely unknown. Most research has focused on theta
46 amplitude, showing that it increases after conflicts and errors, causing subsequent neural adaptation
47 leading to better task performance¹.

48 Critically, a second essential aspect of theta oscillations, their peak frequency within the 4-7Hz
49 range, has occasionally been reported to vary across tasks and participants¹³⁻¹⁵. However, most studies
50 report band-average theta power per condition which precludes observing changes in peak theta frequency
51 across conditions. Moreover, estimating shifts in peak frequency from conventional representation of
52 spectral data (e.g. power spectra or time-frequency maps) is non-trivial and must avoid confounding factors
53 such as changes in the aperiodic component of the power spectra¹⁶. It therefore remains unclear whether
54 reliable theta peak differences exist. Finally, this variability and its mechanistic consequences are
55 commonly ignored and no theoretical account has considered its role in cognitive control. To address this
56 gap, we draw from two prominent frameworks: biased competition (BC)¹⁷ and communication-through-
57 coherence (CTC)¹². We built a computational model where theta oscillations orchestrate competition

58 between task representations, which in turn guides CTC to set up task-relevant functional networks. Model
59 simulations show that, depending on task demands, different theta frequencies are optimal for task
60 performance. We tested model predictions on behavioural and electrophysiological data and confirmed
61 that the frequency of theta oscillations adaptively shifts towards optimal frequency depending on task
62 demands.

63

64 **Results**

65 **Theta frequency controls reliable task implementation.**

66 We designed a stimulus-action mapping task (Figure 1a) wherein on each trial, a different mapping
67 (i.e. a rule, with variable difficulty) needs to be established. The task consists of reporting the tilt of one of
68 two gratings, clock-wise (CW) or counter-CW from the vertical axis, using the index or middle finger of one
69 of both hands. On each trial a two-letter cue instructed the rule: which was the target grating (Left (L) or
70 Right (R), top-letter) and which hand to use (L or R, bottom-letter). We thus manipulated task difficulty:
71 same-side cues (i.e. (top-letter – bottom-letter) RR and LL) were easier than different-side cues (LR and RL).

72 Our model consists of five units (Figure 1b): two control units (Lateral and Medial Frontal Cortex,
73 respectively LFC and MFC), two processing units (Sensory and Action), and an Integrator unit. In LFC, cues
74 activate instruction nodes, which themselves activate rule nodes. Rule nodes form a competitive
75 accumulator network¹⁸ that implements BC: In a Stroop-like manner, the connectivity between instruction
76 nodes induces stronger competition between rule nodes for different-side than same-side rules.
77 Importantly, rule node competition is orchestrated by theta oscillations generated by the MFC unit:
78 competition is (re-)initiated when MFC theta exceeds a processing threshold (Figure 2a, b). Each rule node
79 points to rule-relevant processing modules. Processing nodes oscillate at gamma frequency. Rule nodes
80 gate communication between Sensory and Action units through CTC^{12,19}, thereby implementing the
81 instructed mapping, by means of phase-resetting bursts emitted by MFC at theta oscillation peaks^{3,19}
82 (Figure 2b-d). The Integrator unit constitutes a competitive accumulator network¹⁸ that accumulates
83 information received from Action nodes, and triggers a response once one of the Integrator nodes reaches
84 a threshold (Figure 2e).

85

- - - - Figure 1 - - - -

86

87

88

89

90

91

92

93

94

95

Crucially, with a fast theta frequency, e.g. 7Hz, rule nodes gate processing modules frequently, shortening “off”-periods in which rule-relevant processing nodes de-synchronize, at the cost of shorter competition windows. With a slow theta frequency, e.g. 4Hz, gating is imposed less frequently, but competition windows are longer. Due to BC, one rule will win the competition; but for difficult rules, resolving the competition will take more time, i.e. require longer competition windows. In our task different-side rules are more difficult, so the model achieves better performance at slower theta frequencies where competition is long enough for the correct rule node to win (Extended Data Figure 1a-b). In contrast, for easy rules, competition is won quickly, thus higher theta frequencies yield better performance as rule-relevant nodes are frequently gated, reducing “off”-periods. Hence, an optimal agent would shift theta frequency depending on task demands.

96

- - - - Figure 2 - - - -

97

98

99

100

101

102

103

Model simulations (Figure 3a) confirmed that for difficult rules the model achieves optimal accuracy at a slow theta frequency, whereas for easy rules, a fast theta is optimal ($W = 105.5$, $p < 0.001$, $r = 0.64$, 95% CI: (1.00, 2.00); Figure 3b). Fits from the drift diffusion model on model data (see Methods) showed that only drift rate exhibited this theta-frequency – rule-difficulty interaction (Extended Data Figure 2a), refuting a speed-accuracy trade-off (SATO) explanation. Theta amplitude alone could not explain this result as theta amplitude only negligibly affected competition window length relative to frequency (Extended Data Figure 1c-d).

104

105

106

107

108

Furthermore, theta-rhythmic gating of processing nodes should yield better model performance shortly after a burst, i.e. at theta oscillation peaks (Figure 3c). By varying the instruction-stimulus delay (ISD), to sample model performance at different phases of the theta-rhythmic process^{20,21}, we showed that model accuracy oscillates at a frequency closely matching MFC theta frequency (Figure 3d, Supplementary Figure 1).

109

- - - - Figure 3 - - - -

110 These simulations lead to two key behavioural and neural predictions. First, oscillations of accuracy-
111 by-ISD should shift towards optimal theta frequency depending on task demands. Second, frontal theta
112 oscillations should also exhibit this effect, and the degree to which theta frequency shifts according to task
113 demands should be predictive of subsequent task performance.

114

115 **Frequency shift in behavioural performance oscillations.**

116 In an experiment on human participants (Dataset 1), we first confirmed that rules varied in
117 difficulty (Figure 4a). There was a significant target-location – hand interaction in accuracy (RR and LL easier
118 than LR and RL; $F(1, 33) = 27.82, p < 0.001, \eta^2 = 0.236$), and a main effect of hand ($F(1, 33) = 4.33, p = 0.045,$
119 $\eta^2 = 0.012$). Consistent with model simulations, only drift rate exhibited this interaction (Extended Data
120 Figure 2b); we therefore used accuracy as our dependent variable. To test model predictions on
121 behavioural oscillations, we computed peak theta frequency of accuracy-by-ISD (see Methods,
122 Supplementary Figure 2a). As predicted, we found a significant target-location – hand interaction ($F(1, 33) =$
123 $6.51, p = 0.015, \eta^2 = 0.047$), showing that accuracy oscillated at a slower theta frequency for difficult rules
124 (LR, RL; Figure 4b), and no main effect.

125

126 **Frequency shift in frontal theta predicts task performance.**

127 Next, we investigated whether neural theta exhibited this frequency shift due to task demands. We
128 extracted EEG theta peak frequency in a 1s pre-stimulus window from an electrode cluster exhibiting
129 significantly higher theta power in correct than incorrect trials ($p < 0.001$; Figure 4c; see Methods). As
130 predicted, peak theta frequency in correct trials significantly decreased from same-side to different-side
131 rules ($F(1, 33) = 18.96, p < 0.001, \eta^2 = 0.107$; Figure 4d; see also individual participant spectra in Extended
132 Data Figure 3). Although peak theta frequency differed numerically between different-side rules (i.e. LR and
133 RL), this difference was not statistically significant ($W=211, p = 0.139, r = -0.29, 95\% \text{ CI: } (-0.72, -0.14)$).
134 Furthermore, contrasting correct and incorrect trials revealed that higher theta frequency improved
135 performance in same-side rules, whereas a lower theta frequency improved performances in different-side
136 rules ($F(1, 33) = 4.62, p = 0.039, \eta^2 = 0.036$; Figure 4d). Finally, across participants, the degree to which
137 theta frequency shifted from difficult to easy rules positively correlated with overall accuracy ($r(32) = 0.49,$

138 $p = 0.004$, 95% CI: (0.17, 0.71); Figure 4e), indicating that a higher sensitivity of theta frequency to rule
139 difficulty improved task performance. These analyses were carried out using the FOOOF toolbox¹⁶ to
140 estimate peak and power of theta oscillations. Additional control analyses revealed that our results were
141 robust and observable without using this toolbox (i.e. by estimating theta peak frequency on raw spectra,
142 see Methods and Supplementary Figure 7). These results cannot be explained by changes in theta power
143 alone as both peak and power were estimated independently over the 1/f spectrum (Supplementary Figure
144 2b, see also the Control analyses section below).

145 - - - - Figure 4 - - - -

146

147 **Theta frequency shift generalizes to other tasks.**

148 Having established a robust effect of task demands on theta frequency in our stimulus-action
149 mapping task, we tested the generality of this mechanism, namely a decrease of theta frequency for
150 difficult tasks, to other cognitive control tasks. First, we reanalysed previously published data²² from an
151 experiment in which seventeen participants performed an arithmetic task, preceded by a cue indicating
152 whether the arithmetic operation was going to be easy or difficult (Dataset 2; Figure 5a). Different from our
153 original experiment (i.e., Dataset 1), in Dataset 2, only two levels of difficulty were used, thus allowing us to
154 test whether theta frequency is lower following a difficult compared to an easy cue. There was a significant
155 effect of difficulty on error rates and on reaction times (see original article for details²²). In order to use a
156 comparable time window for the analysis of the EEG data (relative to Dataset 1), we selected a 1 second
157 segment of EEG data in the post-cue interval. This segment was centered around the time point in which
158 the difference in theta power between difficult and easy conditions was the highest (2,000ms post-cue
159 onset, see Figure 4a in ²²). Thus, we considered EEG data in the 1,500 to 2,500ms segment post-cue onset.
160 Furthermore, because of the low number of incorrect responses (error rates of 1% and 6% for the easy and
161 difficult conditions respectively) we decided not to use the correct-incorrect contrast as in Dataset 1 (Figure
162 4c) and chose an a-priori electrode, FCz, based on prior findings in theta oscillations in cognitive control^{1,23-}
163 ²⁵. Due to the absence of identified theta oscillations using the FOOOF toolbox in the easy condition of
164 three participants, we analysed 14 participants in total in Dataset 2. Confirming the model predictions and

165 the observation from Dataset 1, we found a lower peak theta frequency in correct trials in the difficult
166 compared to easy condition ($W = 86$, $p = 0.017$, $r = 0.64$, 95% CI: (0.01, 0.14); Figure 5b).

167 Second, we reanalysed another published dataset²⁶ from an experiment in which thirty-three
168 participants performed an Go – No-Go task, where each trial was preceded by a cue indicating whether the
169 upcoming stimulus was a certain-go (i.e. a Go stimulus with 100% certainty) or a maybe-go (i.e. a No-Go
170 stimulus with 25% certainty; Dataset 3; Figure 5c). There was a significant effect of cue type on error rates
171 (see original article for details²⁶). We tested whether theta frequency is lower following a maybe-go cue
172 compared to a certain-go cue (followed by a No-Go stimulus). As for Datasets 1 and 2, we estimated peak
173 theta frequency in a 1s segment preceding stimulus onset. Similar to Dataset 2, the number of incorrect
174 responses was low (1.7% error rates in the certain-go condition), we thus used electrode FCz. Again,
175 confirming our findings from model and Datasets 1 and 2, we found a higher peak theta frequency
176 following certain-go cues compared to maybe-go cues ($W = 358$, $p = 0.039$, $r = 0.36$, 95% CI: (-0.01, 0.08);
177 Figure 5d)).

178 - - - - Figure 5 - - - -

179 **Control analyses.**

180 We showed that peak theta frequency decreases with task difficulty. A recent study has
181 demonstrated that, in the case of posterior alpha oscillations, amplitude and frequency are intrinsically
182 related²⁷ due to the thalamo-cortical circuits thought to generate alpha oscillations²⁸. One concern could be
183 that such a relationship also exists in the case of midfrontal theta oscillations, thereby confounding
184 frequency and power. We thus verified whether peak theta amplitude exhibited the same pattern of
185 decrease with task difficulty. We carried out the same analysis procedure that was used for peak theta
186 frequency on peak theta amplitude (Figure 6) and showed that there was no statistically significant
187 difference in peak theta amplitude between easy and difficult conditions in any of the three Datasets (all ps
188 > 0.078 , uncorrected for multiple comparisons). This result shows that peak theta amplitude could not
189 account for the decrease in peak theta frequency across conditions.

190 Additionally, we tested whether the shift in peak theta frequency could be confounded by
191 amplitude or frequency of nearby frequency bands. For both the delta and alpha frequency bands, we

192 followed the same procedure as for the main results on the theta band but instead we analyzed peaks in
193 the 1-3Hz range (delta) or 8-12Hz range (alpha band). No statistically significant decrease with task
194 difficulty was found for the delta or alpha band, neither in peak frequency (all ps > 0.091, uncorrected for
195 multiple comparisons, Extended Data Figure 3a, c, e, g, i, k) nor in peak amplitude (all ps > 0.200,
196 uncorrected for multiple comparisons, Extended Data Figure 3b, d, f, h, j, l).

197 These control analyses therefore suggest that the shift of peak theta frequency with task difficulty
198 happens independently of changes in theta amplitude or changes in nearby frequency bands.

199 - - - - Figure 6 - - - -

200

201 Discussion

202 In this study, we have identified an adaptive mechanism allowing flexible cognitive control. We
203 propose a computational model, test its predictions in behavioural and electrophysiological data, and show
204 that theta oscillations lawfully adapt to task demands by shifting towards optimal frequency for task
205 performance. Moreover, we replicate this finding in two independent datasets implementing entirely
206 different tasks (arithmetic operations and response inhibition) and show that the shift of theta frequency
207 according to task demands is a general mechanism involved in a wide range of cognitive processes. Finally,
208 we controlled for possible confounding factors such as amplitude modulations and changes in nearby
209 frequency bands and showed that the shift in theta peak frequency took place independently from other
210 changes in oscillatory activity.

211

212 These findings are in line with evidence that frequency of neural oscillations adapts to external
213 demands, e.g. perceptual demands in alpha band^{29,30}, and is related to short-term memory capacity in theta
214 band³¹. Our study complements and extends our understanding of how neural oscillations support
215 cognitive processes by providing a mechanistic account allowing to simulate and test further hypotheses.
216 An exciting avenue for future research lies in characterizing how adaptive shifts in theta frequency relate to
217 cross-frequency coupling dynamics³², e.g. between theta and gamma oscillations.

218 A related body of work has investigated the role of theta peak frequency in working memory
219 processes. Indeed, theta oscillations originating from medial temporal lobe and basal forebrain structures
220 (e.g. hippocampus, septum) have been hypothesized to support the maintenance of ordinal information in
221 an item sequence in working memory³¹. According to this theory, the phase of theta oscillations structures
222 the (re-)activation of distinct neural populations oscillating at gamma frequency, each representing an item
223 of the maintained sequence. This theory thus predicts that a slower theta frequency, leading to longer
224 periods in which items could be nested, would increase working memory capacity (see also ³³ for a
225 discussion of oscillatory frequency and cognitive resources). Some studies have confirmed this prediction
226 empirically by showing that higher working memory loads led to a reduction of theta frequency^{34,35}.
227 Moreover, a recent study causally tested this prediction using tACS³⁶ and showed that stimulating a fronto-
228 parietal network at a slow (i.e. 4 Hz) versus fast (i.e. 7 Hz) theta frequency led to increase in working
229 memory capacity. Although theta oscillations that support working memory and cognitive control serve
230 different purported roles (i.e. structuring maintained information in working memory versus synchronizing
231 for communication in cognitive control), and have distinct neural origins (hippocampus/septum in working
232 memory versus medial frontal cortex in cognitive control), both views on theta oscillations highlight the
233 importance of peak oscillatory frequency. One exciting avenue of research concerns the interplay between
234 theta oscillations supporting cognitive control and supporting working memory. Indeed, the control over
235 memorized items in a working memory task, for instance after retro-cuing a subset of maintained items or
236 during manipulation of a memorized sequence of items, has been shown to depend on midfrontal theta
237 oscillations^{37,38}, which are plausibly homologous to the ones observed in the current experiment and in
238 cognitive control more generally^{1,39}. Finally, some studies have demonstrated that midfrontal and
239 hippocampal theta oscillations can phase-lock or exhibit coherence to each other in certain contexts^{39,40},
240 suggesting that the two theta-generating systems can interact. For now, more studies are needed, for
241 instance using intracranial recordings in humans, to better understand the relationship between midfrontal
242 and hippocampal theta oscillations.

243

244 In our model, the MFC unit generates theta oscillations when a rule is instructed. These oscillations
245 orchestrate rule node competition and generate bursts synchronizing rule-relevant sensory and action

246 nodes. This mechanism based on theta oscillations is coherent with an energizing, or more generally
247 modulatory, role of the dorsal Anterior Cingulate Cortex (dACC), in line with the Expected Value of Control
248 (EVC) theory^{41,42}. According to the EVC theory, the dACC specifies the intensity of the control signal. This
249 has also been described as a motivational function of the dACC, in line with the observation that lesion of
250 dACC can lead to deficits in motivated behaviour⁴³. However, these accounts do not discuss the specific role
251 or importance of theta oscillations. Another line of work demonstrated that the exertion of cognitive
252 control critically relies on theta oscillations^{1,2} to create task-relevant functional networks^{3,4}. Most of these
253 studies showed that the amplitude of theta oscillations generated in the dACC, increases after conflicts and
254 errors, and that it predicts improvement in task performance¹. Our study thus extends our knowledge on
255 the energizing role of dACC by showing that, in addition to theta amplitude, another dimension of theta
256 oscillations is crucial for optimal control of task representations: theta frequency. This generalization of the
257 energizing role to a modulatory one, allows for an extra degree of freedom in control. Specifically, it posits
258 that two separate aspects of this control signal can be independently manipulated (by the dACC): the
259 intensity of the control signal through theta amplitude, and the time window of the control signal's effect
260 on the task representations through theta frequency. Previous studies have shown that adaptive changes in
261 theta amplitude (i.e. the intensity of the control signal) are critical for cognitive control, e.g. in conflict
262 adaptation^{44,45}. On the other hand, adaptively changing the processing time window through theta
263 frequency allows to adjust a trade-off in the orchestration of task representations by theta oscillations. In
264 easy task rules, a faster theta frequency is optimal as these representations are set up quickly and reliably,
265 whereas a slower theta frequency is necessary for difficult rules. This observation opens new avenues for
266 research to understand the functional role of both theta amplitude and frequency in dACC.

267 Prior models have ascribed some aspects of rule or action representations to the dACC (e.g. ^{46,47}),
268 and it has been shown empirically that dACC represents certain aspects of task sets^{41,48-51}. It could thus be
269 argued that our MFC unit should represent some aspects of actions or task sets. Here, we would like to
270 underline that our anatomical labelling (e.g. LFC, MFC) was rather broad, in part because the functional
271 architecture is not fully known, especially with respect to the division of labour between lateral and medial
272 prefrontal cortices. Thus, the two theories are not necessarily in contradiction, and may simply highlight
273 different functional roles of lateral and medial frontal cortex. We believe that it will be critically important

274 for future modelling studies to investigate how the modulation of task representations can be implemented
275 through targeted theta oscillations, and that future experimental work should aim at disentangling how
276 dACC and LFC modulate and represent and task information.

277

278 Theta oscillations have also been shown to support attention in a several previous studies (e.g. ^{20,52-}
279 ⁵⁶). Many studies showed that the amplitude of theta oscillations increases when attention is endogenously
280 oriented (sometimes referred to as sustained attention^{52,55}), or when it needs to be re-oriented^{20,53}. These
281 studies report changes of amplitude in neural oscillations in the theta frequency band or theta band
282 fluctuations in behavioural performance⁵⁵⁻⁵⁸ (see also ²¹ for a review). It has been proposed that theta
283 oscillations supporting attentional processes reflect rhythmic sampling of visual information at the
284 attended location and across the visual field. These theta oscillations are thought to be supported by a
285 network comprising the lateral intraparietal cortex, the pulvinar nucleus of the thalamus and the frontal
286 eye fields^{59,60} (but see also evidence that interaction between local receptive fields in V4 can induce theta
287 oscillations⁶¹). But to date, none of these studies reported a shift in theta frequency across conditions. One
288 possibility is that such shifts have been overlooked due to averaging of spectral amplitude across
289 frequencies in the theta frequency band that is commonly performed to test for difference in amplitude of
290 theta band oscillations, or contamination of peak frequency by other factors (e.g. the aperiodic component
291 of the spectrum).

292 However, it seems unlikely that differences in attentional demands underlie our findings. In Dataset
293 1, the difficulty between same-side and different-side instructions (which induced the theta frequency
294 shift) was situated at the stimulus-action mapping level, in contrast with attentional demand manipulations
295 of stimulus discriminability or identity (e.g. simple feature versus feature conjunction searches in ⁵⁷, or the
296 number of stimuli to track in ⁵⁸). Indeed, the tilt of grating stimuli in Dataset 1 was determined in a separate
297 block before the main experiment and kept constant throughout the main experiment blocks. Furthermore,
298 there was no difference in the validity of rule instructions relative to target location (i.e., all instructions
299 were 100% valid) and thus, no uncertainty in stimuli location that would differentially affect sampling of
300 visual information by attentional processes. Similarly, no differences specific to attentional orientation or
301 attentional sampling demands distinguished the conditions in Datasets 2 and 3. Thus, differences in

302 attentional demands alone cannot explain the shift in theta frequency we observed here. Nevertheless, it is
303 possible that different theta oscillation-generating systems co-exist and interact to support attention and
304 cognitive control. In fact, in our model, the activity of rule-relevant Sensory nodes oscillates at theta
305 frequency due to the bursts sent from the MFC unit. It would therefore be interesting in future studies to
306 investigate how midfrontal theta oscillations supporting cognitive control interact with other generators of
307 theta oscillations shown to support attentional processes⁶².

308
309 Our study also provides a potential explanation for the discrepancy in reported oscillatory
310 frequencies contributing to top-down cognitive processes⁶³. Indeed, several studies have reported the
311 involvement of different low-frequency bands^{14,64-67} during top-down control processes (e.g. decision
312 making, working memory, hierarchical task implementation). The intrinsic frequency range of theta
313 oscillations poses limits on the processes that they can orchestrate. Therefore, based on the overwhelming
314 evidence that theta oscillations support task rule implementation and action monitoring^{1,68,69}, it would
315 seem that such processes must take place within a theta cycle. In our model, we chose to use the canonical
316 4-7 Hz limits of theta oscillations, thus a task rule that would require a longer build-up time than the
317 slowest theta frequency (4Hz, period of 250ms) could not be reliably instantiated. However, several recent
318 studies have shown that the implementation of complex task rules (e.g. multiple simultaneous novel rules,
319 nested task rules) elicit frontal midline slow oscillations in the delta range (1-3Hz; e.g. ^{65,66}). Although we
320 have not found such a spread into lower or higher frequency bands in our own Datasets (see Extended Data
321 Figure 3), these results potentially question the conventional frequency limits of oscillations supporting
322 cognitive control (usually attributed to the theta band). More abstract task rules (e.g. ^{65,70}) recruit a larger
323 extent of frontal areas⁷¹ and more rostral frontal areas, in line with accounts of a hierarchical organization
324 of the frontal cortex ^{70,72}. This larger network of areas might thus require longer periods to build up stable
325 task representations and necessitate slower oscillations (in the low theta or delta bands) to efficiently and
326 reliably implement such abstract task rules.

327 It would therefore be of interest to further test whether a gradual increase in task complexity or
328 abstractness could elicit a slowing of neural oscillations generated by the medial frontal cortex towards the
329 delta range. For instance, based on the stimulus-action mapping task we developed for our model and

330 Dataset 1, it would be interesting for future research to see whether we observe further slowing of
331 midfrontal theta oscillations (i.e. into the delta frequency band) if we increase the number of response
332 options from two (i.e. clockwise, counter-clockwise) to three or four (i.e. different angles of grating rotation
333 relative to the vertical). Another possibility to study the effect of gradual increase in task complexity would
334 be to use multi-step tasks, such as hierarchical and/or temporally-extended tasks^{73,74}. Such studies would
335 inform our understanding of the interaction between task complexity and the flexibility of the temporal
336 scale of neural operations.

337 Our model predicts that a decrease in MFC theta frequency from easy to difficult tasks is beneficial
338 for behavioural performance. Several studies have tested the causal role of theta oscillations in cognitive
339 control using transcranial alternating current stimulation (tACS) in the theta frequency band⁷⁵⁻⁷⁷. In these
340 studies, a fixed theta frequency (e.g. 6 Hz) is used across participants and conditions. It would therefore be
341 interesting to test this prediction from our model by varying tACS frequency across the theta frequency
342 range.

343
344 The Integrator unit in our model aggregates inputs from the Action unit, which is itself activated by
345 the Sensory unit; the Integrator unit commits to a response when the activity of any node in this unit
346 reaches a (collapsing) threshold (see Methods). The slope of the information accumulation of the correct
347 node (for a particular trial) thus reflects the strength of the sensory signal or the difficulty of the instructed
348 rule; or, more generally, the task difficulty. This relationship with task difficulty is consistent with the
349 relationship observed empirically between the P3b ERP component (also referred to as Centro-Parietal
350 Positivity, CPP) and the difficulty of perceptual decisions⁷⁸. Indeed, the Integrator unit of our model
351 constitutes a leaky competing accumulator network¹⁸, which is thought to capture essential dynamics of
352 perceptual decision making as studied in⁷⁸. One interesting avenue for future research is the observation
353 that our model accumulation in the Integrator node associated with the correct response, is locked to
354 gamma oscillations that modulated the activity of rate code neurons in each neural triplet. It has been
355 previously shown that evidence accumulation is subject to slow-rhythmic fluctuations in the delta
356 frequency band⁷⁹. It would therefore be interesting to test whether additional fast-rhythmic dynamics exist
357 in the upslope of the P3b component, which could have been hidden in previous studies. Indeed, if these

358 gamma oscillations were not phase-locked across trials, the ERP averaging procedure would average out
359 such fluctuations.

360

361 We observed a large interindividual variability in peak theta frequency across conditions (see
362 individual spectra in Supplementary Figures 3-6). Although interindividual variability of the absolute theta
363 peak (in Hertz) can be partly attributed to non-functional sources of variance such as skull conductivity and
364 thickness^{80,81}, variations in individual peak frequency correlate with cognitive performance in the alpha
365 band⁸²⁻⁸⁴ and this variability is related to properties of cortico-thalamic white matter projections⁸⁵. This
366 suggests that individual peak frequencies, in different frequency bands can be a stable neurophysiological
367 trait⁸⁶ and that this variability of peak theta frequency, in itself, would be an interesting topic of
368 investigation for future studies. For instance, investigating the causes and consequences of individual peak
369 theta frequency could have an important impact on the development of personalized neurostimulation
370 interventions using TMS or tACS⁷⁷. Indeed, targeting peak theta frequency could allow to optimally
371 modulate functional connectivity which has been shown to be dysregulated in Alzheimer's disease⁸⁷⁻⁸⁹.
372 Moreover, significant response variability exists in repetitive TMS treatment using intermittent Theta Burst
373 Stimulation (iTBS) for treatment of major depressive disorder⁹⁰. It would thus be interesting to test whether
374 individualized iTBS frequency, estimated in a separate experimental procedure (e.g. ⁹¹), could, at least
375 partly, reduce this response variability.

376

377 Despite the robust and replicable association of theta oscillations and cognitive control, the
378 neurobiological underpinnings of theta generation and modulation remain unclear. Microcircuit models of
379 theta generation in anterior cingulate cortex (ACC) have been proposed⁹², and although the relevance of
380 peak frequency fluctuations has been mentioned, no clear mechanism driving such fluctuations was
381 proposed yet. One candidate mechanism could be a reinforcement-learning system based on ACC-
382 brainstem structures involving the locus coeruleus (LC) and noradrenergic neuromodulation of ACC
383 circuits⁹³. Indeed, the LC heavily innervates medial frontal cortex, has been shown to modulate cortical
384 oscillations, and its activity increases with task demands⁹⁴. It would thus be interesting to test whether
385 noradrenergic pathways modulate the frequency of ACC-generated theta oscillations in response to task

386 demands. Future studies investigating these candidate neurobiological mechanisms allowing adaptive
387 cognitive control will be crucial to better understand pathogenesis of several psychiatric disorders, e.g.
388 attention-deficit/hyperactivity disorder^{93,95}.

389

390 Neural oscillations may address the fundamental binding problem in cognition by gating
391 information flow in the brain to support cognitive flexibility^{33,96}. Our results provide critical insights into the
392 adaptive nature of theta oscillations supporting cognitive control, and call for a more systematic evaluation
393 of theta characteristics, at computational, behavioural, and neurophysiological levels.

394 **Methods**

395 **Model**

396 **Overview**

397 The model implements biased competition (BC) and communication-through-coherence (CTC) and consists
398 of five units: two control units (lateral and medial frontal cortex, LFC and MFC respectively), two processing
399 units (Sensory and Action units), and an Integrator unit accumulating evidence from the Action unit, and
400 producing a response. We will first briefly describe how BC and CTC are implemented in the model, and then
401 proceed to a detailed description of each unit, and the nodes composing them.

402 BC proposes that task representations compete, biased by top-down input. We implemented BC in
403 the LFC unit, which was composed of rule nodes that pointed to specific processing nodes. Each rule node
404 pointed to processing modules composing the rule. This allows a rule node to gate task representations
405 (encoded via an input-output matrix), relevant for that particular rule. For instance, a rule node could
406 implement the rule “report sensory feature 1 using action set 2” (see this example in Figure 1b). We used
407 location (Left (L) or Right (R)) as a sensory feature. We used two action sets, namely Left (L) and Right (R)
408 hand (see Action unit in Figure 1b). Rule nodes were interconnected to create a competitive accumulator
409 network. Each rule node also received a biasing input throughout a trial from instructions in the form of two
410 letters presented simultaneously and modelled as a top letter instructing which stimulus feature was the
411 target (L or R) and bottom letter instructing which action set to use (L or R). We refer to these instructions,
412 or rules, in this manner: RL for “Right-Left”, in which the first letter refers to the top instruction letter,
413 instructing the target stimulus feature (Right grating), and the second letter refer to the bottom instruction
414 letter, instructing the action set to use (Left hand). Each rule in the task (i.e. RR, LL, LR, RL) activated a unique
415 set of instruction nodes (see in Figure 1b, LFC unit). Two nodes represented the top letter of an instruction,
416 and two others the bottom letter. This network of instruction nodes created a congruency effect between
417 instruction letters: top and bottom “Left” nodes were connected, thereby activating each other, and similarly
418 for “Right” nodes. In a Stroop-like manner, the connectivity in instruction nodes induced a stronger input to
419 rule nodes for same-side (LL, RR) than for different-side (LR, RL) rules. Furthermore, different-side rules also

420 activated non-instructed instruction nodes more than same-side rules due to the lateral excitation in
421 instruction nodes, thereby making the BC between rule nodes more difficult for different-side rules to win.

422 The top-down bias signal from control units was implemented through CTC. The MFC unit generated
423 theta oscillations. During a temporal window whose size depended on the specific theta frequency (i.e. the
424 slower theta, the longer the temporal window), a competition was initiated between rule nodes. During this
425 competition window, MFC unit sent bursts of activity^{11,19,97}. The most active rule node (i.e. the one “winning”
426 the competition) amplified the burst and sent it to Sensory and Action nodes it points to. All Sensory and
427 Action nodes oscillated at gamma frequency. These bursts reset the phase of Sensory and Action nodes
428 selected by the LFC unit, and increased synchrony between them, allowing for efficient communication, i.e.
429 gating. Through this selective routing of bursts to Sensory and Action nodes, the model implements CTC by
430 creating functional networks to implement a rule.

431 As a result of the BC, one rule (typically, the correct one) will win the competition; but in cases in
432 which the competition is stronger, it will require a longer competition window for the correct rule to win the
433 competition. The latter are difficult rules. In the model, rule difficulty was implemented through conflicting
434 instructions that activated more rule nodes than easy instructions, making the competition more balanced
435 between the instructed and the other rule nodes. The consequence is that for difficult rules, the model will
436 achieve better performance at a slower theta oscillation frequency because longer competition will permit
437 rule nodes to win the competition and thus gate the rule-relevant processing nodes. In contrast, for easy
438 rules, performance increases with a higher theta frequency because rule nodes quickly win the competition
439 and the faster theta frequency allows to more frequently gate rule-relevant processing nodes. Hence, an
440 adaptive agent would shift theta frequency depending on task demands.

441

442 **Oscillatory nodes: a neuronal triplet**

443 In the MFC unit and processing units, each node i implements a cortical column simplified as a triplet of
444 neurons, as used in previous models^{19,45,97}: a rate code neuron (x_i) and two phase neurons (one excitatory
445 (E_i) and one inhibitory (I_i) neuron), see Figure 1b-c and Figure 2b, c-d. Phase neurons, i.e. the E-I pair,
446 generate oscillations with a frequency defined by the E-I pair’s coupling parameter (C). This E-I architecture
447 uses the same basic principles as the pyramidal-interneuron network gamma (PING) model, which is

448 commonly used to model gamma frequency generation^{98–100} but has also been used to simulate neural
449 oscillations in other frequency bands (e.g. ^{101,102}). The activity of each phase neuron is defined by a system of
450 stochastic difference equations, following previous work^{19,97}, for E neurons:

$$E_i(t + \Delta t) = E_i + \Delta t(-CI_i(t) - Damp)(r > r_{\min})E_i(t) + B_i(t) \quad 1$$

451 and for I neurons:

$$I_i(t + \Delta t) = I_i + \Delta t(CE_i(t) - Damp)(r > r_{\min})I_i(t) \quad 2$$

452 In which $E_i(t)$ and $I_i(t)$ denote the activity of the excitatory and inhibitory neurons of node i at time t and
453 model data were simulated at 500Hz, so $\Delta t=0.005s$. The radius r of oscillation ($r = E^2 + I^2$) of an E-I pair,
454 which corresponds to its oscillatory amplitude, is constrained to a radius $r_{\min} = 1$ (except for simulations in
455 which we varied MFC theta amplitude, see Extended Data Figure 1c-d). To implement this constraint, we use
456 an indicator function $J(\cdot)$, which returns 1 when its argument is true (i.e., when $r > r_{\min}$), and 0 otherwise.
457 The parameter $Damp$ represents the strength of the attraction towards r_{\min} and prevents the activity of the
458 E-I pair from growing too large (i.e., it dampens activity of the E and I neurons). In more neurophysiologically
459 realistic models (e.g. ¹⁰³), such dampening of the E-I pair oscillatory amplitude would be implemented via a
460 projection between the E-I pair and a pool of inhibitory neurons that in turn can inhibit the E and I neurons.
461 For convenience, we here implemented the simpler, approximate implementation via the indicator function
462 J and the $Damp$ parameter, in line with previous models^{19,45,97}. The parameter $Damp$ was set to 0.3 for
463 processing nodes. For the MFC node, $Damp$ was set to $0.005 * \theta_frequency$ to scale with the speed of the
464 E-I pair theta oscillations and maintain an equal amplitude across time for all theta frequencies. The term B_i
465 denotes the burst that processing nodes could receive depending on the trial instructions (see Medial Frontal
466 Cortex unit and Lateral Frontal Cortex unit for details). The MFC node did not receive bursts, thus B_{MFC} was
467 set to 0.

468 The frequency of oscillations generated by the E-I pair was defined by the coupling parameter C , and
469 its relation to frequency in Hertz is given by the following equation:

$$C = f2\pi \quad 3$$

470 In which f denotes the frequency in Hertz and C the coupling parameter in the E-I pair.

471 Rate neurons receive, process, and transmit information to other nodes. Their activity (x_i) is
 472 determined by the input to the node (in_i). For instance, in a Sensory node, the input in_i to a rate neuron is
 473 either zero (if its preferred stimulus feature is not presented) or 0.02 (if its preferred stimulus is presented;
 474 see Processing units for more details). This input is then modulated by its excitatory phase neuron (E_i). Thus,
 475 rate neuron activity is updated by:

$$x_i(t + \Delta t) = x_i(t) + \Delta t(-x_i(t) + in_i F(E_i(t))) \quad 4$$

476 with $F(.)$ being a logistic function of E_i :

$$F(E_i) = \frac{1}{1 + e^{-5(E_i(t) - \theta_E)}} \quad 5$$

477

478 Processing units

479 The processing units are a Sensory unit and an Action unit. Each unit is composed of nodes
 480 representing cortical columns (see Oscillatory nodes: a neuronal triplet section).

481 In all nodes of Sensory and Action units, the coupling parameter C was set to generate gamma
 482 oscillations. The gamma oscillations were set to 30 Hz by using a coupling parameter of $C = 188.5$, which, in
 483 the computational implementation of the model, was set to 0.377 to account for the sampling rate at 500Hz
 484 ($C/500 = 0.377$). We used low gamma-band oscillations around 30 Hz as this sub-band of gamma has been
 485 shown to be critically important for visual processes^{104,105} and to be modulated by theta-band oscillations
 486 following task cues¹¹. To test model stability, we also ran simulations using a higher gamma frequency of 50
 487 Hz and found similar results.

488 To further show model stability and induce noise in processing nodes' oscillatory phase, we modified
 489 the neural triplet dynamics used in prior implementations¹⁹ in which noise was introduced by independently
 490 varying the oscillatory frequency of each neural triplet across trials, while fixing it across time for each single
 491 trial. For that purpose, we added random slow fluctuations in the coupling parameter of nodes oscillating at
 492 gamma frequency, thereby mimicking noise in ongoing gamma oscillations as observed in empirical
 493 studies¹⁰⁶. We generated random numbers from a normal distribution with parameters $\mu = 1$, $\sigma = 1$, for each
 494 trial and each processing unit (i.e. Sensory and Action). A low-pass filter was then applied to these coupling
 495 fluctuations time courses, i.e. Gaussian convolution with $\sigma = 1$ (in seconds). Finally, the coupling parameter

496 (i.e. $C = 0.377$, for 30Hz oscillations) was multiplied by the value of these low-frequency coupling fluctuations.
 497 The result of this manipulation was slow random fluctuations of gamma frequencies in phase neurons of
 498 processing units. For example, for one trial, Sensory nodes were oscillating at 32Hz at a certain time t , then
 499 gradually shifting to 27Hz, then to 35Hz, etc. This slow fluctuation was generated independently for Sensory
 500 and Action units.

501 Rate neurons of the Action unit receive input from rate neurons of Sensory nodes in order to
 502 implement the two-alternative orientation discrimination task on gratings. The main task was to report
 503 whether the target grating was tilted clock-wise (CW) or counter-CW (CCW) from the vertical axis. To report
 504 the tilt the rule was to use the index and middle fingers of either the left or right hand, indicated by the
 505 instructions. The left middle finger and right index finger should be used to report a grating tilted CW, and
 506 the left index finger and the right middle finger should be used to report a grating tilted CCW. Therefore, the
 507 connectivity between Sensory and Action nodes' rate neurons implemented this rule.

508

509 **Integrator unit**

510 The Integrator unit accumulates information for each response, and triggers the model response once one
 511 of the Integrator nodes reaches a threshold. There is thus one Integrator node for each Action node. The
 512 Integrator nodes constitute a competitive accumulator network (as implemented in prior work, e.g. ¹⁸) and
 513 followed the following update:

$$\mathbf{y}_{Integ}(t + \Delta t) = \mathbf{y}_{Integ}(t) + \Delta t(\mathbf{W}_{Integ}\mathbf{x}_A + \mathbf{W}_{lat,Integ}\mathbf{y}_{Integ}(t)) + \sigma_{Integ}\mathbf{N}(t) \quad 6$$

514 In which $\mathbf{y}_{Integ}(t)$ is a vector collecting the activity of all Integrator nodes at time t , \mathbf{W}_{Integ} denotes the
 515 weight matrix between Action nodes and Integrator nodes, \mathbf{x}_A denotes input from Action nodes to Integrator
 516 nodes. $\mathbf{W}_{lat,Integ}$ denotes the update matrix of Integrator nodes in which off diagonal cells are set to -0.10
 517 to implement lateral inhibition, and diagonal cells, representing the update rate of the competitive
 518 accumulator network, are set to 1. Finally, noise was added for each of the four variable Integrator nodes
 519 with $\sigma_{Integ} = 0.05$ multiplying a vector $\mathbf{N}(t)$ of four random values drawn from a standard-normal Gaussian
 520 distribution.

521 As stated before, the Integrator unit produces a response when a threshold is reached by one of the
 522 Integrator nodes. To model a speeded task constraint, we modified the classic competitive accumulator
 523 network¹⁸ to implement a collapsing threshold, equivalent to a collapsing bound in the drift diffusion model,
 524 which has been shown to adequately model the dynamics of response threshold in speeded tasks¹⁰⁷. The
 525 threshold θ_y therefore decreased exponentially from stimulus presentation to response deadline following
 526 this equation:

$$\theta_y(t) = 4 - \left(1 - e^{\left(-\frac{t}{0.35}\right)^2}\right) \frac{a}{2} \quad 7$$

527 In which $\theta_y(t)$ denotes the threshold of the Integrator unit at time t , and a denotes the initial starting point
 528 of θ_y . In all simulations a was set to 4. Once one of the four Integrator nodes reached the threshold, we
 529 recorded the accuracy, depending on instruction, stimuli and the Integrator node which reach the threshold,
 530 and the time elapsed from stimuli onset, which provided reaction time for this response (see Figure 2e).

531

532 **Medial Frontal Cortex unit**

533 The MFC unit generates theta oscillations that 1) generate bursts that phase-reset the processing units, as in
 534 prior work^{19,45,97}, and 2) that additionally initiate a competition window in LFC nodes (see Figure 2a-b).

535 The MFC unit is composed of one single node in which the E-I pair generates theta oscillations, whose
 536 frequency depends on the coupling parameter between the E-I pair. The rate neuron of the MFC node follows
 537 a Bernoulli process (Be) with a probability defined by the activity of the node's E neuron:

$$MFC_x(t) = Be\left(\frac{1}{1 + e^{-5(E_{MFC}(t) - \theta_{burst})}}\right) \quad 8$$

538 In which Be denotes the Bernoulli process, E_{MFC} denotes the activity of the MFC E neuron, and θ_{burst} (set to -
 539 1) denotes the offset of the relation between E_{MFC} and p (probability to trigger a burst). $Be(p)$ is 1 with
 540 probability p , it will thus typically be 1 when $E_{MFC}(t)$ oscillation is near its peak. When $MFC_x = 1$, a fixed
 541 amplitude burst = 0.5 is emitted to the LFC unit. The purpose of this burst is to synchronize processing nodes
 542 selected by the LFC, by phase reset of their E neuron (see Processing units for the burst's effect, and Lateral
 543 Frontal Cortex unit for the selection of the processing nodes receiving the burst).

544 In addition to the burst-emitting function of the MFC proposed in earlier work^{3,19}, the MFC in the
 545 present model opens a competition window between rule nodes in the LFC at each cycle of its theta

546 oscillations. At each cycle of theta oscillations in E_{MFC} activity, a competition window is opened in which LFC
547 rules compete; this competition starts when $E_{MFC} > \theta_{comp}$, with $\theta_{comp} = 0.1$. The competition window lasts a
548 fixed temporal interval across cycles defined by (as just defined) θ_{comp} and the crucial C_{MFC} parameter, which
549 determines the theta frequency. To simulate different theta frequencies in the MFC, we varied the MFC
550 coupling parameter (C_{MFC}) from 0.050 (for 4Hz theta), to 0.087 (for 7Hz theta), see equation (3) in the section
551 Oscillatory nodes: a neuronal triplet.

552

553 **Lateral Frontal Cortex unit**

554 In order to implement biased competition in rule implementation, we extended previous models
555 simulating task rules. We considered rule nodes as pointers to processing nodes constituting components of
556 the rule (e.g. ^{41,108}). Such pointers permit to bias processing units according to task rules and to create
557 bindings between task-relevant components (e.g. ¹⁰⁹). In recent computational accounts incorporating
558 oscillations and synchrony, the Lateral Frontal Cortex (LFC) has been hypothesized to contain such pointers
559 which route MFC bursts to processing nodes^{3,19,45,97}. However, in these latter models no competition occurs
560 between rule nodes. In the current model, LFC is composed of rule nodes, where each such node consisted
561 of one rate code neuron only. Together, they form a competitive accumulator network¹⁸, thereby
562 implementing competition between rules. Each rule node receives a constant input throughout a trial from
563 instruction nodes, which themselves are activated by the two instruction letters. Two instruction nodes
564 represent the top letter of an instruction, and two other instruction nodes the bottom letter. This network
565 of instruction nodes implements a congruency effect between instruction letters: top and bottom “Left”
566 nodes were connected with a positive weight, thereby activating each other, and similarly for “Right” nodes
567 (see instruction nodes in Figure 1b). Instructions are represented as a vector of binary values (zeros and ones)
568 in which the first two indices represented a top L and R, respectively, and the two last indices represented
569 the presence of a bottom L and R, respectively. For instance, the rule RL was represented as ***instructions*** =
570 [0, 1, 1, 0]. This was the input to the instruction nodes, which then projected to rule nodes through the
571 following equation:

$$in_{rule} = W_{instruction} \mathbf{instructions}$$

9

572 In which in_{rule} denotes the input to rule nodes (i.e. from instruction nodes). Matrix $\mathbf{W}_{instruction}$ represents the
 573 connectivity between instruction nodes implementing the lateral excitation, i.e. instruction letter congruency
 574 effect. The diagonal of $\mathbf{W}_{instruction}$ was set to 1, and the cells representing the positive weight implementing
 575 the lateral excitation were set to 0.5.

576 The activity of rule nodes is updated through the following equation:

$$\mathbf{y}_{rule}(t + \Delta t) = \mathbf{y}_{rule}(t) + \Delta t(\mathbf{W}_{in}\mathbf{in}_{rule} + \mathbf{W}_{lat,rule}\mathbf{y}_{rule}(t)) + \sigma_{rule}\mathbf{N}(t) \quad 10$$

577 In which $\mathbf{y}_{rule}(t)$ denotes the activity of all rule nodes at time t , \mathbf{in}_{rule} denotes the input to rule nodes (i.e.,
 578 instructions) and \mathbf{W}_{in} denotes the weight matrix between instruction nodes and rule nodes in which weights
 579 between an instruction node of a particular letter and rules containing this letter was set at 0.5. For example,
 580 instruction nodes “top R” and “bottom L” projected to the rule node “RL” with weight 0.5 (see connectivity
 581 between instruction and rule nodes in Figure 1b). $\mathbf{W}_{lat,rule}$ denotes the update matrix of rule nodes in which
 582 off-diagonal cells are set to -0.1 to implement lateral inhibition; the diagonal cells, representing the update
 583 rate of the competitive accumulator network, are set at 0.13. Finally, noise was added for each of the four
 584 rule nodes with $\sigma_{rule} = 0.075$, multiplying a vector $\mathbf{N}(t)$ of four random values drawn from a standard-
 585 normal Gaussian distribution.

586 This architecture from instruction nodes to rule nodes allowed to manipulate task difficulty. For
 587 instance, the same-side rule LL, modelled as $\mathbf{instructions} = [1, 0, 1, 0]$, provided strong input to the LL
 588 rule node, and a small input to the LR and RL rule nodes as they each share the bottom and top letter,
 589 respectively, with the instruction LL. Thus, for $\mathbf{instructions} = [1, 0, 1, 0]$, $\mathbf{in}_{rule} = [0, 1.4, 0.7, 0.7]$, in
 590 which the \mathbf{in}_{rule} indices represent, in this order, RR, LL, LR and RL. On the other hand, a different-side rule
 591 like RL, modeled as $\mathbf{instructions} = [0, 1, 1, 0]$, provided a relatively strong input to the RL rule node, and
 592 a small input to LL, RR and LR nodes. Thus, for $\mathbf{instructions} = [0, 1, 1, 0]$, $\mathbf{in}_{rule} = [0.7, 0.7, 0.4, 1]$,
 593 creating a stronger competition between the instructed rule (RL) and the other rules (RR, LL and LR), see
 594 Figure 2a.

595 Finally, the most activated rule node at each time t , amplified and routed the burst emitted at time
 596 t by the MFC ($MFC_x(t)$) to the processing nodes it points to:

$$\mathbf{B}(t) = \mathbf{LFC}_{pointers} [\mathbf{y}_{rule}(t) \circ \mathbf{J}(\mathbf{y}_{rule}(t) = \max(\mathbf{y}_{rule}(t)))] MFC_x(t) \quad 11$$

597 In which $\mathbf{B}(t)$ is a vector of burst values arriving at each processing node's E neuron (to reset its phase).
598 $y_{rule}(t)$ is the activity of rule nodes at time t and \circ represents point-wise product. $J(\cdot)$ is an indicator
599 function that returns an array of 0 and 1, with 1 only for the most activated rule node at time t . $LFC_{pointers}$
600 is a matrix containing the processing nodes each rule node is pointing to. $MFC_x(t)$ is the activity of the MFC
601 rate neuron at time t . This could be 0 or 0.5 (activity values were fixed), depending on whether the MFC is
602 emitting a burst or not at that particular time point. Critically, equation (11) shows that only processing nodes
603 corresponding to the most activated rule node received the burst, while all other processing nodes did not.
604 For instance, if the instructed rule is RR and the most activated rule node at time t is RR, the Sensory module
605 "Right grating" and the Action module "Right hand" received the burst, thereby synchronizing their gamma
606 oscillations.

607 As a result of the congruency in instruction letters and BC between rule nodes, the instructed rule
608 will win the competition more quickly for same-letter rules, i.e. easy rules, than for different-letter rules, i.e.
609 difficult rules (Extended Data Figure 1a-b). Therefore, same-side rules will succeed to synchronize rule-
610 relevant processing nodes more quickly. One consequence is that, for difficult rules, the model will achieve
611 better performance at a slower theta oscillation frequency when competition lasts longer. In contrast, for
612 easy rules, model performance increases with a slightly higher theta frequency. Hence, an optimal agent
613 would shift theta frequency depending on task demands.

614

615 Simulations

616 We ran simulations of the model on the task depicted in Figure 1a. Instructions are shown for 200ms (two
617 letters), then a variable ISD between 1,700 to 2,200ms, in 11 steps of 50ms, allows to prepare the instructed
618 mapping, and subsequently two gratings are shown for 50ms. There were four possible instructions: RR, LL,
619 LR and RL.

620 The presentation of each possible stimulus was modelled as constant input set to a value of 0.02 to
621 the corresponding Sensory node. There were four possible stimulus configurations because each of the two
622 gratings could be tilted either CW or CCW. For each combination of task parameters, we ran 100 repetitions,
623 which amounts to: (11 ISD + 4 instructions + 4 stimuli configurations) * 100 = 17600 trials. We then grouped
624 repetitions into 34 groups of ~500 trials each, each representing one participant.

625

626 **Effect of amplitude on competition window**

627 To verify that high and low theta frequencies are optimal for easy and difficult tasks, respectively, we
628 independently varied MFC theta amplitude and frequency, and computed the competition window lengths
629 for each combination (Extended Data Figure 1c). Higher theta amplitudes increased the competition window
630 length but quickly reached a ceiling (around an amplitude of 3). Theta frequency on the other hand produced
631 larger increases in competition window, indicating that effects of theta frequency on model performance
632 cannot be explained by theta amplitude alone. Furthermore, we replicated our main simulation at different
633 MFC theta amplitudes and obtained similar results (i.e. the difference in optimal frequency for easy and
634 difficult rules, see Extended Data Figure 1d).

635

636 **Stimulus-action mapping experiment (Dataset 1)**

637 **Participants**

638 Thirty-nine human participants were recruited for this experiment ($M \pm \text{STD} = 23.7 \pm 4.5$ years old, range: 18-
639 41 years old; 27 females). All participants had normal or corrected-to-normal vision and no history of
640 neurological problems. All participants provided written informed consent and received monetary
641 compensation for their participation. Five participants were excluded from the analysis: two completed less
642 than 5 blocks, one had less than 200 trials after trial rejection based on eye-tracking data, one had poor
643 overall behavioural performances (i.e. less than 50% overall accuracy), and one participant was left handed.
644 The experiment was approved by the local ethics committee (Faculty of Psychology and Educational Sciences,
645 Ghent University). Sample size was not computed a priori: we aimed for more than 30 participants. First, we
646 recruited 35 participants to reach a total number of more than 30 participants after drop-out, considering a
647 ~10% drop-out rate due to noise-corrupted data or other issues related to participants' task performance.
648 The sample size after exclusions dropped at 30 participants, we thus tested 4 more participants, which were
649 all included, bringing the sample size to 34 participants. Assuming a medium effect size and aiming for a
650 power of 0.8 in a within-subject repeated measures ANOVA analysis, the study would require a sample of 32
651 participants. Data collection and analysis were not performed blind to the conditions of the experiments.

652

653 **Apparatus and stimuli**

654 Participants sat in a dimly lit room, 60 cm from a 24in LCD monitor (refresh rate: 60 Hz; resolution: 1280 ×
655 1080 pixels). A chinrest was used to stabilize head position and distance from the screen. The experiment
656 was implemented using Python 2.7 and the PsychoPy toolbox¹¹⁰.

657

658 **Experimental design**

659 Participants were instructed to perform a 2-alternative forced choice (2-AFC) orientation discrimination task
660 on two sinusoidal gratings presented simultaneously on each side of a central fixation cross, as depicted in
661 Figure 1a. Each grating was randomly tilted either CW or CCW relative to the vertical axis. The stimuli were
662 sinusoidal gratings windowed by a raised cosine (size: 5° of visual angle, 10% contrast, 3 cycles per degree,
663 at 5° eccentricity, on a gray background). The tilt angle was calculated for each participant using a staircase
664 procedure (see below) to avoid ceiling accuracy. Participants were instructed at the beginning of every trial
665 to perform the 2-AFC task on the right or left grating, and respond using their right or left hand (index and
666 middle finger respectively for CW and CCW tilt).

667 Instructions letters were presented for 200ms with a size of 0.75° of visual angle, and positioned
668 above and below the central fixation cross (vertical eccentricity: 1° of visual angle). The letter above the
669 fixation cross instructed which grating was the target, i.e. on which grating the discrimination should be
670 performed, and the letter below the fixation cross instructed which hand to use to respond. After instructions
671 a preparation interval followed to allow participants to process instructions and prepare the stimulus-action
672 mapping to perform the task. We used a dense behavioural sampling paradigm with multiple, densely
673 distributed, instruction-stimulus delays (ISD)²⁰: the duration of the ISD, between instructions and stimuli, was
674 randomly chosen on each trial from 11 possible durations going from 1,700 to 2,200ms in 11 steps of 50ms.
675 The variation in ISD was introduced to measure oscillations in behavioural performance and test predictions
676 of the model (see Figure 3c).

677 A trial time course consisted of a 1000ms baseline period, followed by instruction presentation for
678 200ms, then the ISD, and finally the stimuli presentation for 50ms. After stimuli onset, the fixation cross
679 turned blue, indicating the beginning of the 700ms response window. If a correct response was given, the

680 fixation cross turned green; if an incorrect response was given, the fixation cross turned red. If no response
681 was given during the response window, a message indicated that the participant was too slow and the
682 experiment was paused, prompting the participant to take a break if needed, and press “Space” to resume
683 the experiment. Every trial that was missed, i.e. not responded to, was added to the trial queue, and
684 presented again at the end of the block. Participants performed one training block to familiarize them with
685 the experimental design, one staircase block to compute the participant’s grating tilt angle, and between 5
686 and 8 blocks of the task depending on the number of missed trials (i.e. participants who missed more
687 response deadlines had longer blocks because trials were queued at the end of the block). The practice block
688 consisted of 80 trials, the stimulus was shown for 100ms and the response window lasted 1000ms to make
689 the practice task easier.

690 Following the practice block, participants completed a block implementing a staircase procedure on
691 the tilts of the gratings. The staircase was done across all instructions and all ISDs to find a tilt level that would
692 avoid ceiling performance and thus allow for variability across ISDs. We used a one-up two-down staircase
693 procedure consisting of 80 trials. The event timings and stimulus properties were the same as in the main
694 task. Only the tilt of the gratings varied throughout the trials. Initially, a wide tilt (7°) was set. The procedure
695 started with a step size of 3° , which was divided by 2 every other reversal starting at the second reversal. The
696 reversal corresponded to switches in participants’ response accuracy, i.e. from a sequence of correct
697 responses to an incorrect response or the other way around. When a participant switched from a correct
698 response to an incorrect response, the difficulty of the task decreased by increasing the tilt of the gratings.
699 Conversely, when a participant responded correctly after a sequence of errors, the difficulty of the task
700 increased, i.e. the tilt of the gratings decreased. The minimum tilt step size was set at 0.1° , the maximum
701 final tilt of the gratings was 30° and the minimum was 0.5° . The final tilt was the average of the last 10 tilts.

702 After the staircase block, participants completed between 5 and 8 blocks of the main task depending
703 on the number of trials missed, i.e. participants who missed the response deadline more often, had longer
704 blocks (because of queued trials), and therefore completed less blocks. In total the experiment lasted ~ 3
705 hours from explanation of the task to removing the EEG cap.

706

707 **Eye-tracking acquisition and processing**

708 We recorded eye movements using a SMI eye-tracker with a sampling rate of 250Hz (RED250 mobile system;
709 SensoMotoric Instruments, Teltow, Germany). The eye-tracker camera using infrared optics was attached to
710 the bottom of the computer screen. We used the PyGaze Python toolbox¹¹¹ to control the eye-tracker
711 through the experiment's script. Each block of the experiment started with a calibration procedure in which
712 participants had to follow a moving red dot with their eyes to nine locations on a grey background, the
713 success of which was validated before continuing. Gaze position was epoched from instructions onset to
714 stimulus presentation. To epoch gaze position data and align them with EEG data, we aligned the trial onset
715 (instructions presentation) using the trial onset trigger in eye-tracking data and the trial onset trigger in EEG
716 data. We then calculated the distance from the fixation cross in degrees of visual angle at each time point in
717 the epoch. Any trial in which the gaze was outside a 1.5° radius centered on the fixation cross at any moment
718 in the ISD, was rejected in the behavioural and EEG data.

719

720 **Behavioural data analysis**

721 As described above, trials in which gaze position distance from the fixation cross exceeded 1.5° of visual angle
722 were discarded. Trials were grouped by instruction and by ISD. Model simulations showed a theta-frequency
723 – rule-difficulty interaction in accuracy but not in reaction times (Figure 3a and Extended Data Figure 2a). We
724 therefore used accuracy as our dependent variable.

725 To compute spectra of behavioural accuracy oscillations across ISDs we first average-padded
726 accuracy values (Supplementary Figure 2a). Average-padding was performed for each participant and for
727 each instruction independently to increase frequency resolution to 1Hz²⁰. To pad the data, values
728 corresponding to average accuracy across ISDs (by instructions) were added on either side of the empirical
729 data points. Specifically, the 11 time points, spanning 500ms, were padded to get a 1,000ms segment, thus
730 adding 5 data points before the first data point and 5 after the last one.

731 Then we computed a fast Fourier transform (FFT) to obtain frequency spectra of each accuracy-by-
732 ISD time course for each participant and each instruction. FFT allows to decompose the behavioural data
733 from the time domain into frequency components to estimate an amplitude spectrum, i.e., the amplitude of
734 oscillations at each frequency present in the original data. We then extracted peak theta frequency by
735 selecting the frequency with the largest amplitude. Finally, we z-scored the peak frequency value across

736 rules, separately for each participant, to discard any difference in offset or range of theta peak frequencies
737 across participants. This procedure was carried out to specifically test the model prediction that theta peak
738 frequency decreases with task difficulty, thus inter-individual differences in theta peak frequency for each
739 instruction were not of interest in this specific analysis. The raw peak frequencies for behavioural oscillations
740 are also available in Supplementary Figure 7a (left panel).

741

742 **EEG acquisition and preprocessing**

743 EEG was recorded using a Brain Products actiChamp system with 64 active scalp electrodes positioned
744 according to the standard international 10–20 system at a sampling rate of 512 Hz. Four electrooculographic
745 (EOG) channels were used to record eye-movements and blinks: two were placed on the outer canthi of the
746 eyes, and two were placed above and below the right eye. All preprocessing steps were carried out with the
747 Python MNE toolbox v.0.21¹¹². Raw EEG data were downsampled offline to 200Hz, re-referenced to the
748 average reference and low-pass filtered at 48Hz using a FIR filter with a Hamming window. The analysis of
749 the pre-stimulus interval was performed on epochs from -1000ms to 0ms relative to stimulus onset, yielding
750 epochs of 1000ms. A linear detrend was performed on each epoch individually. After trial rejection based on
751 eye-tracking data (see Eye-tracking acquisition and processing) raw EEG and EOG time courses were visually
752 inspected on a trial-by-trial basis to reject visible artifacts, eye movements or blinks. The average percentage
753 of rejected trials across participants was $26\% \pm 14$ (mean \pm standard deviation).

754

755 **EEG spectral analysis**

756 To estimate peak frequency of theta oscillations we first computed power spectral density over the 1000ms
757 window using Welch's method provided in the Scipy toolbox v.1.3.1¹¹³. The Welch power density estimation
758 was performed using a Hann window and zero-padding to obtain 400 time points of data in order to
759 smoothen the spectra to improve estimation of peak frequency in the following analysis step. We then used
760 a recent method that allows to parametrize neural spectra by fitting the $1/f$ pattern in electrophysiological
761 recordings spectra (also called the aperiodic component), and subsequently identifies spectral peaks by
762 fitting Gaussians on the flattened spectrum (i.e. after removing the aperiodic component). This method

763 thereby provides a sensitive identification and estimation of oscillatory processes in neural activity (FOOOF
764 toolbox, version 1.0.0¹⁶).

765 Indeed, this method permits to de-confound several factors that can mask shifts in peak theta
766 frequency in grand average spectra (Supplementary Figure 3a). First and foremost, interindividual differences
767 in the 1/f structure (also called aperiodic component) of the spectrum can mask shifts of peak theta
768 frequency across conditions. More specifically, the offset and slope (also called exponent) of the aperiodic
769 component have been shown to vary across participants^{16,114–116}, see Supplementary Figure 3b. This
770 variability can therefore affect the apparent peak frequency in the grand average spectrum. Second,
771 interindividual variability in the height of theta band peaks makes it more difficult to compare the grand
772 average (as can be seen in the grand average spectra in Supplementary Figure 3a). And third, relatively large
773 peaks in the alpha band (which can be as much as 6 times larger in power than the theta frequency peaks in
774 some participants) vary in peak frequency and width across participants (see Supplementary Figure 3b
775 (middle panel), individual participants' spectra in Extended Data Figure 3 and Supplementary Figure 5-6).
776 These large peaks in the alpha band can alter the shape of the grand average spectra and mask changes in
777 peak theta frequency. Together these confounding factors require the estimation of the aperiodic
778 component of the spectrum and the independent estimation of oscillatory peaks over the aperiodic
779 component, as is performed in the FOOOF toolbox¹⁶ (but see below for control analyses in which we show
780 that our main results are visible in raw power spectra, and robust and statistically significant when estimating
781 theta peak frequency based on the raw power spectra, i.e. without the FOOOF toolbox).

782 This algorithm yields several measures, including the peak frequency and amplitude of oscillations
783 detected over the 1/f pattern in the spectra (i.e. by reporting the mean and height of the Gaussian fitted to
784 each identified spectral peak in the flattened spectrum, see Supplementary Figure 2b). Using this algorithm,
785 we computed separately for every participant, trial and electrode, whether a peak was detected in the theta
786 frequency range (i.e. higher than 3Hz, and lower than 8Hz) and we saved the estimated peak (in Hertz) and
787 the amplitude of the peak (in $\mu\text{V}^2/\text{Hz}$). Settings for the FOOOF algorithm were set as follows. To obtain peak
788 frequency and amplitude in the theta frequency range, the power spectra were parameterized across the
789 frequency range 2 to 20Hz. The peak width limits were set between 0.5 and 2, to find peaks that were
790 frequency-specific. The maximum number of peaks was set at 4, under the assumption that in the 2-20Hz

791 frequency range there could be four meaningful peaks, i.e. one in each band (delta, theta, alpha and beta).
792 No minimum peak height was set, peak threshold was set at 2 (default), and aperiodic mode was fixed
793 (default).

794 To test model predictions in theta peak frequency we separated trials according to the instruction
795 and accuracy for each participant and each electrode. As a sanity check, replication of previous findings on
796 theta amplitude, and as an independent electrode selection procedure, we investigated the scalp distribution
797 of theta oscillation power for correct versus incorrect trials for Dataset 1 (FCz was selected based on previous
798 findings¹ for Datasets 2 and 3 since there were not enough incorrect trials, see Results). In order to perform
799 this analysis, we extracted the power of peaks found in the theta band using the FOOOF toolbox, for each
800 participant, for each electrode, for each instruction, and for each trial separately. We then z-scored these
801 values across electrodes. This allowed to highlight the specific theta power topography elicited by proactive
802 cognitive control, i.e. preparing to implement an instructed stimulus-action mapping. We performed a
803 cluster-based permutation test¹¹⁷ with 10,000 permutations on scalp topographies to test whether a cluster
804 of electrodes showed relatively higher theta power in correct versus incorrect trials (across all instructions).
805 This analysis revealed a significant cluster of electrodes in fronto-central sites (permutation cluster test: $p <$
806 0.001 , Figure 4c). We then computed the average peak theta frequency (in Hertz), extracted using the FOOOF
807 toolbox, in the selected cluster of electrodes (Figure 4c). In Dataset 1, we z-scored the peak frequency value
808 across rules, separately for each participant, to discard any difference in offset or range of the EEG theta peak
809 frequencies across participants. In Dataset 2 and 3, we centered the peak frequency value across conditions,
810 separately for each participant, for the same reasons and because there were only two conditions. This
811 procedure was carried out to specifically test the model prediction that theta peak frequency decreases with
812 task difficulty, thus inter-individual differences in theta peak frequency for each instruction were not of
813 interest in this specific analysis. The raw peak frequencies for EEG oscillations in all three Datasets are also
814 available in Supplementary Figure 7a-c.

815 Because of the challenging nature of characterizing power spectra (see for instance ¹¹⁸), we also
816 carried out additional control analyses directly on the raw power spectra to test whether our main result was
817 observable without using the FOOOF toolbox (i.e. no aperiodic component estimation nor Gaussian fitting).
818 First, we sought to better illustrate the shift in theta peak frequency that can be confounded by inter-

819 individual differences and other factors (see Supplementary Figure 3a). To do so, we re-plotted the grand
820 average spectra (from Supplementary Figure 3a) after aligning each participant spectra to its own peak in the
821 most difficult rule (i.e. RL) and scaling each grand average spectrum between 0 and 1. This analysis, anchored
822 in the raw spectra (without using the FOOOF toolbox) showed that a clear shift towards higher frequencies
823 was visible for easier rules, confirming that our results are robust and observable at the group level.

824 Second, we showed that our main finding that frontal peak theta frequency decreases with rule
825 difficulty is also present (both substantially and statistically) when estimating theta peak frequency directly
826 in the raw amplitude spectra as the theta-band frequency exhibiting the highest power (in $\mu\text{V}^2/\text{Hz}$, see
827 Supplementary Figure 8). More specifically, for each participant, condition and electrode in the identified
828 fronto-central cluster, we extracted the theta-band frequency with the highest power, then averaged these
829 peak frequencies across electrodes, resulting in a theta peak frequency per participant per condition.
830 Moreover, to evaluate the presence of our predicted effect, i.e. a decrease in peak theta frequency from
831 same-side to different-side rules, we computed the proportion of participants exhibiting a positive difference
832 in peak theta frequency between same-side and different-side rules. We carried out this analysis in both peak
833 frequencies obtained using the FOOOF toolbox, and peak frequencies obtained directly from the raw spectra.
834 This analysis revealed that a majority of participants exhibited the predicted effect (see Supplementary Figure
835 8 and discussion thereof), showing once again that our results are replicated without the FOOOF toolbox,
836 thereby alleviating any concern about the challenges in the estimation of the aperiodic component
837 performed by the FOOOF toolbox (see ¹¹⁸). However, we still believe that condition- and participant-wise
838 estimation of the aperiodic component of the spectra, and frequency and power of peaks (as performed by
839 the FOOOF toolbox) is necessary to avoid confounding factors that could mask the effect or result in spurious
840 differences in peak frequency or power.

841 Finally, for the control analyses on theta amplitude (Figure 6) and nearby frequency bands (Extended
842 Data Figure 4), we followed the same procedure as for the main analyses on theta frequency band (i.e. using
843 the FOOOF toolbox). For the alpha frequency band, we followed the same procedure as for the main results
844 on the theta band but instead we considered peaks in the 8-12Hz range. For the delta frequency band, we
845 used frequency limit parameters equal to [0, 20] Hz in the parametrization of the power spectra, and

846 considered peaks in the 1-3Hz range. For both delta and alpha frequency bands, we then followed the same
847 procedure as for the main results on the theta frequency band.

848

849 **Statistical analyses**

850 To compute the optimal theta frequency per rule difficulty (same-side (RR, LL) versus different-side
851 (LR, RL) rules) in the model we calculated the MFC theta frequency yielding the highest accuracy for each
852 group of simulations (see Simulations). We then compared the two samples of optimal theta frequencies per
853 rule difficulty using a two-sided Wilcoxon signed-rank test (Figure 3b). For this, and all other Wilcoxon signed-
854 rank tests, we computed non-parametric confidence intervals using bootstrapping of the difference in
855 medians between conditions. The reported effect size for Wilcoxon signed-rank tests is the matched pairs
856 rank-biserial correlation (r)¹¹⁹.

857 Reaction times and DDM parameters estimated on model data (Extended Data Figure 2a) were
858 analyzed using a 2-by-7 repeated measure ANOVA with factors rule difficulty (two levels: same-side and
859 different-side) and theta frequency (seven levels: from 4 to 7Hz in steps of 0.5Hz), using the StatsModels
860 v0.10.1 (<https://www.statsmodels.org/v0.10.1/>) and the Pingouin v0.5.0¹²⁰ (<https://pingouin-stats.org/>)
861 Python packages. Data distribution was assumed to be normal but this was not formally tested.

862 Participants' behavioural and EEG data from the stimulus-action mapping experiment (Dataset 1)
863 were entered into two-way repeated measure ANOVAs with factors target-location (two levels: Left, Right)
864 and hand (two levels: Left, Right) using the StatsModels and Pingouin Python packages. For behavioural data
865 it consisted of accuracies, reaction times, and DDM parameters per rule (Figure 4a; Extended Data Figure 2a),
866 and peak theta frequency (i.e. of accuracy-by-ISD) per rule (Figure 4b). For EEG data it consisted of the
867 average peak theta frequency from the selected electrode cluster (see Methods) of correct trials per rule
868 (Figure 4d; circles); and the difference between average peak theta frequency of correct and incorrect trials.

869 For Datasets 2 and 3 we reported the statistical results from their behavioural data analyses. For the
870 analysis of their EEG data we followed the same procedure as for Dataset 1 and, because only two difficulty
871 levels were available in these Datasets, we performed a one-sided Wilcoxon signed-rank test to test the
872 hypothesis that theta frequency decreased from an easy to a difficult condition. For all control analyses, i.e.

873 effect of task difficulty on peak theta amplitude (Figure 6), and on peak frequency and amplitude in the delta
874 and alpha frequency bands (Extended Data Figure 4), we performed two-sided Wilcoxon signed-rank tests.

875 To investigate inter-individual differences in the sensitivity of EEG peak theta frequency to rule
876 difficulty in Dataset 1, we performed a linear regression of each participant's raw EEG peak frequency (in
877 Hertz) in correct trials ordered by each rule's overall accuracy across participants (i.e. rule was treated as a
878 linear predictor: RR = 79.16%, LL = 76.70%, LR = 71.87%, RL = 71.44%). In a second step, individual-participant
879 slopes were correlated with overall accuracy, collapsed across rules (Figure 4e). We used a robust Spearman
880 correlation (i.e. skipped correlation¹²¹) implemented in the Pingouin Python package, which identifies
881 outliers based on the minimum covariance determinant. The effect was also significant using a simple
882 Spearman correlation. For illustration purposes we computed a linear regression, excluding the identified
883 outlier, to plot it as a regression line in Figure 4e.

884

885 **Data availability:**

886 Raw behavioural, eye-tracking and EEG data can be found at this Open Science Framework repository:

887 https://osf.io/nwh87/?view_only=b11ee1f860804da582c816fe8acdecad

888

889 **Code availability:**

890 Code of the model, the behavioural experiment and analysis scripts to reproduce all results and figures

891 from the study can be found on this Github repository:

892 https://github.com/mehdisenoussi/theta_shift_cog_control

893

894 **Acknowledgments**

895 The authors want to thank Cristian Buc Calderon for fruitful discussions and comments on the manuscript.

896 MS and TV were supported by grant G012816 from Research Foundation Flanders. MS, TV and EDL were

897 supported by grant BOF17-GOA-004 from the Research Council of Ghent University. PV was supported by

898 grant 1102519N from Research Foundation Flanders. KD was supported by the FWO [PEGASUS]² Marie

899 Skłodowska-Curie fellowship 12T9717N. The funders had no role in study design, data collection and

900 analysis, decision to publish or preparation of the manuscript.

901

902 **Author contribution**

903 M.S., D.T. and T.V. designed the study. M.S., P.V. and T.V. developed the model. M.S. and E.D.L. collected
904 the data. M.S. analyzed model simulations, behavioural and EEG data. M.S. and T.V. wrote the manuscript.

905 All of the authors discussed the results and commented on the manuscript.

906

907 **Competing financial interests**

908 The authors declare no competing interests.

909

910 **Figure legends**

911

912 Figure 1. Task and model structure. a Stimulus-action mapping task. Each trial starts with a cue instructing
913 the mapping to use. In this example the rule is “LR” instructing to report the left grating’s tilt with the right
914 hand. b Model architecture. c Each node of MFC and processing (Sensory and Action) units is a neural
915 triplet composed of one excitatory (E), one inhibitory (I), and one rate neuron (x). The E-I pair generates
916 oscillations (whose frequency depends on their coupling parameter). MFC Bursts are sent to E neurons.
917 Rate neurons receive input from, and send output to, other nodes’ rate neurons. The activity (output) of a
918 rate neuron is modulated by its E neuron.

919

920 Figure 2. Model dynamics. a The top two panels represent rhythmic BC in rule nodes at instructions onset
921 (from two different trials): 3 cycles at a fast MFC theta frequency (7Hz) to illustrate difference in dynamics
922 between easy and difficult rules. Top left panel (Easy rule): the rule node corresponding to the instructed
923 rule in this trial (green curve) rapidly wins the competition over other rule nodes (grey curves). Top right
924 panel (Difficult rule): the rule node corresponding to the instructed rule in this trial (orange curve) struggles
925 to win the competition and often loses to other rule nodes (grey curves). Yellow areas represent
926 competition window opened by MFC theta oscillations. Dashed yellow line represent MFC E neuron activity
927 (see panel b). Each dot above curves represent a time point at which the instructed rule’s node was
928 winning the competition. The bottom panel represents the biasing signal sent to the processing nodes. This
929 corresponds to the burst sent by MFC (bottom plot in panel b) multiplied by the most activated rule node’s
930 activity (note that the MFC theta frequency was set at 5Hz for this panel). b Activity of the MFC neural
931 triplet at a theta frequency of 5Hz. Top panel represents activity of the E-I pair. The phase of both neurons
932 is reset at instructions onset. c-d Time course of neural triplet activity of the rule-relevant Sensory (c) and
933 Action (d) nodes around stimuli presentation with a MFC theta frequency of 5Hz. The yellow curve
934 representing MFC theta oscillations is for illustration only, its activity is scaled to fit within these plots. e
935 Time course of the four integrator nodes, representing each of the four possible responses to the task,
936 around stimuli presentation. The green line represents the correct response in the simulated trial.

937

938 Figure 3. Model simulations. a Model accuracy by rule difficulty across theta frequencies. Data are
939 presented as mean values, error bars represent standard deviation computed over $n = 34$ simulations per
940 frequency. Green curve represent easy (same-side) rules, orange curve represent difficult (different-side)
941 rules. b Violin plots representing optimal theta frequency, i.e. yielding highest model accuracy, per rule
942 difficulty. For difficult rules the model achieves optimal accuracy at a slow theta frequency, whereas for
943 easy rules, a fast theta is optimal ($n = 34$ simulations per theta frequency, two-sided Wilcoxon signed-rank
944 test: $W = 105.5$, $p < 0.001$, $r = 0.64$, $95\% \text{ CI} = (1.00, 2.00)$). Data are presented as violin plots, left- and right-
945 most bars represent extrema, middle bar represent the median. Distribution density is represented by
946 violin plot width. c Measuring theta oscillations in behaviour through densely-distributed Instruction-
947 Stimuli Delays (ISD). d Accuracy-by-ISD by rule difficulty (left), and estimated peak frequency (right). Data
948 are presented as mean values, error bars represent standard deviation computed over $n = 34$ simulations.

949

950 Figure 4. Testing model predictions in behaviour and EEG. a Overall accuracy: There was a significant target-
951 location – hand interaction in accuracy (RR and LL easier than LR and RL; two-way repeated measure
952 ANOVA: $F(1, 33) = 27.82$, $p < 0.001$, $\eta^2 = 0.236$), and a main effect of hand ($F(1, 33) = 4.33$, $p = 0.045$, $\eta^2 =$
953 0.012). b Peak frequency of oscillations in accuracy-by-ISD: there was a significant target-location – hand
954 interaction (two-way repeated measure ANOVA: $F(1, 33) = 6.51$, $p = 0.015$, $\eta^2 = 0.047$). c Frontal cluster of
955 electrodes with increased theta amplitude: significantly higher theta amplitude in correct than incorrect
956 trials (cluster test, $p < 0.001$). d Theta peak frequency by rule in frontal electrodes cluster. Peak theta
957 frequency in correct trials significantly decreased from same-side to different-side rules (two-way repeated
958 measure ANOVA: $F(1, 33) = 18.96$, $p < 0.001$, $\eta^2 = 0.107$). Comparing correct and incorrect trials, we found
959 that higher theta frequency improved performance in same-side rules, whereas a lower theta frequency
960 improved performances in different-side rules (two-way repeated measure ANOVA: $F(1, 33) = 4.62$, $p =$
961 0.039 , $\eta^2 = 0.036$). e Correlation between theta peak slope across rule difficulty and overall accuracy: the
962 degree to which theta frequency shifted from difficult to easy rules positively correlated with overall
963 accuracy (robust Spearman correlation: $r(32) = 0.49$, $p = 0.004$, $95\% \text{ CI} = (0.17, 0.71)$). Data are presented as

964 mean values, error bars represent s.e.m. computed over $n = 34$ participants. Smaller gray dots (and small
965 colored dots in panel d) represent individual participants' data.

966

967 Figure 5. Testing model predictions in other datasets. a Experimental protocol in Dataset 2. In this study
968 participants performed a cued serial mental calculation task in which a cue indicated whether the following
969 calculations will be easy, i.e. only +1 additions were used, or difficult, i.e. addition or subtraction of
970 different numbers. b Theta peak frequency at FCz for easy and difficult cues. Circles represent correct trials,
971 triangles represent incorrect trials. Peak theta frequency was lower in correct trials in the easy compared to
972 difficult condition (one-sided Wilcoxon sign-rank test: $W = 86$, $p = 0.018$, $r = 0.63$, 95% CI = (0.01, 0.14)). c
973 Experimental protocol in Dataset 3. In this study participants reacted to an action signal (Stimulus; white
974 square, circle or triangle). Each shape was randomly assigned towards one action (Go, No-Go, Switch-Go);
975 only the Go and No-Go actions were analysed because Switch-go data were unavailable. On half of the
976 trials, a *certain-go* cue (e.g. brown cross) indicated that the Stimulus was going to be a Go shape with 100%
977 certainty. On 50% of the trials a *maybe-go* cue (e.g. blue cross) indicated that the Stimulus had a 25%
978 chance of being a No-Go shape. d Theta peak frequency at FCz for *certain-go* and *maybe-go* cues. Circles
979 represent correct trials, triangles represent incorrect trials. Peak theta frequency was lower in correct trials
980 following *certain-go* cues compared to *maybe-go* cues (one-sided Wilcoxon sign-rank test: $W = 358$, $p =$
981 0.039 , $r = 0.35$, 95% CI = (-0.01, 0.08)). The yellow framing around the Cue and Instruction stages in a and c
982 represent the time window from which we extracted peak theta frequency. Data are presented as mean
983 values, error bars represent s.e.m. computed over $n = 14$ participants in Dataset 2, and $n = 33$ participants
984 in Dataset 3. Smaller gray dots represent individual participants' data.

985

986 Figure 6. Peak theta amplitude per condition in each Dataset. a Peak theta amplitude by condition for
987 correct trials in Dataset 1. There was no significant main effect or interaction ($n = 34$ participants, two-way
988 repeated measure ANOVA: all $F_s(1, 33) < 2.21$, all $p_s > 0.146$). b Peak theta amplitude by condition for
989 correct trials in Dataset 2. There was no statistically significant difference between conditions ($n = 14$
990 participants, two-sided Wilcoxon sign-rank test: $W = 24$, $p = 0.079$, $r = 0.54$, 95% CI = (-0.01, 0.03)). c Peak
991 theta amplitude by condition for correct trials in Dataset 3. There was no significant difference between

992 conditions (n = 33 participants, two-sided Wilcoxon sign-rank test: $W = 257$, $p = 0.896$, $r = -0.02$, 95% CI = (-
993 0.02, 0.02)). Data are presented as mean values, error bars represent s.e.m. Smaller gray dots represent
994 individual participants' data.
995

996 **References**

- 997 1. Cavanagh, J. F. & Frank, M. J. Frontal theta as a mechanism for cognitive control. *Trends Cogn. Sci.* 18,
998 414–421 (2014).
- 999 2. Sauseng, P., Tschentscher, N. & Biel, A. L. Be Prepared: Tune to FM-Theta for Cognitive Control. *Trends*
1000 *Neurosci.* 42, 307–309 (2019).
- 1001 3. Voloh, B. & Womelsdorf, T. A Role of Phase-Resetting in Coordinating Large Scale Neural Networks
1002 During Attention and Goal-Directed Behavior. *Front. Syst. Neurosci.* 10, (2016).
- 1003 4. Canolty, R. T. et al. High Gamma Power Is Phase-Locked to Theta Oscillations in Human Neocortex.
1004 *Science* 313, 1626–1628 (2006).
- 1005 5. Varela, F., Lachaux, J.-P., Rodriguez, E. & Martinerie, J. The brainweb: Phase synchronization and large-
1006 scale integration. *Nat. Rev. Neurosci.* 2, 229–239 (2001).
- 1007 6. Bressler, S. L., Coppola, R. & Nakamura, R. Episodic multiregional cortical coherence at multiple
1008 frequencies during visual task performance. *Nature* 366, 153–156 (1993).
- 1009 7. Palva, J. M., Palva, S. & Kaila, K. Phase Synchrony among Neuronal Oscillations in the Human Cortex. *J.*
1010 *Neurosci.* 25, 3962–3972 (2005).
- 1011 8. Wallis, J. D. & Miller, E. K. From Rule to Response: Neuronal Processes in the Premotor and Prefrontal
1012 Cortex. *J. Neurophysiol.* 90, 1790–1806 (2003).
- 1013 9. Stokes, M. G. et al. Dynamic Coding for Cognitive Control in Prefrontal Cortex. *Neuron* 78, 364–375
1014 (2013).
- 1015 10. Mansouri, F. A., Freedman, D. J. & Buckley, M. J. Emergence of abstract rules in the primate brain. *Nat.*
1016 *Rev. Neurosci.* 21, 595–610 (2020).
- 1017 11. Voloh, B., Valiante, T. A., Everling, S. & Womelsdorf, T. Theta–gamma coordination between anterior
1018 cingulate and prefrontal cortex indexes correct attention shifts. *Proc. Natl. Acad. Sci.* 112, 8457–8462
1019 (2015).
- 1020 12. Fries, P. Rhythms for Cognition: Communication through Coherence. *Neuron* 88, 220–235 (2015).
- 1021 13. Klimesch, W., Schack, B. & Sauseng, P. The Functional Significance of Theta and Upper Alpha
1022 Oscillations. *Exp. Psychol.* 52, 99–108 (2005).

- 1023 14. Cooper, P. S. et al. Frontal theta predicts specific cognitive control-induced behavioural changes
1024 beyond general reaction time slowing. *NeuroImage* 189, 130–140 (2019).
- 1025 15. Nigbur, R., Cohen, M. X., Ridderinkhof, K. R. & Stürmer, B. Theta Dynamics Reveal Domain-specific
1026 Control over Stimulus and Response Conflict. *J. Cogn. Neurosci.* 24, 1264–1274 (2011).
- 1027 16. Donoghue, T. et al. Parameterizing neural power spectra into periodic and aperiodic components. *Nat.*
1028 *Neurosci.* 23, 1655–1665 (2020).
- 1029 17. Desimone, R. & Duncan, J. Neural Mechanisms of Selective Visual Attention. *Annu. Rev. Neurosci.* 18,
1030 193–222 (1995).
- 1031 18. Usher, M. & McClelland, J. L. The time course of perceptual choice: The leaky, competing accumulator
1032 model. *Psychol. Rev.* 108, 550–592 (2001).
- 1033 19. Verguts, T. Binding by Random Bursts: A Computational Model of Cognitive Control. *J. Cogn. Neurosci.*
1034 29, 1103–1118 (2017).
- 1035 20. Senoussi, M., Moreland, J. C., Busch, N. A. & Dugué, L. Attention explores space periodically at the
1036 theta frequency. *J. Vis.* 19, 22–22 (2019).
- 1037 21. Kienitz, R., Schmid, M. C. & Dugué, L. Rhythmic sampling revisited: Experimental paradigms and neural
1038 mechanisms. *Eur. J. Neurosci.* n/a,.
- 1039 22. De Loof, E. et al. Preparing for hard times: Scalp and intracranial physiological signatures of proactive
1040 cognitive control. *Psychophysiology* 56, e13417 (2019).
- 1041 23. Cavanagh, J. F., Cohen, M. X. & Allen, J. J. B. Prelude to and Resolution of an Error: EEG Phase
1042 Synchrony Reveals Cognitive Control Dynamics during Action Monitoring. *J. Neurosci.* 29, 98–105
1043 (2009).
- 1044 24. Luu, P., Tucker, D. M. & Makeig, S. Frontal midline theta and the error-related negativity:
1045 neurophysiological mechanisms of action regulation. *Clin. Neurophysiol.* 115, 1821–1835 (2004).
- 1046 25. Senoussi, M. et al. Pre-stimulus antero-posterior EEG connectivity predicts performance in a UAV
1047 monitoring task. in 2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC) 1167–
1048 1172 (2017). doi:10.1109/SMC.2017.8122770.
- 1049 26. Kaiser, J. & Schütz-Bosbach, S. Proactive control without midfrontal control signals? The role of
1050 midfrontal oscillations in preparatory conflict adjustments. *Biol. Psychol.* 148, 107747 (2019).

- 1051 27. Nelli, S., Itthipuripat, S., Srinivasan, R. & Serences, J. T. Fluctuations in instantaneous frequency predict
1052 alpha amplitude during visual perception. *Nat. Commun.* 8, 1–12 (2017).
- 1053 28. Lopes da Silva, F. H., Vos, J. E., Mooibroek, J. & van Rotterdam, A. Relative contributions of intracortical
1054 and thalamo-cortical processes in the generation of alpha rhythms, revealed by partial coherence
1055 analysis. *Electroencephalogr. Clin. Neurophysiol.* 50, 449–456 (1980).
- 1056 29. Wutz, A., Melcher, D. & Samaha, J. Frequency modulation of neural oscillations according to visual task
1057 demands. *Proc. Natl. Acad. Sci.* 115, 1346–1351 (2018).
- 1058 30. Samaha, J., Bauer, P., Cimaroli, S. & Postle, B. R. Top-down control of the phase of alpha-band
1059 oscillations as a mechanism for temporal prediction. *Proc. Natl. Acad. Sci.* 112, 8439–8444 (2015).
- 1060 31. Lisman, J. E. & Jensen, O. The Theta-Gamma Neural Code. *Neuron* 77, 1002–1016 (2013).
- 1061 32. Siebenhühner, F., Wang, S. H., Palva, J. M. & Palva, S. Cross-frequency synchronization connects
1062 networks of fast and slow oscillations during visual working memory maintenance. *eLife* 5, e13451
1063 (2016).
- 1064 33. Senoussi, M., Verbeke, P. & Verguts, T. Time-Based Binding as a Solution to and a Limitation for Flexible
1065 Cognition. *Front. Psychol.* 12, (2022).
- 1066 34. Axmacher, N. et al. Cross-frequency coupling supports multi-item working memory in the human
1067 hippocampus. *Proc. Natl. Acad. Sci.* 107, 3228–3233 (2010).
- 1068 35. Kosciessa, J. Q., Grandy, T. H., Garrett, D. D. & Werkle-Bergner, M. Single-trial characterization of
1069 neural rhythms: Potential and challenges. *NeuroImage* 206, 116331 (2020).
- 1070 36. Wolinski, N., Cooper, N. R., Sauseng, P. & Romei, V. The speed of parietal theta frequency drives
1071 visuospatial working memory capacity. *PLOS Biol.* 16, e2005348 (2018).
- 1072 37. Riddle, J., Scimeca, J. M., Cellier, D., Dhanani, S. & D’Esposito, M. Causal Evidence for a Role of Theta
1073 and Alpha Oscillations in the Control of Working Memory. *Curr. Biol.* 30, 1748-1754.e4 (2020).
- 1074 38. Itthipuripat, S., Wessel, J. R. & Aron, A. R. Frontal theta is a signature of successful working memory
1075 manipulation. *Exp. Brain Res.* 224, 255–262 (2013).
- 1076 39. Mitchell, D. J., McNaughton, N., Flanagan, D. & Kirk, I. J. Frontal-midline theta from the perspective of
1077 hippocampal “theta”. *Prog. Neurobiol.* 86, 156–185 (2008).

- 1078 40. Siapas, A. G., Lubenov, E. V. & Wilson, M. A. Prefrontal Phase Locking to Hippocampal Theta
1079 Oscillations. *Neuron* 46, 141–151 (2005).
- 1080 41. Shenhav, A., Botvinick, M. M. & Cohen, J. D. The Expected Value of Control: An Integrative Theory of
1081 Anterior Cingulate Cortex Function. *Neuron* 79, 217–240 (2013).
- 1082 42. Shenhav, A. et al. Toward a Rational and Mechanistic Account of Mental Effort. *Annu. Rev. Neurosci.*
1083 40, 99–124 (2017).
- 1084 43. Holroyd, C. B. & Yeung, N. Motivation of extended behaviors by anterior cingulate cortex. *Trends Cogn.*
1085 *Sci.* 16, 122–128 (2012).
- 1086 44. Pastötter, B., Dreisbach, G. & Bäuml, K.-H. T. Dynamic Adjustments of Cognitive Control: Oscillatory
1087 Correlates of the Conflict Adaptation Effect. *J. Cogn. Neurosci.* 25, 2167–2178 (2013).
- 1088 45. Verbeke, P. & Verguts, T. Neural Synchrony for Adaptive Control. *J. Cogn. Neurosci.* 1–19 (2021)
1089 doi:10.1162/jocn_a_01766.
- 1090 46. Holroyd, C. B. & McClure, S. M. Hierarchical control over effortful behavior by rodent medial frontal
1091 cortex: A computational model. *Psychol. Rev.* 122, 54–83 (2015).
- 1092 47. Holroyd, C. B. & Verguts, T. The Best Laid Plans: Computational Principles of Anterior Cingulate Cortex.
1093 *Trends Cogn. Sci.* 25, 316–329 (2021).
- 1094 48. Holroyd, C. B., Ribas-Fernandes, J. J. F., Shahnazian, D., Silvetti, M. & Verguts, T. Human midcingulate
1095 cortex encodes distributed representations of task progress. *Proc. Natl. Acad. Sci.* 115, 6398–6403
1096 (2018).
- 1097 49. Womelsdorf, T., Johnston, K., Vinck, M. & Everling, S. Theta-activity in anterior cingulate cortex predicts
1098 task rules and their adjustments following errors. *Proc. Natl. Acad. Sci.* 107, 5248–5253 (2010).
- 1099 50. Haynes, J.-D. et al. Reading Hidden Intentions in the Human Brain. *Curr. Biol.* 17, 323–328 (2007).
- 1100 51. Smith, E. H. et al. Widespread temporal coding of cognitive control in the human prefrontal cortex. *Nat.*
1101 *Neurosci.* 22, 1883–1891 (2019).
- 1102 52. Helfrich, R. F. et al. Neural Mechanisms of Sustained Attention Are Rhythmic. *Neuron* 99, 854-865.e5
1103 (2018).
- 1104 53. Dugué, L., Roberts, M. & Carrasco, M. Attention Reorients Periodically. *Curr. Biol.* 26, 1595–1601
1105 (2016).

- 1106 54. Dugué, L., McLelland, D., Lajous, M. & VanRullen, R. Attention searches nonuniformly in space and in
1107 time. *Proc. Natl. Acad. Sci.* 112, 15214–15219 (2015).
- 1108 55. Fiebelkorn, I. C., Saalman, Y. B. & Kastner, S. Rhythmic Sampling within and between Objects despite
1109 Sustained Attention at a Cued Location. *Curr. Biol.* 23, 2553–2558 (2013).
- 1110 56. Landau, A. N. & Fries, P. Attention Samples Stimuli Rhythmically. *Curr. Biol.* 22, 1000–1004 (2012).
- 1111 57. Dugué, L., Xue, A. M. & Carrasco, M. Distinct perceptual rhythms for feature and conjunction searches.
1112 *J. Vis.* 17, 22 (2017).
- 1113 58. Holcombe, A. O. & Chen, W.-Y. Splitting attention reduces temporal resolution from 7 Hz for tracking
1114 one object to <3 Hz when tracking three. *J. Vis.* 13, 12–12 (2013).
- 1115 59. VanRullen, R. Perceptual Cycles. *Trends Cogn. Sci.* 20, 723–735 (2016).
- 1116 60. Fiebelkorn, I. C. & Kastner, S. A Rhythmic Theory of Attention. *Trends Cogn. Sci.* (2018)
1117 doi:10.1016/j.tics.2018.11.009.
- 1118 61. Kienitz, R. et al. Theta Rhythmic Neuronal Activity and Reaction Times Arising from Cortical Receptive
1119 Field Interactions during Distributed Attention. *Curr. Biol.* 28, 2377-2387.e5 (2018).
- 1120 62. Fiebelkorn, I. C. & Kastner, S. A Rhythmic Theory of Attention. *Trends Cogn. Sci.* 23, 87–101 (2019).
- 1121 63. Helfrich, R. F. & Knight, R. T. Oscillatory Dynamics of Prefrontal Cognitive Control. *Trends Cogn. Sci.* 20,
1122 916–930 (2016).
- 1123 64. de Vries, I. E. J., Slagter, H. A. & Olivers, C. N. L. Oscillatory Control over Representational States in
1124 Working Memory. *Trends Cogn. Sci.* 24, 150–162 (2020).
- 1125 65. Riddle, J., Vogelsang, D. A., Hwang, K., Cellier, D. & D’Esposito, M. Distinct Oscillatory Dynamics
1126 Underlie Different Components of Hierarchical Cognitive Control. *J. Neurosci.* 40, 4945–4953 (2020).
- 1127 66. Formica, S., González-García, C., Senoussi, M. & Brass, M. Neural oscillations track the maintenance
1128 and proceduralization of novel instructions. *NeuroImage* 232, 117870 (2021).
- 1129 67. Formica, S., González-García, C., Senoussi, M., Marinazzo, D. & Brass, M. Theta-phase connectivity
1130 between medial prefrontal and posterior areas underlies novel instructions implementation. *bioRxiv*
1131 2022.02.23.481594 (2022) doi:10.1101/2022.02.23.481594.
- 1132 68. Cavanagh, J. F., Zambrano-Vazquez, L. & Allen, J. J. B. Theta lingua franca: A common mid-frontal
1133 substrate for action monitoring processes. *Psychophysiology* 49, 220–238 (2012).

- 1134 69. Cohen, M. X. Midfrontal theta tracks action monitoring over multiple interactive time scales.
1135 *NeuroImage* 141, 262–272 (2016).
- 1136 70. Nee, D. E. & D’Esposito, M. The hierarchical organization of the lateral prefrontal cortex. *eLife* 5,
1137 e12112 (2016).
- 1138 71. Voytek, B. et al. Oscillatory dynamics coordinating human frontal networks in support of goal
1139 maintenance. *Nat. Neurosci.* 18, 1318–1324 (2015).
- 1140 72. Badre, D. & D’Esposito, M. Is the rostro-caudal axis of the frontal lobe hierarchical? *Nat. Rev. Neurosci.*
1141 10, 659–669 (2009).
- 1142 73. Shahnazian, D., Senoussi, M., Krebs, R. M., Verguts, T. & Holroyd, C. B. Neural Representations of Task
1143 Context and Temporal Order During Action Sequence Execution. *Top. Cogn. Sci.* n/a, (2021).
- 1144 74. Balaguer, J., Spiers, H., Hassabis, D. & Summerfield, C. Neural Mechanisms of Hierarchical Planning in a
1145 Virtual Subway Network. *Neuron* 90, 893–903 (2016).
- 1146 75. Driel, J. van, Sligte, I. G., Linders, J., Elport, D. & Cohen, M. X. Frequency Band-Specific Electrical Brain
1147 Stimulation Modulates Cognitive Control Processes. *PLOS ONE* 10, e0138984 (2015).
- 1148 76. Lehr, A., Henneberg, N., Nigam, T., Paulus, W. & Antal, A. Modulation of Conflict Processing by Theta-
1149 Range tACS over the Dorsolateral Prefrontal Cortex. *Neural Plast.* 2019, e6747049 (2019).
- 1150 77. Riddle, J. & Frohlich, F. Targeting neural oscillations with transcranial alternating current stimulation.
1151 *Brain Res.* 1765, 147491 (2021).
- 1152 78. O’Connell, R. G., Dockree, P. M. & Kelly, S. P. A supramodal accumulation-to-bound signal that
1153 determines perceptual decisions in humans. *Nat. Neurosci.* 15, 1729–1735 (2012).
- 1154 79. Wyart, V., de Gardelle, V., Scholl, J. & Summerfield, C. Rhythmic Fluctuations in Evidence Accumulation
1155 during Decision Making in the Human Brain. *Neuron* 76, 847–858 (2012).
- 1156 80. Hagemann, D., Hewig, J., Walter, C. & Naumann, E. Skull thickness and magnitude of EEG alpha activity.
1157 *Clin. Neurophysiol.* 119, 1271–1280 (2008).
- 1158 81. Voytek, B. et al. Hemicraniectomy: A New Model for Human Electrophysiology with High Spatio-
1159 temporal Resolution. *J. Cogn. Neurosci.* 22, 2491–2502 (2010).
- 1160 82. Klimesch, W. EEG alpha and theta oscillations reflect cognitive and memory performance: a review and
1161 analysis. *Brain Res. Rev.* 29, 169–195 (1999).

- 1162 83. Mierau, A., Klimesch, W. & Lefebvre, J. State-dependent alpha peak frequency shifts: Experimental
1163 evidence, potential mechanisms and functional implications. *Neuroscience* 360, 146–154 (2017).
- 1164 84. Haegens, S., Cousijn, H., Wallis, G., Harrison, P. J. & Nobre, A. C. Inter- and intra-individual variability in
1165 alpha peak frequency. *NeuroImage* 92, 46–55 (2014).
- 1166 85. Minami, S., Oishi, H., Takemura, H. & Amano, K. Inter-individual Differences in Occipital Alpha
1167 Oscillations Correlate with White Matter Tissue Properties of the Optic Radiation. *eNeuro* 7, (2020).
- 1168 86. Grandy, T. H. et al. Peak individual alpha frequency qualifies as a stable neurophysiological trait marker
1169 in healthy younger and older adults. *Psychophysiology* 50, 570–582 (2013).
- 1170 87. Jafari, Z., Kolb, B. E. & Mohajerani, M. H. Neural oscillations and brain stimulation in Alzheimer’s
1171 disease. *Prog. Neurobiol.* 194, 101878 (2020).
- 1172 88. Pistono, A. et al. Language Network Connectivity Increases in Early Alzheimer’s Disease. *J. Alzheimers*
1173 *Dis.* 82, 447–460 (2021).
- 1174 89. Brem, A.-K. & Sensi, S. L. Towards Combinatorial Approaches for Preserving Cognitive Fitness in Aging.
1175 *Trends Neurosci.* 41, 885–897 (2018).
- 1176 90. Chen, L., Chung, S. W., Hoy, K. E. & Fitzgerald, P. B. Is theta burst stimulation ready as a clinical
1177 treatment for depression? *Expert Rev. Neurother.* 19, 1089–1102 (2019).
- 1178 91. Slobodskoy-Plusnin, J. Behavioral and brain oscillatory correlates of affective processing in subclinical
1179 depression. *J. Clin. Exp. Neuropsychol.* 40, 437–448 (2018).
- 1180 92. Cohen, M. X. A neural microcircuit for cognitive conflict detection and signaling. *Trends Neurosci.* 37,
1181 480–490 (2014).
- 1182 93. Silvetti, M., Vassena, E., Abrahamse, E. & Verguts, T. Dorsal anterior cingulate-brainstem ensemble as a
1183 reinforcement meta-learner. *PLOS Comput. Biol.* 14, e1006370 (2018).
- 1184 94. Sara, S. J. Locus Coeruleus in time with the making of memories. *Curr. Opin. Neurobiol.* 35, 87–94
1185 (2015).
- 1186 95. Silvetti, M., Wiersema, J. R., Sonuga-Barke, E. & Verguts, T. Deficient reinforcement learning in medial
1187 frontal cortex as a model of dopamine-related motivational deficits in ADHD. *Neural Netw.* 46, 199–
1188 209 (2013).

- 1189 96. Bonnefond, M., Kastner, S. & Jensen, O. Communication between Brain Areas Based on Nested
1190 Oscillations. *eNeuro* 4, (2017).
- 1191 97. Verbeke, P. & Verguts, T. Learning to synchronize: How biological agents can couple neural task
1192 modules for dealing with the stability-plasticity dilemma. *PLOS Comput. Biol.* 15, e1006604 (2019).
- 1193 98. Bartos, M., Vida, I. & Jonas, P. Synaptic mechanisms of synchronized gamma oscillations in inhibitory
1194 interneuron networks. *Nat. Rev. Neurosci.* 8, 45–56 (2007).
- 1195 99. Whittington, M. A., Cunningham, M. O., LeBeau, F. E. N., Racca, C. & Traub, R. D. Multiple origins of the
1196 cortical gamma rhythm. *Dev. Neurobiol.* 71, 92–106 (2011).
- 1197 100. Tiesinga, P. & Sejnowski, T. J. Cortical Enlightenment: Are Attentional Gamma Oscillations Driven by
1198 ING or PING? *Neuron* 63, 727–732 (2009).
- 1199 101. Wang, X. J., Golomb, D. & Rinzel, J. Emergent spindle oscillations and intermittent burst firing in a
1200 thalamic model: specific neuronal mechanisms. *Proc. Natl. Acad. Sci.* 92, 5577–5581 (1995).
- 1201 102. Gips, B., Eerden, J. P. J. M. van der & Jensen, O. A biologically plausible mechanism for neuronal
1202 coding organized by the phase of alpha oscillations. *Eur. J. Neurosci.* 44, 2147–2161 (2016).
- 1203 103. Wong, K.-F. & Wang, X.-J. A Recurrent Network Mechanism of Time Integration in Perceptual
1204 Decisions. *J. Neurosci.* 26, 1314–1328 (2006).
- 1205 104. Jia, X., Xing, D. & Kohn, A. No Consistent Relationship between Gamma Power and Peak Frequency
1206 in Macaque Primary Visual Cortex. *J. Neurosci.* 33, 17–25 (2013).
- 1207 105. Siegel, M., Warden, M. R. & Miller, E. K. Phase-dependent neuronal coding of objects in short-term
1208 memory. *Proc. Natl. Acad. Sci.* 106, 21341–21346 (2009).
- 1209 106. Atallah, B. V. & Scanziani, M. Instantaneous Modulation of Gamma Oscillation Frequency by
1210 Balancing Excitation with Inhibition. *Neuron* 62, 566–577 (2009).
- 1211 107. Palestro, J. J., Weichart, E., Sederberg, P. B. & Turner, B. M. Some task demands induce collapsing
1212 bounds: Evidence from a behavioral analysis. *Psychon. Bull. Rev.* 25, 1225–1248 (2018).
- 1213 108. Botvinick, M. M. & Cohen, J. D. The Computational and Neural Basis of Cognitive Control: Charted
1214 Territory and New Frontiers. *Cogn. Sci.* 38, 1249–1285 (2014).
- 1215 109. Müller, M. G., Papadimitriou, C. H., Maass, W. & Legenstein, R. A model for structured information
1216 representation in neural networks. *ArXiv161103698 Q-Bio* (2019).

- 1217 110. Peirce, J. et al. PsychoPy2: Experiments in behavior made easy. *Behav. Res. Methods* 51, 195–203
1218 (2019).
- 1219 111. Dalmaijer, E. S., Mathôt, S. & Van der Stigchel, S. PyGaze: An open-source, cross-platform toolbox
1220 for minimal-effort programming of eyetracking experiments. *Behav. Res. Methods* 46, 913–921 (2014).
- 1221 112. Gramfort, A. et al. MEG and EEG data analysis with MNE-Python. *Front. Neurosci.* 7, (2013).
- 1222 113. Virtanen, P. et al. SciPy 1.0: fundamental algorithms for scientific computing in Python. *Nat.*
1223 *Methods* 17, 261–272 (2020).
- 1224 114. Manning, J. R., Jacobs, J., Fried, I. & Kahana, M. J. Broadband Shifts in Local Field Potential Power
1225 Spectra Are Correlated with Single-Neuron Spiking in Humans. *J. Neurosci.* 29, 13613–13620 (2009).
- 1226 115. He, B. J. Scale-free brain activity: past, present, and future. *Trends Cogn. Sci.* 18, 480–487 (2014).
- 1227 116. Voytek, B. et al. Age-Related Changes in 1/f Neural Electrophysiological Noise. *J. Neurosci.* 35,
1228 13257–13265 (2015).
- 1229 117. Maris, E. & Oostenveld, R. Nonparametric statistical testing of EEG- and MEG-data. *J. Neurosci.*
1230 *Methods* 164, 177–190 (2007).
- 1231 118. Gerster, M. et al. Separating neural oscillations from aperiodic 1/f activity: challenges and
1232 recommendations. 2021.10.15.464483
1233 <https://www.biorxiv.org/content/10.1101/2021.10.15.464483v1> (2021)
1234 doi:10.1101/2021.10.15.464483.
- 1235 119. Kerby, D. S. The Simple Difference Formula: An Approach to Teaching Nonparametric Correlation.
1236 *Compr. Psychol.* 3, 11.IT.3.1 (2014).
- 1237 120. Vallat, R. Pingouin: statistics in Python. *J. Open Source Softw.* 3, 1026 (2018).
- 1238 121. Rousselet, G. A. & Pernet, C. R. Improving standards in brain-behavior correlation analyses. *Front.*
1239 *Hum. Neurosci.* 6, (2012).
- 1240