A computational study about the effects of ply cracking and delamination on the stiffness reduction of damaged lamina and laminate

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Abstract

Accurate prediction of stiffness degradation in the damaged plies (laminae) is a fundamental requirement for developing damage models for composite materials. In this study, a mesoscale analysis is proposed to predict all the effective thermo-elastic constants of damaged plies containing ply cracks and delaminations based on a numerical homogenization method. To do so, the in-plane and out-of-plane loading conditions are imposed on the three-dimensional representative volume elements (RVEs) using the periodic boundary conditions (PBCs). Considering the stress/strain fields obtained from the numerical simulations, the effective behavior of the damaged plies is evaluated. Moreover, the effects of various damage configurations, material properties, orientations of adjacent plies and ply thicknesses are studied to address the dependency of the effective elastic constants to those parameters. Numerical results reveal that the ply cracking and delaminations significantly reduce the in-plane and out-of-plane properties (E_{22}, E_{33}, G_{12}, G_{13}, G_{23} and ν_{23}) of the damaged plies. Furthermore, to demonstrate the accuracy and capability of the developed homogenization method, the homogenized properties of the damaged plies are used in the intact laminates where the stiffness of such laminates are compared with direct FE simulation and available experimental data of the laminates containing ply cracks and local delamination.

Keywords: Ply cracking, Local delamination, Stiffness reduction, Homogenization, Periodic boundary conditions (PBCs), Out-of-plane behavior

1. Introduction

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The interesting mechanical properties of composite laminates have led to the growing demands for these materials in various industrial applications (Galos, 2020). However, composite materials demonstrate complex damage behaviors under static and fatigue loading conditions (Van Paepegem et al., 2001; Kalteremidou et al., 2020). These damage mechanisms are mainly known as fiber-matrix debonding, ply cracking, delamination and fiber breakage (Zhuang et al., 2018; Onodera and Okabe, 2020; Sharma and Daggumati, 2020; Hajikazemi et al., 2015; Nairn, 2000). Ply cracking is the first ply-level damage mode that causes a notable stiffness degradation in composite laminates and can initiate other damages such as delamination and fiber breakage (Loukil and Varna, 2019; Hajikazemi et al., 2020b; Hajikazemi and McCartney, 2018). On the other hand, the delamination mechanism is considered as a critical scenario that remarkably influences the thermo-mechanical response of laminates, sometimes leading to a catastrophic failure in composite structures. Delamination is mainly generated from the edge-effect as well as the ply cracks and highly affects the out-of-plane performance of composite structures (Zubillaga et al., 2015). Therefore, it is essential to investigate the mechanical response of composite laminates including the mentioned damage modes (ply cracking and delamination) to develop accurate models for predicting the performance of composite structures.

The effects of ply cracking and local delamination on the in-plane axial properties of symmetric laminates have been extensively investigated experimentally (Carraro et al., 2019; Berglund et al., 1992; Ogihara and Takeda, 1995; Wang and Karihaloo, 1997; Shahkhosravi et al., 2019; Kahla et al., 2020). However, experimental approaches involve time-consuming and expensive processes. Moreover, it is very challenging to experimentally measure the out-of-plane properties of damaged laminates. As an alternative, the finite element method is a powerful technique for calculating the thermo-elastic constants of laminates and it is capable of analyzing a variety of possible damage scenarios under different loading conditions. This popular approach has been widely employed by different researchers to predict the stiffness degradation of cracked composite laminates (Noh and Whitcomb, 2001; Li et al., 2009; Akula and Garnich, 2012; Barbero et al., 2013; Barulich et al., 2018; Montesano et al., 2018; Ahmadi et al., 2020; Loukil et al., 2013b). In this regard, Adolfsson and Gudmundson (1995) presented the influence of ply cracking on the in-plane and out-of-plane mechanical properties of composite laminates. Recently, Garoz et al. (2020) studied the stiffness reduction and the stress distribution of cracked composite laminates under out-of-plane loading conditions by considering the relaxed 3D periodic boundary conditions (PBCs). Moreover, implementing the second-order homogenization method, the flexural behavior of cracked composite laminates was investigated by Dinh et al. (2019). In addition to the FE studies, different analytical investigations indicate the impacts of damages on the out-of-plane and flexural behavior of composite laminates (Talreja, 1985; Hajikazemi et al., 2020a; Li et al., 2019; McCartney, 2000; Pupurs et al., 2016).

Only a limited number of studies have been focused on the stiffness degradation of composite laminates in the presence of combined ply cracking and local delamination using finite element modeling. To this end, the effective mechanical properties and energy release rates of composite laminates including ply cracking and delamination were analyzed by Maragoni et al. (2018). They demonstrated the effects of ply cracking and local delamination on the in-plane stiffness degradation of a $[0/45/-45]_s$ laminate. In another study, Östlund and Gudmundson (1992) calculated the stiffness reduction and energy release rates of a $[55/-55]_s$ laminate in the presence of ply cracking and local delamination by considering two types of idealized local delamination. On the other hand, many analytical studies reported the effects of ply cracking and local delamination on the mechanical behavior of damaged laminates (Kashtalyan and Soutis, 2000; Dharani et al., 2003; Kashtalyan and Soutis, 2002; Zhang et al., 1999; Maimı´ et al., 2011; Berthelot and Le Corre, 2000; Nairn and Hu, 1992; Carraro et al., 2021). In this regard, Farrokhabadi et al. (2011) proposed a generalized micromechanical approach to predict the stress distribution, displacement fields and the energy release rate in the damaged laminates containing ply cracks and local delamination. They employed finite element modeling to verify their analytical solution. Also, Pakdel and Mohammadi (2019) used a unit cell based stress analysis to extract the stiffness reductions of damaged laminates under uniaxial tension-tension fatigue loading conditions where the experimental observation was carried out to demonstrate the accuracy of their results for longitudinal stiffness degradation. Moreover, Hosseini-Toudeshky et al. (2012) linked the micromechanical approach to the meso-scale analyses of damaged laminates having ply cracks and delamination by reducing the in-plane effective elastic properties (Young's modulus E_{22} and shear modulus G_{12}) of the damaged lamina. Additionally, various ply-level continuum damage models have been proposed based on energy concepts to estimate the mechanical behavior of damaged composite laminates using damage state variables, affecting different elastic constants (Ladevèze et al., 2017; Ladeveze and LeDantec, 1992; Burgarella et al., 2019; Ladevèze and Lubineau, 2001). According to these methods, three damage indicators ($\bar{d}_{22}, \bar{d}_{12}$ and $\bar{d}_{23}$) are required to consider the effects of ply cracks in three dimensional models where these parameters can be identified using experimental and numerical tests (Ladevèze et al., 2017). Also, in order to approximately consider the effects of delamination, interlaminar interfaces with three damage variables (d_1, d_2 and d_3) are needed, which are associated with the different delamination modes (Ladeveze, 1995; Abisset et al., 2011). In another study, Ladevèze et al. (2016) proposed a link between micro- and meso-mechanics to investigate out-of-plane normal behavior of laminates including ply cracks and local delamination based on energy concepts. In this regard, a potential energy was defined for homogenized damaged plies (including ply cracks and local delamination) using five variables ($\bar{I}_{22}, \bar{I}_{12}, \bar{I}_{23}, \bar{I}_{13}, \bar{I}_{33}$), where these parameters can be obtained via integration of the strain energy and linked to the variables of continuum damage mechanics ($\bar{D}_{22}, \bar{D}_{12}, \bar{D}_{23}, \bar{D}_{13}, \bar{D}_{33}$). However, while evaluating the potential energy, they have considered only the diagonal terms by ignoring the effects of Poisson's ratios (both the in-plane and out-of-plane terms) as well as coupling terms (important for unbalanced laminates). This means that even for simple cross-ply laminates (where coupling terms are zero) the degradation of Poisson's ratio's was not considered. As we will show later, ply cracking and delamination can largely degrade one of the Poisson's ratio's (υ_{23}). Moreover, neglecting the effects of Poisson's ratios and coupling terms in calculation of the strain energy and consequently the homogenized ply properties will make considerable errors in calculation of diagonal terms as well. It is crucial to sufficiently and accurately degrade (in the presences of different damage modes) all ply properties to be able to deal with multiaxial loading conditions. Moreover, it should be noted that the mentioned study (Ladevèze et al., 2006) mainly highlights the damage variables and strain energy based on micromechanical models where the cracking density ratio's and delamination ratio's (normalized by thickness of damaged ply) are limited to 0.7 and 0.4, respectively. This is mainly due to the fact that in the Ladevèze model (Ladevèze et al., 2006), the damage parameters will be later calibrated from several experiments and there is no attempt to directly provide the values of degraded material properties based on modeling. Another assumption of the mentioned approach is that the thickness of adjacent plies should be equal or thicker than the homogenized plies. However, it is possible to consider any thickness for homogenization of damaged plies in finite element modeling.

Due to the high computational cost of damage modeling in finite element simulation, the homogenized behavior of damaged plies can be used as an alternative approach for structural level analyses of damages within affordable computational cost and acceptable accuracy. Also, understanding the mechanical response of a damaged ply including ply cracking and delamination helps to investigate the stiffness reduction of composite laminates including damages in multiple plies with different ply orientations without the requirement for complicated damage modeling in FE software. Most of the available FEA studies only reported the total stiffness degradation of the damaged laminates and the stiffness degradation of damaged plies are rarely reported in the literature. In this regard, Noh and Whitcomb (2005) studied the in-plane (mainly the in-plane transverse Young's modulus E_{22}) stiffness degradation of damaged plies including ply cracks and delamination for different configurations and suggested a simple analytical formula to estimate the stiffness reduction of I-shape configuration of ply cracks and delamination.

The main aim of this study is to calculate all the effective thermo-elastic constants and the stiffness reduction of damaged plies and laminates including ply cracks and delamination under in-plane and out-of-plane loading conditions based on a fully numerical procedure (three-dimensional finite element simulation) and providing benchmark solutions for validity of future analytical models. This approach can also be used directly to develop anisotropic physic-based damage models based on finite element analyses. For this purpose, the average stress and strain response of damaged plies and laminates are calculated using numerical homogenization approaches. According to the results, it is clear that six elastic constants of damaged plies should be degraded in the presence of ply cracks and delaminations (E_{22}, E_{33}, G_{12}, G_{13}, G_{23} and ν_{23}).

The main focus of this study can be summarized as follows:

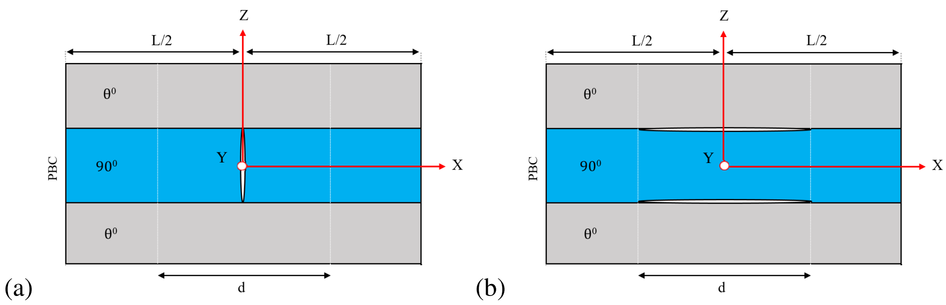
- Determination of all the effective ply properties that should be changed due to the presence of ply cracks and local delamination based on finite element modeling.
- Consideration of both the in-plane and out-of-plane stiffness reductions in a damaged lamina and laminate.
- Investigation of the influence of material properties and damage configurations on the homogenized mechanical behavior of damaged plies including ply cracks and local delamination.
- Evaluating the effects of thickness and orientations of adjacent plies on the stiffness reduction of damaged plies.
- Implementation of a numerical homogenization scheme for damaged lamina containing ply cracks and local delamination.
- Consideration of local delamination damages in the three-dimensional finite element model without the requirement for a cohesive zone.

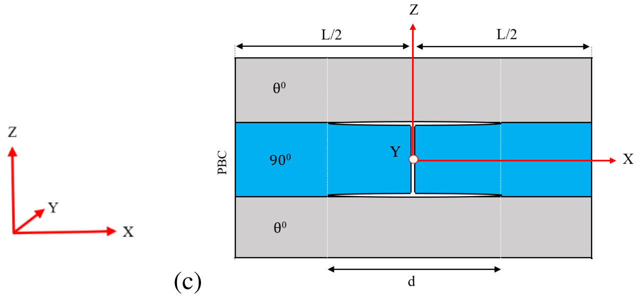
This article has been structured as follows: Section 2 provides details of the finite element model such as the periodic boundary conditions, homogenization method, specified material properties and selected RVEs. Section 3 presents the numerical results for damaged plies and their affecting parameters. The stiffness reduction of damaged laminates and the validity of results are provided in section 4 by comparing the results with direct FE simulations and available experimental data (Carraro et al., 2019). In the end, some concluding remarks are proposed.

2. Methodology

A three-dimensional finite element simulation is performed to study the thermo-mechanical behavior of symmetric composite laminates containing ply cracking and local delamination subjected to multiaxial loading conditions. To do so, the representative volume elements (RVE) have been employed to obtain the mechanical characteristics of damaged composite laminates with affordable computational time.

Three damage scenarios have been considered to study the influences of ply cracking and delamination on the stiffness reduction of composite plies and laminates. Fig. 1 indicates the schematic configuration of the mentioned scenarios which are assigned to the middle ply of a $[\theta/90_{0.5}]_S$ laminate. As illustrated in this figure, the reference coordinate system is placed in the center of the RVE where the x and y directions demonstrate, respectively, the in-plane axial and transverse axes of the RVEs. The composite laminates are symmetric with respect to the x-y plane passing through the center of the RVEs (mid-plane). Moreover, the angle θ_i specifies the orientation of the i_{th} ply, measured counter clockwise between the x-axis and the fiber direction of the ply. The ply thickness is defined by t_i for each ply. It is assumed that the uniformly distributed ply cracks initiate parallel to the direction of the fibers and span the entire thickness and depth of the damaged plies. Furthermore, the crack density is expressed as $\rho = 1/L$, where L refers to the axial length of the RVEs. On the other hand, the delamination mechanisms are assumed to be propagated at the interface of cracked plies and fully span the RVEs in the y-direction. It should be noted that the second scenario (Fig. 1b) may not happen in reality and it is presented herein to solely investigate the influence of delamination on stiffness degradation of damaged composite ply when there are no ply cracks. The dimension of the model in y direction is selected as 0.2 mm and due to the consideration of PBCs, the chosen value will not have effects on the obtained results.





(c)

Figure 1. Schematic demonstration of different idealized damage scenarios in a $[\theta/90_{0.5}]_S$ laminate; (a) ply cracking, (b) delamination and (c) ply cracks and delamination.

2.1. Material properties and mesh details

In this research, the linear elastic behavior has been utilized to describe the mechanical characteristics of unidirectional plies in the mesoscale framework based on small deformation assumption. Table 1 gives the values of the selected material parameters with transversely isotropic property. Moreover, employing full integration elements, the 8-noded brick elements have been used to mesh the RVEs in all simulations. Fig. 2 demonstrates a sample of the defined mesh utilized in this study. As indicated, a mesh refinement strategy is performed for the RVEs to properly capture the stress fields in the damaged zones. Furthermore, a mesh convergence study was carried out before calculating the final results of the damaged RVEs to ensure the accuracy of the computed results. It should be mentioned that in order to model the ply crack and delamination (Fig. 1 (a and b)), the seam crack (Ahmadi et al., 2020; Hajikazemi et al., 2020a) option in Abaqus (Manual, 2020) is utilized. For the RVEs with both ply crack and delamination (Fig. 1(c)) the seam cracks are used for the delamination damages and the ply cracks are modeled using an infinity small cut in the RVE. It should be noted that the use of this strategy will make it possible to define both damages inside the model. Moreover, it is assumed that damages are contact-free for all the finite element simulations. Assuming contact-free crack surfaces might not be realistic for some loading conditions and lay-ups, however, it leads to an overestimation of stiffness reduction which is more conservative and thus, more appropriate for design purposes.

Table 1. Mechanical properties of undamaged composite plies.

Sets	***M-1***	***M-2***	***M-3***
Material	*GFRP* (Smith and Ogin, 1999)	*CFRP* (Siulie and Nairn, 1992)	*GFRP* (Maragoni et al., 2017)
$\boldsymbol{E_{11}(GPa)}$	*45.6*	*134*	*48.830*
$\boldsymbol{E_{22}(GPa)}$	*16.2*	*9.8*	*14.07*
$\boldsymbol{G_{12}(GPa)}$	*5.83*	*5.5*	*5.2*
$\boldsymbol{\nu_{12}}$	*0.278*	*0.3*	*0.308*
$\boldsymbol{\nu_{23}}$	*0.4*	*0.361*	*0.4*
$\boldsymbol{a_{11}(1/°C)}$	8.6×10^{-6}		
$\boldsymbol{a_{22}(1/°C)}$	26.4×10^{-6}		

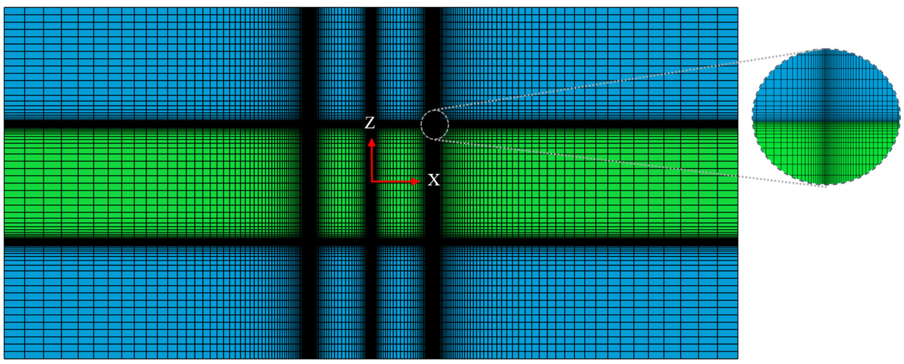


Figure 2. Finite element mesh details of composite laminate including ply cracks and local delamination.

2.2 Periodic Boundary conditions

The use of periodic boundary conditions (PBCs) while calculating the mechanical response of damaged composites decreases the computational costs of numerical analysis when the damage patterns are periodic (Jahanshahi et al., 2020; Garoz et al., 2019; Tian et al., 2019). Based on this approach, it is possible to model a large system using data from a much smaller part which is called the representative volume element (RVE). This method is used to correlate the macroscopic displacement fields of counterpart nodes in the parallel boundaries. Thus, the node-to-node PBCs can be governed by the following equation:

$$u_j^{i+} - u_j^{i-} = \varepsilon_{ij} l_i \; with \; (i, j = x, y, z), \tag{1}$$

where u refers to the nodal displacement in the opposite surfaces and it is related to the macroscopic strain ε of the RVE. Moreover, l_i denotes the RVE length in i direction. To impose different in-plane and out-of-plane loading conditions, six degrees of freedom have been defined in accordance with the reference points. It should be noted that the *EQUATION command can be used in Abaqus finite element software to implement the linear constraint equations of PBCs. For further description of the PBCs, the reader is referred to (Nguyen et al., 2012; Garoz et al., 2019). The RVE under consideration involves both ply cracks and delamination where a refined FEM meshing is implemented to consider the interaction between these damage modes. By applying periodic boundary conditions (PBCs) at the boundaries of the RVE, the interactions between neighboring ply cracks and delaminations are also taken into account.

2.3 Homogenization methodology

When the ply-level damage appears in the composite laminates, it leads to the degradation of effective properties in both the damaged ply and the whole laminate. For this reason, different homogenization approaches have been applied to extract the thermo-mechanical behavior of the damaged ply and laminate.

The average stress tensor $\bar{\sigma}_{ij}$ at the macroscopic scale can be defined in the following form:

$$\bar{\sigma}_{ij} = \frac{1}{V}\int_V \sigma_{ij} dV \tag{2}$$

where V represents the volume of the selected ply/RVE.

Also, it is possible to determine the average strain tensor $\bar{\epsilon}_{ij}$ over the volume by the following expression:

$$\bar{\epsilon}_{ij} = \frac{1}{V}\int_V \varepsilon_{ij} dV \tag{3}$$

However, Eq. 3 can only be used for damage-free models and it is not appropriate for RVEs including ply cracks and local delamination.

The linear relation between the macro-stress and macro-strain forms the basis for computing the homogenized compliance matrix S_{ij} of damaged lamina/laminate as follows:

$$\varepsilon_i = S_{ij}\sigma_j + \alpha_i \Delta T \tag{4}$$

where α_i and ΔT are the thermal expansion coefficients and the temperature difference parameter, respectively.

It is necessary to individually apply six unit strain loading cases with a single nonzero component on the RVEs to calculate the coefficients of the compliance tensor. In doing so, three load cases are considered for uniaxial strains in x,y and z directions and three other load cases for shear strains in xy, xz and yz planes. The strains have unit value for all the loading conditions and thus the resulting stresses correspond to the different columns of the laminate compliance matrix. It should be noted that it is possible to extract the average stress components (six values based on Voigt notation) for each strain loading condition and, therefore, 36 components for the average strain and stress for all the loading conditions. Due to the fact that the laminate stiffness/compliance matrix is symmetric, less loading cases are required to obtain all members of the matrix. Having all the components of the average stress and strain tensors from the applied loads (six individual cases), it is possible to evaluate all the components of the compliance tensor (36 components) and calculate the elastic constants.

2.3.1 Homogenization of damaged ply

In order to obtain the effective properties of a damaged ply, it is essential to evaluate the stress-strain response of that ply under different loading conditions. To do so, the average stress tensor of the damaged ply can be determined by implementing Eq. 2 into the finite element solver. In this case, the components of the macro-stress tensor are calculated by the average of local stresses in the integration points of the damaged ply. On the other hand, to extract the macro-strain tensor of the damaged ply, the nodal displacements of the boundaries have been used to estimate the average macroscopic strain from the displacement fields. Thus, the homogenized strain components of the damaged ply can be explicitly written based on displacements as follow:

$$\bar{\epsilon}_{ij} = \frac{1}{2V}\int_S (u_i n_j + u_j n_i) dS \tag{5}$$

where u and n denote, respectively, the displacement vector and the unit normal vector over the damaged ply. Also, V is the volume of damaged ply and S is the selected surface for calculating the average strain components, respectively. Using a Python script, it is possible to implement such equations in the FE solver (Abaqus) to extract the displacement fields in the specified surfaces of the damaged ply. Having the macroscopic stress-strain data, the homogenized properties of the damaged ply are determined based on Hooke's law (Eq. 4). The thermo-elastic constants related to the equivalent properties of the damaged ply can be presented as follow:

$$
\left[S_{ij}\right]=\begin{bmatrix}
\frac{1}{E_{11}} & -\frac{\nu_{12}}{E_{11}} & -\frac{\nu_{13}}{E_{11}} & 0 & 0 & 0 \\
-\frac{\nu_{12}}{E_{11}} & \frac{1}{\boldsymbol{E_{22}}} & -\frac{\boldsymbol{\nu_{23}}}{\boldsymbol{E_{22}}} & 0 & 0 & 0 \\
-\frac{\nu_{13}}{E_{11}} & -\frac{\boldsymbol{\nu_{23}}}{\boldsymbol{E_{22}}} & \frac{1}{\boldsymbol{E_{33}}} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{\boldsymbol{G_{23}}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{\boldsymbol{G_{13}}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{\boldsymbol{G_{12}}}
\end{bmatrix} \tag{6}
$$

$$
[\alpha_i]=\begin{bmatrix}\alpha_{11} \\ \alpha_{22} \\ \alpha_{33} \\ 0 \\ 0 \\ 0\end{bmatrix} \tag{7}
$$

where the Young's moduli (E), shear moduli (G), Poisson's ratio (ν) and thermal expansion coefficients (α) have been introduced in the local coordinate system. The 1-direction is defined parallel to the fibers, while directions 2 and 3 are considered to be transverse in the plane and perpendicular to the plane, respectively.

It is assumed that the local delamination scenario reduces the effective mechanical properties of the ply containing ply cracks (Fig. 1c), because this type of damage initiates due to the propagation of ply cracks to the interface with neighboring plies.

2.3.2 Homogenization of damaged laminate

Although it is possible to implement the first homogenization approach for whole RVEs, computationally it is more realistic to use the displacement/force fields of reference nodes for calculating the effective properties of the damaged laminate based on the kinematics of PBCs (Garoz et al., 2020). Considering the macroscopic stress-strain results which are obtained based on the reference node data, the elastic constants of damaged RVEs are defined as:

$$
\left[S_{ij} \right] = \begin{bmatrix}
\frac{1}{E_{xx}} & -\frac{\nu_{xy}}{E_{xx}} & -\frac{\nu_{xz}}{E_{xx}} & 0 & 0 & -\frac{\lambda_x}{E_{xx}} \\
-\frac{\nu_{xy}}{E_{xx}} & \frac{1}{E_{yy}} & -\frac{\nu_{yz}}{E_{yy}} & 0 & 0 & -\frac{\lambda_y}{E_{xx}} \\
-\frac{\nu_{xz}}{E_{xx}} & -\frac{\nu_{yz}}{E_{yy}} & \frac{1}{E_{zz}} & 0 & 0 & -\frac{\lambda_z}{E_{xx}} \\
0 & 0 & 0 & \frac{1}{G_{yz}} & -\frac{\lambda_s}{G_{xz}} & 0 \\
0 & 0 & 0 & -\frac{\lambda_s}{G_{xz}} & \frac{1}{G_{xz}} & 0 \\
-\frac{\lambda_x}{E_{xx}} & -\frac{\lambda_y}{E_{xx}} & -\frac{\lambda_z}{E_{xx}} & 0 & 0 & \frac{1}{G_{xy}}
\end{bmatrix} \tag{8}
$$

$$
[\alpha_i] = \begin{bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ \alpha_{zz} \\ 0 \\ 0 \\ \alpha_{xy} \end{bmatrix} \tag{9}
$$

where the Young's moduli (E), shear moduli (G), Poisson's ratio (ν), coupling terms (λ) and thermal expansion coefficients (α) have been introduced in the global coordinate system.

3. Stiffness reduction of damaged lamina

The homogenized behavior of damaged plies can be affected by different factors such as damage configurations, crack density, delamination length, ply thickness, orientations of adjacent plies and initial material properties. Using the developed finite element strategy, it is possible to obtain the effective elastic constants of damaged plies under in-plane and out-of-plane loading conditions.

In order to reveal the effects of various damage scenarios on the stiffness reduction of the damaged ply, several $[0/90_{0.5}]_s$ RVEs made of GFRP (M-1) are studied. The damages are specified in the middle ply where the thickness of each ply is selected as $t_{ply} = 0.2\ mm$.

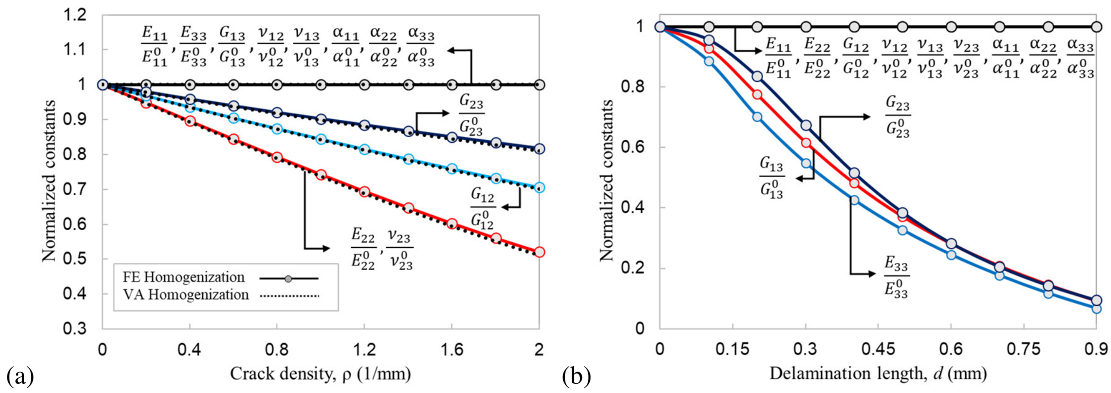


Figure 3. Normalized effective thermo-elastic properties of the damaged 90º ply in $[0/90_{0.5}]_s$ laminates including; (a) only ply cracks (Fig. 1a), (b) only delamination (Fig. 1b).

Fig. 3 (a) shows the normalized effective properties of the 90º cracked ply as a function of crack density (Fig. 1a). It is clear that only four ply properties (E_{22}, ν_{23}, G_{12} and G_{23}) must be decreased due to the ply cracking damage where the highest stiffness degradations are related to E_{22} and ν_{23} which have the same reduction trend (Ahmadi et al., 2020). Also, the validity of the FE results is investigated using a recently developed homogenization method based on the variational approach (Hajikazemi et al., 2020b). The perfect agreement between the results of the current numerical approach with those of Ref. (Hajikazemi et al., 2020b) (based on highly accurate variational stress transfer models and an analytical homogenization scheme) for homogenization at ply level, verifies the accuracy of the proposed numerical homogenization approach. It also shows that the current results are based on very refined meshes and are thus, completely converged. A very interesting point in Fig. 3 is that the *ply* thermal expansion coefficients remain constant while it is shown in several works (Hajikazemi et al., 2020a; Ahmadi et al., 2020; Loukil et al., 2013a) that the *laminate* thermal expansion coefficients can be degraded largely in the presence of damage modes. It is a natural result of the Levin's theorem (Rosen and Hashin, 1970) where he elegantly provides a means for relating composite thermal expansion coefficient to composite elastic moduli. Therefore, the degradation of ply stiffness properties is sufficient to consider the degradation of the laminate thermal expansion coefficients (Hajikazemi et al., 2020b). On the other hand, specifying the length of RVEs as $L = 1$, the variation of the normalized homogenized properties of the 90^0 ply for the delamination scenario (Fig. 1b) are demonstrated in Fig. 3 (b). As it is already mentioned, the second scenario (delamination without ply cracking) may not happen in real composite structures. However, it is considered here to identify the parameters that can be affected by such damage alone and to have a better understanding about the combination of ply cracks and delamination. For this purpose, it is assumed that such damage only reduces the stiffness of the middle ply. It can be clearly seen that the delamination mechanism has significant effects on the out-of-plane (E_{33}, G_{13} and G_{23}) ply properties of the lamina. It is also worth noting that among various elastic constants that get reduced due to the presence of ply cracking and delamination, G_{23} is degraded under both damage configurations. Considering the crack density as $\rho = 1\ (1/mm)$, the homogenized properties of the 90º ply containing ply cracking and delamination (Fig. 1c) are presented in Fig. 4 as a function of delamination length. As it can be observed, six elastic ply properties (E_{22}, E_{33}, ν_{23}, G_{12}, G_{13} and G_{23}) are affected by this damage scenario. Besides, as apposed to the delamination mechanism (second scenario), the in-plane properties of the damaged 90º ply are decreased by increasing the length of delamination.

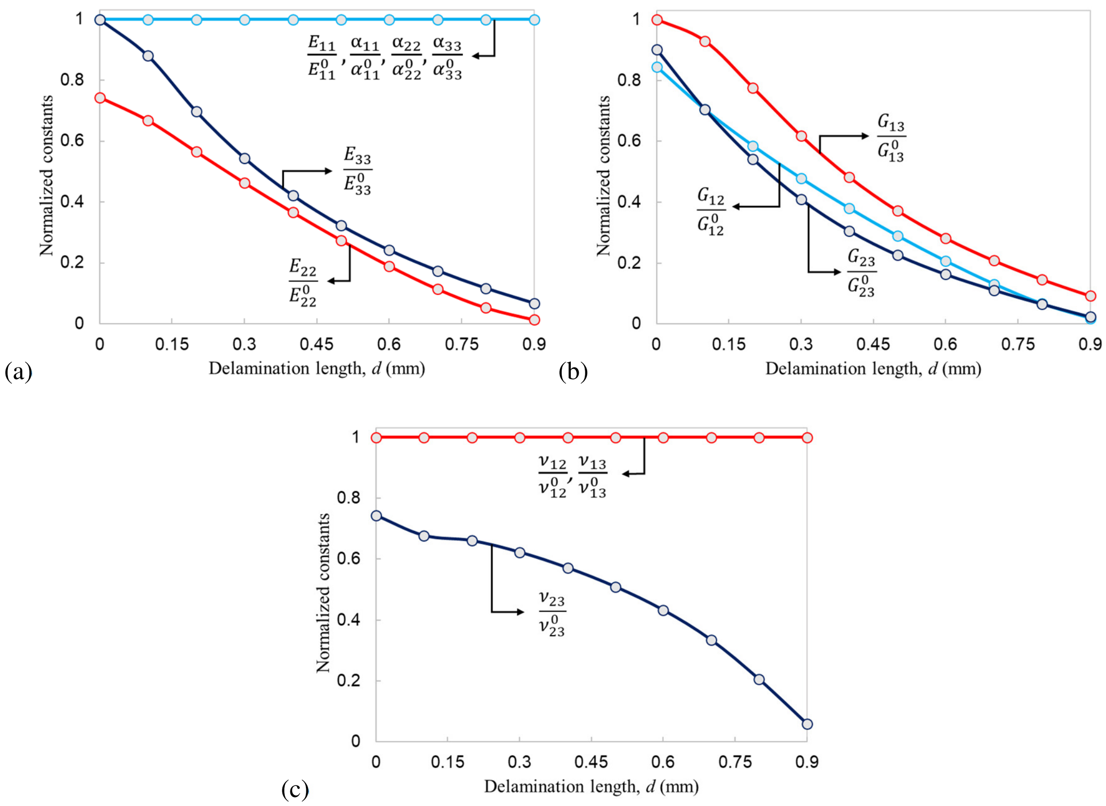


Figure 4. Normalized effective thermo-elastic properties of the damaged 90º ply in $[0/90_{0.5}]_s$ laminates as a function of delamination length having ply cracks with $\rho = 1\ (1/mm)$; (a) Elastic modulus and thermal expansion coefficient, (b) Shear modulus, (c) Poisson's ratios.

Similarly, Fig. 5 shows the variation of the normalized homogenized properties of the damaged 90º ply in terms of the crack density by choosing the delamination length as $d = 0.2$ mm (Fig. 1c). The results show an approximately linear stiffness reduction for all the six elastic properties which is mainly due to the rather small thickness of the damaged ply (0.2 mm).

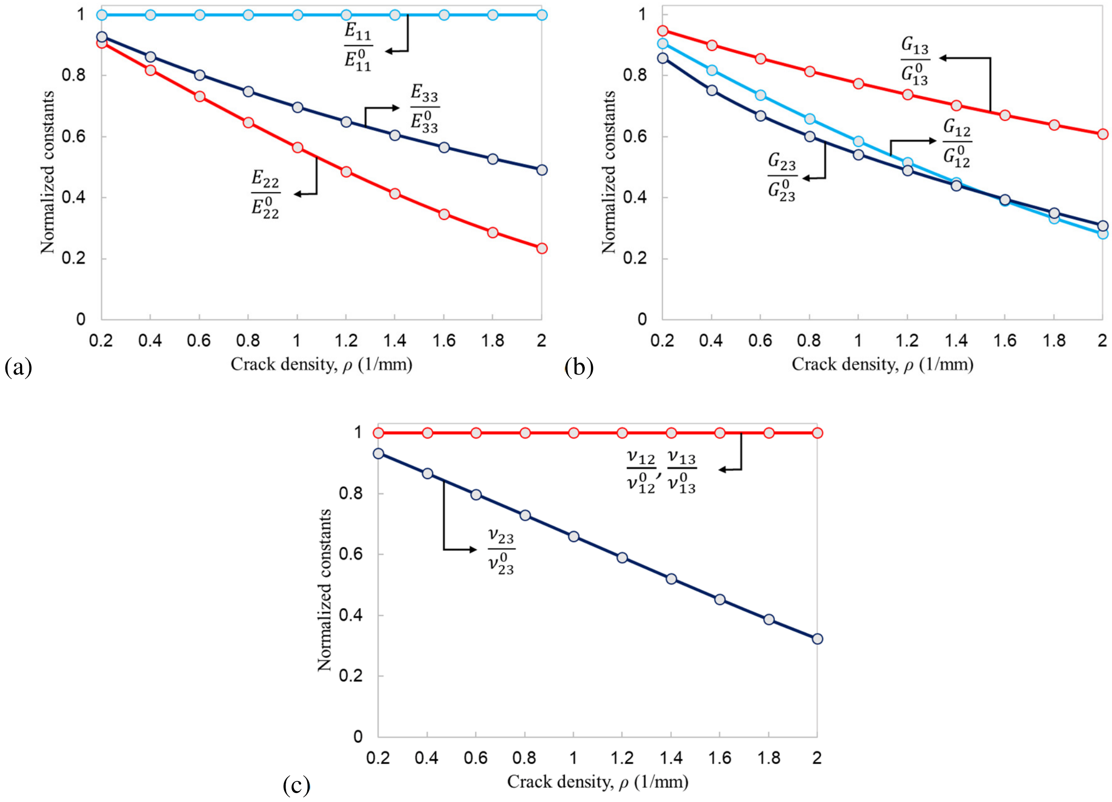
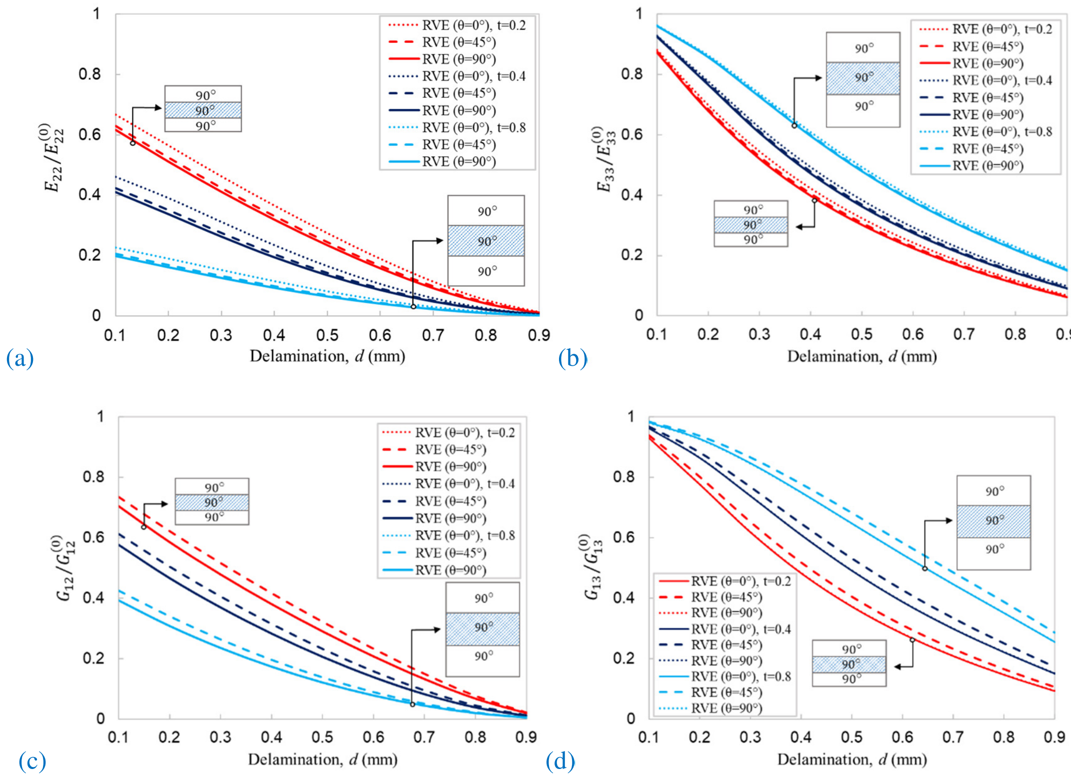


Figure 5. Normalized effective elastic properties of the damaged 90º ply in $[0/90_{0.5}]_s$ laminates as a function of crack density having delamination length $d = 0.2$ mm; (a) Elastic modulus and thermal expansion coefficient, (b) Shear modulus, (c) Poisson's ratios.

In addition to the crack density and delamination length, other factors can also affect the homogenized properties of damaged plies containing ply cracking and local delamination. Thus, using the crack density of ρ=1 (1/mm), the effects of ply thickness and adjacent plies orientations are evaluated. The effective elastic constant of the damaged 90º ply with adjacent ply orientations of ($\theta_{RVEs\ 1} = 0^{\circ}$, $\theta_{RVEs\ 2} = 45^{\circ}$, $\theta_{RVEs\ 3} = 90^{\circ}$) and various ply thicknesses are plotted in terms of delamination length in Fig. 6. This figure highlights the effects of ply thickness on the stiffness reduction of the damaged ply. The results demonstrate that by increasing the thickness of plies, the in-plane properties (E_{22} and G_{12}) of the damaged ply will highly reduce. The main reason for such behavior is the direct relationship between the crack density and the thickness of the ply which means that by increasing the thickness in the cracked ply we will expect more stiffness reduction in the ply. However, for the out-of-plane behavior (E_{33} and G_{13}), the results illustrate an inverse correlation between the delamination length and thickness of the damaged ply. In this regard, increasing the thickness of the ply can improve the out-of-plane (E_{33} and G_{23}) performance of the damaged ply. As discussed above, G_{23} is the only effective elastic property that can be affected by both ply cracking and delamination damage modes. With respect to the opposite effects of thickness to the stiffness reduction of both damage scenarios, the degradation slopes have been decreased by increasing the ply thickness in the damaged ply. Moreover, the obtained results demonstrate the effects of adjacent ply ordinations to the stiffness degradation of damaged plies where the highest stiffness reduction is relevant to the damaged ply in the $[90/90_{0.5}]_s$ stacking sequence. It can be seen in Figure 6 that in the presence of both ply cracks and delamination, the homogenized properties of the damage ply are affected by both adjacent ply orientations and ply thickness. In the previous study (Ahmadi et al., 2020) by the authors which was focused on ply cracking damage mode only, it was shown that the adjacent ply orientation has a negligible effect on the homogenized ply properties. It was also concluded that the effects of ply thickness can be taken into account by introducing a normalized crack density (t_{ply}×ρ). Fig. 6 shows that the effect of adjacent ply orientation is rather small especially for major ply properties (E_{22}, E_{33}, G_{12}, G_{13} and G_{23}). However, the effect of ply thickness cannot be explained by a simple fitting function and a more sophisticated fitting function in terms of ply thickness is required to address the variation of ply properties as a function of crack density and delamination length. The numerical results presented in this paper can provide a data base for finding a fitting function which is not in the scope of the current paper.



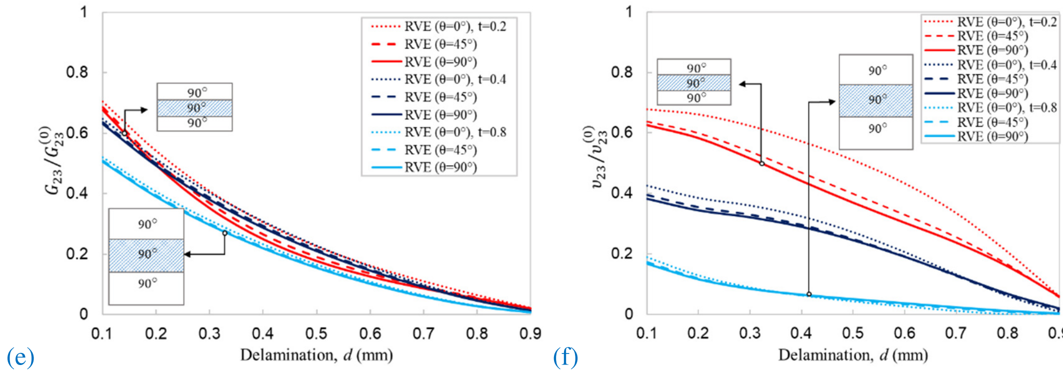
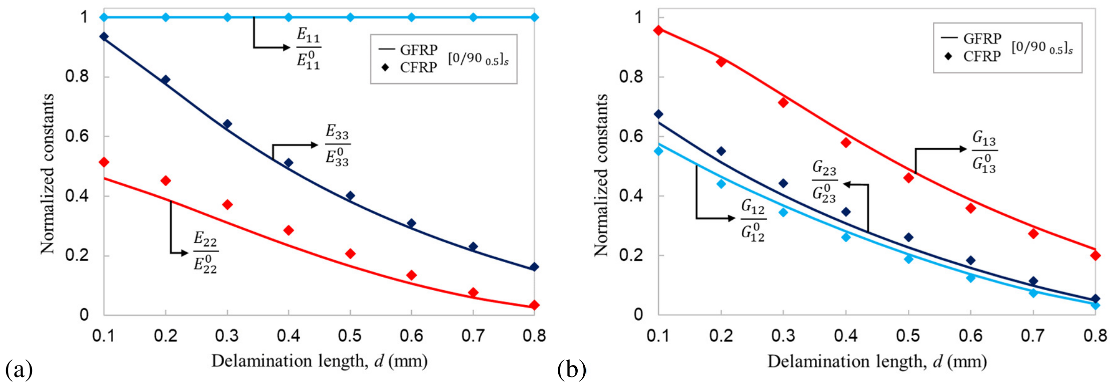
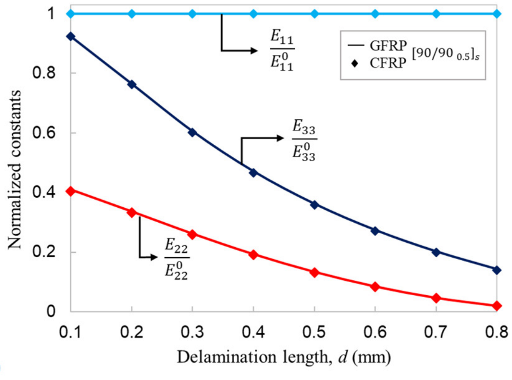


Figure 6. Normalized elastic properties of the damaged central 90º ply in $[\theta/90_{0.5}]_s$ laminates including ply cracking and delamination with $\rho = 1\ (1/mm)$; (a) Normalized E_{22} , (b) Normalized E_{33}, (c) Normalized G_{12}, (d) Normalized G_{13}, (e) Normalized G_{23}, (f) Normalized υ_{23}.

Using a similar strategy, the effects of initial material properties are studied for GFRP and CFRP damaged composite laminates. To this purpose, the stacking sequence of $[\theta/90_{0.5}]_s$ and ply thickness of $t_{ply} = 0.4\ mm$ are selected to identify the influence of material properties on the stiffness reduction of the 90° damaged ply. The normalized homogenized properties of the damaged ply have been determined for $[0/90_{0.5}]_s$ and $[90/90_{0.5}]_s$ laminates and are shown in Fig. 7. Interestingly, there is a rather similar accordance between the normalized stiffness degradation of GFRP and CFRP plies in the $[90/90_{0.5}]_s$ layup. However, regarding the $[0/90_{0.5}]_s$ stacking sequence, the results show some discrepancies in the stiffness reduction of damaged plies made of different material properties.





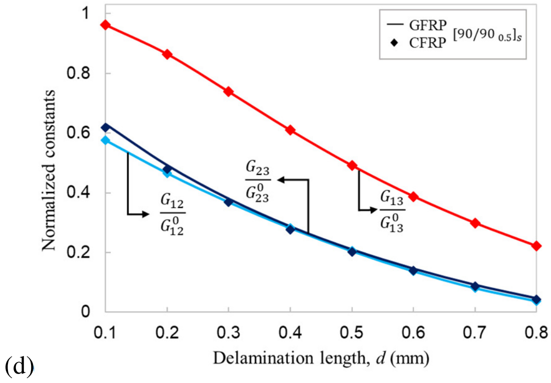


Figure 7. Normalized effective elastic properties of the damaged 90º ply in $[\theta/90_{0.5}]_s$ laminates including ply cracking and delamination with $\rho = 1\ (1/mm)$; (a-b) $[0/90_{0.5}]_s$and (c-d) $[90/90_{0.5}]_s$ layup.

It is possible to use the homogenized behavior of the damaged plies as a material property of layers in an intact laminate to predict the elastic constants of damaged laminates based on higher-order laminate plate theories or FEM simulation. It should be noted that this strategy has been employed herein to verify the accuracy of the obtained homogenized behavior and also to show the efficiency of the suggested approach to capture the mechanical response of damaged laminates.

The use of homogenized behavior of damaged plies has some advantages compared to the direct explicit modeling of damage modes in finite element analyses. Besides the obvious computational efficiency, one advantage of the proposed strategy is the possibility of developing physics-based damage functions for stiffness reduction of plies with a variety of crack densities and delamination lengths where the interactions between ply cracks and delamination are accurately considered. It should be noted that in the design of composite components and structures there is a need to account for the effects of damage development in regions of multiaxial stress concentration. FEM methods based on homogenized anisotropic properties will be needed and there is, therefore, a requirement for a knowledge of the full 3D effective properties (in-plane, out-of-plane and shear properties) of undamaged and damaged laminated materials. In addition, these properties need to be correctly degraded when damage develops. The current strategy provides predictions for values of the degraded 3D properties which are needed for an anisotropic damage mechanics approach (Nairn, 2018). This approach can be later linked to damage evolution laws for prediction of damage propagation at structural level. In addition, the proposed homogenization approach can consider ply cracks and delamination in multiple plies/interfaces with different orientations where it is absolutely cumbersome to model such complex cases directly in finite element modeling with periodic boundary conditions (Ahmadi et al., 2020).

The aim of the current study is in line with the work of Ladevèze et al. (2006), in which the strain energy was used to calculate five damage variables based on micromechanics ignoring multiaxial deformations (only diagonal terms) and by using the superposition of the effects of ply cracking and delamination, separately. However, in this study an accurate computational approach is employed to calculate all stiffness terms (considering multiaxial deformations) in damaged plies (six damage variables) without using any superposition assumption in the homogenization approach. The use of superposition for the effects of different damage modes (mainly ply cracks and delaminations) cannot be trusted and this is due to the fact that the stress fields in the presence of these two damage modes are very different than the ones in the presence of these damage modes separately. Moreover, in the current work, the stiffness reduction is investigated for many conditions without any limitation to the size of delamination, crack density nor laminate geometry.

4. Validation of results

In order to examine the validity and efficiency of the proposed strategy in predicting the in-plane behavior of damaged plies, the homogenized behavior of damaged laminates is evaluated and compared with available macroscopic FE data (Maragoni et al., 2018). In this regard, a $[0/45/-45]_s$ stacking sequence made of glass/epoxy (M-3) with the ply thickness of $t_{ply} = 0.6\ mm$ is considered to depict the capability of the developed method in obtaining the stiffness reduction of a damaged composite laminate including ply cracking and local delamination in the off-axis (45°) ply. To estimate the homogenized properties of the 45° ply, first the effective elastic properties of the 90° ply are calculated from a $[45/90/0]_s$ laminate. It should be noted that the main reason for the 45° rotation of damaged laminate is to have the crack planes in the 90º ply parallel to the y-axis and to simplify the damage modeling in the off-axis plies. The calculated results have been used as effective material properties for the 45° ply in an intact composite laminate with the $[0/45/-45]_s$ layup. Fig. 8 provides the variation of normalized in-plane elastic properties of the laminate in terms of delamination length for the crack densities of $\rho = 1\ and\ 2\ /mm$. Comparing the results of the present homogenization approach and the FEM results (Maragoni et al., 2018), it is clear that there is a perfect accordance between these two data and the stiffness reduction is successfully predicted by the implemented homogenization method. It is worth mentioning that the work of Maragoni et al. (2018) was limited to the in-plane behavior of damaged laminate where the ply cracking and local delamination were directly modeled in the finite element simulation. However, in this study, the homogenized elastic properties of the damaged ply were employed to predict the overall mechanical behavior of the damaged laminate. Fig. 9 shows the out-of-plane behavior of the mentioned damaged laminate. Comparing the stiffness reductions for both the in-plane and out-of-plane properties of damaged laminates reveal that by increasing the normalized delamination length the out-of-plane properties of laminate highly decrease.

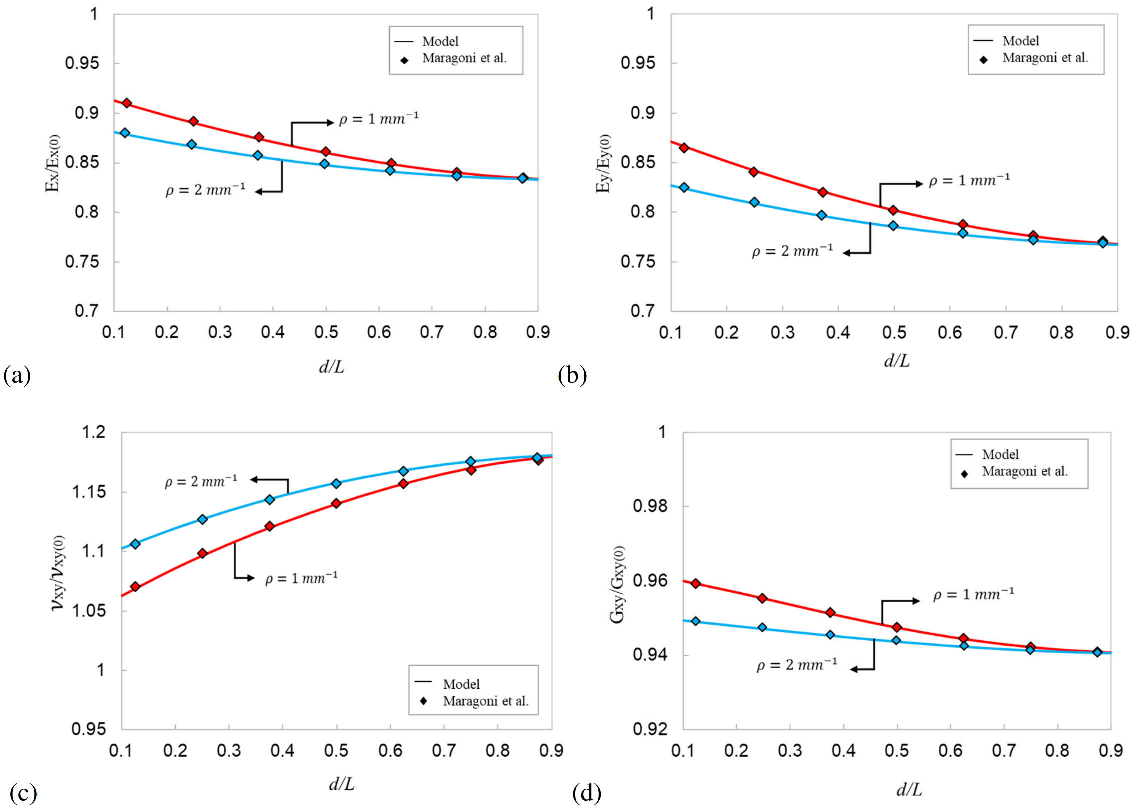
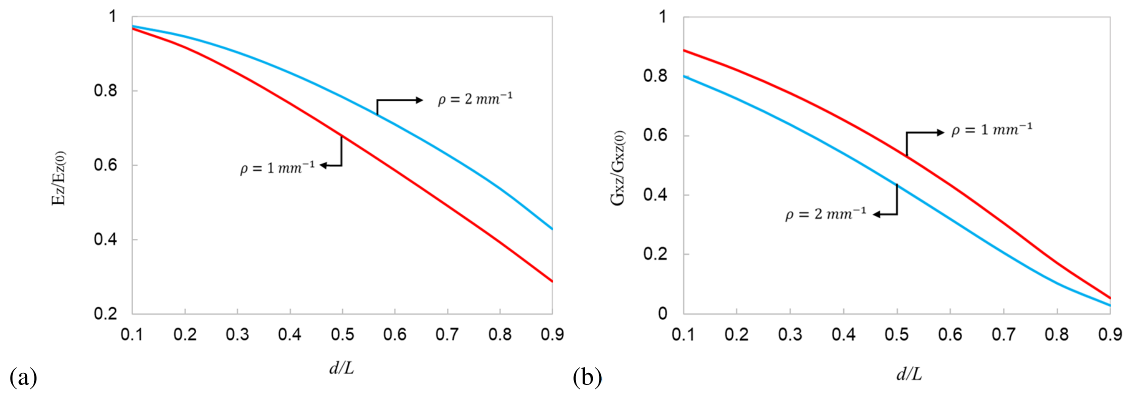
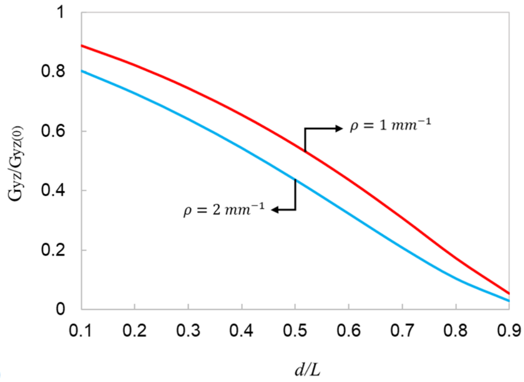


Figure 8. Normalized in-plane elastic constants of damaged $[0/45/-45]_s$ laminate including ply cracks and delamination with $\rho = 1$ and 2 $(1/mm)$.



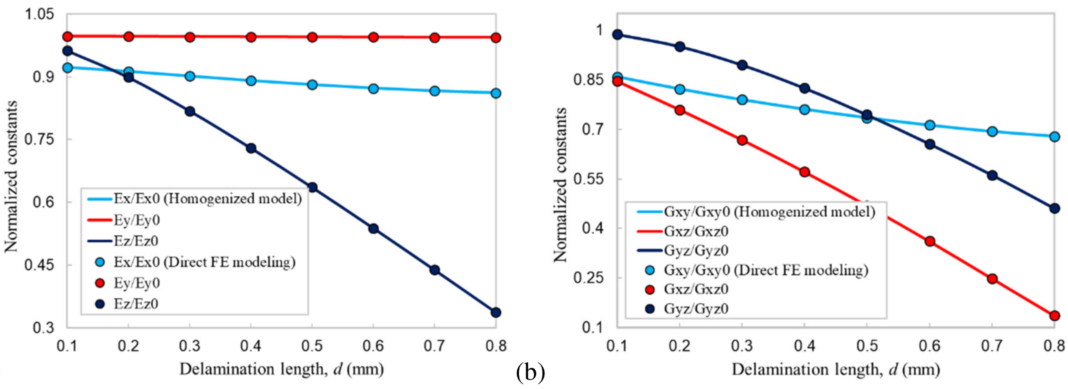


(c)

Figure 9. Normalized out-of-plane elastic constants of damaged $[0/45/-45]_s$ laminate including ply cracks and delamination with $\rho = 1$ and 2 $(1/mm)$.

To further assess the accuracy and robustness of the developed strategy in capturing the in-plane and out-of-plane mechanical behavior of damaged laminates, the thermo-elastic constants obtained by the proposed technique (use of homogenized ply properties) have been verified by direct FE simulation of the damaged laminate. To do so, the finite element analysis is employed to model the damages in the composite laminate and the homogenized ply properties are implemented in an intact FE model of the laminate. A $[0/90/0]_s$, $t_{ply} = 0.4\ mm$ laminate made of GFRP (M-1) is considered where the 90° ply contains ply cracking and delamination. Fig. 10 demonstrates the prediction of laminate effective elastic constants as a function of delamination length. According to the obtained results, there exists a perfect agreement between the FEM and homogenization approach for all the thermo-elastic constants.

(a) (b)



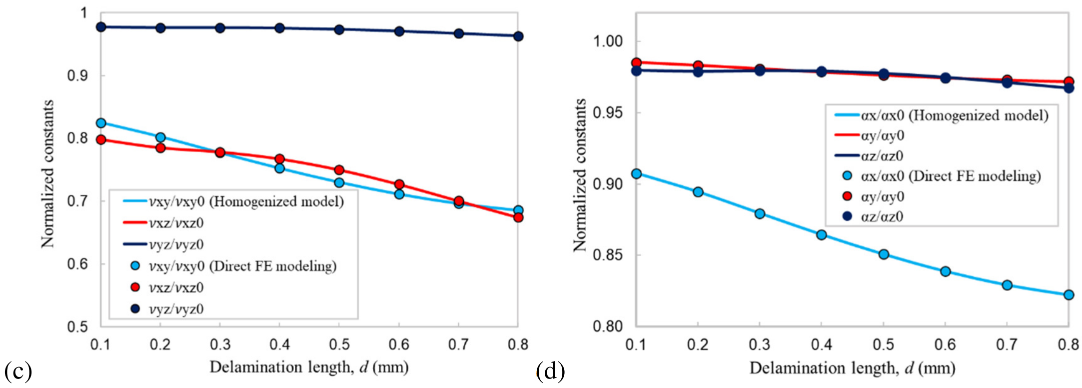


Figure 10. Normalized thermo-elastic constants of the damaged $[0/90/0]_s$ laminate including ply cracks and delamination with $\rho = 1\ (1/mm)$.

As a last validation step, the experimental results of Carraro et al. (2019) with microscopic observations are considered to evaluate the accuracy of the developed approach while predicting the mechanical behavior of laminates containing both ply cracks and delamination. In this regard, a $[0_2/90_4]_s$ laminate (Carraro et al., 2019) made of GFRP (M-3) with ply thickness 0.34 mm is selected to study the stiffness reduction under tension-tension fatigue tests with $\sigma_{x,max} =$ 90 and 120 MPa. Carraro et al. (2019) have reported both the crack density and delamination length at each cycle which are used here for calculation of stiffness reduction. Fig. 11 demonstrates the normalized Young's modulus, weighted crack densities and normalized delamination length for various number of fatigue cycles. Considering the crack density and delamination length from the experimental data, the developed strategy is employed to determine the elastic constants for the damaged laminate. The obtained results are in a good agreement with the experimental observations showing that the assumptions made in the current methodology are capturing the physics of the underlying damage mechanisms.

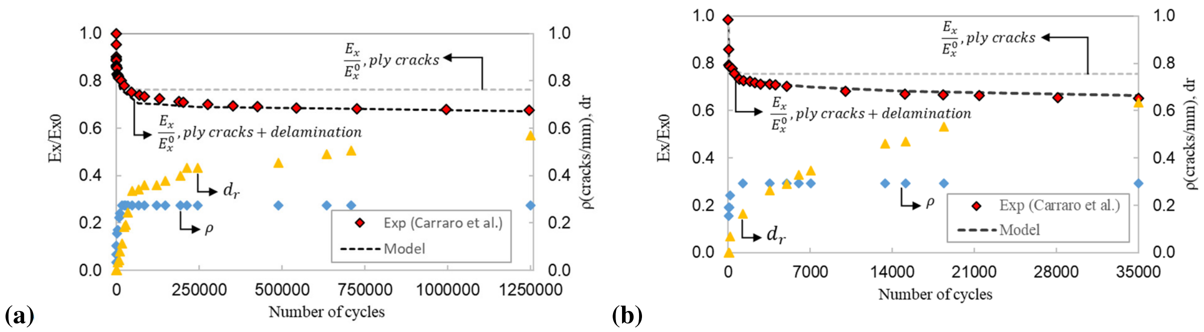


Figure 11. Normalized Young's modulus of $[0_2/90_4]_s$ laminate containing ply cracks and delamination; (a) $\sigma_{x,max} = 90\ MPa$, (b) $\sigma_{x,max} = 120\ MPa$.

It can be seen in Fig. 11(a) that at early stages of loading (<250,000 cycles) the predicted stiffness values are slightly lower than the experimental measured values. This can be explained by the fact that in the modelling, ply cracks and delaminations are assumed to be open (contact free) with a uniform spacing. It is discussed in (Hajikazemi et al., 2018; Hajikazemi et al., 2020a) that both these assumptions lead to a slight overprediction of stiffness reduction. However, by applying further cycles, damage modes tend to have more uniform distributions with larger crack opening displacements which both justify the very good agreement between the calculated results and those of experiments.

5. Conclusion

The main conclusions of the current study can be drawn as follows:

- A finite element model was implemented to analyze the stiffness reduction of damaged lamina and laminates containing ply cracks and local delamination under in-plane and out-of-plane loading conditions.
- Various damage scenarios were studied to reveal the effects of ply cracking and delamination on the homogenized behavior of damaged plies.
- It was found that the delamination damage scenario mainly degrades the out-of-plane (E_{33}, G_{13} and G_{23}) behavior of the damaged lamina while the ply cracking affects both the in-plane and out-of-plane (E_{22}, ν_{23}, G_{12} and G_{23}) response of the damaged lamina. Also, the presence of local delamination at the tips of ply cracks leads to degradation of the both in-plane (E_{22} and G_{12}) and out-of-plane properties (E_{33}, G_{13}, ν_{23}, and G_{23}) of the damaged plies.
- Considering different damage configurations, material properties, ply thicknesses and adjacent plies orientations, insightful analyses were performed to investigate the influence of different parameters on the effective elastic constants of damaged plies. This research has shown that the homogenized behavior of damaged plies can be affected by all these factors. Moreover, the damaged lamina in the $[90/90_{0.5}]_s$ stacking sequence provides the most conservative results of the elastic constants and indicates approximately similar stiffness reduction for both GFRP and CFRP material properties.
- Using a hierarchical framework, the homogenized elastic properties of damaged plies were employed in the undamaged composite laminate to calculate the stiffness degradation of damaged laminates by only modifying the elastic constants of damaged plies without modeling the damage in the laminates. The validity of results was investigated by comparing the obtained data with available FEM studies.

- The suggested method can be used in order to calculate the stiffness reduction for other configurations of ply cracking and local delamination.
- Provided results can be utilized as a benchmark solution for future enhancement of physics-based damage models.
- The results show good agreement with available high quality experimental results in the literature.
- Additional research is needed to analyze the effects of contact and friction in the homogenized behavior of damaged plies.
- It is noteworthy that the reported results concerning stiffness reduction of lamina/laminate correspond to the cases where ply cracks and delaminations are open without any contact. It is evident that under some circumstances such as multiaxial compressive loads, ply cracks and delaminations are not necessarily open where a complex contact between damage surfaces should be considered. However, the authors think that the assumption of contact free damage surfaces lead to a higher stiffness reduction that might be useful for safe design strategies.

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