Parametric Study of a Wing Fence on a UAV using Surrogate Modeling

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In this paper the application of a wing fence on an unmanned aerial vehicle (UAV) is examined. The UAV under consideration is characterized by flow separation initiating at the tip, leading to a loss of lift and controllability and the appearance of a nose-up pitching moment. A possible solution to this problem is the use of wing fences: plates placed on top of the wing aligned with the flow and developed from the idea of stopping the transverse component of the boundary layer flow. Firstly, existing theories in regard to the working of wing fences are brought together. Secondly, the sensitivity of stall speed and controllability to the design variables of the wing fence are laid bare. Finally, the aerodynamic and stability characteristics of the UAV as a function of the design variables are assessed. To accomplish the aforementioned three objectives in both an affordable and accurate manner, computational fluid dynamics (CFD) simulations using the γ − Reθ model to correctly model the low Reynolds effects that characterize the flow over a UAV and surrogate modeling in the form of regressive universal co-Kriging are brought together.

Nomenclature

\( c \) Chord length, \([m]\)

\( C_D \) Drag coefficient; \( D/0.5\rho U_{ref}^2 \), \([-\] \)

\( C_L \) Lift coefficient; \( L/0.5\rho U_{ref}^2 \), \([-\] \)

\( C_M \) Moment coefficient; \( M/0.5\rho U_{ref}^2 \), \([-\] \)

\( C_P \) Pressure coefficient; \( P/0.5\rho U_{ref}^2 \), \([-\] \)

\( d \) Number of design parameters

\( D \) Drag, \([N]\)

\( f(x) = [f_1(x), ..., f_j(x)] \) Trend function

\( F \) Model matrix

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\( g \) Grid level
\( h \) Height, [\( m \)]
\( k \) Turbulent kinetic energy, [\( m^2/s^2 \)]
\( l \) Length, [\( m \)]
\( L \) Lift, [\( N \)]
\( L \) Likelihood function
\( \mathcal{L} \) Characteristic length, [\( m \)]
\( n \) Sample size, [-]
\( M \) Moment, [\( Nm \)]
\( P \) Static pressure, [\( Pa \)]
\( Re_c \) Chord-based Reynolds Number; \( \rho U_{ref} c/\mu \), [-]
\( Re_\theta \) Momentum-thickness Reynolds number; \( \rho U_{ref} \theta/\mu \), [-]
\( s \) Span, [\( m \)]
\( S \) Projected area, [\( m^2 \)]
\( s^2(x) \) Predicted variance of the stochastic process
\( S_i \) Sobol index for design parameter \( x_i \)
\( U_{ref} \) Free-stream velocity, [\( m/s \)]
\( x \) Chordwise position, [\( m \)]
\( \mathbf{x} = [x_1, \ldots, x_i, \ldots, x_d] \) Sample point, vector containing the design parameters
\( \mathbf{X} = [\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \ldots \mathbf{x}^{(n)}] \) Training set
\( y \) Objective function
\( Y(\mathbf{x}) \) Stochastic process
\( y^+ \) Distance in wall coordinates; \( \rho y u_c / \mu \), [-]
\( Z(\mathbf{x}) \) Gaussian process

**Greek symbols**
\( \alpha \) Angle of attack, [\( ^\circ \)]
\( \beta \) Trend function coefficients
\( \delta \) Elevon angle, [\( ^\circ \)]
\( \gamma \) Intermittency, [-]
\( \lambda \) Regression constant
\( \mu(\mathbf{x}) \) Predicted mean of the stochastic process
\( \nu \) Matern smoothness parameter
\( \omega \) Specific turbulence dissipation rate, \([1/s]\)

\( \phi \) Gaussian probability density function

\( \phi_p \) Maximin distance

\( \Phi \) Gaussian cumulative distribution function

\( \Psi \) Correlation matrix

\( \rho \) Density of air, \([kg/m^3]\)

\( \rho_d \) Scaling factor

\( \sigma \) Process variance

\( \theta_j \) Width parameter

**Acronyms**

\( AoA \) Angle of Attack, \( [^\circ] \)

\( AR \) Aspect Ratio

\( BLUP \) Best Linear Unbiased Prediction

\( BWB \) Blended Wing Body

\( CFD \) Computational Fluid Dynamics

\( CoG \) Center of Gravity

\( DoE \) Design of Experiments

\( (C/MO)EI \) (Constraint/Multi-Objective) Expected Improvement

\( GCI \) Grid Convergence Index

\( GP \) Gaussian Process

\( LHS \) Latin Hypercube Sampling

\( MLE \) Maximum Likelihood Estimation

\( OBD \) Overall Better Design

\( ooDACE \) object oriented Design and Analysis of Computer Experiments

\( (U)RANS \) (Unsteady) Reynolds-Averages Navier-Stokes

\( SST \) Shear Stress Transport

\( UAV \) Unmanned Aerial Vehicle

**Subscripts**

\( c \) Cheap/Low fidelity

\( d \) Difference/Residual

\( down \) Down/pressure side

\( e \) Expensive/Accurate/High fidelity
I. Introduction

The widespread use of unmanned aerial vehicles (UAV) has become clear over recent years, thanks to its increasing ability to be deployed for a series of comprehensive tasks: from the more well-known military up to and including its humanitarian counterpart. This has led to an enormous research boost in that field. Within the extensive range of UAVs that exists nowadays, this paper focuses on those that operate at a chord-based Reynolds numbers ($Re_c$) below $5 \times 10^5$, the condition which is referred to as low Reynolds number flow [1, 2].

The UAV which will serve as the base of our study is characterized by its blended wing body (BWB) design: a tailless aircraft with its fuselage, the aircraft's main body, integrated in the wing [3]. This is a typical concept for remotely piloted aircraft designed for mapping and surveying, characterized by a compact shape and a high lifting surface, making up for the low aspect ratio (AR). The absence of a horizontal tailplane combined with profiles that have positive camber enforces the sweeping and twisting of the wing to obtain longitudinal static stability, the intrinsic desire of the airplane to correct minor changes in its angle of attack (AoA). Firstly however, the sweeping of the wings introduces a pressure gradient on the wing normal to the free stream, decreasing from the root to the tip. This effect will result in a more rapid growth of the thickness of the boundary layer towards the tip and results in a higher likelihood of flow separation. Secondly, the vortex-shedding caused by the inboard sections results in a decreased downwash and consequently a higher loading at the tip. Thirdly, tapering of the wing is present in order to reduce the structural load caused by the lift at the tip. This results in a smaller sweep angle of the trailing edge, improving the effectiveness of the control surfaces. However, this also implies that, due to the shorter chord length, the pressure has to increase much faster on the upper surface. Thus the pressure gradient becomes much larger, leading up to an increased likelihood of separation. Fourthly, the BWB design is typically characterized by wing tip devices that take up the job of providing lateral and directional stability. These winglets or end-plates introduce secondary flows at the tip due to the interaction of the boundary layers on them and on the wing, also resulting in a flow more prone to separation. This leads to a special case of stall: tip-stall [4]. As the name indicates, the tips of the wings start stalling first. This is a common but unwanted characteristic of swept wings. Since the tips are generally located behind the center of gravity (CoG), tip stalling will result in a pitch-up moment, thus pulling the plane further into stall. Even more dangerous is the situation where tip stalling occurs only on one side. The airplane then makes a roll movement due to the loss of lift on this side and increases the AoA of the stalling side even further while the other side keeps developing lift. As a result, the drag
on one side will significantly increase from which a yawing maneuver follows. This phenomenon is referred to as spin and is hard to recover from: it requires the nose to be pushed down.

Since before that time a number of solutions were introduced to tackle the problem of tip stalling, such as slats, slots, dogtooth, notched leading-edge, leading-edge cuff, leading-edge root extension, vortex generators, stall strips, pylons, vortilons and wing fences. Based on the solutions presented above, only a handful can be used on the blended wing body aerial vehicle after its manufacturing (typically addressed as aerodynamic afterthoughts). Therefore, the device chosen was the wing fence. The wing fence, boundary layer fence or potential fence can be defined as a plate which is placed on top of the wing aligned with air flow and, depending on its design, extending up to the trailing edge or extending over the leading-edge to the lower surface. It was the first aerodynamic device introduced on swept wings to tackle the phenomenon of tip stalling. It has a straight-forward concept: stopping the transverse component of the boundary layer flow. It is easily installed on the wing without making any modifications to the shape of the wing itself, making it an attractive device. The wing fence was invented in 1938 by Wolfgang Liebe to tackle the unfavorable stall behavior of the Messerschmitt Bf-109 B, a German World War II fighter aircraft. Later studies on unswept wings found the inability of the wing fence to alter the pre-stall behavior. Liebe’s initial concept (Fig. 1a), the disruption of the flow separation moving from root to tip on a straight wing, has changed over the years with its introduction on swept wings (most noteworthy on the MiG-15: Fig. 1b) leading to different design and theories in regard to its working principles. In the early 50s, the wing fence became a means to stop the spanwise flow component moving from root to tip on backward swept wings and consequently, fences were designed to do exactly that (Fig. 1c). Das discovered that the wing fence not only avoids or greatly reduces the flow separation on the wing tips, but that on the inside of the fence another small separation area is formed with hardly any change in total lift and total drag. This leads to a change of the pitching moment, which is in particular crucial for the improvement of the aerodynamic characteristics. Rudenko & Ryzhkova found that a trailing vortex is formed on the inboard side following the lift loss in the region of the fence and that it has the same sense of rotation as the tip vortex. On the other hand Haines, Fozard and Williams et al. (Fig. 1d) found the vortex on the other side rotating in the opposite sense.
A difficulty that comes up when examining the different studies is the inconsistency in not only shape and position of the wing fence, but also the stall behavior (leading-edge versus trailing edge stall) and aircraft design (swept versus delta wing). Firstly, this obstructs the formulation of a closing theory on the working of a wing fence on a BWB UAV. Secondly, this makes the assessment of the sensitivity of flight mechanics to the design variables hard to predict. Thirdly, an assessment of the influence of the different design variables on a BWB UAV becomes ambiguous. In this paper, we will address these three issues through an aerodynamic examination using unsteady Reynolds-Averaged Navier-Stokes (URANS) simulations employing the $γ – Re_θ$ model to account for the low-$Re$ effect that characterizes the flow around the UAV, as discussed in § II.B, in combination with a surrogate model, as discussed in § II.C.

Consequently, there are three objectives: a unification of the working principle of a wing fence on a BWB UAV (see § III.B), determining the sensitivity of stall speed and controllability to the design variables of the wing fence (see § III.C) and analyzing the aerodynamic and stability characteristics of the UAV as a function of the design variables (see § III.D).

II. Methodology

A. Parametric Model

The UAV under consideration is a tailless aircraft, which implies that the devices that guarantee the rotations around the principal axes of the aircraft (roll, yaw and pitch) are all to be incorporated in the wing. The classical flight control surfaces (ailerons, elevators and rudder) are contracted to a single set of elevons, accomplishing roll and pitch by means of the same surfaces. Yawing is obtained through the addition of drag on one side of the aircraft by deflecting the control surface and thus typically accompanied by a rolling movement. The lever arm of the pitching moment is the distance between center of mass and center of pressure of the elevon along the principal x-axis (typically aligned with the chord...
on the symmetry plane). Since it is fairly small, the elevons are required to be quite big. This translates in a strong influence on the lift and drag. Therefore, the elevon deflection $\delta$ is added as a design parameter. Furthermore, since it is our intent to evaluate the characteristics of the UAV when fitted with various fence designs as a function of the angle of attack, the latter is also added as a design variable.

The parameters of the wing fence design are chosen such that every combination can be meshed in a structured manner as to avoid altering the grid discretization error (see Fig. 2b):

1) The spanwise position of the fence ($s$), ranging from the fuselage to the tip.
2) The height of the fence perpendicular to the wing ($h$), expressed as a percentage of the aerodynamic chord, ranging from 0 to 10%.
3) The length on the suction side of the wing ($l_{up}$), measured from the leading-edge to the hinge point of the elevon, expressed as a percentage of the constrained local chord, ranging from 0 to 100%. The length does not extend to the trailing edge, because it was found that this introduces a strong buffeting on the elevon [15].
4) The length on the pressure side of the wing ($l_{down}$), measured from the leading-edge to the hinge point of the elevon, expressed as a percentage of the constrained local chord, ranging from 0 to 100%.

This creates a total of $d = 6$ design variables.

![Fig. 2 Parameterization of the wing fence](image)

**B. Computational Fluid Dynamics**

Airfoils operating at low Reynolds number conditions are characterized by the appearance of a transitional separation bubble [16]. This bubble is often detrimental to the performance of the airfoil and is preferably avoided (for example by means of turbulators and bubble ramps). It is nevertheless of importance to correctly resolve this phenomenon to assure a correct estimation of the flight behavior of the UAV. This can be obtained by means of computational fluid dynamics (CFD) if appropriate turbulence modeling is applied. The relatively low computational cost that is attributed to Reynolds-Averaged Navier-Stokes (RANS) simulations allows its use in increasingly complex 3D geometries. The assumption of a fully turbulent flow that goes hand in hand with classic turbulence models makes their use in low
Reynolds applications somewhat ambiguous. The last couple of decades have however seen the birth of a number of turbulence models that attempt to model the transition phenomena that are attributed to low Reynolds number flow. Here Menter and Langtry’s correlation-based $\gamma - Re_\theta$ model $[17, 18]$ is used as it gives the best result for a profile in low $Re$ flow in a comparative study of transition models $[19]$. The model builds on the $k - \omega$ SST model, but distinguishes itself through the addition of a supplementary transport equation for the intermittency, $\gamma$, and the momentum thickness Reynolds number at transition, $Re_\theta$. The former represents the time fraction the flow is turbulent and allows transition to be spread in space. The latter assures that the model captures strong variations of the turbulent intensity that may occur due to turbulence decay, the influence of the free-stream and the pressure gradient. The transition onset Reynolds number is established in the free-stream through experimental correlations and is diffused into the boundary layer. The production term of $k$ as used in the model is obtained by multiplication of $\gamma$ with the production term of $k$ as used in the traditional $k - \omega$ SST model. In this way it distinguishes itself from other $\gamma$ models that use $\gamma$ to limit the turbulent viscosity, $\mu_t$. The advantage of the former is found in its ability to capture the effect of large free-stream turbulence levels on laminar boundary layers and the related increase in the laminar skin friction and heat transfer. Yet it was found that the production of turbulent kinetic energy in case of separation induced transition occurred too slowly, leading to a reattachment too far downstream. To counter this feature an effective intermittency, $\gamma_{eff}$, was introduced, which can obtain the value of 2 inside a separation bubble, accelerating the production of $k$ (Fig. 3b) and forcing the transition bubble to an earlier reattachment.

**Fig. 3** Flow modeling presented on the suction side of the UAV

All calculations are performed using the CFD-code ANSYS Fluent 16.2 with a second-order upwind discretization for convective terms, second-order central discretization for diffusive terms, gradients from a least squares cell based discretization, a transient second-order implicit formulation and the SIMPLE pressure-velocity coupling.
The mesh is created in a hybrid manner: a structured hexahedral mesh is created in the close proximity of the body and extending in the wake through the generation of blocks. Defining a wall between these blocks allows the creation of the wing fences. Outside of the structured region an unstructured grid is created composed of tetrahedral cells. A minimum of 10 cells is found in the boundary layer on the coarsest level (g3, Table 1) with a stretching factor equal to 1.1. This stretching factor is held constant for the different meshes.

Roache’s Grid Convergence Index [20] is used to quantify the discretization error and is based on a Richardson extrapolation of any quantity of interest as a function of the grid spacing, $h$, around the theoretical solution with an infinitely small grid ($g = 0$). The idea of a GCI is to accomplish an estimate of the relative error that would be obtained in case of grid doubling with a second-order method, even if that was not performed during the study.

The refinement study has been performed on three levels for the mesh size, refining equally in all directions. Two of them make up the low and high-fidelity levels used during the surrogate modeling. One finer mesh has been added to establish whether they lie within the asymptotic range such that the mesh is fine enough to resolve the physical phenomena correctly. The meshes have been assessed for the clean geometry (absence of wing fence) at $\alpha = 15^\circ$, which approximately corresponds to the stall angle. The time-step size corresponds to $1 \times 10^{-4}$ and the residual size to $1 \times 10^{-5}$. The functionals that are evaluated are the lift coefficient $C_L$, drag coefficient $C_D$, pitching moment coefficient $C_{M,pitch}$ and rolling moment coefficient $C_{M,roll}$. The results are found in Table 1 with $g_0$ the index of the Richardson extrapolation and $g_1$, $g_2$ and $g_3$ representing the fine, medium and coarse setting.
As will be discussed in the next section, the low-fidelity case is only used to accelerate the prediction process based on the vector of coefficients and function between two objectives and is a function of their inputs, typically written as covariance function process:

\[ \text{function between two objectives} = \text{function of their inputs} \]

In this work, a methodology to answer the problem at hand is found in the field of surrogate modeling, which is actively used for aerospace applications: calibration of turbulence models [21], modeling of flight data [22,23], uncertainty quantification [24–26] and robust [27,28], multi-objective [5], shape [29] and multi-disciplinary [30] optimization. This implies that, after defining the objective function and the design space, a *design of experiments* (DoE) is set up to select samples in the design space, for which the objective function is subsequently calculated and of which a surrogate is defined. This cheap to evaluate surrogate or meta model can subsequently be sampled to define the entire characteristics in function of the geometric design variables and allows a direct assessment of derivatives without the need to perform additional costly simulations. In this work, a *regressive universal co-Kriging* model is used.

*Kriging* is a popular surrogate modeling technique. It can be seen as the sum of a trend function and Gaussian process: \( Y(x) = f(x)^T \beta + \mathcal{Z}(x) \) with \( \mathbb{E}[\mathcal{Z}(x)] = 0 \) and \( f(x) = [f_i(x), i = 1, ..., m] \) the vector of basis functions, \( \beta \) the vector of coefficients and \( \mathcal{Z}(x) \) a Gaussian process \( \mathcal{GP}(0, \text{cov}(y^{(i)}, y^{(j)})) \), with zero mean and fully described by the covariance function \( \text{cov}(y^{(i)}, y^{(j)}) = \sigma^2 \text{cor}(y^{(i)}, y^{(j)}) \). \( \sigma \) is the process variance and \( \text{cor}(y^{(i)}, y^{(j)}) \) is the correlation function between two objectives and is a function of their inputs, typically written as \( \psi(x^{(i)}, x^{(j)}) \). The notation \( x = [x_1, ..., x_d] \) is the input, while \( y \) is called the output. Samples are indicated as \( x^{(i)}, y^{(i)}, i = 1, ..., n \). The trend is

### Table 1  Grid convergence index (CI) for lift, drag, pitching moment and rolling moment coefficient

<table>
<thead>
<tr>
<th>Level</th>
<th>Grid size</th>
<th>( C_L )</th>
<th>GCI (%)</th>
<th>( C_D )</th>
<th>GCI (%)</th>
<th>( C_{M,pitch} )</th>
<th>GCI (%)</th>
<th>( C_{M,roll} )</th>
<th>GCI (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_0 )</td>
<td>( \infty )</td>
<td>0.6660</td>
<td>-</td>
<td>0.0601</td>
<td>-</td>
<td>-0.0015</td>
<td>-</td>
<td>-0.2900</td>
<td>-</td>
</tr>
<tr>
<td>( g_1 )</td>
<td>( 20.7 \times 10^6 )</td>
<td>0.6611</td>
<td>0.9242</td>
<td>0.0647</td>
<td>9.5611</td>
<td>-0.0016</td>
<td>26.4246</td>
<td>-0.2777</td>
<td>5.2696</td>
</tr>
<tr>
<td>( g_2 )</td>
<td>( 10.5 \times 10^6 )</td>
<td>0.6553</td>
<td>1.9951</td>
<td>0.0665</td>
<td>13.4941</td>
<td>-0.0017</td>
<td>36.6113</td>
<td>-0.2702</td>
<td>8.5280</td>
</tr>
<tr>
<td>( g_3 )</td>
<td>( 1.5 \times 10^6 )</td>
<td>0.5698</td>
<td>18.0529</td>
<td>0.0763</td>
<td>33.1778</td>
<td>-0.0025</td>
<td>112.894</td>
<td>-0.2097</td>
<td>34.5877</td>
</tr>
</tbody>
</table>

The orders of convergence for \( C_L, C_D, C_{M,pitch} \) and \( C_{M,roll} \) are respectively 1.7, 1.4, 1.6 and 2.2. While the GCI of the coarest mesh can be considered fairly large, it lies in the asymptotic range such that it resolves the physics correctly. As will be discussed in the next section, the low-fidelity case is only used to accelerate the prediction process based on the high-fidelity case. Each simulation is characterized by \( 2 \times 10^5 \) time steps with an averaging over the last \( 1 \times 10^5 \) time steps to obtain the quantities of interest. For the medium grid this corresponds to 500 core hours, while for the fine grid this corresponds to 5000 core hours on 2 Intel Xeon E5 2680v4-processors.
typically the solution of a regression problem and the Gaussian process captures the variation on this trend to exactly interpolate the evaluated data. The strength of the inclusion of the Gaussian process is found in that it does not restrict the class of functions considered, but, from a Bayesian point of view, gives a prior probability to every possible function, where higher probabilities are given to functions that we consider to be more likely. This is done by correctly defining the covariance function. Consequently, the supervised learning problem comes down to finding the correct covariance function and the values of its parameters, the hyperparameters. The combination of the prior and the data leads to the posterior distribution. This can intuitively be understood as the sampling of functions that pass through the evaluated points.

Predicted points that lay closer to evaluated points are prone to be correcter. This notion of similarity is defined by the covariance function. Here the Matérn covariance function is used with $\nu = 3/2$, defined as

$$
cor[Y(x^{(i)}), Y(x^{(j)})] = \frac{2^{1-\nu}}{\Gamma(\nu)} \left(\sqrt{2\nu L}\right)^{\nu} K_{\nu} \left(\sqrt{2\nu L}\right)
$$

with $\Gamma(\nu)$ the Gamma function, $K_{\nu}$ the modified Bessel function of the second kind and $L$ the characteristic length and typically expressed in a Mahalanobis distance-like manner $L = \sqrt{\sum_{j=1}^{d} \theta_j (x_j^{(i)} - x_j^{(j)})^2}$.

The parameter $\theta_j$ of the Matérn function is a width parameter and $\nu$ is a smoothness parameter. In order to determine the hyperparameters, we maximize the likelihood, $L$, that the aforementioned surrogate can reproduce the evaluated data. Solving the maximum likelihood estimation (MLE) problem, we can define the Best Linear Unbiased Prediction (BLUP), which allows the prediction of unsampled locations $x$ with respectively the predicted mean and predicted variance:

$$
Y(x) = f(x)^T \beta + \psi(x)^T \Psi^{-1} (y - F \beta)
$$

$$
s^2(x) = \sigma^2 \left(1 - \psi(x)^T \Psi^{-1} \psi(x) + \left(F^T \Psi^{-1} \psi(x) - f(x)\right)^T \left(F^T \Psi^{-1} F\right)^{-1} \left(F^T \Psi^{-1} \psi(x) - f(x)\right)\right)
$$

with $F$ the model matrix: $F_{i,j} = f_i(x^{(j)})$ and $\Psi$ the correlation matrix: $\Psi_{i,j} = \psi(x^{(i)}, x^{(j)}), \psi(x) = [\psi(x^{(1)}, x), ..., \psi(x^{(n)}, x)]$, 

$y = [y^{(1)}, ..., y^{(n)}]$ and the MLE of the coefficient vector and the process variance defined by: $\beta = (FP^{-1}F)^{-1}F^T\Psi^{-1}y$ and $\sigma^2 = \frac{1}{n}(y - F\beta)^T\Psi^{-1}(y - F\beta)$.

The accuracy of the surrogate can be significantly enhanced for the same computational budget if multiple fidelity models or grid levels are available [31–33]. Co-Kriging can be considered a powerful correction process which makes use of the correlation between cheap and expensive data to enhance the prediction accuracy. We refer to the most accurate expensive data values $y_e$ at points $X_e$ (corresponding to the medium grid from the previous section) and the less accurate cheap data $y_c$ at points $X_c$ (corresponding to the coarse grid from the previous section). Here the auto-regressive model of Kennedy and O’Hagen [34] is used. Conform the notion of correction processes the co-Kriging formulation presents the surrogate of the expensive model, $Y_e$, as the sum of the surrogate $Y_c$ of the cheap data ($X_c, y_c$) and the surrogate $Y_d$ of the residuals ($X_e, y_e - \rho_d Y_e(X_e)$), with $\rho_d$ the scaling factor. The construction of the model occurs similar to the manner by which it was defined above: with a sequential construction of the two surrogates, where the scaling factor is determined through MLE along with the second surrogate.

The Kriging model introduced above builds on the assumption that the physical phenomenon that we are modeling is smooth and continuous. While this is often the case, especially for aeronautical applications (of course, exceptions exist, such as shocks at higher Mach numbers and bursting at lower Reynolds number), the evaluation and/or experimental observation is often subjected to scatter. Luckily, a key advantage of using a surrogate is that the latter can be filtered out. The filtering of noise can be achieved through the introduction of a regression constant $\lambda$ that is added to the diagonal of the covariance matrix $\Psi$ [35]. Consequently, the data is not interpolated as for $|x^{(i)} - x| \to 0$, the correlation becomes $\text{cor}(x^{(i)}, x) = 1 + \lambda$. The regression constant $\lambda$ is determined using maximum likelihood estimation in a similar manner as the other model parameters.

The quality of the surrogate is highly dependent on the available data. This somewhat paradoxical statement is countered by the design of experiments, which attempts to create an initial training set, $X = [x^{(1)}, ..., x^{(n)}]$ containing $n$ sampling locations, to capture the behavior of the black box model with a minimal amount of training data. A uniform level of accuracy throughout the design space enforces a space-filling initial sampling plan. Within a stochastic context this can be obtained through the use of stratification: the process of dividing members of the population into homogeneous subgroups before sampling in order to achieve uniformity when projecting the sampling plan onto the variable axes. The Latin Hypercube sampling (LHS) is based on this principle [36]. Here we use Morris and Mitchell’s criterion $\phi_d$ to quantify the space-filling property, optimized by simulated annealing [37].

The surrogates that are used to perform this study are the result of the constrained multi-objective optimization of stall speed $V$ and controllability $M'$ subjected to pitching moment equilibrium ($C_{M,pitch}(x) = 0$) and longitudinal static stability ($\partial C_{M,pitch}(x) / \partial \alpha < 0$) [38]. The speed (and thus angle of attack $\alpha$ and the elevon deflection $\delta$) can be determined from the lift, drag and moment coefficient by solving the equilibrium equations. The controllability coefficient is defined as the derivative of the roll moment coefficient to the elevon deflection around the equilibrium
position times its velocity squared \( M' = \frac{\partial C_{M,\text{roll}}}{\partial \delta} \cdot V^2 \). This corresponds to the aircraft’s ability to perform a roll maneuver through an elevon deflection. Each CFD-simulation is characterized by a certain \((\alpha, \delta)\)-setting, which does not necessarily correspond with \( C_{M,\text{pitch}}(x) = 0 \). The latter is obtained by searching the surrogate of \( C_{M,\text{pitch}} \) for the appropriate elevon deflection \( \delta \) that gives \( C_{M,\text{pitch}} = 0 \) instead of iterating \( \delta \) for each \( \alpha \). This leads to a much more efficient approach to solving the problem.

A summary of the optimization framework is presented here. First, a LHS of 10\( d \) points (with \( d \) the dimensionality of the problem, equal to 6) is created to be evaluated by the low-fidelity model and from this LHS a space-filling subset of 3\( d \) points is selected to be evaluated by the high-fidelity model. The subset is selected through an exchange algorithm: a random subset is selected of which each element is sequentially replaced by the remaining members of the original set and for which \( \phi_q \) is calculated. The exchange with the highest score is retained [39]. Furthermore, the corner-points of the design space are also evaluated by both the low and high-fidelity models (2\( d \) points). The evaluation of the design sets through URANS simulations, using the \( \gamma - Re_\theta \) model performed at a fixed velocity \( (U_{ref} = 15m/s) \) close to the stall speed, gives us \( C_L(x), C_D(x), C_{M,\text{pitch}}(x) \) and \( C_{M,\text{roll}}(x) \). We assume a this point that the variations in \( \gamma \) are small enough to be negligible. The roll moment coefficient of the entire aircraft during equilibrium flight is equal to zero. Here we simulate only half of the aircraft and calculate the coefficient around the symmetry plane using this half plane, which leads to a non-zero coefficient. However, since we are interested in the derivative of the coefficient due to an elevon deflection, this is not an issue. A separate surrogate is constructed for each of these coefficients that allows the analysis of the design variables and the calculation of the descent speed and controllability coefficient. Subsequently, 40 additional high-fidelity infills are selected, evaluated and used to retrain the surrogates in an iterative manner using Keane’s multi-objective formulation of the expected improvement [40]. This leads to a total of 10\( d + 2^d = 124 \) low-fidelity and 3\( d + 2^d + 40 = 122 \) high-fidelity points.

Amongst the number of methods that exist to assess the accuracy of the Kriging model two criteria are presented here: the leave-one-out cross validated prediction error (CVPE) [41] and the integrated mean square error (IMSE) [42]. They are given by

\[
CVPE = \frac{1}{n} \sum_{i=1}^{n} \left( \psi^{-1}_{i,:} \left( \mathbf{d} + \mathbf{H}_{i,:} \mathbf{d} - \mathbf{H}_{i,:} \right) \left( \psi^{-1}_{i,:} \right)^{-1} \right)^2 \quad \text{with} \quad \mathbf{H} = \mathbf{F}^T \mathbf{F}^{-1} \mathbf{F}^T \quad \text{and} \quad \mathbf{d} = \mathbf{y} - \mathbf{F} \mathbf{\beta} \quad (4)
\]

\[
IMSE = \int_{\Omega} \hat{s}^2(x) \, dx \approx A \frac{1}{n} \sum_{i=1}^{n} \hat{s}^2(x_i) \quad \text{with} \quad A = \int_{\Omega} dx \quad (5)
\]

where \( \mathbf{H}_{i,:} \) and \( \mathbf{H}_{i,:} \) respectively represent the \( i^{th} \) row and column of matrix \( \mathbf{H} \). The CVPE can be directly evaluated following the surrogate construction. The IMSE can be approximated through Monte Carlo integration using for example LHS.
The construction of the Kriging model is performed using an in house toolbox ooDACE (object-orientated Design and Analysis of Computer Experiments) [43]. The maximization of the concentrated ln-likelihood function is performed through a multi-start sequential quadratic programming (SQP) methodology.

III. Results and Discussion

The CVPE and IMSE of the surrogates after the final iteration are given in table 2 normalized by dividing them by the objective range of the respective surrogates. They indicate good convergence of the entire space.

Table 2 Quality assessment of the surrogates examined

<table>
<thead>
<tr>
<th></th>
<th>( C_L )</th>
<th>( C_D )</th>
<th>( C_{M,pitch} )</th>
<th>( C_{M,roll} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CVPE_{norm} )</td>
<td>( 6.13e-4 )</td>
<td>( 5.60e-5 )</td>
<td>( 3.80e-4 )</td>
<td>( 1.70e-4 )</td>
</tr>
<tr>
<td>( IMSE_{norm} )</td>
<td>( 1.50e-3 )</td>
<td>( 1.20e-4 )</td>
<td>( 3.90e-4 )</td>
<td>( 5.15e-4 )</td>
</tr>
</tbody>
</table>

A. Aerodynamics of the BWB UAV at High Angle of Attack

We start the study by examining the flow around the baseline model and compare it with the surrogate predictions in an attempt to quantify the appearance of tip stall. On the one hand, the analysis of the chordwise component of the wall-shear stress (Fig. 5a) allows a qualitative assessment of progression of separation. Across the span and near the leading-edge a separated region is present. This corresponds with the separation bubble that is modeled by the \( \gamma - Re_p \) model. Furthermore, the region near the tip is fully separated, corresponding to the decreased lift seen at the tip in Fig. 5c which shows the spanwise lift distribution for a fixed elevon deflection with increasing angle of attack, with 0% representing the symmetry plane and 100% the tip. At \( AoA = 15^\circ \) the tip sees an abrupt decrease of local lift coefficient, which progresses inward with increasing angle of attack, while the region near the symmetry plane sees an increase of lift. On the other hand, the analysis of the pathlines on the surface (Fig. 5b) indicates the presence of a cross flow component building up towards the trailing edge and tip. The transverse flow becomes more severe with increasing \( AoA \) caused by a more severe spanwise pressure gradient accompanying the sweeping of the wing. This accumulates towards a thicker boundary layer near the tip and greater proneness to separation.
The characteristics predicted by the surrogates of the clean wing are presented in Fig. 6. This corresponds to a full factorial sampling of $\alpha$ and $\delta$ while the height $h$ of the fence is set to zero, which corresponds to the absence of a fence. The red line indicates the pitching moment equilibrium position. Up to $\alpha = 15^o$, $\delta$ increases with increasing $\alpha$ to meet $C_{M,pitch} = 0$ which indicates static longitudinal stability. However, between $\alpha = 15^o$ and $\alpha = 17^o$ it shows a fast decrease indicating instability attributed to the separation of the wing tips and accompanying occurrence of tip-stall (Fig. 5a, 5c). The surrogate of $C_L$ illustrates the well-known near linear increase with angle of attack up to stall, found around $\alpha = 15^o$ after which a gradual decrease follows. Furthermore noticeable and non-negligible is the decrease of lift coefficient with increasing elevon deflection (Fig. 6a moving from bottom to top). This validates the statement of its importance as design parameter in the parametric study. Finally, the roll moment coefficient’s behavior is similar to that of the lift coefficient, but slightly shifted to lower angles of attack: the roll moment coefficient is much more strongly influenced by the lift produced at the tip through its lever arm, thus a decrease of lift at the tip will present itself earlier in the roll moment coefficient, before becoming noticeable in the lift coefficient. Fig. 6 and Fig. 13 show a slightly
wobbly behavior of $C_{M,\text{pitch}}$. This is unphysical and mainly caused by the low-fidelity data. Increasing the regression coefficient could diminish this effect, but might filter out physical behavior, such as tip stall.

If we eliminate $\delta$ from the $C_L$ and $C_D$ through $C_{M,\text{pitch}} = 0$ we obtain the characteristics of the UAV as a function of $\alpha$. Furthermore, $\partial C_{M,\text{roll}}/\partial \delta$ can be evaluated as a function of $\alpha$ for $C_{M,\text{pitch}} = 0$. The results are presented in Fig. 7. While the maximum value of $C_L$ is found between $\alpha = 15^\circ$ and $\alpha = 16^\circ$ (Fig. 6b), it has slightly moved up to $\alpha = 16^\circ$ caused by the sudden decrease of elevon deflection that is required to maintain an equilibrium position following the occurrence of flow separation at the tip. From $\partial C_{M,\text{roll}}(\alpha|C_{M,\text{pitch}}(\alpha, \delta) = 0)/\partial \delta$ we observe a decrease in the lower $\alpha$-range with increasing angle of attack, attributed to the increasing elevon deflection. In the unstable region, the partial derivative of the roll moment is seen to have a strong increase. After which it strongly decreases as the elevon deflection increases. Noticeable is the parallel behavior between elevon deflection and partial derivative of the roll moment coefficient to the elevon deflection. Imposing the constraint of longitudinal static stability enforces the elevon deflection to increase with increasing angle of attack.
B. Unification of Theories for the Working of a Wing Fence on a BWB UAV

The first objective of this paper is the working principles of a wing fence when applied to a BWB UAV and unify the existing theories. This is achieved by comparing the initial design (ID) with a design optimized for stall speed $V$ and controllability $M'$ while maintaining stability, called the overall better design (OBD) (see Table 3) [38].

<table>
<thead>
<tr>
<th>Design</th>
<th>$\alpha$ [$^\circ$]</th>
<th>$\delta$ [$^\circ$]</th>
<th>$s$ [%]</th>
<th>$h$ [%]</th>
<th>$l_{up}$ [%]</th>
<th>$l_{down}$ [%]</th>
<th>$V$ [m/s]</th>
<th>$M'$ [$m^2/s^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ID</td>
<td>14.6</td>
<td>3.607</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>13.58</td>
<td>7.4186</td>
</tr>
<tr>
<td>OBD</td>
<td>15.185</td>
<td>5.008</td>
<td>48.85</td>
<td>79.13</td>
<td>7.13</td>
<td>13.50</td>
<td>12.87</td>
<td>13.36</td>
</tr>
</tbody>
</table>

The design parameter settings, stall speed and controllability coefficient of the ID and OBD are presented in Table 3. It can be clearly noted that the OBD has a respectively lower stall speed and higher controllability coefficient, outperforming the ID on both fronts. The chord-wise component of the wall shear stress is once more examined to evaluate the separation behavior (Fig. 8a). In comparison with the initial design (Fig. 5a) the separated region near the tip is gone, which proves the effectiveness of the wing fence altering the stall characteristics of the UAV.
The spanwise lift distribution in Fig. 8c shows that the lift reduces at the fence compared to a clean wing. However, at the tip the lift increases strongly thanks to the fence. This can be explained as follows. The sweeping of the wing backwards introduces a spanwise pressure gradient that leads to an outward component of the boundary layer on the suction side. On the other hand, it also introduces an inward flow component outside of the boundary layer on the suction side because the flow in front of the leading-edge is first subjected to the suction peak of the inboard side positioned closer. The consequence is that, while the fence is placed aligned with the chord, the part of the fence outside of the wing’s boundary layer can be seen as a small wing placed under an angle of attack, leading to the formation of a tip vortex (as depicted in Fig. 9a). On the pressure side of the wing a mirrored behavior is observed: the formation of a vortex on the outboard side of the fence rotating in counter-clockwise direction (see Fig. 9b) \{s = 0.53, h = 0.46, l_{up} = 0.87, l_{down} = 0.54\}, part of the DoE of Wauters et al. [38] using the $\lambda_2$-criterion*. This observation is confirm the observation by Rudenko & Ryzhkov [11] but contrary to the observations by Haines, Fozard

*The $\lambda_2$-criterion measures the excess of rotation rate over the strain rate magnitude by considering the second eigenvalue of the symmetric tensor $S^2 + \Omega^2$; where $S$ and $\Omega$ are the symmetric and antisymmetric components of the gradient of the velocity defined as $S = \frac{1}{2}(\nabla v + (\nabla v)^T)$ and $\Omega = \frac{1}{2}(\nabla v - (\nabla v)^T)$. The presence of a vortex translates itself in a negative value of $\lambda_2$.
and Williams et al. [12,14]. The origin of this difference lies in the formation of the leading-edge vortex due to the
sharp leading-edge and larger sweep angles in the settings investigated in the latter works and not present in this work.

The wing fence was originally designed by Liebe to stop the progression of stall from the root to the tip of the
wing. Since its introduction on swept wings it is typically addressed as a means to stop the spanwise boundary layer
flow component that leads to a thickening of the boundary layer at the tip. However, the OBD has a very short length
on the suction side of the wing (table 3). Fig. 8b shows the oil flow on the suction side of the wing at $\alpha = 15^\circ$ in
case of the OBD. While the fence does not extend to the trailing edge, the region of wing that is characterized by a
spanwise flow component is noteworthy smaller than the region of the initial design (Fig. 5b). This is caused by the
vortex that acts as an ‘aerodynamic barrier’. Selan & Bandettini experimentally found a similar result and attributed the
delayed separation to the re-energizing ability of the vortex by introducing air from the stream outside into the boundary
layer [45]. Completely abandoning the term boundary layer fence as proposed by Nickel & Wohlfahrt [10] is therefore
considered extreme.

![Vortex visualization around the fence using the $\lambda_2$-criterion at $\alpha = 10^\circ$.](image)

Fig. 9  Vortex visualization around the fence using the $\lambda_2$-criterion at $\alpha = 10^\circ$

At lower AoAs, the influence of the fence is minimal, conform Newson’s observations [7]. However, as the
AoA increases the apparent AoA (considered from the tip) under which the fence is placed when acting as a wing
itself increases. This leads to an increased suction peak on the inboard side (Fig. 10a, $\alpha = 5^\circ$ and $\alpha = 10^\circ$) and a
decreased suction peak on the outboard side (Fig. 10b, $\alpha = 5^\circ$). No change in pressure on the pressure side is observed.
Subsequently, the fence introduces a change in spanwise lift distribution and thus acts as a device to introduce an
aerodynamic washout, corresponding to the Nickel & Wohlfahrt formulation of the wing fence as ‘potential’ fence [10].
With increasing AoA, a region of separated flow appears on the inboard side of the fence near the leading-edge (Fig. 8a and Fig. 8c) conform Das his findings [9]. The separation of the inboard region results in an outward flow component, which translates itself in the formation of a vortex on the outboard side, also observed by Schlichting [9] and Haines [12]. However, this vortex is rather weak and of little importance (Fig. 11) $\{s = 0.46, h = 0.31, l_{up} = 0.51, l_{down} = 0.33\}$, part of the DoE of Wauters et al. [38], looking from upstream to the left side of the wing: wing tip to the left, center of the plane to the right.)
The outward flow next to the fence and the inward flow will meet each other creating additional suction. This was also a conclusion drawn by Williams [14]. However, in his case, the wing was characterized by a lifting vortex and the outboard vortex was already present before in the inboard side separated. As the AoA is increased further, the turbulent wake of the separated flow on the inboard side of the fence will lean over to the outboard side. This results in a strongly decelerated flow towards the trailing-edge and thus a more severe adverse pressure gradient towards the leading-edge. This aspect leads to leading-edge separation (Fig. [12]) pulling the remaining area outboard of the fence into separation. The wing fence is thus also a method to stop the progression of separation, for which it was designed by Liebe [9].
Fig. 12 Chordwise component of the wall-shear (red=positive, yellow=negative) at $\alpha = 17^\circ$ and $\delta = 3^\circ$

The characteristics of the OBD are presented in Fig. 13. It can be seen that the elevon deflection is characterized by a continuous growth with increasing angle of attack, which implies longitudinal static stability across the entire examined parameter range and the absence of tip stall. The constraint limits the characteristics of the ID to a maximum angle of attack approximately equal to $15^\circ$, while $C_{L,max}$ is found at $\approx 16^\circ$ and $(\partial C_{M,roll}/\partial \delta)_{max}$ at $\approx 17^\circ$ (Fig. 6a). This constraint is at all times met in case of the OBD such that the UAV can operate safely at higher AoAs and thus higher $C_L$ and $\partial C_{M,roll}/\partial \delta$. Between $\alpha = 15^\circ$ and $\alpha = 16^\circ$ a stronger increase of $\delta(\alpha \mid C_{M,pitch} = 0)$ is observed, caused by the appearance of a separation front on the inboard side of the fence, which progresses inward with increasing angle of attack conform the observations drawn by Haines [12].

In conclusion, it can be stated that a wing fence on a BWB UAV, characterized by the absence of a leading-edge vortex, acts as both a vortex generator inducing an aerodynamic barrier and as a device inducing an aerodynamic washout and a stall barrier at even higher AoAs. Unique to this specific application is the interaction with the elevon.
setting, which strongly influences the aerodynamic coefficients and thus performance and stability.

C. Global Sensitivity Study of Stall Speed and Controllability to Design Variables

The second objective of this paper, to examine the sensitivity of the stall speed and controllability to the design variables, is assessed through the calculation of the Sobol Indices \[46\] (using Saltelli’s pick-freeze method \[47\], see appendix). The extent of the influence is furthermore determined by the range of operating conditions, which might give a distorted view on the impact of the remaining geometry parameters. In the case of the velocity\[†\] with the angle of attack ranging from cruise to deep stall\[‡\] and elevon deflection between its maximal operational settings, the first order Sobol indices of the aforementioned operation parameters account for 92% of the variance. The total order index of the elevon deflection and angle of attack accounts for 96% of the variance, indicating that 4% of the variability of the objective is defined by the geometry parameters of the fence. By removing the total-order index\[§\] of the elevon deflection and angle of attack from the variance, we can assess the influence of the design parameters alone. The results are presented in Fig. 14a. From this we see that the influence of the position and the interaction between the position and the height on the one hand and the length on the suction side on the other hand are the most dominant contributors to the velocity.

In regard to the controllability, the first order indices of the angle of attack and elevon deflection contribute 51% of the variance. The former contributing most strongly (43%), followed by the interaction between the two (27%) and the latter (8%). This behavior follows from the physical explanation given for the clean wing: controllability is proportional to the square of the velocity, which in turn is mainly determined by the angle of attack and the interaction with the elevon deflection, as discussed above. Secondly, also noted was the increased magnitude of the derivative of the roll moment to elevon deflection, which is found in the controllability coefficient, with decreasing elevon deflection. Thirdly, the interaction is a consequence of the appearance of separation at the tip. Even when delayed through the addition of a fence, the outer part of the wing will separate with increasing angle of attack, at which point the elevon deflection must decrease to maintain longitudinal equilibrium and velocity must increase to uphold the vertical equilibrium, which leads to an increased controllability but longitudinal unstable state. Furthermore, the total order index of the elevon deflection and angle of attack accounts for 95% of the variance of the controllability, indicating that 5% of the variability of the objective is defined by the geometry parameters of the fence. On the other hand, this also implies that the interaction terms of the design parameters with the operation parameters are significant. By again removing the total-order index of the elevon deflection and angle of attack from the variance, we can assess the influence of the design parameters alone. The results are presented in Fig. 14b. From this we see that the influence of the position completely dominates the other

†By considering the velocity, calculated from the lift and drag coefficient, subjected to pitching equilibrium, calculated from the pitching moment coefficient, we assess the physical behavior of the UAV, rather than the decomposition of the forces, lift and drag, and moments, pitch and roll.

‡With ‘deep stall’, sometimes also referred to as ‘stable stall’ and ‘super stall’, we refer to the state past the stall angle where the UAV is again longitudinally statically stable \[48\].

§The total-order index corresponds to the sum of all indices containing the considered parameter: first order and higher order interactions.
parameters, as also experimentally observed by Schuringa [49].

In conclusion, it can be stated that the stall speed is characterized by a strong nonlinear dependence on the design variables, especially on the interaction between position and fence height and position and fence length on the suction side. On the other hand, the controllability is almost solely a function of position. We see this for example materialized on the Boeing EA-18G Growler and its Russian ‘counterpart’, the Sukhoi Su-34, which are used for electronic warfare and are both mounted with electronic countermeasure devices mounted on their wingtips. Shock induced separation leads to buffeting, which undermines the effectiveness of these devices. However, this can be countered by a well-positioned wing fence.
D. Surrogate-Assisted Study of Aerodynamic and Stability Characteristics as Function of Design Variables

The final objective of this paper is to gain an aerodynamic understanding the influence of each design parameter individually (corresponding to the first order Sobol indices) by varying each of them 10% around the overall better design and evaluating the characteristics that most strongly influence the objectives and constraints while keeping the others constant: $C_L(\alpha \mid C_{M,pitch} = 0)$, $\delta(\alpha \mid C_{M,pitch} = 0)$, $\partial C_{M,roll} / \partial \delta(\alpha \mid C_{M,pitch} = 0)$ and $C_L(s \mid \alpha = 15^\circ, \delta = 3^\circ)$. 

Fig. 14  Sobol indices of the geometrical parameters of the wing fence (with 1: spanwise position, 2: height, 3: suction side length, 4: pressure side length)
When examining the influence on the elevon angle as a function of the angle of attack when subjected to variations of the spanwise position of the fence (Fig. 15a), it can be observed that the outward movement of the fence leads to higher elevon deflections for $\alpha > 15^\circ$ and thus a stronger stability characteristic of the UAV. This can be understood from Fig. 15d: the outboard movement of the fence leads to a greater lift production near the tip. Since the tip is swept back, this introduces a stronger nose-down pitching moment which must be countered by a greater elevon deflection. The region outboard of the fence becomes smaller and the flow more strongly aligned with the chord, which leads to the outboard sections approximating more closely the better performing 2D-characteristics. Nevertheless, the increased elevon deflection slightly decreases $C_L$ (Fig. 15b).

However, the outward movement is limited: if the region of separation on the inboard side of the fence moves closer to the tip, it will eventually lead to destabilization as the separated flow is found behind the center of gravity and the
region outboard of the fence has become to small to compensate for this. The roll coefficient derivative (Fig. 15c) decreases across nearly the entire angle of attack range with outward movement, which implies that the increase of lift on the outboard section of the wing cannot make up for the increase of elevon deflection and loss of lift on the inboard section, both of which lead to a decrease in the derivative. Moving the fence inward leads to the appearance of an unstable region, here seen between $\alpha = 13^\circ$ and $\alpha = 16^\circ$ (Fig. 15a). This is caused by the inability of the fence to avoid the appearance of separation at the tip (Fig. 15d). On the other hand, the decreased elevon deflection will lead to a higher $C_{L,max}$ (Fig. 15b), but due to the constraint of static longitudinal stability, this setting is not of interest.

The analysis shows that a small region exists where the fence proves effective. Moving beyond the bounds of the region will not only lead to the appearance of tip stall, but may even lead to a design that falls behind the initial design.

2. Height

![Pressure coefficient distribution as a function of chordwise position for different fence heights](image)

**Fig. 16** Pressure coefficient distribution as a function of chordwise position for different fence heights

Increasing the height of the fence leads to a stronger pressure difference across the fence and stronger adverse pressure gradient on the inboard side of the fence (Fig. 16). This immediately implies that a minimal fence height is required in order for the inner side of the fence to separate before the tip. This can be seen from Fig. 17a: the smaller fence is noted for a region of instability between $\alpha = 15^\circ$ and $\alpha = 17^\circ$. From the spanwise lift distribution (Fig. 17d) it can also be deduced that increasing the fence height results in a greater effectiveness of delaying the separation of the tip. From Fig. 17b it can be seen that both an increase and decrease of fence height lead to loss of the lift coefficient. Consequently an ideal height exists for which the stability is maintained without an excessive loss of lift on the inboard side. An assessment of the roll moment derivative (Fig. 17c) is harder to make: the behavior is mainly determined by the elevon setting and the state of separation, the former a consequence of the latter. For example, at $\alpha = 20^\circ$ the largest fence has the smallest derivative, followed by the overall better design, the initial design and the smallest fence.
While the elevon deflection is lowest for the initial design followed by the smallest fence, the required elevon deflection to uphold the moment equilibrium is bigger in the presence of a fence. This is caused by the presence of a region of attached flow. The same region allows for a larger derivative to be obtained.

Fig. 17 Assessment of the influence of fence height
3. Length Suction Side

Since the OBD is noted for the near absence of a wing fence on the suction side, we examine here two designs characterized by a longer length. The increase of the fence’s length results in an extremely stable state up until $\alpha = 15^\circ$, after which the design becomes unstable until the end of the angle of attack range examined (Fig. 18a). The high elevon angle that is required results in a severe decrease of the corresponding lift coefficient (Fig. 18b) with the appearance of a second stall angle. This particular behavior shows how the fence seemingly splits the wing in two independent wing sections. The ability of the fence to avoid separation at the tip is not influenced by the length of the fence on the suction side (Fig. 18d). However, the increase of the fence length leads to an earlier and more severe separation on the inboard side of the fence and decreased lift on the outboard of the fence. The earlier separation on the inboard side of the fence can be attributed to the interaction of the boundary layers on the inboard side of the fence and the wing leading to corner separation. The shorter fence allows the flow on the outboard side to energize the flow on the inner side, ensuring an
attached flow up until a higher angle of attack.

Taking into account the observations drawn above in regard to the separation behavior on the inboard side of the fence: a smaller fence leads to a later separation on the inboard side. To maintain a similar trend, while decreasing the height of the fence, its length should increase.

4. Length Pressure Side

![Figure 19: Assessment of the influence of fence length on the pressure side](image)

The least dominant design parameter, the length of the fence on the pressure side, shows a particular behavior that appears counterintuitive when compared with the aforementioned: while the increase of the length results in an increase of the elevon deflection (Fig. 19a) it also leads to an increase of the roll moment derivative (Fig. 19c). Where the change of the length of the fence on the suction side influenced the separation behavior on the inboard side, increasing the length on the pressure side leads to an increase of the pressure peak on the outboard side of the fence (Fig. 20). This
results in an increased lift coefficient in the region outboard of the fence (Fig. 19d), which then again requires a higher elevon deflection and reduces $C_{M,max}$ (Fig. 19b).
Appendix A: Roache’s Grid Convergence Index

Roache’s Grid Convergence Index is based on a Richardson extrapolation of any quantity of interest as a function of the grid spacing, $h$, around the theoretical solution with an infinitely small grid ($h = 0$). The idea of a GCI is to accomplish an estimate of the relative error that would be obtained in case of grid doubling with a second-order method, even if the former was not performed during the study.

It is assumed that errors due to time-step convergence or round-off are much smaller and only to a minimal extent influence the solution. When the latter statement cannot be made, a GCI can be performed for each of the former and under the worst-case-scenario be assumed uncorrelated and summed.

The Richardson Extrapolation, also known as “$h^2$ extrapolation”, “the deferred approach to the limit” and “iterated extrapolation” can be formally written as a series representation of the discrete solution, in this case as a function of the grid size:

$$f = f[exact] + g_1 h + g_2 h^2 + g_3 h^3 + ...$$

With $g_1, g_2, ...$ defined in the continuum and independent on discretization. For infinitely differentiable solutions, they are related to all orders of the derivatives of the exact solution and reproduce the well known Taylor expansion. However, the latter is not a necessary assumption for the Richardson Extrapolation. Furthermore, an attractive aspect of the Extrapolation is that it applies not only to point-by-point solution values, but also to integrated quantities, such as the drag coefficient, $C_D$, provided that higher-order methods are used in the evaluation.

In case of a second-order discretization, $g_1 = 0$, two solutions, $f_1$ and $f_2$, with different grid sizing, $h_1$ and $h_2$ respectively the fine and coarse grid, allow an estimate of the exact solution. Furthermore, a grid refinement ratio, $r = h_2/h_1$ is introduced, which gives us

$$f[exact] = (h_2^2 f_1 - h_1^2 f_2) / (h_2^2 - h_1^2) + H.O.T.$$  
$$\approx f_1 + (f_1 - f_2) / (r^2 - 1)$$

with H.O.T. the higher order terms. The solution above is a third-order accurate estimate if second-order differences
are used, both centered and uncentered. The Richardson Extrapolation depends on the assumption of asymptotic convergence and thus that the considered grids are within the asymptotic range. For elliptic solutions, this is easy to obtain. However, for Reynolds numbers \( \gg 1 \) more than two grids are required to assess the validity of the statement.

In case of a constant refinement ratio: \( r = h_2/h_1 = h_3/h_2 \) with \( h_1 < h_2 < h_3 \) and corresponding discrete solutions \( f_1, f_2 \) and \( f_3 \) and the order of convergence, \( p \), can be determined from

\[
p = \ln \left( \frac{f_3 - f_2}{f_2 - f_1} \right) / \ln(r).
\] (8)

Hybrid grids, such as used in this particular case, are characterized by an unstructured refinement and even more important by a non-uniform grid size over the domain. Consequently, an effective grid refinement ratio is defined as

\[
r_{ef} = \left( \frac{N_1}{N_2} \right)^{(1/D)}
\] (9)

with \( N \) the total number of grid points used for the grid and \( D \) the dimensionality of the problem. However, this brings forth the difficulty of obtaining a uniform grid refinement. The latter can be overcome by solving the transcendental equation below

\[
p = \frac{\left| \ln(|f_{32}/f_{21}|) + q(p) \right|}{\ln(r_{21})} \quad \text{with}
\]

\[
q(p) = \ln \left( \frac{r_{21}^p - s}{r_{32}^p - s} \right)
\] (11)

\[
s = \text{sign}(f_{32}/f_{21})
\] (12)

with \( f_{ij} = f_i - f_j \) and \( r_{ij} = h_i/h_j \). In the ideal case it holds that \( p = 2 \), as presented in Equation (7). However, the value may deviate, caused by the higher order terms and the approximation of the grid refinement ratio with the effective one.

The generalized Richardson Extrapolation can now be rewritten as an error estimator of \( f_1 \):

\[
A_1 = E_1 + O(h^{p+1}, E_1^2)
\] (13)

Where \( A_1 \) is the actual fractional error defined as:

\[
A_1 = \frac{f_1}{f_{\text{exact}}} - 1
\] (14)

And \( E_1 \) is the estimated fractional error defined as:
\[ E_1 = \frac{\varepsilon}{r^p - 1} \quad (15) \]

With \( \varepsilon \) the relative error:

\[ \varepsilon = \frac{f_2 - f_1}{f_1} \quad (16) \]

It should be stressed at this point that \( \varepsilon \) should not be used as an error indicator because it does not take into account \( r \) or \( p \) and may lead to a completely wrongful interpretation of the former depending on the values of the latter. The Richardson Extrapolation and to a further extent the Grid Convergence Index account for this deficit. The latter is defined as

\[ GCI = \frac{F_x |\varepsilon|}{(r^p - 1)} \quad (17) \]

with \( F_x \) a safety factor, which accounts for difference between the actual and estimated fractional error. Roache proposed \( F_x = 3 \), such that in the case of grid doubling \( (r = 2) \) with a second-order method \( (p = 2) \): \( GCI = |\varepsilon| \). He did, however, recognize this as a very conservative estimate. When \( F_x = 1 \), we obtain \( GCI = E_1 \). Surely, a reasonable value will lie somewhere in between. We follow NASA in this matter, by taking \( F_x = 1.25 \) if three meshes have been used to establish the \( p \).

**Appendix B: Variance Based Sensitivity Analysis**

To assess the influence of the different design parameters on the outputs a variance based sensitivity analysis is performed. This method builds on the Sobol-Hoeffding decomposition \([46, 50]\) of the objective function under the assumption that the function is part of \( L_2(U^d) \), defined as the space of real-valued squared-integrable functions over the \( d \)-dimensional hypercube \( U^d \).

\[ f : x \in U^d \mapsto f(x) \in \mathbb{R}, \quad f \in L_2(U^d) \iff \int_{U^d} f(x)^2 dx < \infty \quad (18) \]

Any \( f \in L_2(U^d) \) has a unique hierarchical orthogonal decomposition of the form
\[ Y = f(x) = f(x_1, \ldots, x_d) = \]
\[ f_0 + \sum_{i=1}^{d} f_i(x_i) + \sum_{i=1}^{d} \sum_{j=i+1}^{d} f_{i,j}(x_i, x_j) + \sum_{i=1}^{d} \sum_{j=i+1}^{d} \sum_{k=j+1}^{d} f_{i,j,k}(x_i, x_j, x_k) + \ldots + f_{1,\ldots,d}(x_1, \ldots, x_d) \quad (19) \]

where hierarchical refers to the structure of the function with \( f_i \) being 1\(^{st}\)-order functionals, \( f_{i,j} \) being 2\(^{nd}\)-order functionals and so on. Orthogonal refers to the decomposition satisfying the following relations:

\[ \int_{\mathcal{U}}^{d} f_i(x) dx = 0, \quad \forall i \subseteq \mathcal{U}, \ j \in i, \quad (20) \]
\[ \int_{\mathcal{U} \mathcal{U}^d} f_i(x) f_j(x) dx = 0, \quad \forall i, j \subseteq \mathcal{U}, \ i \neq j, \quad (21) \]

introducing the ensemble notations: let \( \mathcal{D}^d = \{1, 2, \ldots, d\} \) and given \( i \subseteq \mathcal{D}^d \), we denote \( i_+ = \mathcal{D}^d \setminus i \) its complement set in \( \mathcal{D}^d \), such that \( i \cup i_+ = \mathcal{D}^d \) and \( i \cap i_+ = \emptyset \).

In view of a global sensitivity analysis, it is assumed that the objective function is a 2\(^{nd}\)-order random variable:

\[ f \in L^2(\mathcal{U}^d), \text{ depending on } x, \text{ which is considered as a set of } d \text{ independent random parameters uniformly distributed on } \mathcal{D}^d. \]

Because of the orthogonality of the decomposition the variance \( \mathbb{V}[Y] \) can be decomposed as

\[ \mathbb{V}[Y] = \sum_{i=1}^{d} V_i + \sum_{i=1}^{d} \sum_{j=i+1}^{d} V_{i,j} + \ldots + V_{1,2,\ldots,d}, \quad (22) \]

where \( V_i = \mathbb{V}_{X_i}[\mathbb{E}_{X_{-i}}[Y \mid X_i]] \). This corresponds to the calculation of the variance over all possible values of \( X_i \), where the mean value of \( Y \) is calculated over all possible values of \( X_{-i} \), while keeping the corresponding \( X_i \) fixed. This can be interpreted as the contribution to the total variance of the interaction between parameters \( x_i \). Similarly, the second-order conditional variance \( V_{i,j} \) is defined as \( \mathbb{V}_{X_i}[\mathbb{E}_{X_{-i,j}}[Y \mid X_i, X_j]] - V_i - V_j \) and expresses the variance of \( Y \) explained by the interaction of factors \( X_i \) and \( X_j \).

Normalization of the afore-defined with the total model variance produces the global variance-based sensitivity indices and more commonly known as the Sobol indices:

\[ S_i = \frac{V_i}{\mathbb{V}[Y]}, \quad \text{with} \quad \sum_{i=1}^{d} S_i + \sum_{i=1}^{d} \sum_{j=i+1}^{d} S_{i,j} + \ldots + S_{1,2,\ldots,d} = 1 \quad (23) \]

Homma & Saltelli [51] introduced an additional index, the total-order sensitivity index, \( S_{T,i} \), that accounts for
first-order plus all interaction contributions to the output variation due to factor $X_i$. Furthermore, taking into account the law of total variance \[52\]

$$\begin{align*}
V(Y) &= \mathbb{V}_{X_{-i}}[\mathbb{E}_{X_i}[Y \mid X_{-i}]] + \mathbb{E}_{X_{-i}}[\mathbb{V}_{X_i}[Y \mid X_{-i}]] \\
\text{(24)}
\end{align*}$$

we obtain

$$S_{T_i} = 1 - \frac{\mathbb{V}_{X_{-i}}[\mathbb{E}_{X_i}[Y \mid X_{-i}]]}{V(Y)} = \frac{\mathbb{E}_{X_{-i}}[\mathbb{V}_{X_i}[Y \mid X_{-i}]]}{V(Y)}.$$  \hspace{1cm} (25)

The physical significance of the indices above is found in their ability to assess the non-importance of a parameter: for $x_i$ to be deemed non-influential on the model-output, both $S_i$ and $S_{T_i}$ have to be negligible. Furthermore, it follows that: $0 \leq S_i \leq S_{T_i} \leq 1$.

While, the $1^{st}$-order sensitivity indices characterize the fraction of the variance due to parameter $X_i$ alone, thus without the interaction with any other parameter, the $2^{nd}$-order indices, $V_{i,j}$, represent the fraction of the variance due to the interaction of two parameters $X_i$ and $X_j$ alone. Typically, this is the only form of interaction screening that is performed, due to the computationally high budget of calculating the higher order indices (as discussed below). If we furthermore include any interaction of the two parameters with the other parameters, we obtain the closed index \[53\].

By analogy with the total-order index, the total interaction index, $TII$ can be defined as the superset importance of a pair of variables \[54\]. $TII_{i,j}$ can be understood as the variance of the output explained by the variables $X_i$ and $X_j$ simultaneously.

The difficulty lies in the assessment of these indices. Among the different methods that exist, there are spectral methods, Polynomial Chaos (PC) based methods \[55\] and Pick and Freeze schemes.

Among the spectral methods, estimation of the first order indices is typically performed with the Fourier amplitude sensitivity test (FAST). Here the sample points are chosen such that the amplitudes obtained by the Fourier analysis of the objective correspond to the indices. Extensions of this method, amongst others the Extended (E)FAST \[56\] and the Random Balance Design (RBD-)FAST \[53\], allow the computation of total indices. The PC method relies on the construction of a surrogate of the objective using polynomials, based on the input distribution. Again, the sample points are chosen specifically to determine the coefficients of the polynomials. These can then be used to determine the sensitivity indices. The pick and freeze scheme relies on numerical integration through sample methods, such as Monte Carlo, to solve the analytical definition of the indices.

The Pick and Freeze scheme \[57\] \[58\] relies on the lemma presented by Sobol (taken from \[59\]) \[57\]:
Lemma 1  Let \( U = (X, Y) \). If \( X \) and \( Y \) are independent:

\[
\nabla \mathbb{E}[Y | X] = \text{Cov}[Y, Y^X]
\]

with \( Y^X = f(X, Z') \), \( Y = (X, Y) \) where \( Z' \) is an independent copy of \( Y \)

Consequently, a Sobol index is viewed as the correlation coefficient between the output of the model and its pick-freeze replication. This replication is obtained by holding the value of the variable of interest (frozen variable) and by sampling the other variables (picked variables). This allows the derivation of estimations of the indices: following Saltelli’s notation [47], the estimation of \( V_i \), as derived by Sobol et al. [60] is given by:

\[
\nabla X_i [\mathbb{E}_{X_i} [Y | X_i]] = \frac{1}{N} \sum_{j=1}^{N} f(A)_j (f(B_A^{(i)})_j - f(B)_j)
\]

(26)

And an estimation of \( S_{Ti} \) according to Jansen [61] is given by:

\[
\mathbb{E}_{X,i} [\nabla X_i [Y | X_i]] = \frac{1}{2N} \sum_{j=1}^{N} (f(A)_j - f(B_A^{(i)})_j)^2
\]

(27)

Here \( A \) and \( B \) represent two independent sampling matrices and \( A_B^{(i)} \) corresponds to matrix \( A \) where the \( i \)-th column is replaced by the \( i \)-th column of matrix \( B \). To generate two independent matrices, a single \( N \times 2d \) sample matrix corresponding to the input distribution is generated by Monte-Carlo sampling or any quasi-Monte Carlo method such as Sobol sequence or Latin Hypercube sampling. The first \( d \) columns of the aforementioned matrix form matrix \( A \) and the remaining \( d \) columns form matrix \( B \).

References


[59] Grandjacques, M., Janon, A., Delinchant, B., and Adrot, O., “Pick and freeze estimation of sensitivity indices for models with dependent and dynamic input processes,” 2014. URL https://hal.inria.fr/hal-00963649
