

ARE JUDGES MORE TRANSPARENT THAN BLACK BOXES? A SCHEME TO IMPROVE JUDICIAL DECISION-MAKING BY ESTABLISHING A RELATIONSHIP WITH MATHEMATICAL FUNCTION MAXIMIZATION

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I

INTRODUCTION

It can be safely considered common knowledge that it is difficult to consistently predict a judicial outcome. The term by which these situations are best described is legal uncertainty. Professor Anthony D'Amato defines legal uncertainty in a mathematical sense, as meaning that a legal rule is expected by informed attorneys to have an official outcome at or near the 0.5 level of predictability, which means that the outcome is barely indistinguishable from random prediction.¹ Other definitions have been introduced, but they all refer to legal uncertainty as a vice.² Legal uncertainty is routinely blamed for undermining both the rule of law in general and the law's ability to achieve specific objectives, such as deterring anti-social conduct or encouraging trade and investment.³

Research has pointed to diverse explanations for this lack of predictability, such as the inherent ambiguity of human language that is used to describe legal rules, the use of open norms (for example, the reasonable person standard in tort

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1. D'Amato, *Legal Uncertainty*, 71 CAL. L. REV. 1, 2 (1983).

2. See generally John F. Davis & William L. Reynolds, *Juridical Cripples: Plurality Opinions in the Supreme Court*, 23 DUKE L.J. 59 (1974) (arguing that legal uncertainty inhibits a court's leadership role and its ability to develop laws).

3. Arnaud Raynouard & Anne-Julie Kerhuel, *Measuring the Law: Legal Certainty as a Watermark*, 8 INT'L J. DISCLOSURE & GOVERNANCE, 360, 362 (2010).

claims), and a natural process of obsolescence due to continual changes in society and technology.⁴ Although each of these explanations might contribute to our understanding of the non-deterministic character of judicial decisions, it is useful to consider this phenomenon in terms of two formal principles introduced in Part II, Section A: incompleteness and uncertainty.⁵

In this Article we put forward an alternative formulation of legal uncertainty that is even more mathematically oriented than the one proposed by D'Amato. This requires us to introduce some mathematical background, although we do so in a rather non-rigorous manner.⁶ What both conceptualizations of legal uncertainty have in common, however, is that they merely provide mental pictures. It is often impossible to predict with a high degree of accuracy the expected outcome of the application of a certain rule to a specific act or transaction, which can limit the practicability of defining legal uncertainty. Nevertheless, we believe that the introduced analogy with mathematical function maximization is useful as a new conceptual definition of legal uncertainty. Even more importantly, once legal uncertainty is properly visualized, we argue that it can be reduced by a certain scheme in judicial decision-making.⁷ These concepts and ideas are then illustrated in a fictitious case study.⁸

In short, the proposed way to improve judicial decision-making entails that the judge communicates all legally valid solutions to the litigants and, more broadly, to the outside world. One of the major benefits of this proposed way of judging cases would be the reduction of the human black box character that is currently present in many, if not most, judicial decision-making settings around the world. For example, judges typically produce a single outcome that resolves the case at hand, although they probably thought of other possible solutions. Even though these other solutions might also be legally defensible, judges restrict their judgment to that single outcome. Reasons for not mentioning other outcomes might include the fear that some alternative outcomes are socially not acceptable, the urge to maintain the appearance that the law is unambiguous, or the lack of willingness to thoroughly ascertain all legally valid solutions and compare them to each other.

In contrast, black boxes from the domain of artificial intelligence are easily able to produce a set of alternative solutions to a given problem and produce all reasonable predictions in a forecast study. In the debate on the applicability of artificial intelligence tools in legal practice, it is frequently argued that because

4. H. KELSEN, PURE THEORY OF LAW (M. Knight trans., The Lawbook Exchange 2d ed. 2009); Giuseppe Dari-Mattiacci & Bruno Deffains, *Uncertainty of Law and the Legal Process*, 163 J. INST. & THEORETICAL ECON., 627, 629 (2007) (arguing that the difficulty of predicting law can be attributable to “the natural process of obsolescence due to continual changes in society and technology . . .”).

5. See *infra* Part II.

6. See *infra* Part II, Section A.

7. See *infra* Part II.B.

8. See *infra* Part II.C.

most of these tools are black boxes, they are inferior to human judges.⁹ The given considerations show, however, that reality is much more nuanced, as artificial black boxes communicate a wide range of plausible outcomes, in contrast to most human judges. The central theme of this Article is as follows: If judges are expected to be at least as transparent as artificial black boxes, they should communicate all legally valid solutions to a given dispute.

II

INCOMPLETENESS AND AMBIGUITY

Every legal practitioner has encountered the frustrating situation where the law does not provide a clear solution to the problem at hand. To such a person it is evident that law is a system that is unable to ensure a perfectly tailored legal solution to every dispute.

Those with a background in mathematics may recognize the imperfections of the law as being the result of an incomplete system. Incompleteness was formally proven as a property of mathematical machinery by the famous logician, Gödel.¹⁰ Ignoring the many technicalities underlying his proposition, Gödel essentially demonstrated that regardless of the number of axioms (which are statements that are taken to be true) introduced into the mathematical system, there will always exist statements that can neither be proved nor disproved. Put another way, one can always find claims that cannot merely be labelled true or false. The mathematical system is simply unable to provide any definite answer as to the truth or falsity of every proposition. Thus, if even the highly formalized world of mathematical abstractness struggles with incompleteness, how can the much less formal rules and principles of law hope to overcome this challenge?

Whereas scholars Katharina Pistor and Chenggang Xu define incompleteness as meaning that some relevant applications of the law are missing or ambiguously stipulated in the law,¹¹ we prefer the concept of incompleteness to be restrictively related to missingness. A simple example of incompleteness is a rule stating that some laws will be no longer valid as from a certain date D_1 , and that a new set of legal rules will be in force from date D_2 , but there is some period between D_1 and D_2 for which the rule does not stipulate which laws are applicable.

9. Jason Tashea, *Courts Are Using AI to Sentence Criminals. That Must Stop Now*, WIRED (Apr. 17, 2017, 7:00 AM), <https://www.wired.com/2017/04/courts-using-ai-sentence-criminals-must-stop-now/> [https://perma.cc/Z74F-Y865]. (arguing that such artificial intelligence lacks transparency in its decision-making process as compared to human judges).

10. See generally TORKEL FRANZÉN, GÖDEL'S THEOREM: AN INCOMPLETE GUIDE TO ITS USE AND ABUSE (2005) (explaining the mathematical proof of Gödel's Incompleteness Theorem).

11. Chenggang Xu & Katharina Pistor, *Law Enforcement Under Incomplete Law: Theory and Evidence from Financial Market Regulation* 3 (COLUM. L. SCH. CENTER FOR LAW & ECON. STUD., Working Paper No. 222, 2001), https://papers.ssrn.com/sol3/papers.cfm?abstract_id=396141 [https://perma.cc/RGE2-AZC4]. ("Incomplete law means that some relevant applications of the law are missing, or that they are ambiguously stipulated in the law.").

Another characteristic of some judicial rules is that they are ambiguous.¹² While ambiguity has certain similarities to incompleteness, it should be considered a distinct concept. Although there is no agreement on the exact definition of this notion,¹³ ambiguity is often understood as to mean that some rules are susceptible to more than one reasonable interpretation.¹⁴ Ambiguity is very apparent in the case of open norms, which are behavioral norms with an open texture, such as the norm of good faith, the prohibition of the abuse of power, and notions of trust and confidence.¹⁵

Incompleteness and ambiguity are catalysts for biases in judicial decision making. Biases are highly prevalent in the reasoning processes of humans in general and of judges in particular.¹⁶ The more ambiguous or incomplete the rules are, the more flexibility there is to use these rules in a way that is not entirely rational. Ambiguity and incompleteness may, therefore, serve as an occasion for judges to consult their own views of policy, whether openly, quietly, or unconsciously.¹⁷ Judges may also use the flexibility offered by incompleteness and ambiguity to rationalize their decisions by strategically interpreting a legal standard.¹⁸ This is not to say that judges deliberately misuse the deficiencies of the legal system to arrive at decisions that seem illogical from a legal or rational point of view. Psychologists have introduced the notion of cognitive dissonance to explain how people hold inconsistent, sometimes even foolish, beliefs, while internally being convinced that their behavior relies on a rational and logical reasoning process.¹⁹ Even elite decision makers, such as U.S. Supreme Court

12. Mark Moller, *Procedure's Ambiguity*, 86 IND. L.J. 645, 646 (2011) (discussing various cases which generated "fuzzy and indeterminate" legal tests).

13. See generally Ward Farnsworth, Dustin Guziar, Anup Malani, *Ambiguity About Ambiguity: An Empirical Inquiry into Legal Interpretation*, 2 J. LEGAL ANALYSIS 257, 257 (2010) (discussing the lack of consensus behind the meaning of ambiguity).

14. *Ambiguity*, BLACK'S LAW DICTIONARY (11th ed. 2019).

15. Aline Van Bever, *Open Norms and the Foundations of Employment Law: A Legal Examination of Hidden Patterns in Employment Relationships*, 3 EUR. LABOUR L. J. 173, 173 (2012).

16. See generally Allison P. Harris & Maya Sen, *Bias and Judging*, 22 ANN. REV. POL. SCI. 241, 259 (2019) (finding race, gender, and ethnicity can sometimes be used to predict judicial decisions); Jenny Yourstone, Torum Lindholm, Martin Grann, Ola Svenson, *Evidence of Gender Bias in Legal Insanity Evaluations: A Case Vignette Study of Clinicians, Judges and Students*, 62 NORDIC J. PSYCHIATRY 273 (2008) (finding evidence of gender bias among judges in legal insanity evaluations); Yevgeny Mugerman, Neta Nadiv, Moran Ofir, *Are Courts Biased? The Anchoring Heuristic and Judicial Decisions in Personal Bankruptcy Proceedings*, (May 29, 2020) (unpublished manuscript) https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3590544 [https://perma.cc/Q2X9-YNFE] (finding a strong correlation between the payment plan initially recommendation and the court's final ruling); Masua Sagiv, *Cultural Bias in Judicial Decision Making*, 35 B.C. J. L. & SOC. JUST. 229 (2015) (finding a cultural bias to be present in judicial decision making).

17. Cass R. Sunstein, *Beyond Marbury: The Executive's Power to Say What the Law Is*, 115 YALE L. J. 2580, 2610 (2006) ("[T]he political convictions of federal judges continue to play a role in judicial review of agency interpretations.").

18. John Zhuang Liu & Xueyao Li, *Legal Techniques for Rationalizing Biased Judicial Decisions: Evidence from Experiments with Real Judges*, 16 J. EMPIRICAL L. STUD. 630, 631–32 (2019) (finding judges rationalize their decision making to align with their implicit biases).

19. See generally LEON FESTINGER, *A THEORY OF COGNITIVE DISSONANCE* (1957) (introducing

Justices, are not free from cognitive dissonances.²⁰

The fact that any legal system is, in general, ambiguous and incomplete, does not imply that all cases are subject to legal uncertainty. Given a certain legal dispute, the set of legal rules that is feasible to resolve that specific case is a subset of all rules that exist within the legal system. It may well be that a specific subset is complete and unambiguous with respect to the case at hand. In such a situation, there is a clear legally permissible outcome. For example, if a person is using their cell phone while driving, and the law states that using a cell phone while driving will be punished with a fine of \$50, then the set of feasible legal rules that applies to this case is both complete and unambiguous. Only one verdict is legally acceptable, and thus legal uncertainty is zero.

However, due to the properties of incompleteness and ambiguity in the legal system, there exist many cases where the subset of legal rules that are feasible to decide the case do not result in a clear outcome. For these cases, multiple legally permissible decisions exist. In this Article, we argue that judges should attempt to describe all these potential outcomes. That is to say, judges should not only outline different interpretations of ambiguous rules and incomplete laws, but also state what the resulting judicial outcome would be under these alternative reasoning processes. Thus, in contrast to the many verdicts that contain a single decision, we propose that the final opinion of the court should contain all legally permissible decisions. Of course, the court should ultimately settle the dispute, which requires an unequivocal decision. Our proposal is not in contradiction with this task, as we argue that the judge should simply select the most legally valid outcome from the set of all legally permissible decisions. We refer to this last decision as the final decision. All other legally permissible decisions found by the judge are referred to as alternative decisions.

The advantages of communicating a final decision with a set of alternative decisions are outlined below. Before elaborating on the consequences of this proposed method of judicial decision-making, we will conceptualize the problem of finding the set of legally permissible solutions for any given case. We do this by establishing a relationship with mathematical function maximization.

the theory of cognitive dissonance); CAROL TAVRIS & ELLIOT ARONSON, *MISTAKES WERE MADE (BUT NOT BY ME): WHY WE JUSTIFY FOOLISH BELIEFS, BAD DECISIONS, AND HURTFUL ACTS* (2015).

20. See generally Paul M. Collins, Jr., *Cognitive Dissonance on the U.S. Supreme Court*, 64 POL. RES. Q. 362 (2009) (examining the applicability of dissonance theory to elite decision makers).

III

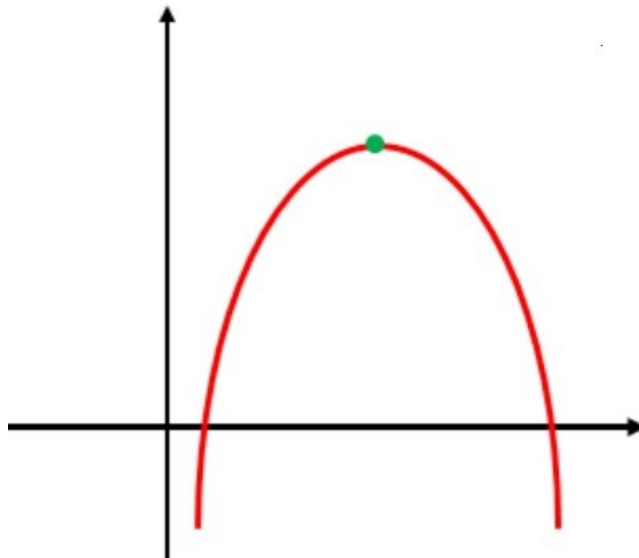
JUDICIAL DECISION-MAKING AND MATHEMATICAL FUNCTION MAXIMIZATION

The problem of finding all legally permissible solutions for a given case can be illustrated in terms of a well-established mathematical issue, namely the task of finding points where a mathematical function reaches a maximum. Before describing this connection, it is necessary to first introduce some basic concepts related to mathematical functions and their maxima.

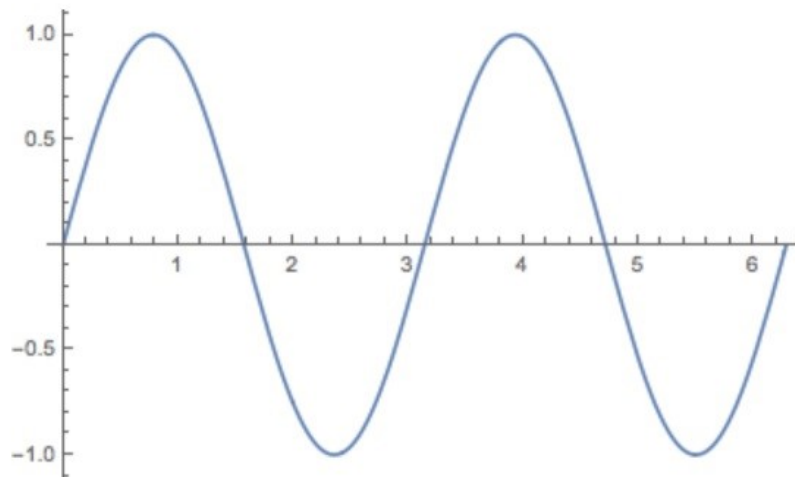
A. Mathematical Function Maximization

A maximum is defined as a value that is larger (or at least not smaller) than all other values in a neighborhood, or specific range of values. Figure 1 is an example of a mathematical function with one maximum. The green dot denotes a value that is larger than all values nearby, and thus it is a maximum.

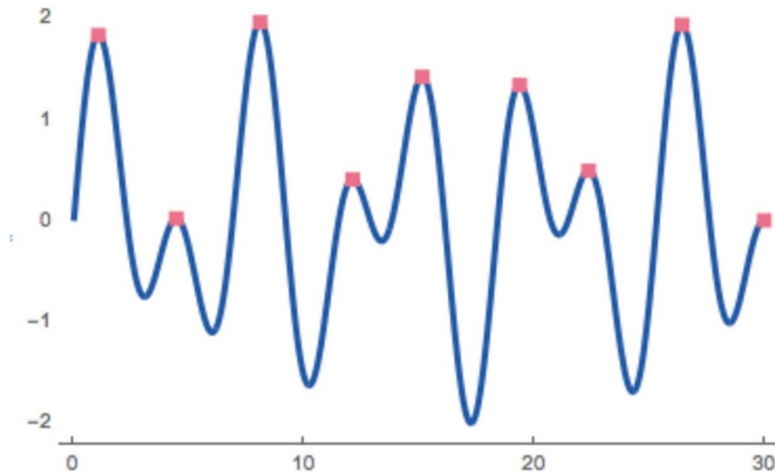
Figure 1. Illustration of a Maximum (Green Dot) of a Mathematical Function



A function can have more than one maximum, as illustrated in Figure 2. Although in this figure the two maxima have the same value, this need not be the case. The definition of maximum only requires that the corresponding value be larger than values nearby.

Figure 2. A Mathematical Function with Two Maxima

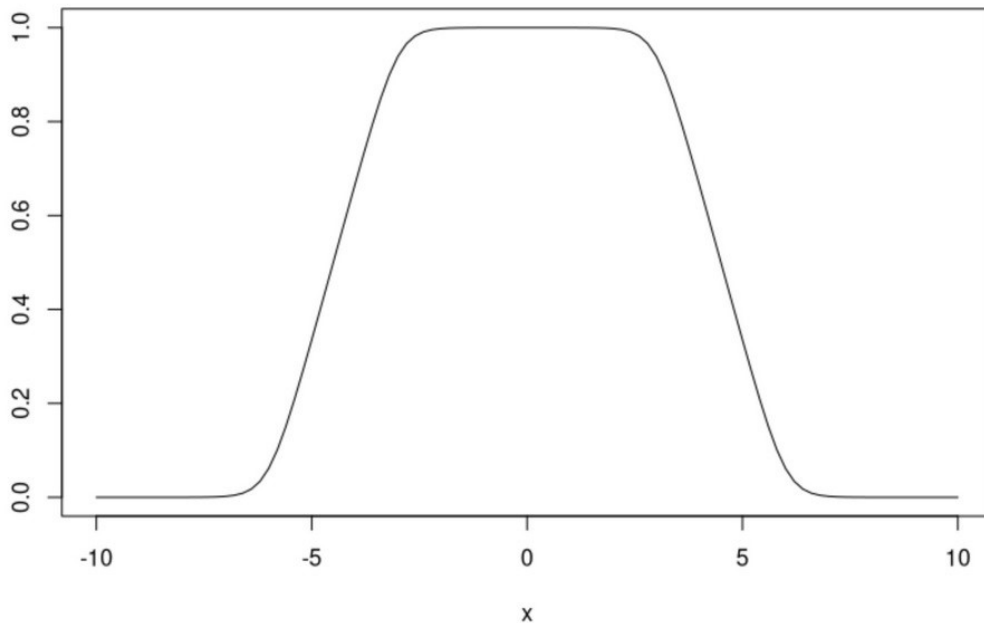
Consequently, a value might be larger than all values in the immediate neighborhood, and thus be a maximum, although it is smaller than values further away. In such a case there will be multiple maxima having different values. This is illustrated in Figure 3, which shows a function containing nine maxima, which are denoted by red dots. Notice that the second maximum from the left has a value of zero and is much smaller than other values of the function, even values that are not maxima. Nonetheless, because the value is larger than values nearby, it is a maximum.

Figure 3. A Mathematical Function with Nine Maxima²¹

It is convenient to divide the set of maxima into two sets, namely global maxima and local maxima. While a global maximum corresponds to the largest value among all values of the function, a local maximum is smaller than the global maximum but the largest value in the neighborhood of values. For example, in Figure 3, the second maximum from the left is a local maximum, while the third maximum is a global maximum. A function has at least one global maximum, and zero or more local maxima. As an example, the function that was displayed in Figure 2 has two global maxima, while it does not contain any local maximum.

The figures shown so far contain a finite number of maxima, but this is not always the case, as demonstrated by Figure 4. This figure displays a plateau and has an infinite number of maxima.

21. Image adapted from <https://brilliant.org/wiki/extrema/> [<https://perma.cc/6U4E-M5L5>].

Figure 4. A Mathematical Function with an Infinite Number of Maxima

For many mathematical functions, it is a non-trivial problem to locate all the points where that function reaches a local or global maximum.²² If the function can be easily plotted, then the maxima can be located by pure visual inspection, and thus the corresponding value can be easily determined, at least approximately. But for high dimensional functions, the problem becomes much more complex. Consider, for example, the four-dimensional Styblinski-Tang function, defined as:

$$f(x_1, x_2, x_3, x_4) = \frac{1}{2} \sum_{i=1}^4 (x_i^4 - 16x_i^2 + 5x_i)$$

This is one of the many functions used as benchmarks for optimization algorithms.²³ Because this function has four input variables and one output

22. See e.g., Robert E. Dorsey & Walter J. Mayer, *Genetic Algorithms for Estimation Problems with Multiple Optima, Nondifferentiability, and Other Irregular Features*, 13 J. BUS. & ECON. STAT. 53, 66 (1995) (examining the general algorithm method); Donald R. Jones, *A Taxonomy of Global Optimization Methods Based on Response Surfaces*, 21 J. GLOBAL OPTIMIZATION 345, 383 (2001) (describing classifications of global optimization methods); DAVID G. LUENBERGER, *OPTIMIZATION BY VECTOR SPACE METHODS* (1997) (analyzing different vector space methods in relation to optimization theory).

23. See generally Igor Grigoryev & Svetlana Mustafina, *Global Optimization of Functions of Several Variables Using Parallel Technologies*, 106 INT'L J. PURE & APPLIED MATHEMATICS 301 (2016) (testing benchmark optimization problems); Hannan A.R. Akkar & Firas R. Mahdi, *Evolutionary Algorithms*

variable, it lives in five-dimensional space and is impossible to plot. In this case, it is necessary to rely on sophisticated algorithms to find the maxima, such as the famous Levenberg-Marquardt algorithm.²⁴

B. Maxima in Judicial Decision-Making

Although perhaps not obvious at first sight, locating the maxima of a function is useful for conceptualizing the task of a judge ruling on a case. This is done by equating the solutions to a dispute with a certain mathematical function. Just as each function has a very specific behavior, and just as its graph differs from that of any other function, each dispute is unique in its own way. Disputes that slightly differ from each other—for example a case where person X violated the speed limit of fifty miles per hour by driving fifty-five miles per hour and a case where person Y violated the same speed limit by driving fifty-six miles per hour—would correspond to mathematical functions that are very similar, e.g., $f(x) = x^2$ and $g(x) = x^2 + 0.01$. We will refer to the function that corresponds to a case as the case-function.

The independent variable is assumed to take as values all possible solutions to the case, even resolutions that are not legally permissible, while the dependent variable corresponds to a measure of legal correctness. Only local maxima are considered legally permissible. This means that outcomes close to, but still different from, a local maximum, are, in a certain sense, close to a legally valid solution. For example, if in a certain dispute the law stipulates that compensation between 500 and 1000 euros may be awarded, all compensations ranging from 500 to 1000 euros will be maxima of the case-function. A compensation of 1001 euros will not correspond to a maximum, but it is literally close to the case function maximum corresponding to 1000 euros. Values that are gradually larger than 1000 euros correspond to gradually smaller values of the dependent variable.

Obviously, this conceptualization is merely a mental picture of a process that is very different in nature. Yet, the visualization of a case by a function is useful in many respects. In particular, each maximum of the case-function can be interpreted as a legally valid solution. The simple case above, where a person is using his cell phone while driving, might be associated with the function that was shown in Figure 1. This function has one maximum, corresponding to the single legally permissible judicial decision. More generally, a case for which the subset of applicable legal rules is unambiguous and complete corresponds to a function with a unique maximum. Cases to which ambiguous or incomplete rules apply

Performance Comparison for Optimizing Unimodal and Multimodal Test Functions, 5 INT'L J. SCI. & TECH. RES. 38, 45 (2016).

24. See generally Donald W. Marquardt, *An Algorithm for Least-Squares Estimation of Nonlinear Parameters*, 11 J. SOC'Y FOR INDUS. & APPLIED MATHEMATICS 431, 441 (1963) (developing the *maximum neighborhood* method and applying it to solving nonlinear algebraic equations); Kenneth Levenberg, *A Method for the Solution of Certain Nonlinear Problems in Least Squares*, 2 Q. APPLIED MATHEMATICS 164, 166 (1944) (explaining a solution to some non-linear problems in least squares).

may be associated with functions having multiple maxima. A global maximum then corresponds to a verdict that is legally best, while a local maximum is also legally valid but is, in some sense, inferior to a global maximum solution, perhaps because it relies on a minority opinion among the judiciary.

A function with a plateau, as is shown in Figure 4, is also useful to describe cases with quantifiable outcomes. For example, a plaintiff may be granted compensation due to a breach of contract by a defendant, but it is typically impossible to compute the exact loss of a plaintiff's future earnings owed as damages.²⁵ The case-function will then contain a plateau, where each value of the plateau corresponds to a specific amount of compensation that might be granted under the applicable legal rules. This plateau may be represented as an interval $[a, b]$ where a denotes the compensation that should at the very least be granted, while b represents the largest amount that can be granted in accordance with the given facts and the law. Every amount between a and b is legally valid. As another example, in a criminal procedure there might be a plateau representing the length of imprisonment accorded to a guilty verdict. There will also be a local maximum corresponding to the verdict of not guilty. If there is convincing evidence that the accused is guilty, the plateau will correspond to a larger value than the local maximum that corresponds to the non-guilty case. If, however, the standard of proof would not be met, the plateau will be located below the non-guilty local maximum. An interesting question is whether the height of the case-function could be used as a concrete measure of the standard of proof. This is, however, a question that is outside the scope of the current Article, since in this work the focus is on the local maxima. It is also possible for a case-function to contain multiple plateaus, some of which may contain global maxima, while others may correspond to local maxima.

It is reasonable to expect that case-functions with many maxima are prone to appeal, as judges will have selected a single maximum as the final decision, although they might have selected other maxima. An appellant is basically hoping that the court of appeal will select one of these other maxima. Furthermore, the value of the selected maximum will also influence the probability that one of the parties will lodge an appeal. If the judge has selected a local maximum with a very low value, such as the local maximum with value zero in Figure 2, parties will be more keen to lodge an appeal than in the case the judge selects a maximum with a much higher value. Even for case-functions where a global maximum has been selected as final decision, it is still quite possible that a party will lodge an appeal. This holds in particular for case-functions with multiple global maxima, such as case-functions with a plateau. In this case, the party that is lodging an appeal is hoping that the appellate court will select one of the other global maxima.

25. Elmer J. Schaefer, *Uncertainty and the Law of Damages*, 19 WM. & MARY L. REV. 719, 719 (1978).

IV

IMPROVING JUDICIAL DECISION-MAKING BY FINDING AND COMMUNICATING ALL MAXIMA OF THE CASE-FUNCTION

“Legal certainty represents the public interest in clear, equal and foreseeable rules of law,”²⁶ or, as we have called them, complete and unambiguous rules. Having established the relationship with mathematical function maximization, we can define legal uncertainty as the amount of global and local maxima of the case-function. In cases where the number of maxima is finite, legal uncertainty can simply be defined as the number of maxima. Things become more complicated if an infinite number of maxima are involved, especially if one wants to compare the degree of legal uncertainty of two case-functions having an infinite number of maxima. While there are many hypotheticals that make legal uncertainty even more complicated, we can suffice here by stating that legal uncertainty is related to the number of maxima of the case-function.

We next propose that judicial decision-making involves finding and communicating all maxima of the case-function, which would improve judicial decision-making in several respects.

A. Reduction of Inconsistent Verdicts on Similar Cases

First, inconsistencies among verdicts by different judges who have ruled on similar cases would be reduced. This might seem counterintuitive, as communicating multiple decisions can create confusion, but discussing all the potential maxima makes explicit what everyone already knows: that the legal system does not always provide a unique and unquestionable outcome for a given case.

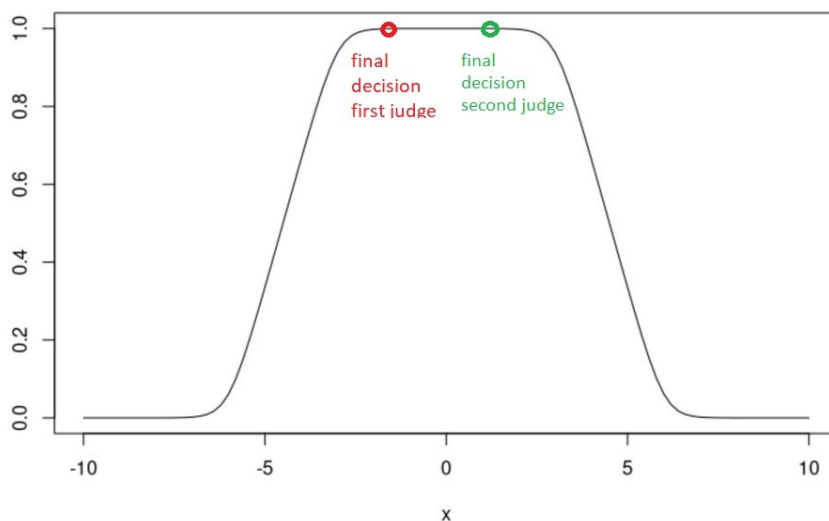
Now, suppose that Judge X and Judge Y rule on very similar cases. Both have determined all maxima, and both have selected what they consider the best maximum as the final decision. Of course, if one judge missed certain maxima, the best maximum for each judge, and thus their final decisions, may differ. This is an irregularity in legal ruling, since the public expects that similar cases should result in almost identical decisions. However, if many of the found maxima are the same, it holds that the judges’ perspectives on the set of legally permissible outcomes are very similar. In other words, the case-function they identified is similar, suggesting they have analyzed the case in a similar way. On the other hand, if only their different final decisions were communicated through the verdict, then one might conclude that their respective legal reasoning is different. That is to say, in the traditional view of judicial decision-making, the comparison between two verdicts amounts to comparing two outcomes, and they are either the same or different.

26. Magdalena Pfeiffer, *Legal Certainty and Predictability in International Succession Law*, 12 J. PRIV. INT’L L. 566, 566 (2016).

In our proposed view, there are a lot more variables to compare: All maxima that have been found by Judge X can be compared to all maxima found by Judge Y. The more maxima they have in common, the smaller the inconsistency between their verdicts. Ideally, their final decisions are the same, but inconsistency is still rather low if the final decision by the one judge corresponds to an alternative decision by the other judge.

This subtle difference between legal uncertainty on the one hand, and inconsistent or irregular verdicts on the other hand, is very apparent if the involved case-functions have a plateau. Even if two judges pick the same plateau, it is very possible that their final decision is different. Recall that the plateau consists of an infinite number of legally permissible solutions,²⁷ and thus it would not be remarkable if two judges picked a different number from the plateau as their final decision. Under our approach to judicial decision-making, however, as long as their final decision belongs to the same plateau, the two verdicts are perfectly consistent.

Figure 5. A Case-Function Where Two Judges Reach Different Decisions



B. Counteracting Biases in Judicial Decision-Making

As outlined previously, biases are highly prevalent in judicial decision-making. One plausible explanation is that people in general rely on heuristics in processing information. Heuristics are defined as “cognitive shortcuts, or rules of thumb, by which people generate judgments and make decisions without having to consider all the relevant information, relying instead on a limited set of cues

27. See Fig. 5.

that aid their decision making.”²⁸ Inevitably, judges start the analysis of a case from their own background experience and personality, and from their interpretation of the law and the given facts. This can be referred to as the initial state of that particular judge.

Once again, there is a connection between the initial state and finding maxima of a mathematical function. Many computer algorithms that have been developed to perform function maximization also start from an initial state. The initial state is a point of the function, often randomly chosen, that is used as the seed point by the algorithm, after which heuristic rules are applied to find points nearby the initial state that have a higher value. This procedure is repeatedly applied until the subsequent values do not grow in magnitude anymore. The final value is then a maximum. This is illustrated in Figure 6, which shows an initial state together with the direction of subsequent points of increasing value that are selected by the algorithm.

Figure 6. A Typical Search Algorithm’s Process from an Initial State in Finding a Maximum

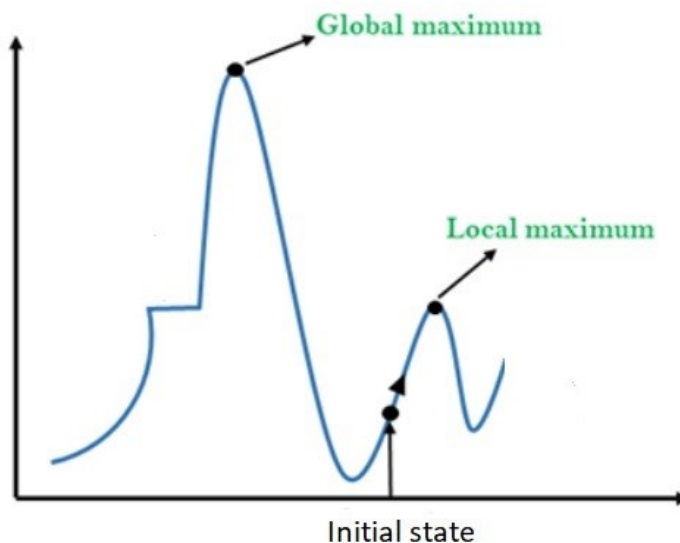


Figure 6 clearly shows that the algorithm will end in a local maximum. By applying the heuristic rules, the algorithm proceeds in the direction where the points have a higher value, and the global maximum is missed. To reach the global maximum, the algorithm would need to apply the counterintuitive approach of first passing through many points with a lower (that is, a worse)

28. Eyal Peer & Eyal Gamliel, *Heuristics and Biases in Judicial Decisions*, 49 CT. REV. 114, 114 (2013).

value. Judicial decision-making can be understood in the same way: starting from their initial state, judges apply the rules they consider best for the case at hand, to gradually arrive at a legally acceptable decision. This will often be a local maximum, just as in the algorithmic function maximization.

This idea is clearly reflected in the phenomenon of anchoring.²⁹ Anchoring occurs when judges are highly influenced by certain suggestions of how the case might be resolved. One typical example of a suggestion influencing a judge's reasoning is a plaintiffs' claim for a certain amount of damages.³⁰ Another well-known anchor is the initial plea offer made by the prosecutor.³¹ The anchor, together with legal experience and personal background, puts judges in a certain initial state, from which they start their search for a maximum of the involved case-function.

To avoid being trapped in a local maximum, the best strategy the judge can employ is the same one algorithms employ for function maximization. After having found a certain maximum from an initial state, a computer algorithm simply starts from another initial state, and the same heuristic rule is again applied to arrive at another maximum. This procedure can then be repeated a predefined number of times, resulting in a set of maxima. Some maxima might be equal to each other, particularly when the initial states are similar. Then, the maxima can be ranked according to their magnitude, with the largest value corresponding to the best maximum. Of course, it is possible the best maximum is still a local, rather than global, maximum, but the maximum will not be lower than the initial individual maximum.

The judge is, as a human being, less flexible than a computer algorithm that can be easily programmed to search for a maximum from multiple initial states. Yet even judges are in a position to vary their initial states by utilizing certain mechanisms, such as a what-if strategy. For example, judges may ponder "What if I were to adhere to an alternate interpretation of the law?" or "What if a more liberal judge than me were to consider this factual situation?" By trying different initial states, the judges might arrive at multiple solutions for the case at hand.

29. See generally Christoph K. Winter, *The Value of Behavioral Economics for EU Judicial Decision-Making*, 21 German L.J. 240, 264 (2020); Jungwook Kim & Subok Chae, *Anchoring Effect of the Prosecutor's Demand on Sentence: Evidence from Korean Sexual Crime Cases*, 39 KDI J. ECON. POL'Y 1, 18 (2017); Adrian Furnham & Hua Chu Boo, *A Literature Review of the Anchoring Effect*, 40 J. SOCIO-ECON. 35, 42 (2011); Francisca Fariña, Ramón Arce & Mercedes Novo, *Anchoring in Judicial Decision-Making*, 7 PSYCHOL. SPAIN 56, 65 (2003); Yun-chien Chang, Kong-pin Chen, Chang Lin & Yu-sheng Liu, *Anchoring Effect in Real Litigation: An Empirical Study*, SSRN ELEC. J. (2016).

30. Mollie W. Marti & Roselle L. Wissler, *Be Careful What You Ask For: The Effect of Anchors on Personal-Injury Damages Awards*, 6 J. EXPERIMENTAL PSYCHOL.: APPLIED 91, 94–95 (2000); see generally John Campbell, Bernard Chao, Christopher Robertson & David V. Yokum, *Countering the Plaintiff's Anchor: Jury Simulations to Evaluate Damages Arguments*, 101 IOWA L. REV. 543 (2016) (explaining that studies show that damage decisions by juries are strongly affected by the plaintiff's requested amount).

31. Colin Miller, *Anchors Away: Why the Anchoring Effect Suggests That Judges Should Be Able to Participate in Plea Discussions*, 54 B.C. L. REV. 1667, 1667 (2019); see also Y. Chang et al., *supra* note 29 (discussing the effect of anchoring on judicial decision making).

Perhaps they consider some of these solutions better than the one initially found. They may even attain a global maximum of the case-function that would have been overlooked if the search strategy were restricted to one initial state. Trying different initial states thereby circumvents the anchoring effect, and perhaps many other biases.

C. Influence on Legislation

If judges consistently find many maxima for a certain class of case-functions, the degree of incompleteness or ambiguity in certain areas of law would become more readily apparent. This would indicate a need for legislative intervention, for example by introducing additional or more unambiguous rules. Interestingly, the success of the legislative intervention is easily measured, at least in principle, by comparing the number of maxima that are found by judges after the legislative intervention with the number of maxima that predated the legislative intervention.

Thus, judicial decision-making using maxima has advantages for legislation. Although a verdict is primarily meant to settle a dispute between litigants, the impact of judicial decisions on other actors should not be underrated. Indeed, a judicial opinion that underlies a verdict is “simultaneously addressed to several, layered audiences: the parties to the case, potential litigants, and even politicians and the broader public.”³² Furthermore, as Professor Martin Shapiro expressed, “. . . opinions themselves, not who won or lost, are the crucial form of political behavior by the appellate courts, since it is the opinions which provide the constraining directions to the public and private decision-makers who determine 99 per cent of the conduct that never reaches the courts.”³³

D. Increasing Transparency

Communicating multiple legally permissible decisions may be akin to expressing uncertainty about how a dispute should be resolved. After all, if the alternative decisions are legally permissible, they may have been chosen as the final decision by the judge. Yet are explicitly expressed uncertainties truly undesirably? In the empirical sciences, it is tradition to express uncertainties about outputs generated by a model. In fact, it is a cornerstone of the scientific domain that results that are not solid and indisputable should also not be reported as such. Philosopher and mathematician Bertrand Russell expressed that “not to be absolutely certain is, I think, one of the essential things in rationality.”³⁴ Communicating uncertainties actually enhances the credibility in outcomes

32. Arthur Dyevre, *The Promise and Pitfall of Automated Text-Scaling Techniques for the Analysis of Jurisprudential Change*, ARTIFICIAL INTELLIGENCE & L., 1, 10 (2020).

33. THE SUPREME COURT AND ADMINISTRATIVE AGENCIES 39 (Martin Shapiro, ed., Free Press, 1968).

34. BERTRAND RUSSELL, AM I AN ATHEIST OR AN AGNOSTIC? A PLEA FOR TOLERANCE IN THE FACE OF NEW DOGMAS (1947).

produced by a model.³⁵

To be sure, reporting uncertainties in scientific results not only delivers additional information, but also useful information.³⁶ For example, research has shown that visualizing location uncertainty in GPS positioning is beneficial to users, as this helps them to better cope with situations arising from GPS degradation.³⁷ In the same way, credibility of the judicial decision-making process would be increased if courts communicated all legally permissible solutions, thereby expressing the uncertainty about the best legal outcome to the case.

Besides, if machine learning techniques that are applied to judicial decision-making are required to be transparent,³⁸ should this also not apply to the main actors in judicial decision-making? In this respect it is crucial to remark that even black boxes, considered to be the most unexplainable type of machine learning systems, are able to describe uncertainties about produced outcomes, for example by employing the bootstrap method.³⁹ Given this observation, it is hard

35. See Kwang-Il Ahn & Young-Ho Jin, *A Formal Approach for Quantitative Treatment of Modeling Uncertainties in Safety Analysis*, 116 NUCLEAR TECH., no. 2, 1996, at 146, 147 (describing the advantages of “explicit consideration of modeling uncertainty”).

36. See Joanna IntHout, John Ioannidis, Maroeska Rovers, Jelle Goeman, *Plea for Routinely Presenting Prediction Intervals in Meta-Analysis*, 6 BMJ OPEN, no. 2 at 1, 2 (2016), <http://dx.doi.org/10.1136/bmjopen-2015-010247> [<https://perma.cc/BDV4-P5QN>]. (“Therefore, summarising [sic] the findings of a meta-analysis in a single summary value sacrifices potentially informative variation”); L. H. Kaack, Jay Apt, Granger Morgan, Patrick McSharry, *Empirical Prediction Intervals Improve Energy Forecasting*, 114 PROC. NAT’L ACAD. SCI., no. 33, 2017, at 8752, 8752 (“Providing information on the likely uncertainty associated with such projections would help individuals and organizations use them in a more informed manner.”).

37. See S. Burigat & L. Chittaro, *Pedestrian Navigation with Degraded GPS Signal: Investigating the Effects of Visualizing Position Uncertainty*, PROC. 13TH INT’L CONF. HUM. COMPUT. INTERACTION MOBILE DEVICES & SERV., 221–230 (2011) (discussing a study on how visualizations of uncertainty assisted users of GPS technology); see generally Petr Aksenov, Kris Luyten, Karin Coninx, *O Brother, Where Art Thou Located?: Raising Awareness of Variability in Location Tracking for Users of Location-Based Pervasive Applications*, 6 J. LOCATION BASED SERV., 211–233 (2012) (for an overview of location uncertainty variation and its impact on users of tracking technology).

38. See R. Yu & G. Spina Ali, *What’s Inside the Black Box? AI Challenges for Lawyers and Researchers*, 19 LEGAL INFO. MGMT. 2, 6–7 (2019) (for a list of possible solutions to AI’s drawbacks, including heightened transparency); see M. Buiten, *Towards Intelligent Regulation of Artificial Intelligence*, 10 EUR. J. RISK REGUL., 41, 45 (2019) (“We may thus need new rules to mitigate the risks of AI. One proposal is to introduce transparency requirements for AI . . .”). But see S. Piano, *Ethical Principles in Machine Learning and Artificial Intelligence: Cases from the Field and Possible Ways Forward*, 7 HUMANITIES & SOC. SCI. COMM’N 2, 3 (2020) (considering some ethical issues with transparency, such as data privacy concerns); SARA GERKE, TIMO MINSEN, GLENN COHEN, *Ethical and Legal Challenges of Artificial Intelligence-Driven Healthcare*, in ARTIFICIAL INTELLIGENCE IN HEALTHCARE 295, 302–03 (Adam Bohr & Kaveh Memarzadeh eds., 2020) (discussing the ethical complications between transparency and privacy concerns). See generally S. Larsson and F. Heintz, *Transparency in Artificial Intelligence*, 9 INTERNET POL’Y REV., no. 2, May 2020 (for a deeper discussion of transparency in A.I.); Heike Felzmann, Eduard Villaronga, Christoph Lutz, *Transparency You Can Trust: Transparency Requirements for Artificial Intelligence Between Legal Norms and Contextual Concerns*, 6 BIG DATA & SOC’Y 1 (2019) (same).

39. See generally B. EFRON & R. J. TIBSHIRANI, AN INTRODUCTION TO THE BOOTSTRAP (1993). See Tomohiro Endo, Tomoaki Watanabe, Akio Yamamoto, *Confidence Interval Estimation by Bootstrap Method for Uncertainty Quantification Using Random Sampling Method*, 52 J. NUCLEAR SCI. & TECH.

to consider judges as transparent when they are often reluctant to be explicit about the uncertainties in analyzing a case.

E. Improving the Quality of the Adversarial Procedure

In an adversarial procedure, the parties are given the chance to present their arguments. In most proceedings this does not hinder the judge from deciding a case in a way that was not foreseen by the parties, such as issuing a decision grounded on legal rules that differ significantly from the ones put forward by the parties. The judge who describes all maxima of the case-function ensures a lively adversarial debate between the parties, since the parties also are urged to thoroughly discuss the case-function. In particular, parties will discuss the number of local and global maxima, the presence of plateaus, the function values that correspond to the maxima of the case-function, and so forth. Together with the judge, the parties will obtain a suitable case-function, resulting in the case being analyzed much more in depth compared to traditional judicial decision-making, where the parties and the judge restrict the discussion to a limited number of legally valid solutions.

V

JUDICIAL DECISION-MAKING IN A FICTITIOUS CASE

In this section we present a fictitious case study that applies the judicial decision-making method outlined in this Article.

A. Factual Situation

Mr. Jones runs a moving company. Recently, he entered into a contract with the Simons family to move all their belongings from their current residence to their new villa. The contract stipulates that all possessions will be moved in three days, at a price of \$1500. There is a damages clause in case he fails to achieve the three-day goal, consisting of a lump sum of \$800. Furthermore, it is agreed that Mr. Jones will start performing the contract on October 26 at 10:30 a.m.

On October 26 at 10:45 a.m., Mr. Jones is present at the old residence. A friend accompanies him, since his plan is to first move the very heavy items. The move continues for the next two days. On October 29 at 9:00 a.m., Mr. Jones arrives at the old residence to pick up some final small things belonging to the Simons family. He is very surprised to see some people waiting outside the house whom he has never seen before. They claim to be the new owners, stating that they signed a contract with the Simons family that stipulates that they are allowed

993, 998 (2015) (“[I]t is expected that the bootstrap method is applicable to estimate the confidence intervals of uncertainties without complicated mathematical procedures in the case of non-normal distribution. Furthermore, the bootstrap method has capability to estimate confidence intervals for other statistical estimates.”). See generally P. Kirk & M. Stumpf, *Gaussian Process Regression Bootstrapping: Exploring the Effects of Uncertainty in Time Course Data*, 25 *BIOINFORMATICS* 1300 (2009) (for an explanation of the bootstrap method’s ability to account for uncertainty, applied here to biology).

to move in on October 29 from 8.00 a.m. Mr. Jones quickly collects all the remaining items, and hurries to the Simons' new residence.

Both Mr. Jones and the Simons claim not to have made any mistake regarding the unfortunate state of affairs under the contract. While the Simons argue that Mr. Jones should have finished the move by October 28, since October 26-28 constitutes a three-day period, Mr. Jones replies that one day consists of 24 hours, so that he had until October 29 at 10:30 a.m. to move everything.

B. Judicial Procedure

A court procedure is subsequently initiated. After hearing the arguments of the parties, the judge takes the matter into consideration. The law stipulates that in a contract between parties, the intention of the parties dominates, but if their intention is unclear, the literal meaning of words is decisive. As the intention of the parties is determined to be unknown, the judge relies on the meaning of the words. According to a widely accepted dictionary, "day" refers to the time between sunrise and sunset, according to which the Simons are right that the move should have finished on October 28. However, the same dictionary offers as alternative description of "day" as "a division of time equal to 24 hours," in which case Mr. Jones finished his task on time. Certain case law states that if contractual matters remain unclear after literal interpretation, the clause should be interpreted against the professional, at least in situations where the contract is concluded between a professional and a non-professional. This case law is criticized, since there is no legal basis to apply such a rule. However, lacking any other criterion to decide on this point of contention, the final decision of the judge is that Mr. Jones should have finished his contractual obligations on October 28.

According to the damage clause, he is obliged to pay \$800 to the Simons. However, the law stipulates that the judge can reduce a damage clause if the specified amount is considered out of proportion to the actual damage. The judge considers \$800 too high, given that the total cost of the move was \$1500 and that Mr. Jones finished his job the following morning. He reduces the damage clause to an amount of \$200, although he is convinced that any amount from \$100 to \$250 would also have been reasonable.

C. Verdict

The judge applies the new method of judicial decision-making, so he considers and communicates all legally permissible outcomes. His verdict, taking into account the considerations of the previous subsection, is summarized as follows:

Concerning the performance of the contract:

Final decision: Mr. Jones failed to perform the contract on time, since the contract stipulated to finish the task in three days, and Mr. Jones did not do so.

Alternate decision: Mr. Jones performed the contract on time based on an alternative definition of "day."

Complete set of decisions: YES/NO concerning the timely execution of the contract.

Concerning the damage clause:

Final decision: Mr. Jones has to pay damages, but the amount of \$800 in the contract is reduced to \$200.

Alternate decision 1: Mr. Jones has to pay damages for any other amount between \$100 and \$250.

Alternate decision 2: Mr. Jones does not have to pay any damages, if the alternative interpretation of “a day” is accepted.

Complete set of decisions: \$0 or [\$100, \$250] has to be paid by Mr. Jones.

It is clear that this verdict is much more informative than a traditional judgment in which only one final decision is communicated. The alternate decisions presented together with the final decision provide an extensive legal analysis of the case at hand. Even though only the final decision is relevant for enforcement of the judgment, the comprehensive court decision ensures that the judge has considered all legally permissible rules. Not only did he consider all rules that are legally acceptable, but he has also applied these rules to assess alternate outcomes.

VI

CONCLUSION

Legal practitioners tend to consider machine learning black boxes as synonymous with models that provide only a single produced outcome. Yet, such models can report an optimal solution with a set of alternative solutions that are also plausible, like the set of all reasonable predictions for a given forecast problem.

Judicial decision makers operate in a remarkably similar setting. The factual situation is frequently vaguely described by the parties, while the legal system is ambiguous and incomplete. The result is that there is typically no clear legal outcome to similar cases. If judges are expected to be at least as transparent as black boxes, they should communicate all reasonable legally valid solutions. In terms of the relationship with mathematical function maximization judges should identify and consider all maxima, whether local or global, of the case-function.

Although our Article has focused on judicial decision-making, there is another particular application where our conceptualization of finding legal solutions to a given case is of main relevance. Lawyers giving legal advice might benefit from our proposed method of constructing legally valid solutions. Lawyers might even use our conceptual underpinnings to visualize the possible legal outcomes for a case described by a client. More concretely, clients might be presented a function with global and local maxima, such that they can easily grasp the complexity of the case, the number of legally admissible solutions, and the

legal validity of solutions represented by the values of the corresponding maxima. In short, lawyers have completely and thoroughly analyzed a case when they have constructed the corresponding case-functions.

As a critique against our new judicial decision-making method, if the global maximum of a case-function is a plateau, the judge may pick any of the points lying on the plateau as the legally optimal decision. It might be useful to improve our conceptualization by adding other techniques that may be used to select points on a plateau. For example, one method is to describe optimal decision in terms of minimizing costs and maximizing benefits.⁴⁰ A prison sentence might be considered in terms of balancing costs and benefits, where costs related to adverse effects on the defendants and his family, while benefits relate to ensuring that the defendant cannot commit a crime as long as he is in jail. Specifying the costs and benefits allows the judges to determine a unique optimal solution. While our method might be used to identify a plateau as the legally best verdict (containing a range of lengths of imprisonment), other optimization methods could be used to subsequently select a unique length of imprisonment in terms of predefined costs and benefits.

Another critique is that it might result in an increase in the workload of judges, which is especially alarming given that courts around the world often face budgetary constraints and barely have time to thoroughly construe a single legal outcome, let alone a range of decisions. Although there is no doubt that our proposed method of judicial decision-making will increase the workload of judges at first, it is not obvious that the total time to handle a case, including the time to decide an appeal, will be increased. Since the judges will have constructed the case-function at the beginning of the case, thereby thoroughly describing all legally valid decisions, the court of appeal will strongly benefit from the additional work performed by the initial judges. The additional workload for judges is also compensated by all the other benefits of this new method of judicial decision-making.⁴¹

Finally, the methodology described in this Article might equally be considered a premature tool that will turn out to be of important use only in a future era, when artificial intelligence will have a major impact in the legal context. The time to construct the set of legally valid decisions will then be reduced dramatically, and the case-function might be considered a transparent and visually attractive concept by litigants, produced in no time by sophisticated artificial intelligence tools.

40. See generally H. Lakkaraju & C. Rudin, *Learning Cost-Effective and Interpretable Treatment Regimes*, PROC. 20TH INT'L CONF. A.I. & STAT., 166 (2017) (for a proposal on how to automate decisions that are made based on weighing costs and benefits).

41. See *supra* Part I.