An analytical model for budget allocation in risk prevention and risk protection

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Abstract

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Risk is inherent and inevitable during project execution. Its occurrence often has an adverse impact on the project or even leads to project failure. In order to ensure the successful project completion, risk prevention used for reducing the risk probability and risk protection used for reducing the risk loss should be considered. Given the different effects and costs of these two strategies and the limited project budget, project managers should be able to reasonably divide the budget among these two strategies. This study proposes a budget allocation method consisting of three modules for mitigating the project risks. First, we model the relation between the effect and cost of each strategy as linear and non-linear relations. Second, two mathematical models are built and the corresponding analytical solutions are obtained. Afterwards, a three-step procedure for budget allocation is proposed and illustrated risk examples are discussed. To validate the analytical results and investigate the impacts of the characteristics of the project risk and response on the optimal budget allocation, numerical experiments are conducted and managerial insights are drawn. Finally, the budget allocation model is extended with multiple risks, secondary risk and risk transfer, and validated using an empirical analysis.

Keywords: Budget allocation, risk prevention, risk protection, analytical solution, project risk management

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1. Introduction

Risk is inherently related to projects since unexpected events will impact the project and potentially change the project outcomes and objectives. This paper will discuss the budget allocation problem in a project risk management (PRM) setting. Risk management in projects entails three phases, i.e. risk identification, risk analysis and risk response. The first two phases identify and evaluate the underlying risks that would affect the project objectives (Chapman, 1997). The risks above a certain level are considered *intolerable*, and will be mitigated in the risk response phase. Since project managers (PMs) are aware of the inherent nature of project risks, the most important issue in PRM is not the identification and evaluation of the risks, but rather how to respond and deal with these underlying risks (Elmaghraby, 2005).

Generally, risk is measured by its expected risk loss, also known as risk magnitude (Kayis et al., 2007), which is computed as the product of its *probability* of occurrence and its *impact* on the project. Hence, two kinds of strategies can be used in the phase of risk response: risk prevention and risk protection. In this paper, *risk prevention* aims at reducing the occurrence probability of the risk, while *risk protection* mitigates risk loss. Although risk prevention and risk protection can contribute to the risk reduction, they both consume project budget and the risk reduction resulting from risk prevention or risk protection highly depends on the invested budget amount. Thus, making trade-offs between risk prevention and risk protection as well as simultaneously determining the budget amount allocated to each strategy are two critical problems of great practical need in achieving an effective risk response in projects.

In order to solve these challenging problems in PRM, we propose an approach for budget allocation among risk prevention and risk protection. First, we analyse two possible relations between the costs and effects of risk prevention and risk protection given the characteristics of the project risk and the response strategy. Then, two mathematical models are built for determining the optimal budget amount for each strategy. Finally, numerical experiments are carried out to validate the results obtained from the analytical models and investigate the impacts of risk parameters on the optimal budget allocation. In order to increase the practical value of our model, possible extensions of the budget allocation problem and an empirical analysis are presented.

To derive analytical solutions, we introduce two types of relations between risk response costs and effects. In the first type, the relation between the response cost and effect is modelled as a *continuous linear curve*. This type of relation is based on the idea that the achieved outcome of the response effect can be better when more budget is invested. However, the linear relation is only applicative in a limited number of situations, such as a fixed return rate of investments. Hence, the second type of relations considers *non-linear relations* between response costs and effects, represented by convex functions. This type of relation is based on the idea that the effect of risk responses would gradually decrease as the risk has already been reduced to some extent. It is noted that risk probability ranges from 0 to 1, while the risk loss is generally a large positive value. This makes it difficult to compare the effects of risk prevention (i.e. focused on risk probability) and protection (i.e. focused on risk loss). In order to address this issue, we introduce two auxiliary variables, i.e. the ratios of probability and loss after risk response, to represent the risk response effect.

The remainder of this paper is structured as follows. In Section 2, we summarise the existing literature on risk response models as well as types and costs of risk response strategies. Section 3 presents the formulation of the problem statement and the response budget allocation models. Section 4 reports the general approach to solve the budget allocation models, the optimality conditions and the analytical results for both the linear and non-linear cases. Numerical experiments and results are shown in Section 5, while three possible extensions of the budget allocation models in Section 4 are discussed in Section 6. In Section 7, we illustrate the practical applicability of the proposed models using real-life project data. Finally, the conclusions and future research are discussed in Section 8.

2. Literature overview

In this section, we present an overview of the existing literature on risk response models in projects (Section 2.1), types of risk response strategies (Section 2.2) and costs of risk response strategies (Section 2.3). Finally, we discuss our contributions to the existing literature in Section 2.4. A summary of the related work in literature is presented in Table 1.

Table 1: A summary of literature reviewed

Cost	References	Methods	Main focus	Risk			Strategy	
				P	L	R	Effect	Relation
Unfixed	Sherali et al. (2008)	INT	BA	×	✓	×	L	logit choice based and linear functions
	Sherali et al. (2011)	INT	RRS	×	✓	×	L	logit choice based and linear functions
	Sato and Hirao (2013)	OPT	BA	✓	✓	×	P	power-law function
	Fan et al. (2008)	OPT	RRS	✓	✓	✓	P, L	logarithmic function for P, linear or exponential function for L
	Kuo et al. (2019)	OPT	Tradeoff	✓	✓	✓	P, L	logarithmic function for P, linear or exponential function for L
	Mizgier et al. (2015)	INT	RRS	✓	✓	×	P, L	No explicit function
Fixed	Miller and Lessard (2001)	ZON	RRS	×	×	×	×	
	Piney (2002)	ZON	RRS	✓	✓	×	R	
	Dey (2002, 2012); Kujawski and Angelis (2010); Marmier et al. (2014, 2013)	DT	RRS	×	×	✓	R	
	Todinov (2013, 2014); Zhang and Fan (2014)	OPT	RRS	×	×	✓	R	
	Ben-David et al. (2002); Ben-David and Raz (2001); Glickman and Khamooshi (2005)	OPT	RRS	✓	✓	✓	P, L	
	Kayis et al. (2007)	OPT	RRS	✓	✓	✓	R	
	Wu et al. (2018)	OPT	RRS	✓	✓	✓	P	
	Ykhlef and Algawiaz (2014)	INT	RRS	✓	✓	✓	P	
	Špačková and Straub (2015)	INT	RRS	×	×	✓	R	
	Fang et al. (2013); Reniers and Sörensen (2013); Zhang and Guan (2018)	INT	RRS	✓	✓	✓	P, L	
Unconsidered	Elkjaer and Felding (1999)	ZON	RRS	✓	✓	×	P, L	
	Lyons and Skitmore (2004)	Survey	RRS	×	×	×	P, L	
Unfixed	Current paper	OPT	BA	✓	✓	✓	P, L	linear and logarithmic functions for both P and L

Note: RRS = evaluating, investigating, determining or selecting risk response strategy. BA = budget allocation. INT = integrated model. OPT = optimization-based model. ZON = zonal-based model. DT = decision tree-based model. P = risk probability. L = risk loss. R = expected loss of the risk.

2.1. Risk response models in projects

The project risk response problems are generally modelled as determining and selecting risk response strategies from a pool of alternatives. These risk response models can be divided into four categories according to the methods used, namely, *zonal-based models*, *decision tree-based models*, *optimization-based models* and *integrated models*.

The *zonal-based models* map the risk response strategies into multiple zones shown in a two

- dimensional diagram, where the two dimensions represent the characteristics of risks or projects, such as the influence degree and predictability of the risk (Elkjaer and Felding, 1999), the acceptability of the impact and the probability of the risk (Piney, 2002), and the controllable degree of the risk and the degree to which the risk is specific to the project (Miller and Lessard, 2001). According to the zone in which the risk is located, the corresponding response strategy is determined. However, the effectiveness of the selected response strategies is not well justified. The *decision tree-based models* (Dey, 2002, 2012; Kujawski and Angelis, 2010; Marmier et al., 2014, 2013) apply a decision tree to depict the paths of possible outcomes, where the root node represents the initial risk event and the branches refer to risk response alternatives followed by possible outcomes. By comparing each branch's outcome, the best risk response strategy is determined. However, it becomes inefficient to make comparisons among branches at the situation of abundant available response strategies. The *optimization-based models* (Ben-David et al., 2002; Ben-David and Raz, 2001; Fan et al., 2008; Kayis et al., 2007; Kuo et al., 2019; Todinov, 2013, 2014; Wu et al., 2018; Zhang and Fan, 2014) formulate the problem of selecting response strategies as a mathematical optimization model taken various objectives and constraints into account. Despite the fact that an optimal solution can be obtained in optimization models, its solution quality partially depends on the reliability of its input. Hence, some studies propose *integrated models* that combine optimization with other approaches, such as event tree analysis (Sherali et al., 2011), cost-benefit analysis (Reniers and Sörensen, 2013; Špačková and Straub, 2015), simulation and empirical analysis (Mizgier et al., 2015) and bow-tie analysis (Zhang and Guan, 2018), to determine the risk response strategies.

The above studies have emphasized the importance of risk responses in projects and provided some generic guidelines for predefining response strategies to cope with the underlying risks in projects. Nonetheless, such models do not deal with the *budget allocation problem* among risk response strategies. Most of the studies assume the costs and effects of response strategies to be known, which might not be the case in reality. Generally, a higher risk reduction can be achieved when investing more budget, but for different strategies, their contributions on risk reduction may be different, even if a same amount of budget is invested. Thus, determining the budget amount for each response strategy and making trade-offs among them are crucial for achieving an effective risk response.

To the best of our knowledge, only several studies have not made the assumption of known costs and effects of response strategy, however, their focus does not lie on the budget allocation among response strategies. Fan et al. (2008) proposed a conceptual framework which describes the quantitative relationships between the costs and effects of risk response strategies as well as project characteristics, and then performed an optimization analysis for choosing a minimum-cost response strategy for a given risk. However, their focus lies on how to select a response strategy under various project contexts. Based on the relationships proposed by Fan et al. (2008), Kuo et al. (2019) constructed a mathematical model aiming at achieving a minimum project cost by allocating the budget among risk response and risk contingency and proposed a particle swarm optimization algorithm to solve it. Even though the probabilities and losses of the risks after risk handling are obtained by the algorithm, the budget allocation decisions among different response strategies are not examined. Sherali et al. (2011, 2008) built mathematical models to allocate risk-related resources among safety measures and consequence events in a safety system, where the failure probability of safety measures and the loss magnitude of consequences depend on the resource amount allocated. Sato and Hirao (2013) proposed a mathematical model to allocate a limited budget among project activities to achieve a maximum risk-based project value, where the risk probability of each activity is related to the invested budget. Špačková and Straub (2015) investigated the trade-offs between the cost of risk mitigation measures and the achieved risk reduction aiming at finding a set of criteria to ensure the optimal risk mitigation, where the risk reduction resulting from mitigation measures is related to their costs. Mizgier et al. (2015) proposed three propositions on whether operational disruption risk can be mitigated by process improvement and capital adequacy (which play a role as response strategies), and verified them by simulations and empirical data. Their results also suggest a mathematical model to optimally allocate capital among different event types, i.e. the sources of the disruption risk, which can provide insights at the firm or industry level. However, the explicit function of capital and the capital allocation among response strategies are not determined. Our paper goes further by considering two ways to deal with one risk, formulating the cost-effect functions of response strategies and developing a three-step decision procedure for budget allocation among response strategies, which can support the budget allocation decision at the project or lower level.

2.2. Types of risk response strategies

A risk response strategy is a measure or an action implemented to cope with project risks (PMI, 2000). The PMBoK (PMI, 2000) has suggested four types of response strategy. *Risk avoidance* is to eliminate the risk by changing project plans. *Risk transference* is to shift the risk to a third party, for example a sub-contractor. *Risk mitigation* refers to actions that reduce the probabilities or impacts of risks to an acceptable threshold. *Risk acceptance* means doing nothing to deal with the risk, and is generally used when the risk is less serious. Among these four strategies, risk mitigation is the most frequently used strategy in practice and then followed by risk transference (Lyons and Skitmore, 2004).

Apart from the four-type response strategy, Glickman and Khamooshi (2005) have classified response strategies into two types according to their effects on the risks. The *prevention strategy* focuses on risk probability reduction, and the *protection strategy* refers to the effort that reduces the risk impacts or consequences. Besides the effects, these two strategies also indicate the timing of strategy implementation, i.e. the prevention strategy is implemented before risk occurs to prevent it from happening while protection strategy is applied after risk happens to alleviate the consequences. Due to these advantages, these two strategies have been used in many studies (Fang et al., 2013; Marmier et al., 2014, 2013; Ykhlef and Algawiaz, 2014; Zhang and Guan, 2018).

Regardless of the number of response strategy types and their names, all these types contain a strategy aiming at reducing the risk probability or the risk impact, where the project risks are essentially decreased. In addition, determining the timing of response strategy implementation is also a concern for effective risk response.

2.3. Costs of risk response strategies

Formulating the relationships between the costs and effects of risk response strategies is the premise of obtaining an optimal budget allocation. As mentioned in Section 2.1, several studies have suggested functions for describing the relationships. Mizgier et al. (2015) suggested a non-negative function to describe the relation between the loss and the capital for each event type, but no explicit expression is provided. Sherali et al. (2008) proposed a logit choice based function to define the relation between the failure probability of safety measure and the corresponding cost, and described the loss magnitude of the consequence as a linear function of its cost. Sato and Hirao (2013) suggested a power-law relationship between the activity risk probability and the invested cost with the consideration of the probabilities of cost-independent and cost-dependent risk drivers. Both of them have indicated an increase in cost with the probability or loss decreases. However, the initial risk value (probability or loss), which might be an influential factor, is not taken into account. Considering the effects of the initial risk value, Fan et al. (2008) formulated the risk preventive cost as a logarithmic function of the initial risk probability and the remaining risk probability, and employed a linear function and an exponential function to model the risk protective cost. These functions have considered the initial risk characteristics and stated that the costs increase when more probability or loss needs to be reduced.

However, two limitations raise when applying them to making trade-offs between risk prevention and risk protection. First, risk is inevitable in projects and cannot be removed completely. Thus, a minimum probability and loss should be reflected in functions. However, only a minimum probability is considered (Fan et al., 2008). Second, we focus on the trade-off between risk prevention and risk protection, the risk probability and risk loss should be comparable for investigating which response strategy is more preferable. However, the different value ranges of probability and loss make it difficult to compare risk prevention and risk protection on a same scale.

2.4. Conclusion literature overview

Based on the above literature overview, our research study has three main contributions to the existing literature on PRM. First, although the above studies determined the budget or resource amount of risk response measures, none of them have focused on the budget allocation among risk response strategies and examined the effects of risk response strategies on budget allocation decisions. Secondly, we will build upon the existing literature in the field of PRM as we consider two-type strategies: risk prevention and risk protection. Finally, trade-offs between risk prevention and risk protection are investigated and the corresponding budget allocation problem is examined in our research study. This dual approach addresses an important gap in the existing literature. Therefore, we investigate both linear and non-linear cost functions, and conduct numerical experiments to validate the results.

3. Problem formulation

This section will first provide the general problem statement of the considered budget allocation problem. Afterwards, two types of budget allocation models are introduced based on the linear and non-linear relations between the response costs and effects.

3.1. Problem statement

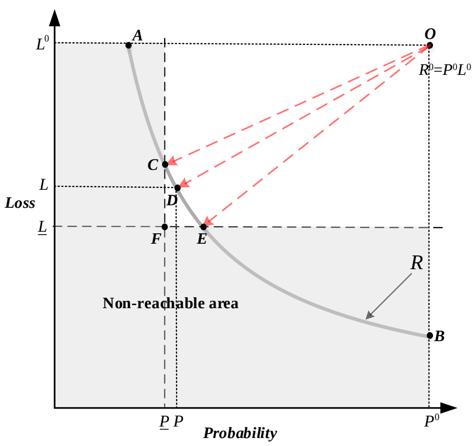

Figure 1: Problem statement

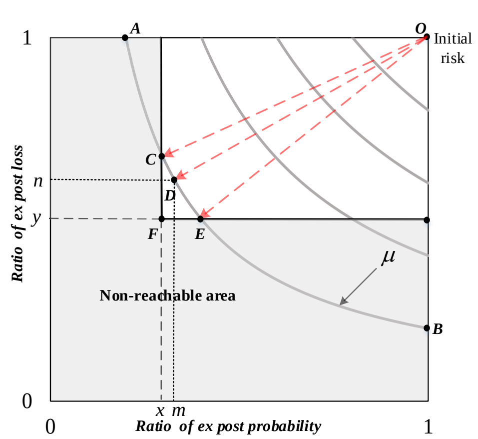

Figure 2: Simplified problem statement

The *budget allocation problem* (BAP) starts with an intolerable project risk R^0, i.e. initial risk magnitude, which is defined as the product of risk probability P^0 and risk loss L^0, represented by point O in Figure 1, where the x-axis and y-axis refer to risk probability and risk loss, respectively. In this research study, we thus follow the general concensus that the risk magnitude is computed as the product of its probability (i.e. likelihood magnitude) and its impact (i.e. severity magnitude). The ex-post risk magnitude (R) is preset by the project manager to capture the target or acceptable risk level in the project and is equal to the product of the ex-post risk probability P and ex-post risk loss L, which is reflected by the curve AB in Figure 1. A lower ex-post risk level implies a stricter requirement for the risk response strategy.

Definition 1. ***Response requirement*** *(μ): The degree of risk reduction that should be achieved by allocating the total risk budget. This reduction is defined as the ratio of the ex-post risk magnitude ($R = P \cdot L$) and the current risk magnitude ($R^0 = P^0 \cdot L^0$). The lower the value for μ, the further R lies below R^0, denoting a more ambitious risk response strategy.*

The goal of risk response is to mitigate the project risk from point O to curve AB in Figure 1. Since the risk probability ranges from 0 to 1 and the risk loss generally takes a relatively large value, it is difficult to compare risk prevention and risk protection. Therefore, we introduce the ratio $m = \frac{P}{P^0}$ for the response requirement of risk probability and the ratio $n = \frac{L}{L^0}$ for the response requirement of risk loss (see Figure 2). A direct implication of the ex-post or target risk level is that an acceptable portion of the risk will remain after the risk response strategy is applied. We refer to this risk as *residual risk.* The more ambituous the target risk level is, the lower the residual risk will be.

Figure 1 shows that risk prevention (path OA), risk protection (path OB), and combinations of them (paths OC, OD, OE) are feasible ways to achieve the risk response goal. In reality, however, the effect of risk prevention or risk protection is restricted to a certain level because a part of risk probability or risk loss (denoted by $\underline{P}$ and $\underline{L}$) cannot be removed feasibly, even with infinite budget investment. This will lead to some infeasible paths since they cannot achieve the risk response goal, shown in Figure 1 as the non-reachable grey area.

Definition 2. ***Risk controllability*** *(x, y): The maximum amount that the current risk level (for probability P^0 and loss L^0) can be reduced. Since this maximum reduction is limited by its uncontrollable risk ($\underline{P}$ and $\underline{L}$), lower values for x (y) denote a higher risk controllability for probability (loss).*

Hence, the remaining feasible strategies for satisfying the risk response requirement are the paths from point O to curve CE in Figure 1. The values $x = \frac{\underline{P}}{P^0}$ and $y = \frac{\underline{L}}{L^0}$ describe the uncontrollable degree of risk probability and risk loss since they capture how much risk probability and risk loss cannot be removed (see Figure 2). Thus, the risk controllability should be taken into account when determining the feasible response strategies for achieving the risk response goal. By definition, this uncontrolable risk will be part of the residual risk that remains after the risk response strategy is implemented. In case of the most ambituous risk level ($R = \underline{P} \cdot \underline{L}$), i.e. point F in Figure 1, the residual risk is equal to the uncontrollable degree of risk. In less ambituous scenarios ($R > \underline{P} \cdot \underline{L}$), the residual risk consists of the uncontrollable degree of risk and the controllable degree of risk under the curve R as shown in Figure 1.

Furthermore, the remaining feasible paths (from point O to curve CE) correspond with different costs that depend on the portion of reduction in risk probability and risk loss as well as the unit cost for risk prevention a and risk protection b. This will result in the total cost for risk prevention q and risk protection r, and the path (m, n) with the lowest cost $(q + r)$ is the best option for risk response. Hence, the relations between the costs (a, b) and the ex-post risk probability (P) or ex-post risk loss (L) should be analysed since a trade-off exists between these two response strategies. The definitions and notations used throughout this paper are summarized in Table 2.

3.2. Response budget allocation models

As explained in Section 3.1, the objective of the budget allocation problem is to determine the optimal ratio of ex-post risk probability $m^\star$ and ratio of ex-post risk loss $n^\star$ such that the total risk response cost $Z^\star$ is minimised given that there is a limited budget for risk prevention and risk protection.

Table 2: Notations and definitions used

Notations	Definitions
Project risk	
• R^0	Intolerable magnitude of project risk, $R^0 = P^0 \cdot L^0$
• R	Ex-post level of the risk magnitude
• μ	Response requirement, $\mu = \frac{R}{R^0}$ with $0 < \mu \leq 1$
Risk Prevention	
• P^0	Initial risk probability
• $\underline{P}$	Minimum value of risk probability with $0 < \underline{P} \leq P^0$
• x	Ratio of uncontrollable risk probability with $x = \frac{\underline{P}}{P^0}, 0 < x \leq 1$
• P	Ex-post risk probability
• m	Auxiliary variable, ratio of ex-post risk probability with $m = \frac{P}{P^0}$
Risk Protection	
• L^0	Initial risk loss
• $\underline{L}$	Minimum value of risk loss with $0 < \underline{L} \leq L^0$
• y	Ratio of uncontrollable risk loss with $y = \frac{\underline{L}}{L^0}$ with $0 < y \leq 1$
• L	Ex-post risk loss
• n	Auxiliary variable, ratio of ex-post risk loss with $n = \frac{L}{L^0}$
Response strategy costs	
• q	Decision variable, budget allocated for risk prevention
• a	Unit cost for risk prevention
• r	Decision variable, budget allocated for risk protection
• b	Unit cost for risk protection

Risk response strategies aim at reducing the risk to an acceptable threshold, referred to as the *ex-post risk level* (R) in this paper. Thus, a mathematical model with the objective of minimizing the risk response cost Z, which is the sum of the costs for risk prevention (q) and risk protection (r) (see Eq.(1)), and the constraint of risk response requirement (see Eq. (2)) is constructed for the BAP. We observe that the risk magnitude R is computed as the product of its probability P (i.e. likelihood magnitude) and its loss L (i.e. severity magnitude). The model is restricted to $mn = \mu$, which is derived from Eq. (2) with specific ranges of m and n.

$$\textbf{BAP} \quad min \quad Z = q + r \tag{1}$$

$$\text{s.t.} \quad P \cdot L = R \tag{2}$$

Linear relation The linear relations are formulated based on the idea that a higher risk reduction (i.e. a lower ex-post probability and/or loss) requires more budget, meaning a decreasing relation between the cost of risk prevention q (cost of risk protection r) and the ex-post risk probability P (loss L). In addition, the cost might be affected by the characteristics of the risk and response strategy. Given a certain ex-post risk (P or L), a higher initial risk value (P^0 or L^0) or unit response cost (a, b) will result in a greater cost (q or r). Thus, the following equations are formulated in terms of risk prevention and risk protection, respectively, where $a > 1$ since the probability ranges from 0 to 1 and $0 < b < 1$ because the risk loss generally takes large positive values.

$$q = a(P^0 - P) = aP^0(1 - m), \qquad a > 1, \quad x \leq m \leq 1 \tag{3}$$

$$r = b(L^0 - L) = bL^0(1 - n), \qquad 0 < b < 1, \quad y \leq n \leq 1 \tag{4}$$

Given Eqs. (3) and (4), the objective function of the Linear BAP (LBAP) can be derived from Eq. (1).

$$\textbf{LBAP} \quad min \quad Z = aP^0(1 - m) + bL^0(1 - n) \tag{5}$$

Non-linear relation In most situations during a risk response process, the relations between the costs and the effects of the risk response strategies are not strictly linear. At the beginning of a risk response, a small investment can yield a significant reduction in risk. However, after the risk has been reduced to a relatively low level, further risk reduction requires a larger investment. This implies that the cost will increase faster and faster as the risk approaches an extremely low value. Hence, the marginal cost (denoted by c_P for risk prevention and c_L for risk protection) is the change in cost when the risk probability or risk loss is decreased by one unit. We assume that the marginal cost increases with the unit response cost and the initial risk value, and decreases with the remaining risk that can be removed. Due to the inverse relations between the costs and the effects of risk response strategies, we can formulate the functions for c_P and c_L as $c_P = -a\frac{P^0}{P - xP^0}$ and $c_L = -b\frac{L^0}{L - yL^0}$, respectively. Accordingly, the non-linear formula for risk prevention cost and risk protection cost can be obtained by integrating c_P and c_L, respectively, as seen below.

$$q = \int_{P^0}^{P} c_P dP = aP^0 \ln \frac{P^0(1 - x)}{P - xP^0} = aP^0 \ln \frac{1 - x}{m - x}, \qquad a > 0, \quad x < m \leq 1 \tag{6}$$

$$r = \int_{L^0}^{L} c_L dL = bL^0 \ln \frac{L^0(1 - y)}{L - yL^0} = bL^0 \ln \frac{1 - y}{n - y}, \qquad b > 0, \quad y < n \leq 1 \tag{7}$$

Given Eqs. (6) and (7), the objective function of the Non-linear BAP (NBAP) can be derived from Eq. (1).

$$\textbf{NBAP} \quad min \quad Z = aP^0 \ln \frac{1-x}{m-x} + bL^0 \ln \frac{1-y}{n-y} \tag{8}$$

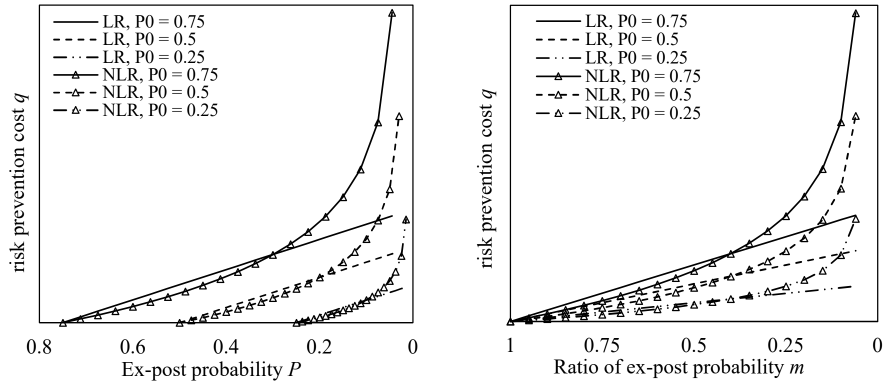

Figure 3: Relations between the cost and effect of risk prevention

Figure 3 displays the linear (LR) and the non-linear (NLR) relations for the risk prevention functions (Eqs. (3) and (6)) against P and m, respectively, where $x = 0.05$ and $a = 1,000$ for the linear relation and $a = 600$ for the non-linear relation (because of the different ranges of a in these functions). Figure 3 shows that all the equations indicate an increase in the cost when a great risk reduction is required (i.e. lower P and m). Also, a higher cost is always needed when the initial probability P^0 is higher, independent of the values for P and m. The functions for risk protection (Eqs. (4) and (7)) show the same trend with that for risk prevention.

4. Analytical results

In Section 4.1, we define different scenarios that show which response strategies can be used to obtain the ex-post risk level given the uncontrollable portion of risk in the project. Based on the BAP models introducted in Section 3.2, we present the optimality conditions for each scenario and discuss the analytical results for the LBAP using a three-step process in Section 4.2. Finally, we extend this discussion to the NBAP in Section 4.3.

4.1. Scenarios

Based on the comparison between the response requirement (μ) and the risk controllability (x, y), restricted strategies can be identified given the following definition:

Definition 3. ***Restricted strategy:*** *A strategy (risk prevention or risk protection) for which $\mu < x$ in case of risk prevention and $\mu < y$ in case of risk protection.*

For example, a strategy that focuses solely on risk prevention or risk protection will not be able to reduce the project risk to the target level in case that, respectively, $\mu < x$ and $\mu < y$. The definition of a restricted strategy results in the following theorem and proof:

Theorem 1. *The response requirement μ cannot be satisfied with a single restricted strategy (either risk prevention or risk protection).*

Proof 1. *The proof assumes that the risk prevention strategy is restricted. Recall that P is defined as the ex-post probability obtained by the risk prevention strategy. Assume that the chosen strategy is risk prevention only, i.e. $P < P^0$ and no risk protection strategy is chosen, i.e. $L = L^0$. Then:*

(a) $\mu = \frac{P \cdot L}{P^0 \cdot L^0}$*, and since no risk protection is used ($L = L^0$),* $\mu = \frac{P}{P^0}$*, and*

(b) $x = \frac{\underline{P}}{P^0}$ *is the risk controllability of risk prevention and denotes the maximum amount that the probability can be reduced (cf. definition 2).*

From definition 3, the risk prevention strategy is restricted if $\mu < x$, and from (a) and (b), it follows that $\frac{P}{P^0} < \frac{\underline{P}}{P^0}$ *and thus $P < \underline{P}$. Since $\underline{P}$ is the minimum value for risk probability, it means that P cannot be reached. Hence, the strategy is restricted.*
A similar logic can be considered for a restricted risk protection strategy, i.e. $\frac{L}{L^0} < \frac{\underline{L}}{L^0}$ *and thus $L < \underline{L}$.*

The response requirement μ can only be reached when (1) only an unrestricted strategy is selected or (2) a combination of two strategies is selected (as shown in Theorem 1). So, if both strategies are restricted, only a combination of strategies will be able to achieve the response requirement. In this case, an infeasible situation might occur. In case that $\mu < xy$, the reponse requirement can never be achieved, independent of the amount of budget that is invested in response strategies. This is shown in the following theorem and proof:

Theorem 2. *The response requirement μ can never be satisfied when $\mu < xy$.*

Proof 2. *In case that $\mu < xy$, it follows that $\mu < x$ and $\mu < y$, and thus risk prevention and protection are restricted strategies. As a result, a feasible strategy should be a combination of risk prevention ($P < P^0$) and risk protection ($L < L^0$) as shown in Theorem 1. Then:*

(a) $\mu = \dfrac{P \cdot L}{P^0 \cdot L^0}$ *and*

(b) $x = \dfrac{P}{P^0}$ *and* $y = \dfrac{L}{L^0}$

If $\mu < xy$, it follows from (a) and (b) that $\dfrac{P \cdot L}{P^0 \cdot L^0} < \dfrac{P}{P^0} \cdot \dfrac{L}{L^0}$ and thus $P \cdot L < \underline{P} \cdot \underline{L}$. Since $P > \underline{P}$ and $L > \underline{L}$, it follows that $P \cdot L > \underline{P} \cdot \underline{L}$.

Based on Theorem 1, four scenarios are distinguished that indicate which strategies will result in a feasible solution based on the lower and upper bound on m and n, labelled as m_l, n_l, m_u, n_u, such that $m_l \leq m \leq m_u$ and $n_l \leq n \leq n_u$:

- **Scenario 1**: $\mu > x$ and $\mu > y$. In this area, m and n range from μ $(= m_l, n_l)$ to 1 $(= m_u, n_u)$. Risk prevention, risk protection and a combination of both are thus feasible for achieving the risk requirement.
- **Scenario 2**: $x < \mu < y$. In this scenario, $\mu \leq m < \frac{\mu}{y}$ and $y < n \leq 1$. Thus, risk response requirement can only be satisfied by risk prevention or a combination of risk prevention and risk protection.
- **Scenario 3**: $y < \mu < x$. In this scenario, $x < m \leq 1$ and $\mu \leq n < \frac{\mu}{x}$, which implies that risk protection or the combination of risk prevention and protection can be opted for ensuring the response requirement to be satisfied.
- **Scenario 4**: $\mu < x$ and $\mu < y$. In this scenario, the response requirement is stricter, $x < m < \frac{\mu}{y}$ and $y < n < \frac{\mu}{x}$. Theorem 1 says that when $\mu < x$ and $\mu < y$, a feasible solution should be a combination of the two restricted strategies. In this case, $\mu < xy$ is possible, but we exclude this scenario as shown in Theorem 2.

The four scenarios are shown using an illustrative example in Figure 4 given that $x = 0.3$ and $y = 0.5$ $(x < y)$ and $x = 0.5$ and $y = 0.3$ $(x > y)$.

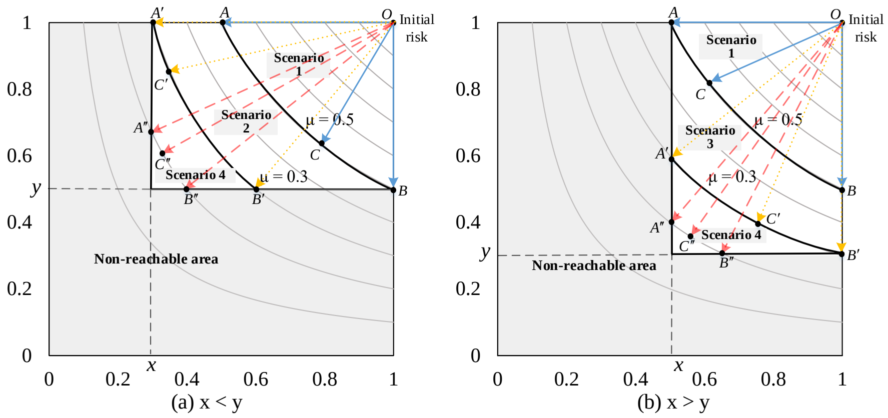

Figure 4: Representation of the different scenarios

4.2. Analytical optimality

For the BAP in Section 3.2, we can derive the optimal solution $Z^\star$ in an analytical way within the specific ranges of m. Therefore, we replace n by $\frac{\mu}{m}$ using Eq. (2). The LBAP model can be transformed into Eq. (9), and the first derivative of Z is shown in Eq. (10).

$$min \quad Z = aP^0(1-m) + bL^0(1-\frac{\mu}{m}) \tag{9}$$

$$\frac{\mathrm{d}Z}{\mathrm{d}m} = \frac{\mu bL^0}{m^2} - aP^0 \tag{10}$$

According to Eq. (10), $\frac{\mathrm{d}Z}{\mathrm{d}m} > 0$ when $m < \sqrt{\frac{\mu bL^0}{aP^0}}$ and $\frac{\mathrm{d}Z}{\mathrm{d}m} < 0$ when $m > \sqrt{\frac{\mu bL^0}{aP^0}}$, indicating that as m increases, Z increases first and then decreases. Thus, $Z^\star$ (i.e. the minimum value of Z) will be located at the bounds of m. Let m_l and m_u denote the lower bound and the upper bound of m, and Z_l and Z_u denote the corresponding objective value, if $Z_l > Z_u$, $m^\star = m_u$. Otherwise, $m^\star = m_l$.

The flowchart in Figure 5 shows the logic that is followed in the LBAP model to derive the optimal budget allocation $Z^\star$ for the different risk response strategies in each scenario, where 'prev' and 'prot' represent risk prevention and risk protection, respectively.

Step 1: Scenario selection The response requirement (μ) will be compared with the controllability of risk prevention (x) and risk protection (y) in order to determine the restricted strategies based on Theorems 1 and 2.

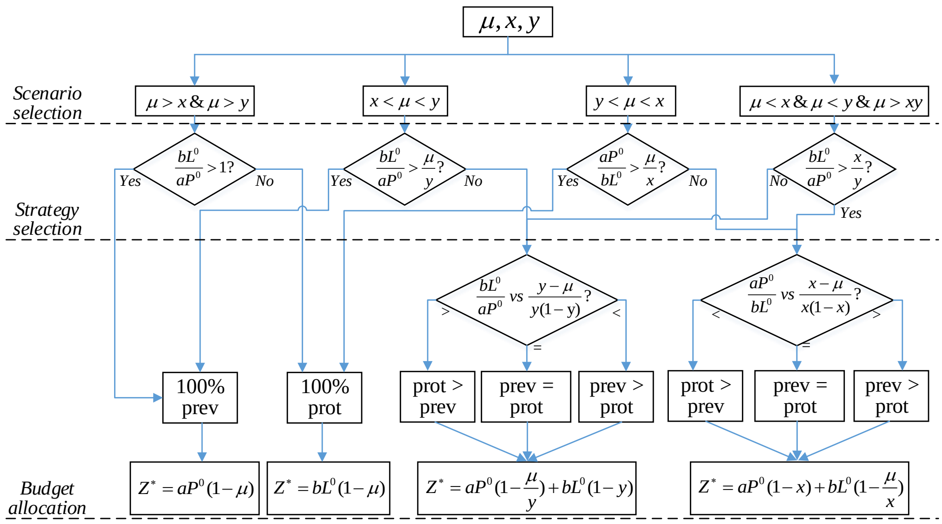

Figure 5: Flowchart for LBAP

Step 2: Risk response strategy selection In this step, we first compare the relative costs of risk prevention and protection, and subsequently analyse the most restricted strategy. In scenario 1, no restricted strategy (prevention or protection) exists and the strategy that can achieve the response requirement using the lowest amount of cost (aP^0, bL^0) will be selected. In contrast to scenario 1, a single restricted strategy exists in both scenarios 2 and 3. In scenario 2 (3), the risk prevention (protection) strategy has the ability to reduce the risk probability (loss) in order to meet the response requirement, however, this ability is much more restricted for the risk protection (prevention) strategy. Since Theorem 1 denotes that a single restricted strategy cannot satisfy the response requirement in scenarios 2 and 3, the choice whether or not to consider the restricted strategy depends on the comparison of the costs (aP^0 and bL^0) with the response requirement (μ) and the controllability of the restrictive strategy (x, y). In case that the relative cost of the restricted strategy is larger than its possibility to meet the response requirement, it is excluded from the risk response strategy. In scenario 4, both strategies are restricted due to the high response requirement (i.e. low μ) or the low controllability of the risk probability or loss (i.e. high x, y) and thus a combination of the restricted strategies is required (see Theorem 1). In order to determine a preference for one of the two restricted strategies, their ability to reduce the risk level given the uncontrollable portion of risk will be compared with their respective cost. The strategy that has the possibility to reduce the most risk at the lowest cost will be preferred.

Based on the above explanation, it is clear that the degree of *restrictiveness* is an important metric to select the optimal risk response strategy from the moment that one strategy is restricted. However, in case that both strategies are restricted, the relative restrictiveness of both strategies is considered. The restrictiveness is measured for each restricted strategy by the distance between x, y and μ:

- Scenario 1: Since scenario 1 does not contain any restricted strategy, no restrictiveness is taken into account and only costs should be used and thus the cost of risk prevention (aP^0) and protection (bL^0) should be compared.
- Scenario 2: $\frac{\mu}{y}$ is always lower than 1 with lower values of $\frac{\mu}{y}$ denoting a stronger restrictiveness.
- Scenario 3: $\frac{\mu}{x}$ is always lower than 1 with lower values of $\frac{\mu}{x}$ denoting a stronger restrictiveness.
- Scenario 4: The relative restrictiveness of risk prevention $\frac{\mu}{x}$ and risk protection $\frac{\mu}{y}$ is measured, which is equal to $\frac{x}{y}$ after elimination of the factor μ.

Step 3: Budget allocation In this step, the optimal budget allocation ($m^\star$, $n^\star$) and the corresponding minimal risk response cost $Z^\star$ differ in the four scenarios. For each scenario, the most restricted strategy should be analysed. In case that there exist no restricted strategy, the complete risk budget will be allocated to the unrestricted strategy (e.g. in scenario 1). In scenario 2 (3), the protection (prevention) strategy is restricted (see Step 2). In order to determine the portion of the budget for the restricted strategy, the costs of both strategies (aP^0 and bL^0) will be compared with the risk controllability of the restricted strategy (x or y) and the response requirement (μ). In scenario 4, both strategies are restricted and, therefore, the above comparison is conducted for the *most* restricted strategy (see Step 2). After this step, the part of the risk budget that is allocated to the restricted and unrestricted strategy is known.

An *adapted restrictiveness* measure is computed from the moment that one strategy is restricted. However, in case that both strategies are restricted, the adapted restrictiveness of the most restricted strategy is computed:

- Restricted prevention strategy: Lower values of $\dfrac{x-\mu}{x(1-x)}$ imply that x lies closer to μ and thus a lower restrictiveness.
- Restricted protection strategy: Lower values of $\dfrac{y-\mu}{y(1-y)}$ imply that y lies closer to μ and thus a lower restrictiveness.

The minimal risk response cost $Z^{\star}$ in a scenario depends on the costs of risk protection and risk prevention (aP^0, bL^0). In case that only risk prevention (protection) is preferred (scenario 1), the allocated budget $Z^{\star}$ increases with the initial risk probability P^0 (loss L^0) and the unit preventive a (protective b) cost, and decreases with the response requirement μ. In case that both risk prevention and risk protection are required (scenarios 2, 3, 4), risk can be considered relatively uncontrollable in terms of probability or loss ($x > \mu$ or $y > \mu$). We observe that $Z^{\star}$ also depends on the uncontrollability of risk (x, y).

A guideline for project managers to find the optimal risk response strategy and the corresponding solution to the budget allocation problem given the input parameters (x, y, μ, aP^0, bL^0) is shown in Figure 6. In other words, we observe that the scenario selection, strategy selection and budget allocation depends on the uncontrollable risk probability (x), the uncontrollable risk loss (y) and the response requirement μ (i.e. the relative reduction in the magnitude of the risk). In conclusion, the proposed budget allocation model compares the required reduction in risk magnitude with the potential reduction in risk probability and impact given their respective costs.

4.3. Non-linearity

In this section, we extend the logic of Section 4.2 to the NBAP discussed in Section 3.2. Similar to the LBAP, the NBAP model is transformed into Eq. (11), and the first derivative of Z is shown in Eq. (12).

$$min \quad Z = aP^0 \ln \frac{1-x}{m-x} + bL^0 \ln \frac{m-my}{\mu-my} \tag{11}$$

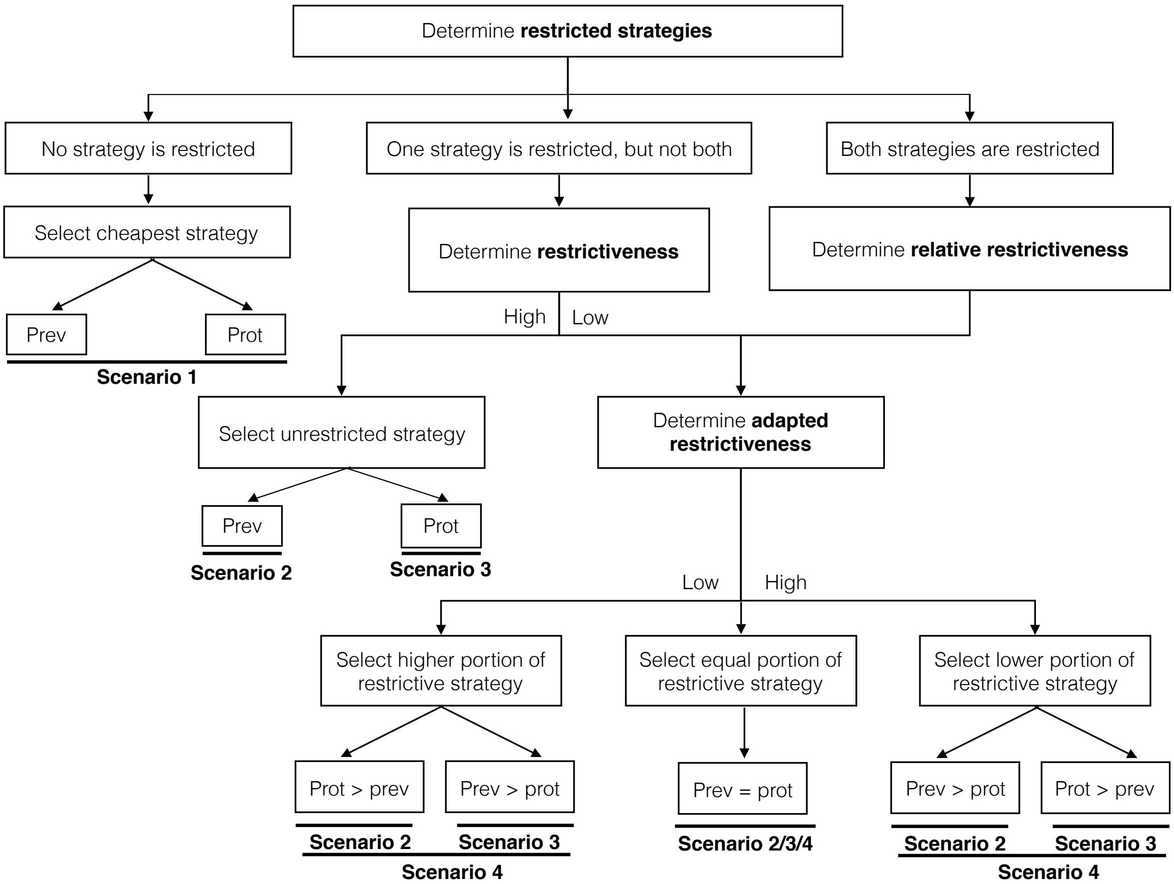

Figure 6: Overview of three-step procedure for optimal budget allocation

$$\frac{\mathrm{d}Z}{\mathrm{d}m} = \frac{aP^0 m(\mu - my) + \mu bL^0(x - m)}{(x - m)(\mu - my)m} \tag{12}$$

Since $x < m$ and $\mu > my$ always hold in each scenario, we can have:

$$\begin{aligned} \frac{\mathrm{d}Z}{\mathrm{d}m} < 0 \quad &\Leftrightarrow \quad aP^0 m(\mu - my) + \mu bL^0(x - m) > 0 \\ &\Leftrightarrow \quad aP^0 ym^2 - (aP^0 - bL^0)\mu m - bL^0 \mu x < 0 \\ &\Leftrightarrow \quad A_1 < m < A_2 \end{aligned} \tag{13}$$

where $A_1 = \frac{(aP^0 - bL^0)\mu - \sqrt{(aP^0 - bL^0)^2\mu^2 + 4abP^0L^0xy\mu}}{2aP^0y}$ and $A_2 = \frac{(aP^0 - bL^0)\mu + \sqrt{(aP^0 - bL^0)^2\mu^2 + 4abP^0L^0xy\mu}}{2aP^0y}$. It is seen that, $A_1 < 0$, and thus $m > A_1$ always holds. Accordingly, Z decreases first and then increases along the range of m, and will achieve its minimum value when $m = A_2$. Given that m_l, m_u, Z_l, Z_u denote the lower and upper bounds of m and the corresponding value of

Z, we can derive:

- If $A_2 < m_l$, $m^\star = m_l$.
- If $m_l < A_2 < m_u$, $m^\star = A_2$.
- If $A_2 > m_u$, $m^\star = m_u$.

By replacing the specific bounds of m in each scenario, the optimal conditions are obtained for the NBAP as shown in the flowchart in Figure 7. The logic that is followed in the NBAP model is very similar to the logic discussed in Figure 5 and the same three-step procedure can be used. First, four scenarios are identified and, subsequently, the restricted strategies are identified in each scenario based on the comparison between the relative costs of risk prevention and protection (aP^0, bL^0) and the response requirement (μ) as well as risk controllability (x, y). Finally, the optimal budget allocation can be derived from the total cost of risk prevention and protection. The optimal allocated budget can be observed for each response strategy in each scenario in Figure 7. Despite the high similarity with the analytical discussion of LBAP, we can derive three main differences with the analytical results of the LBAP based on the analysis of Figure 7:

Remark 1 In case that the two strategies are unrestricted (Scenario 1), the LBAP model focuses either on the risk prevention strategy or the risk protection strategy, but never on both. For the NBAP model, however, this is no longer true since a combination of both can also be a feasible strategy. Due to the non-linear relation between the response costs and effects, the optimal budget allocation in case of two unrestricted strategies is not - by definition - to allocate the total budget to the cheapest strategy. Based on the comparison of the relative costs of risk prevention and risk protection with the response requirement and the risk controllability of both strategies, a combination of the strategies can be optimal.

Remark 2 In case one or more restricted strategies are present (i.e. Scenarios 2, 3 and 4), the risk response strategy selection is based on the comparison of the relative costs $(\frac{bL^0}{aP^0})$ and the restrictiveness of risk prevention $\frac{\mu}{x}$ or risk protection $\frac{\mu}{y}$, similar to the LBAP.

However, the LBAP always takes the restrictiveness of the (most) restricted strategy into account, while the NBAP model considers the restrictiveness of the unrestricted strategy as well as the controllability of the restricted strategy (i.e. the formulas contain both x and y). Because of the non-linear behavior of risk prevention and protection, it no longer suffices to only focus on the (un)restricted strategy.

Remark 3 A similar logic holds for the adapted restrictiveness. Since the focus on a single (un)restricted strategy is insufficient in case of non-linearity (see remark 2), the adapted restrictiveness should not be computed for the most restricted strategy only (as is the case for the LBAP model), but should also contain both strategies. As a result, there exists only one - rather than two - restrictiveness measure that incorporates risk prevention and protection. Consequently, the formula for the adapted restrictiveness is somewhat more complex in comparison to the one for the LBAP model. Despite the difference due to the non-linear relation between risk costs and effects, its logic and interpretation is similar. An additional difference between the NBAP and LBAP models is that the adapted restrictiveness should also be computed for scenario 1 in the NBAP model, even though this scenario consists of only unrestricted strategies ($\mu > x$ and $\mu > y$).

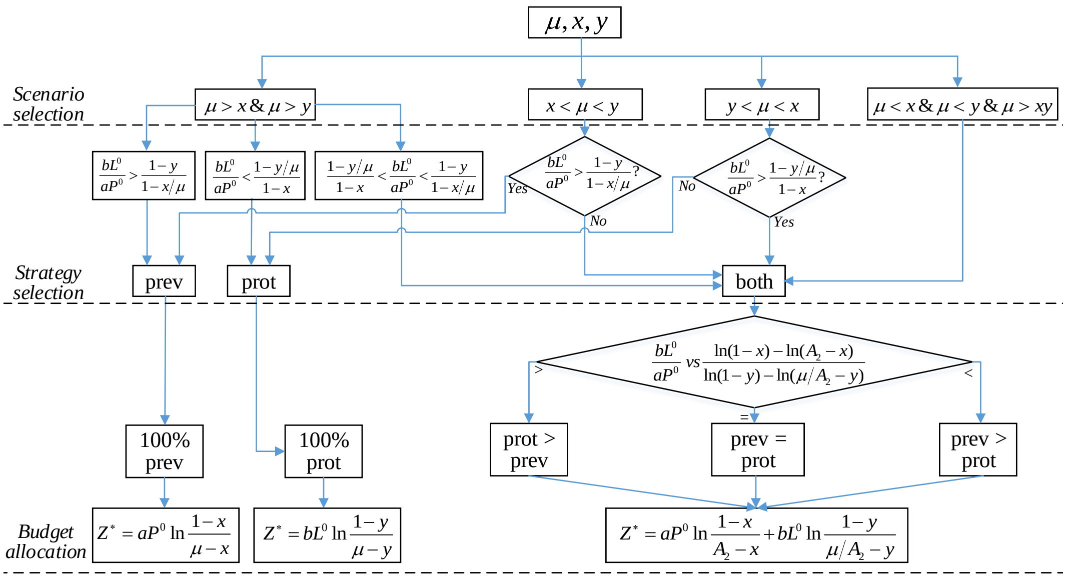

Figure 7: Flowchart for NBAP

5. Numerical experiments

In this section, we will conduct some numerical experiments to validate the analytical results presented in Section 4. First, we will investigate the impact of the controllability (x, y) and response requirement (μ) on the optimal budget allocation $Z^\star$ and the corresponding risk response strategy (risk prevention, protection or combination). Subsequently, we will investigate the effect of the cost of risk prevention (aP^0) and protection (bL^0) on the optimal budget allocation and risk response strategy. In the remainder of this section, we will show the results for LBAP and NBAP separately in Figures 8 - 11. In each figure, the optimal budget allocation $Z^\star$ is illustrated by a line chart and a bar chart is used to show the proportional distribution of this risk budget in the risk response strategy (risk prevention, protection or combination). The optimal budget allocated to risk prevention and risk protection is labelled, respectively, $q^\star$ and $r^\star$ in Figures 8 - 11. In case that the project manager is indifferent to risk prevention or protection, this is also indicated in the bar chart (i.e. labelled $q^\star$ or $r^\star$). When the results are shown for the LBAP (NBAP), however, the optimal budget allocation for the NBAP (LBAP) is also presented using a dotted line as a reference. The impact of the restrictiveness (RT) and adapted restrictiveness (ART) on the optimal risk response strategy is highlighted in Figures 8 - 11 and their exact values are explained along the following lines.

5.1. Impact of x, y and μ

In these numerical experiments, we fix aP^0 and bL^0 at 10,000 and vary the values for x, y and μ independently in order to consider the different scenarios. The values for x, y and μ are considered in the interval $[0.05; 0.95]$ with increments of 0.05. The impact of the controllability x, y and response requirement μ is, respectively, presented in Figures 8, 9 and 10. In each figure, the respective scenarios (labelled as S1, S2, S3 and S4) are shown based on the values for x, y and μ. The fixed values are set at equal distance from each other in order to ensure a fair comparison between the different scenarios. For example, different values of x are shown in Figure 8 with $y = 0.25$ and $\mu = 0.5$ for S1 and S3, while $y = 0.75$ and $\mu = 0.5$ for S2

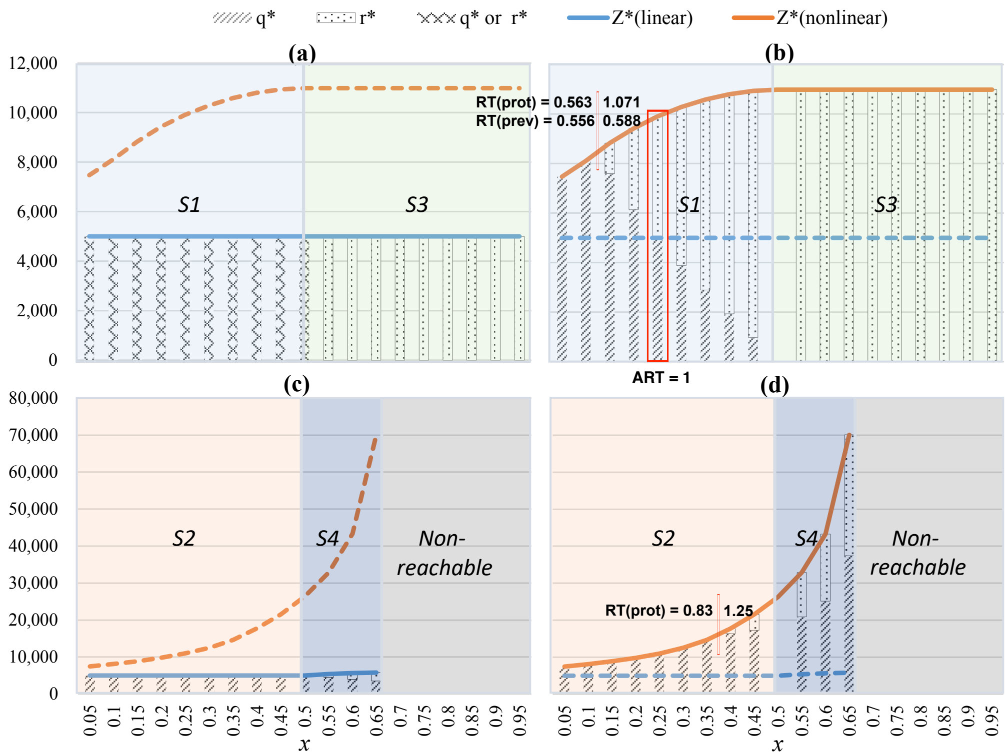

Figure 8: Effect of controllability x on optimal budget allocation $Z^\star$ given that $y = 0.25$ and $\mu = 0.5$ for (a) LBAP and (b) NBAP and $y = 0.75$ and $\mu = 0.5$ for (c) LBAP and (d) NBAP

and S4. First, we will discuss some general conclusions that are derived from the numerical experiments and, subsequently, we will present some detailed observations subdivided for the linear and non-linear cost functions.

In general, the controllability (x, y) has no significant effect on the optimal risk cost in the LBAP as shown in Figures 8 - 9. When only one strategy is restricted, the controllability of the restricted strategy (i.e. x in Scenario 3 and y in Scenario 2) has no effect on the optimal risk cost since the complete budget is allocated to the unrestricted strategy. Otherwise, the optimal risk cost will increase as x and y increases in almost all cases. When the risk controllability or the response requirement reaches a certain level (high x, y or low μ), the response requirement can never be satisfied (non-reachable area in Figures 8 - 10). In both LBAP and NBAP, a stricter risk response requirement (i.e. μ is lower) leads to a higher risk cost (see Figure 10). Some detailed observations for both the LBAP and NBAP will be discussed in the following lines based on the scenarios shown in Figures 8 - 9.

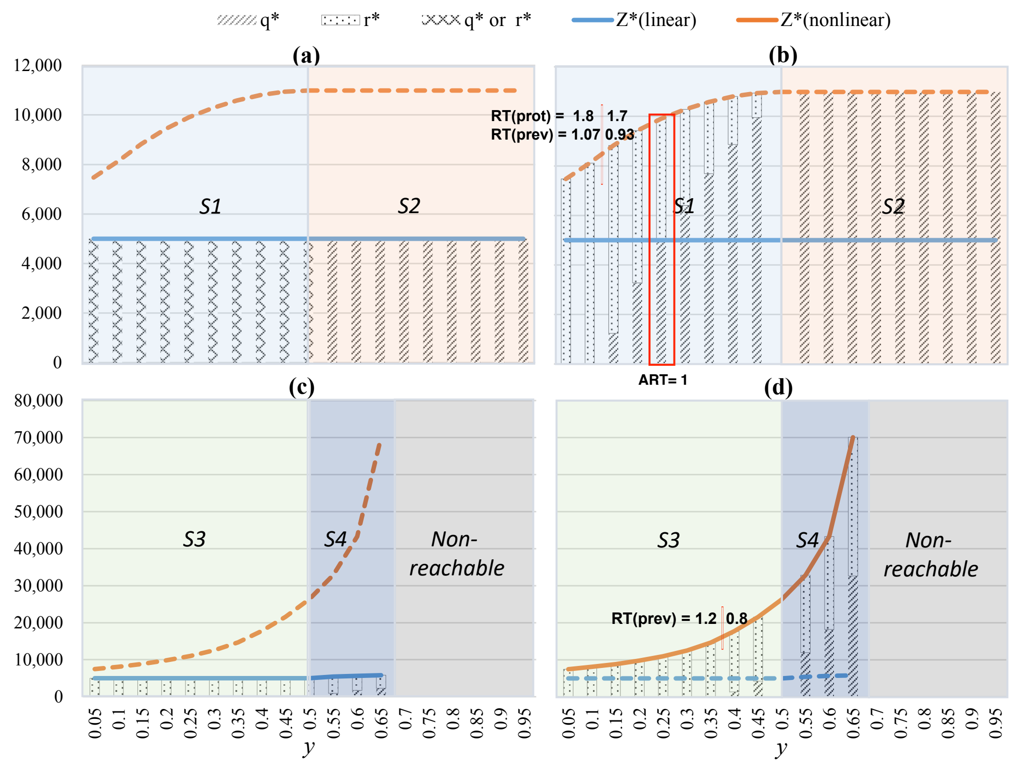

Figure 9: Effect of controllability y on optimal budget allocation $Z^{\star}$ given that $x = 0.25$ and $\mu = 0.5$ for (a) LBAP and (b) NBAP and $x = 0.75$ and $\mu = 0.5$ for (c) LBAP and (d) NBAP

Linear

- In Scenario 1, the project manager is indifferent between risk prevention and protection for different values of x and y since aP^0 is equal to bL^0. The optimal budget allocation $Z^{\star}$ is completely determined by μ and aP^0 or bL^0, i.e. $0.5 * 10,000 = 5,000$.
- In Scenarios 2 and 3, the unrestricted strategy is preferred over a combination of risk prevention and protection because, respectively, $\frac{\mu}{y} < 1$ and $\frac{\mu}{x} < 1$.
- In Scenario 4, a combination of risk prevention and protection is selected.

Non-linear

- In Scenario 1, a 100% risk prevention or protection strategy is optimal when risk is highly controllable (smaller x or y) given that both strategies have the same cost. The choice for risk prevention, risk protection or a combination of them depends on the restrictiveness of both strategies. When risk becomes less controllable (x or y increases),

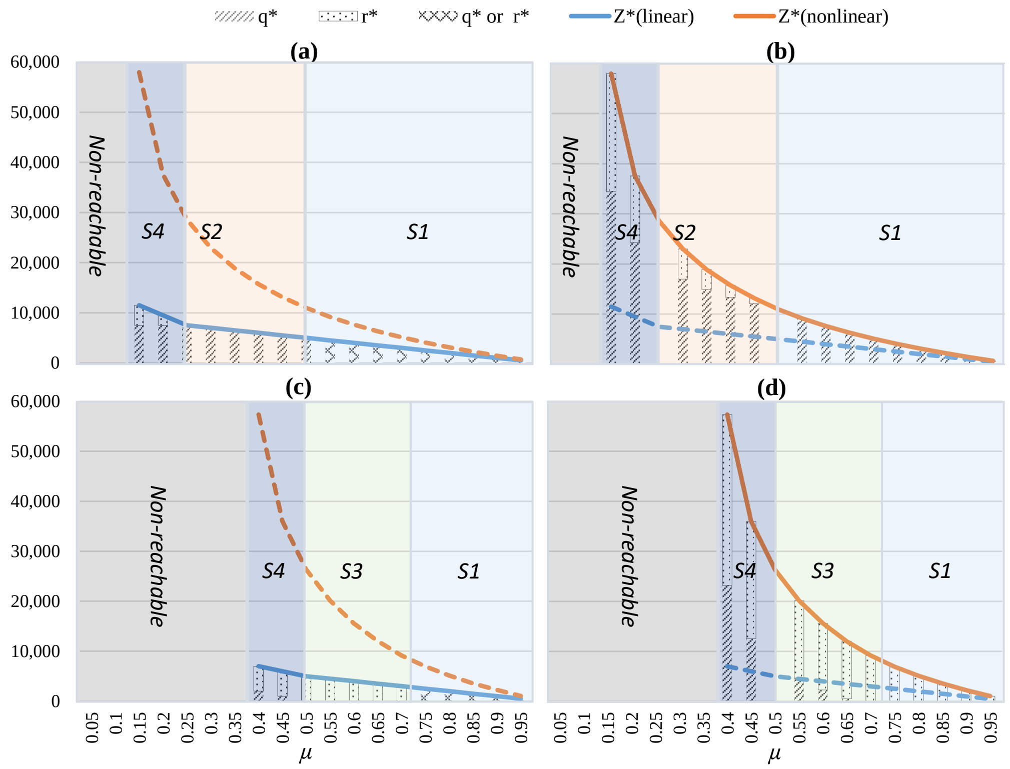

Figure 10: Effect of response requirement μ on optimal budget allocation $Z^{\star}$ given that $x = 0.25$ and $y = 0.5$ for (a) LBAP and (b) NBAP and $x = 0.75$ and $y = 0.5$ for (c) LBAP and (d) NBAP

the optimal strategy changes from risk prevention (protection) into a combination of prevention and protection. When $x = 0.1$, $\frac{bL^0}{aP^0} = 1 > \text{RT(prot)}$ (0.563) resulting in risk prevention, while $x = 0.15$, RT(prev) (0.588) $< \frac{bL^0}{aP^0} = 1 <$ RT(prot) (1.071) leading to a combination of risk prevention and protection (see Figure 8b). Similar results are observed for the effect of y (see Figure 9b). The budget allocated to both risk prevention and protection depends on the adapted restrictiveness. The budget will be equally distributed over risk prevention and protection in case that their controllability is the same (i.e. $x = y = 0.25$, $\frac{bL^0}{aP^0} = \text{ART} = 1$ in Figures 8b and 9b).

- In Scenario 2, the 100% unrestricted strategy is optimal in most cases, although a combined strategy is preferred when the controllability x is much higher since this has an impact on the restrictiveness. In case that a combined strategy is selected, an increasing budget is allocated to the restricted strategy as the risk becomes less controllable. The opposite behaviour is observed in Scenario 3.

- In case that the risk is more controllable in terms of probability (loss) ($\frac{x}{x+y} < 0.5$ or $\frac{x}{x+y} > 0.5$) in Scenario 4, risk prevention (protection) should be allocated more budget, independent of its cost. Risk prevention and protection are allocated the same amount of budget when their response cost and controllability are the same.

5.2. Impact of the relative response cost $\frac{bL^0}{aP^0}$

In these numerical experiments, we fix aP^0 at 10,000 (while varying bL^0 in the interval [2,000; 26,000] with increments of 2,000). The values for x, y and μ are set for the four scenarios as follows: $(x, y, \mu) = (0.25, 0.5, 0.75)$ in Scenario 1, $(x, y, \mu) = (0.25, 0.75, 0.5)$ in Scenario 2, $(x, y, \mu) = (0.75, 0.25, 0.5)$ in Scenario 3 and $(x, y, \mu) = (0.75, 0.5, 0.4)$ in Scenario 4.

In general, we observe that a higher relative response cost leads to a higher optimal risk cost in most cases as shown in Figure 11. However, some detailed observations for both the LBAP and NBAP will be discussed in the following lines.

Linear

- In Scenario 1, the cheapest strategy is selected and, hence, $Z^\star$ initially increases for $\frac{bL^0}{aP^0}$ as long as $\frac{bL^0}{aP^0} \leq 1$) and it levels out at $(1-\mu) * aP^0$ (i.e. 2,500) when $\frac{bL^0}{aP^0} > 1$, which depends on the response requirement $\mu = 0.75$.

- In Scenario 2, the optimal budget allocation depends on the restrictiveness of risk protection (i.e. the restricted strategy) which equals to $\frac{0.5}{0.75} = 0.66$. Hence, a combined approach of risk prevention and protection is preferred when $\frac{bL^0}{aP^0} < 0.66$, while only the unrestricted strategy (risk prevention) is selected when $\frac{bL^0}{aP^0} > 0.66$ and thus $Z^\star$ only depends on aP^0.

- In Scenario 3, the slope of the increase of $Z^\star$ for $\frac{bL^0}{aP^0}$ changes at $\frac{0.75}{0.5} = 1.5$ as the risk response strategy shifts from 100% risk prevention to a combination of risk prevention and protection.

- In Scenario 4, the slope of the increase of $Z^\star$ for aP^0 and bL^0 depends on the relative restrictiveness of x and y and the adapted restrictiveness of the most restricted strategy.

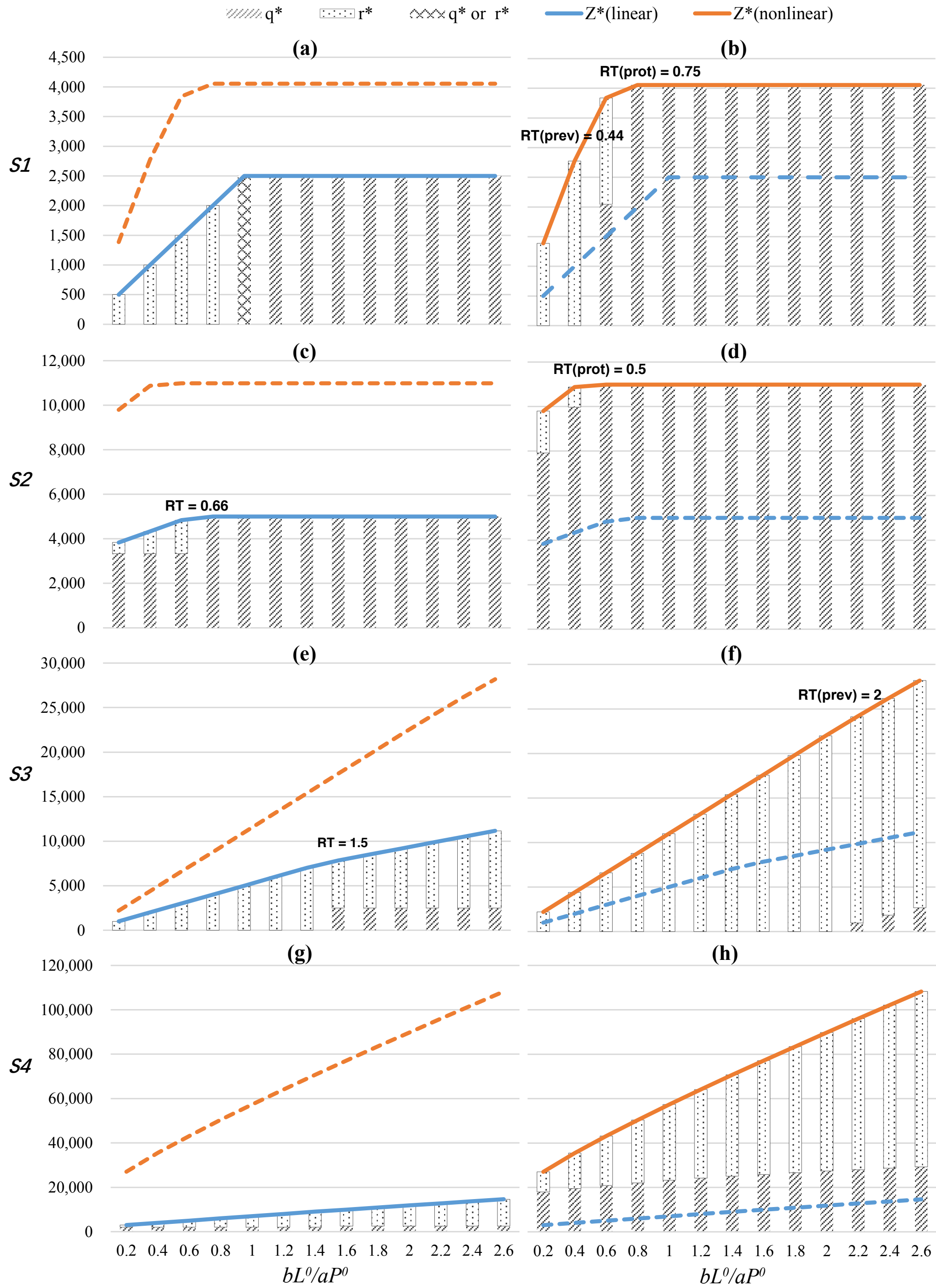

Figure 11: Effect of costs aP^0 and bL^0 on optimal budget allocation $Z^\star$ in different scenarios

Non-linear

- In Scenario 1, as $\frac{bL^0}{aP^0}$ increases, the optimal risk cost $Z^\star$ increases first and then remains unchanged. The slope of the increase of $Z^\star$ depends on decision on strategy selection, which results from the comparison between the relative response cost ($\frac{bL^0}{aP^0}$) and the restrictiveness of both strategies.

- In Scenario 2, the same trend of $Z^\star$ as in Scenario 1 is observed, however, the cut-off point to switch from a combined strategy to a 100% prevention strategy is determined by the restrictiveness of the restricted strategy (risk protection), i.e. $\frac{bL^0}{aP^0} = 0.5$ shown in Figure 11d.

- In Scenario 3, as $\frac{bL^0}{aP^0}$ increases, the optimal strategy changes from a 100% protection strategy to a combined strategy at the point $\frac{bL^0}{aP^0} = 2$. However, the optimal risk cost $Z^\star$ always increases with $\frac{bL^0}{aP^0}$ since both risk preventive and protective costs will increase the risk cost in the situation of a combined strategy.

- In Scenario 4, $Z^\star$ always increases with $\frac{bL^0}{aP^0}$ and its slope depends on the adapted restrictiveness.

5.3. Managerial implications

Based on the numerical experiments, we can draw some general conclusions and present a summary of managerial implications for constructing an effective PRM strategy. The decision about the optimal risk budget in general and the allocation of this budget over risk prevention and risk protection in particular is heavily influenced by the parameters that determine the risk environment, such as risk controllability, response requirement, relative costs of risk prevention and protection. This is shown in both the analytical results (Section 4) and the numerical experiments (Section 5). Hence, it is important for project managers to closely monitor these risk parameters since they are crucial for finding the optimal risk response strategy within the entire range of potential risk strategies from complete risk prevention to complete risk protection. Using the proposed analytical model, we summarise the main takeaways and lessons-learned for project managers in practice along the following lines:

1. A low optimal risk budget is obtained with a high controllability of risk prevention and protection (i.e. low x, y) and a low response requirement (i.e. high μ). However, we observe that the adverse effect of a single parameter, or even two parameters, on the optimal risk budget is limited as long as the third parameter is set to a beneficial value. Hence, the project manager can obtain a low optimal risk budget by focusing his/her effort on a single risk parameter or controlling the combined impact of the three risk parameters on the optimal risk budget.
2. In most scenarios, there exists a limit on the increase of the optimal risk budget as the controllability decreases (i.e. x, y increase) or the response requirement increases (μ decreases). This limit can be explained by a constant optimal risk budget or an increasing risk budget with a diminishing slope. However, such a cap on the increase of the risk budget does not exist in Scenario 4. As a result, project managers should be especially alert for adverse trends in the risk parameters in Scenario 4.
3. While the controllability (x, y) has no guarenteed effect on the optimal risk cost, a stricter risk response requirement (μ) leads to a higher risk cost in any situation. However, it is suboptimal to investigate the different risk parameters independently, rather a combination of these parameters should be monitored. More precisely, the (relative) restrictiveness is important for the risk response strategy selection and the adapted restrictiveness is important for the risk budget allocation in any situation, except in Scenario 1 when a linear cost function is assumed.
4. We observe that Scenarios 2 and 3 are inverse, for both the linear and non-linear cost functions, with respect to the optimal budget allocation and risk response strategy selection. This implies that project managers should not disregard their PRM strategy in case that the risk enviroment slightly changes from Scenario 2 to 3 and vice versa. Rather they can build upon the risk knowledge and strategic guidelines in the existing situation, keeping in mind that the risk strategy is inversed. In case that the risk environment changes to Scenario 1 or 4, however, the risk response strategy might drastically change.

Based on the above observations, we are able to highlight the important steps of any PRM strategy, reduce the effort invested by project managers and improve their focus during PRM. Also, we notice the importance of monitoring the project environment when constructing an optimal risk strategy. However, we are well aware that the risk parameters considered in this study do not cover the complete risk environment and other parameters, such as the probability of risk events and their consequences as well as the budget constraints in companies, should be taken into account. These aspects of future research will be further discussed in the next section.

6. Possible extensions

In this section, we will briefly discuss three possible extensions of the models in Section 4. First, we will extend the LBAP and NBAP to consider two interrelated risks rather than only a single risk. Secondly, we will introduce secondary risk that is independent or dependent on the budget allocated to the response strategies. Finally, we will consider when risk transfer is an optimal strategy.

6.1. Multiple risks

In contrast to the models presented in Section 4, we now consider two risks with each their own probability and impact. However, the existence of multiple risks implies that the risk response strategy for one risk might have an impact on the other risk and vice versa. We model this interrelation between both risks by means of a constant θ. The ex-post risk level for the total expected loss $R = \mu(R_1^0 + R_2^0)$ is predefined by a total response requirement for the risks, denoted as μ. The specific response requirement for each risk is labelled as μ_1 ($= m_1 n_1$) and μ_2 ($= m_2 n_2$). For both risks, we identify an initial risk level, respectively, R_1^0 (with probability P_1^0 and impact L_1^0) and R_2^0 (with probability P_2^0 and impact L_2^0). Finally, there exist two response strategies (prevention and protection) for each risk and the allocated budget for each response strategy is referred to as q_1 and q_2 (for prevention), and r_1 and r_2 (for protection).

In order to illustrate the impact of the interrelation between both risks, we show that the ex-post probability of risk 1 depends on the ratio of ex-post risk probability for risk 1 (m_1) and 2 (m_2) as well as the degree of interrelation θ, i.e. $P_1 = (m_1 - \theta + m_2\theta)P_1^0$. The expression of P_1 represents (1) the probability reduction resulting from the prevention strategy for risk 1 (i.e. m_1) and (2) the probability reduction resulting from the prevention strategy for risk 2 (i.e. $\theta(m_2 - 1)$). The model presented in Section 4 shows that, if the response requirements are known, we can obtain the budget allocation by the LBAP or NBAP flowchart. Accordingly, we can get the values of μ_1 and μ_2 as well as the budget allocation among both risks based on Eq.(14) given that $A = (m_1 - \theta + m_2\theta) \cdot (n_1 - \theta + n_2\theta)$ and $B = (m_2 - \theta + m_1\theta) \cdot (n_2 - \theta + n_1\theta)$.

$$AR_1^0 + BR_2^0 = \mu(R_1^0 + R_2^0) \rightarrow \frac{R_1^0}{R_2^0} = \frac{\mu - B}{A - \mu} \tag{14}$$

We conclude that, for the LBAP, the best decision is to reduce only one risk if this can satisfy the response requirement R. Otherwise, we need to focus on both risks. For the NBAP, under some parameter combinations, coping with both risks can be the best decision even if reducing only one risk can meet the response requirement. Nonetheless, the budget allocation decision in case of multiple risks depends on the unit costs for prevention and protection, the magnitude of each risk and the degree of interrelation between the risks.

6.2. Secondary risk

Secondary risk refers to the risk that arises as a direct consequence of implementing a risk response strategy (Zuo and Zhang, 2018). In the models of Section 4, we have neglected this type of risk, however, we will now discuss two cases of secondary risk: the risk is independent of the budget allocated to the risk response strategy (case 1) or the risk depends on the budget allocated to the risk response strategy (case 2).

CASE 1 The expected loss of secondary risk is fixed, which can be regarded as a fixed cost added on the implementation cost of response strategy. This fixed cost should be included into the comparison between prevention and protection costs. Also, the option to combine both types of response strategies will become less interesting compared to a 100% prevention or protection strategy since each response strategy will imply a fixed cost. Therefore, the expression of the restrictiveness should be updated in scenarios 1, 2 and 3, while scenario 4 remains unchanged (see Step 2 in Section 4). In Table 3, we present the updated conditions for strategy selection in the LBAP and NBAP. We denote the expected losses of the secondary risks resulting from the prevention and protection strategies as R_{prev} and R_{prot}, respectively.

Table 3: Conditions of strategy selection for scenarios 1, 2 and 3 with secondary risks

	Strategy	LBAP	NBAP
Scenario 1	prev	$aP^0 - bL^0 < \frac{R_{prot}-R_{prev}}{1-\mu}$	$R_{prot} > aP^0 \ln(\frac{1-x}{\mu-x}) - bL^0 \ln(\frac{1-y}{\mu-y}) + R_{prev}$
			$\& \ R_{prot} > aP^0 \ln(\frac{A_2-x}{\mu-x}) - bL^0 \ln(\frac{1-y}{\mu/A_2-y})$
	prot	$aP^0 - bL^0 > \frac{R_{prot}-R_{prev}}{1-\mu}$	$R_{prev} > bL^0 \ln(\frac{1-y}{\mu-y}) - aP^0 \ln(\frac{1-x}{\mu-x}) + R_{prot}$
			$\& \ R_{prev} > bL^0 \ln(\frac{\mu/A_2-y}{\mu-y}) - aP^0 \ln(\frac{1-x}{A_2-x})$
	both	—	$aP^0 \ln(\frac{A_2-x}{\mu-x}) - bL^0 \ln(\frac{1-y}{\mu/A_2-y}) > R_{prot}$
			$\& \ bL^0 \ln(\frac{\mu/A_2-y}{\mu-y}) - aP^0 \ln(\frac{1-x}{A_2-x}) > R_{prev}$
Scenario 2	prev	$aP^0 \frac{\mu}{y} - bL^0 < \frac{R_{prot}}{1-y}$	$aP^0 \ln(\frac{A_2-x}{\mu-x}) - bL^0 \ln(\frac{1-y}{\mu/A_2-y}) < R_{prot}$
	both	$aP^0 \frac{\mu}{y} - bL^0 > \frac{R_{prot}}{1-y}$	$aP^0 \ln(\frac{A_2-x}{\mu-x}) - bL^0 \ln(\frac{1-y}{\mu/A_2-y}) > R_{prot}$
Scenario 3	prot	$bL^0 \frac{\mu}{x} - aP^0 > \frac{R_{prev}}{1-x}$	$bL^0 \ln(\frac{\mu/A_2-y}{\mu-y}) - aP^0 \ln(\frac{1-x}{A_2-x}) < R_{prev}$
	both	$bL^0 \frac{\mu}{x} - aP^0 < \frac{R_{prev}}{1-x}$	$bL^0 \ln(\frac{\mu/A_2-y}{\mu-y}) - aP^0 \ln(\frac{1-x}{A_2-x}) > R_{prev}$

CASE 2 The magnitude of the secondary risk depends on the budget allocated to the response strategy. In this case, we should add a cost parameter a' for the prevention strategy and b' for the protection strategy. Subsequently, we should replace a by $a + a'$ and b by $b + b'$ in the models of Section 4.

6.3. Risk transfer

Besides risk protection and risk prevention, risk transfer can be considered as a third risk response strategy. The essential characteristic of a risk transfer is that the potential risk loss, if it occurs, is shared with or totally carried by a third party (AlBahar and Crandall, 1990). For example, Mizgier et al. (2018) analyse business interruption (BI) insurance as a technique to transfer the disruption risk. In our study, there are two fundamental differences between risk transfer and risk protection or prevention:

1. The project manager has to pay an upfront premium C to transfer the partial or complete risk loss from L^0 to L^t, where a unit cost a or b is paid to reduce a single unit of

risk using, respectively, a prevention or protection strategy. The value of C depends on the amount of risk loss that is shared with the third party and can be converted into a unit cost c as follows: $c = \frac{C}{L^0 - L^t}$. In case that the project manager chooses the option of risk sharing, a pre-defined $L^t \geq L$ is guaranteed. This reduces the number of possible combinations of (P, L) that can be selected to obtain the target risk level R.

2. In the previous analyses, we have assumed that an unlimited budget was available and the objective was to find the minimal strategy response cost Z $(= q + r)$ that reduces the risk to the target risk level R. Given the pre-defined cost of risk transfer C, it is interesting to analyse this extension under a budget constraint B. We assume that $B = Z + \epsilon$ in order to ensure that the optimal solution (q, r) is budget feasible, given that ϵ indicates the degree of budget tightness.

Based on the above two problem features, we can conclude that the following budget constraint should always hold:

$$Z + C \leq B \leftrightarrow a(P^0 - P) + b(L^t - L) + c(L^0 - L^t) \leq a(P^0 - P) + b(L^0 - L) + \epsilon \tag{15}$$

$$\rightarrow c \leq b + \frac{\epsilon}{L^0 - L^t} \tag{16}$$

In other words, the optimality of the risk transfer strategy depends on four components: the unit cost of risk transfer (c), the unit cost of risk protection (b), the budget tightness (ϵ) and the amount of risk transfer ($L^0 - L^t$). With respect to this last component, we can distinguish between two scenarios. First, a partial risk transfer ($L^t > L$) implies that the remaining risk loss ($= L^t - L$) should be mitigated by means of a prevention or protection strategy. However, the options of risk response strategies are determined by the budget constraint given that only a limited budget $B - C$ is remaining. Secondly, a full risk transfer ($L^t = L$) implies that only risk prevention could be used to meet the target risk level R. In case that $a(P^0 - P) > B - C$, it is impossible to meet the acceptable risk level R that the project manager has set. For the full risk transfer case, L^t can be replaced by L in Eq. (16).

In the above analysis, the unit cost of risk transfer c is considered linear, however, we can also provide a nonlinear unit cost of risk transfer. In this case, the marginal cost $c_t = -c\frac{L^0}{L^t}$ similar to the nonlinear functions shown in Eqs. (6)-(7). The premium $C = \int_{L^0}^{L^t} c_t dL = cL^0 \ln \frac{L^0}{L^t} = cL^0 \ln \frac{1}{t}$, where $t = \frac{L^t}{L^0}$. After applying risk transfer, the cost for risk protection becomes as:

$$r = \int_{L^t}^{L} c_L dL = bL^0 \ln \frac{L^t - yL^0}{L - yL^0} = bL^0 \ln \frac{t-y}{n-y} \quad (17)$$

which is derived from Eq.(7). As a result, we can obtain the following equations for the nonlinear risk transfer cost functions based on Eqs. (15)-(16):

$$Z + C \leq B \leftrightarrow aP^0 \ln \frac{1-x}{m-x} + bL^0 \ln \frac{t-y}{n-y} + cL^0 \ln \frac{1}{t} \leq aP^0 \ln \frac{1-x}{m-x} + bL^0 \ln \frac{1-y}{n-y} + \epsilon \quad (18)$$

$$\rightarrow c \ln \frac{1}{t} \leq b \ln \frac{1-y}{t-y} + \frac{\epsilon}{L^0} \quad (19)$$

In summary, it is always possible to meet the risk target level R in case that an infinite budget is assumed for risk prevention, protection and transfer, i.e. $B \geq Z + C$. However, in case of a limited budget, risk transfer might be a suboptimal strategy depending on the budget tightness and the cost of risk transfer.

7. Empirical analysis

In this section, we present an empirical analysis to validate the applicability of our proposed methodology. In Section 7.1, we analyse a total of 181 real-life projects obtained from the dataset of Bastelier and Vanhoucke (2015). Subsequently, we illustrate the analysis in detail for a single real-life project in Section 7.2.

7.1. Data analysis

Bastelier and Vanhoucke (2015) have started a continuous process of data collection, classification and validation for real-life projects and they have made this information publicly available to stimulate research efforts in the field of project management. Furthermore, they have introduced standardised data files, called project cards, in order to facilitate a more uniform data collection and validation process. Amongst others, these project cards show the quality of the data collection for each real-life project. At the time of our research, the dataset consisted of 181 projects, which can be consulted and downloaded from the website `www.projectmanagement.ugent.be/research/data`. In our research, we will analyse this dataset with respect to the availability and quality of risk information.

First, we can eliminate 116 out of the 181 projects because the project cards indicate that the quality of the risk information was insufficiently high. This means that assumptions were made about the risk profile of the project during data collection (e.g. random distribution) and, hence, no specific information about the probability and impact of risk was provided. For the remaining 65 projects, we have a relative high certainty about the quality of the risk data. In the dataset, we identify two main industries (construction and IT) with the construction industry subdivided into civil, industrial and building construction. The last category consists of commercial, institutional and residential building projects. Besides these two main industries, we also identify projects from the event sector as well as education and engineering projects. A total of 34 projects could not be assigned to a specific industry due to missing data, hence, they will not be considered in the following discussion. In Figure 12, we show the relative number of projects with and without risk information in the dataset subdivided for various (sub)industries. Although these results cannot be used to draw generalisable conclusions, they show that our approach is generally applicable since - at least - one project with sufficient risk information is observed in each (sub)category.

Furthermore, we categorise the empirical projects with risk information in terms of size (number of activities), planned duration and budget. In order to distinguish between small and large projects, we subdivide the 65 projects into groups based on the quartiles (Q1, Q2 and Q3) of the data points in the complete dataset of 181 projects. For example, the projects in the complete dataset are distributed such that the first, second and third quartile corresponds with less than 23, 30 and 56 activities, respectively. The results are summarised in Table 4 and show that both smaller and larger projects in terms of number of activities, duration and budget are represented in the subset of projects with high-quality risk information.

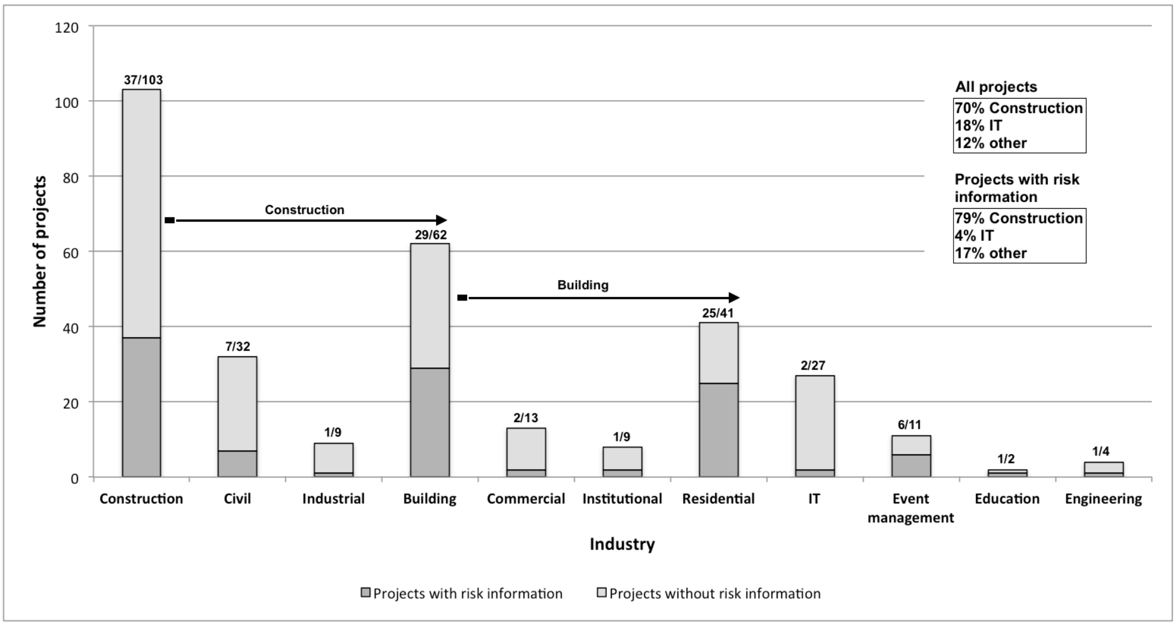

Figure 12: Projects with/without risk information in the dataset of Bastelier and Vanhoucke (2015)

7.2. Case study

The analysis of the budget allocation decisions using the proposed approach from Section 4 can be conducted on each of the 65 projects identified in Section 7.1. In the remainder of this section, we illustrate the analysis using a single project out of this set of 65 projects. The case study is a fuel tank filter project that consists of two risks (see Section 6.1) with a non-linear cost function (see Section 4.3). The two possible risk events that may affect the successful project completion are *material shortage* and *welding accident*. Further project details are available in the empirical database from the website `www.projectmanagement.ugent.be/research/data` (ID = 133; code = C2019-05).

The initial probability P_1^0 and P_2^0, and initial loss L_1^0 and L_2^0 are estimated based on the project description and summarised in Table 5. This results in an initial magnitude of risk equal to 11,600 for the material shortage and 50,000 for the welding accident. The minimal probabilities ($\underline{P}_1$ and $\underline{P}_2$) are assessed based on the information acquired from the supplier market (risk 1) and the reports on industrial accidents and safety in manufacturing sector (risk 2). The minimal loss ($\underline{L}_1$ and $\underline{L}_2$) is estimated based on the administration expenses that should be paid when the risk occurs. The degree of interrelation (θ) is evaluated by the

Property	Interval	#Projects
Number of activities		
$x < Q1$	< 23	18
$Q1 \leq x < Q2$	≥ 23 & < 30	26
$Q2 \leq x < Q3$	≥ 30 & < 56	14
$Q3 \leq x$	≥ 56	7
Planned duration (days)		
$x < Q1$	< 101	28
$Q1 \leq x < Q2$	≥ 101 & < 191	36
$Q2 \leq x < Q3$	≥ 191 & < 320	0
$Q3 \leq x$	≥ 320	1
Planned budget (€)		
$x < Q1$	< 43.170	17
$Q1 \leq x < Q2$	≥ 43.170 & < 220.271	22
$Q2 \leq x < Q3$	≥ 220.271 & $< 1.102.537$	17
$Q3 \leq x$	$\geq 1.102.537$	9

Table 4: Data analysis of risk information in dataset of Bastelier and Vanhoucke (2015)

ID	P_0	L_0	R_0	$\underline{P}$	$\underline{L}$	x	y	R	μ
1	0.2	58,000	11,600	0.01	2,000	0.05	0.0345	3,000	0.2586
2	0.1	500,000	50,000	0.005	5,000	0.05	0.01	10,000	0.2

Table 5: Summary of case study data

project manager, however, no interrelation between both risks is assumed in this case study (i.e. $\theta = 0$) in order to allow a fair comparison with other projects in the dataset (see Section 7.1). Also, the accepted expected loss for risk 1 and 2 were set by the project manager as 3,000 and 10,000, respectively, which resulted in the following response requirements: $\mu_1 = \dfrac{3,000}{0.2 * 58,000} = 0.2586$ and $\mu_2 = \dfrac{10,000}{0.1 * 500,000} = 0.2$.

In Table 6, we list a prevention and protection strategy for each risk as well as the unit preventive and protective cost. For example, a a strict supplier evaluation as a prevention strategy for risk 1 costs 2,000, but it can reduce the probability of material shortage from 20% to 10% (i.e. $m = 0.5$). In case that we assume a nonlinear cost-effect function, we can estimate the unit prevention cost as

$$a_1 = \frac{q}{P^0 * ln\frac{1-x}{m-x}} = \frac{2,000}{0.2 * ln\frac{1-0.25}{0.5-0.25}} = 13,383 \tag{20}$$

Based on the above information, we apply the three step procedure of Section 4.2 adjusted

ID	Description	Prevention	Protection	a	b
1	Material shortage	Strict supplier evaluation	New supplier	13,383	0.038
2	Welding accident	Experienced team/update calls	Insurance	3,396	0.0007

Table 6: Two risks in the fuel tank filter project

for multiple risks (Section 6.1). For the scenario selection (Step 1), we classify both risks in scenario 1 since $\mu_1 > x_1$ and $\mu_1 > y_1$ as well as $\mu_2 > x_2$ and $\mu_2 > y_2$. Then, the relative costs of risk prevention and protection $(a_1 P_1^0, b_1 L_1^0)$ and $(a_2 P_2^0, b_2 L_2^0)$ should be compared with the response requirement (μ_1 and μ_2) and the risk controllability (x_1, x_2, y_1 and y_2) for the strategy selection (Step 2). Based on this analysis, we select the 100% protection strategy to mitigate risk 1 (Eq.(21)) and the combined strategy for risk 2 (Eq.(22)) because:

$$\frac{b_1 L_1^0}{a_1 P_1^0}(= 0.82) < \frac{1 - \frac{y_1}{\mu_1}}{1 - x_1}(= 0.91) \tag{21}$$

$$\frac{1 - \frac{y_2}{\mu_2}}{1 - x_2}(= 1) < \frac{b_2 L_2^0}{a_2 P_2^0}(= 1.04) < \frac{1 - y_2}{1 - \frac{x_2}{\mu_2}}(= 1.32) \tag{22}$$

In Step 3, the budget allocated to protect against the material shortage is equal to $r_1 = 3,884$. For the welding accident, the budget allocated for protection is equal to $r_2 = 452$ and the budget allocated for prevention is equal to $q_2 = 128$. This results in an optimal total risk response cost $Z = 4,464$.

Finally, we will compare the model suggestions with the actual actions implemented in the real-life project. First, we observe that a new supplier was contacted after the material shortage occured. This response action is consistent with our model suggestion (i.e. 100% protection strategy), which shows the practical application of the proposed decision model. Second, both risk prevention and protection strategies were implemented in the project, however, the protection strategy (i.e. insurance) was not triggered since no accident occured throughout the project execution. These response actions are again consistent with the results of our model (i.e. a combined strategy).

We identified two main risks in the above case study, however, there exists more than two risks in most real-life projects. As more risks are observed in a project, the number of interrelations between those risks will exponentially increase. Also, we have considered a single project in this research study, while today many project managers have to simultanously deal with multiple projects. In such a project portfolio, risk sharing between different projects becomes an increasingly important risk mitigation strategy. In the above situations, our analytical approach falls short and more complex techniques, such as artificial intelligence and machine learning, are required (Afzal and Bhatti, 2021; Gondia et al., 2020).

8. Conclusions

In this paper, we have presented an analytical approach to reasonably allocate the response budget among risk prevention and risk protection for effective project risk management. The proposed method considers the characteristics of the project risk, risk response strategy as well as risk response requirement and models the relation between the risk response cost and effect as both linear and non-linear. Accordingly, two mathematical models are constructed and the optimality conditions as well as the analytical results are discussed. In order to validate the analytical results, numerical experiments are conducted and managerial insights in terms of the strategy selection and budget allocation are drawn. Furthermore, three extensions of the budget allocation models (multiple risks, secondary risk and risk transfer) are discussed in order to increase the general applicability of our research. Finally, we analyse a set of emperical projects and illustrate the proposed approach on one specific case study in order to validate our models and improve their practical value.

We believe that this paper could be of great value to academics given the proposed mathematical models and the analytical results, and to practitioners given the three-step decision procedure and managerial guidelines. However, we can identify some limitations of this work that could be extended in future research in the following ways. First, we only employ two specific linear and non-linear functions to model the relations between the costs and effects of risk response strategies. Future research can examine the robustness of the results in various situations by collecting empirical data or relaxing the relations to a more general case with random cost variables given various probability distributions. This might provide more realistic managerial insights. Second, we have introduced risk transfers with budget constraints as an extension of the budget allocation models. However, this research area could be further developed into a future research avenue. Third, the case study analysis was conducted on a single project, but this research could be extended to complete project portfolios where risk transfer and risk sharing is possible. Also, our study focuses more on the tactical rather than the strategical level of risk management since we provide models to deal with general risks rather than specific types of projects or industries. Fourth, the number of projects in a portfolio as well as the risk structure for a specific project can increase drastically. For example, we have introduced multiple risks with interrelations, however, this extension could be further analysed from a theoretical perspective. Also, more case study data could be collected in order to ensure that these risk interrelations are modelled in a realistic way. As a result, our analytical models can potentially no longer provide a high-quality solution for these complex budget allocation problems and more advanced techniques such as artificial intelligence and machine learning become interesting. Finally, the proposed models could be extended to incorporate risk sources and consequences when analysing the optimal risk response strategies and the corresponding budget allocation.

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