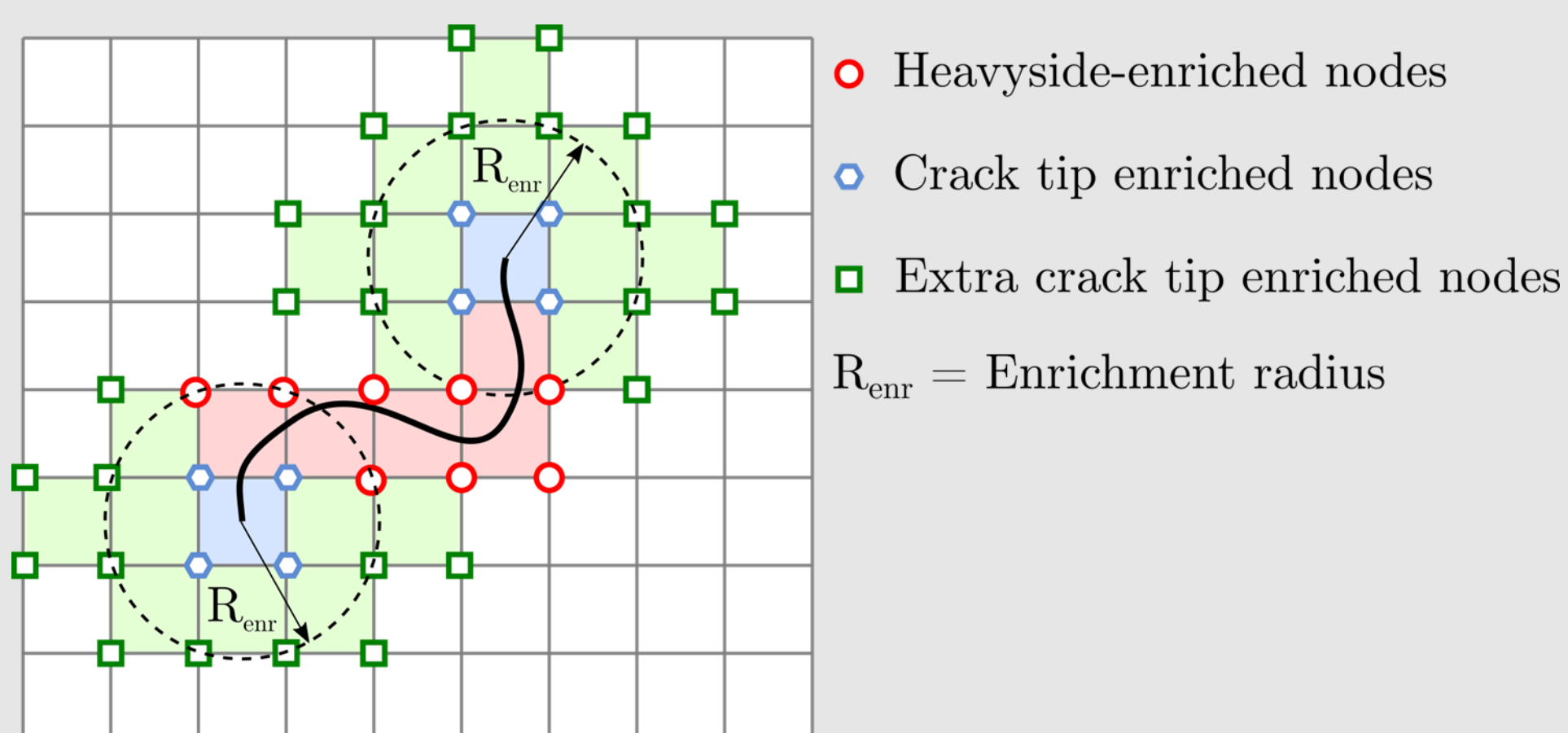


AN X-FEM FATIGUE CRACK SIMULATION FRAMEWORK BASED ON A B-SPLINE CRACK DESCRIPTION

Introduction

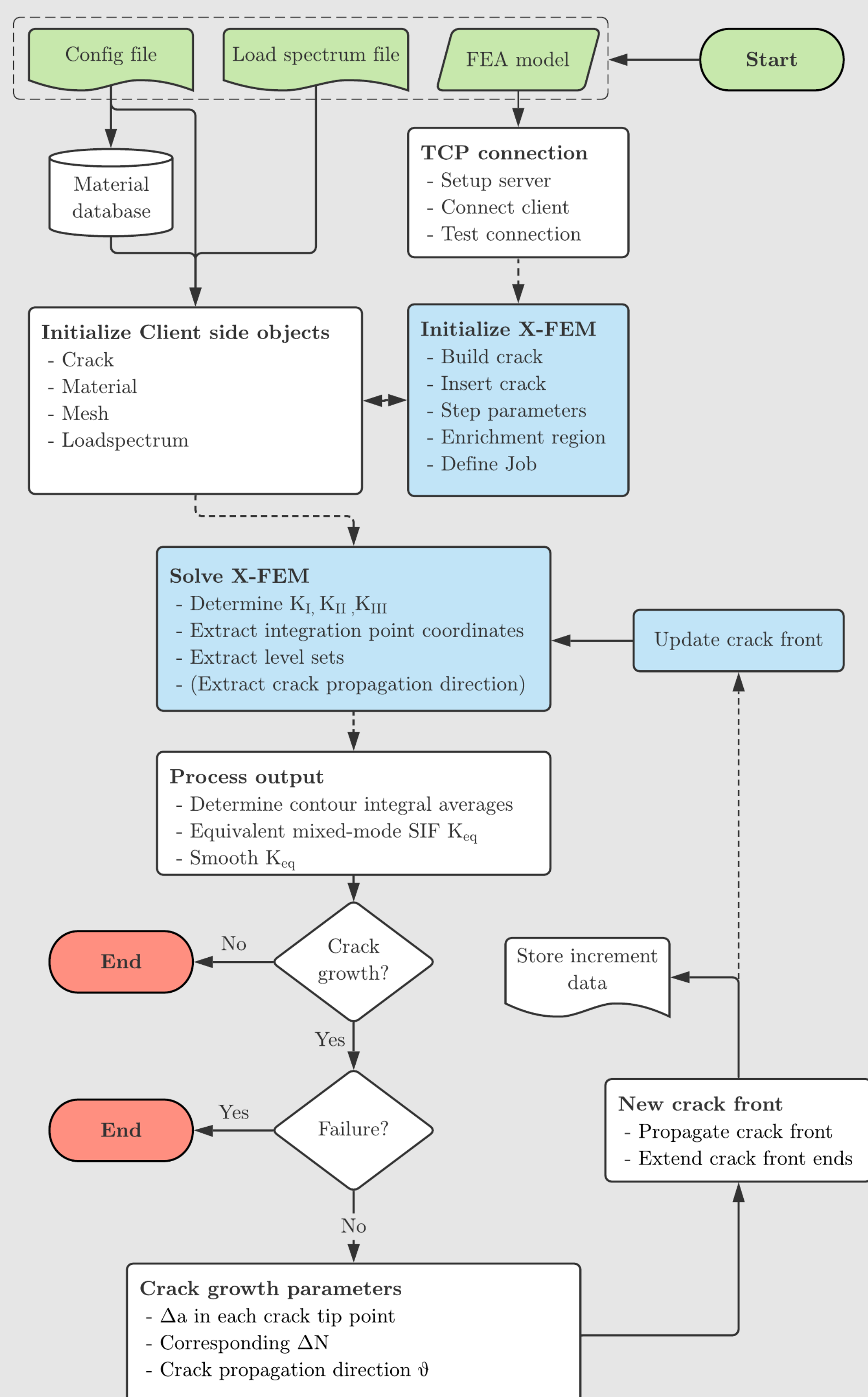
The extended finite element method (X-FEM) was introduced to overcome the need for remeshing when modeling cracks and crack propagation. It allows modeling of an arbitrary crack path in the mesh through the enrichment of the FEM displacement field $\mathbf{u}(x)$ with discontinuous functions. The shape functions are multiplied with the Heaviside function $H(x)$ to represent the crack faces. The crack tip elements are enriched with asymptotic crack tip functions that reproduce the asymptotic LEFM fields.

$$\mathbf{u}(x) = \underbrace{\sum_{i \in S} N_i(x) \mathbf{a}_i}_{\text{Standard}} + \underbrace{\sum_{i \in S_H} N_i(x) H(x) \mathbf{a}_i}_{\text{Discontinuous enrichment}} + \underbrace{\sum_{i \in S_T} N_i(x) \sum_{\alpha=1}^4 F_{\alpha}(x) b_{i\alpha}}_{\text{Crack tip enrichment}}$$



In this work, a purely Python X-FEM based framework for mixed-mode non-planar fatigue crack growth is presented. It makes use of the X-FEM solver of Abaqus. A hybrid implicit/explicit crack description approach is adopted for tracking and extending the crack. The explicit description of the crack front(s) and face(s) is achieved with B-spline curve and surface formulations, respectively.

Framework structure



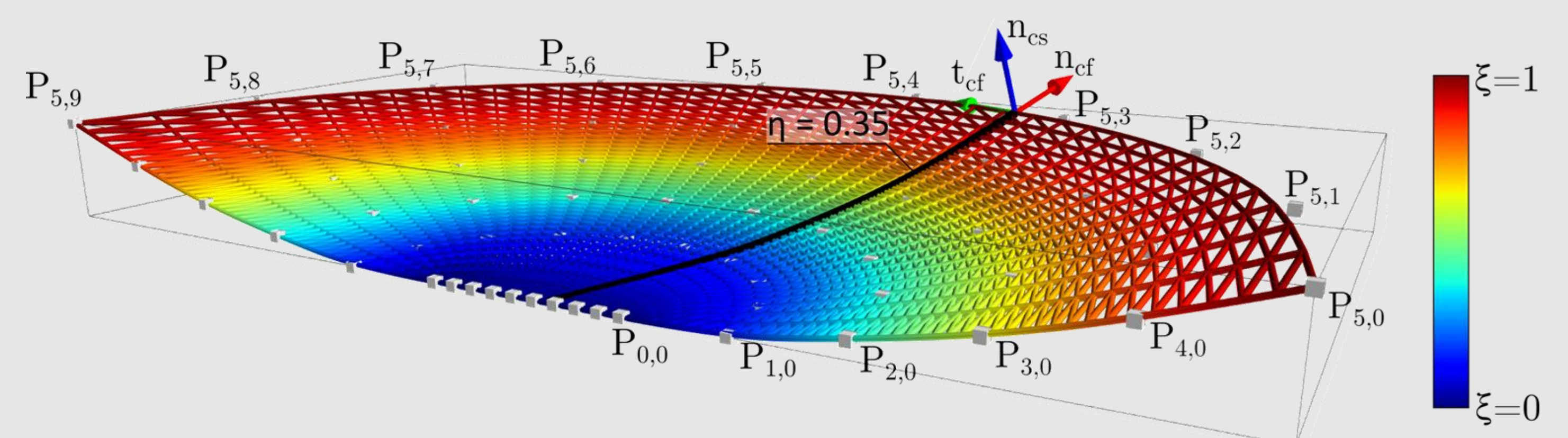
B-spline crack description

The crack face is defined as a B-spline surface to ensure at least C^1 continuity. The crack surface is spatially three-dimensional (x, y, z) and is described in a two-dimensional parametric space (ξ, η) . Considering the two-dimensional parametric space (ξ, η) and given a bidirectional net of control points $\mathbf{P}_{i,j}$, a B-spline surface with polynomial order p in ξ and polynomial order q in η can be described as the tensor product of the basis functions and the points:

$$S(\xi, \eta) = \sum_{i=1}^n \sum_{j=1}^m N_{i,p}(\xi) M_{j,q}(\eta) \mathbf{P}_{i,j}$$

Stress intensity factors (SIFs) are determined along the crack front. The SIFs are determined with respect to local coordinate systems defined as $(\mathbf{t}_{cf}, \mathbf{n}_{cf}, \mathbf{n}_{cs})$ which are determined as:

$$\mathbf{n}_{cf} = \frac{\partial}{\partial \xi} S(\xi, \eta) \quad \mathbf{t}_{cf} = \frac{\partial}{\partial \eta} S(\xi, \eta) \quad \mathbf{n}_{cs} = \mathbf{t}_{cf} \times \mathbf{n}_{cf}$$



Crack front extension

The SIFs are used to determine the crack propagation angle θ_i and the crack growth increment Δa_i in each crack front point i . Every iteration the crack front advances with the crack growth increment Δa_i along the vector that makes an angle θ_j with respect to \mathbf{n}_{cf} . Propagation along the axis \mathbf{n}_{cf} corresponds to $\theta_j = 0^\circ$.

Validation

Numerical simulations performed with the framework are compared to fatigue crack growth experiments. Validation specimens were designed in the form of compact tension specimens with an additional hole. A constant amplitude cyclic load with a maximum value of 13.5 kN and a minimum value of 5 kN at a frequency of 5 Hz, was used. A digital image correlation (DIC) system was used to track the crack path at the specimens' surface. Good agreement with the experimental results was found. The numerical simulations slightly overestimate the experimental crack length.

