Modified Padé-Approximant-Based Wide-Angle Beam Propagators

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I. INTRODUCTION

Efforts to improve the limitations of the paraxial approximation have so far made use of wide-angle formulations. Different treatments of wide-angle BPM (WA-BPM) based on the slowly varying envelope approximation (SVEA) have been developed. There exist rational approximants of the square root operator, conventional Padé approximant operators (referred to as Hadley-(m,n) approximant thereafter in this work) [1]. In addition, treatments of WA-BPM without having to make the SVEAs have also been reported, including the series expansion technique of the propagator, the split-step of beam propagation equation and the recently proposed rational KP approximant operators [2].

The Padé-approximant-based WA-BPM is one of the most commonly used techniques for modeling optical waveguide structures. It is a nonlinear expression in the form of a rational function *N(m)/D(n)*, a ratio of two polynomials in operator *P* [1], where m and n are the highest degree of *P* in the polynomials *N* and *D*, respectively.

However, as the denominator *D(n)* of the rational function of Hadley(m,n) approximant gradually approaches zero, its absolute values are thus indefinite. It physically causes the fact that the conventional Padé approximant incorrectly propagates evanescent modes leading to additional errors to the final solution. To circumvent this problem we proposed the so-called modified Padé approximant operators that not only give evanescent modes the desired damping but also allow more accurate approximations to the Helmholtz equation than conventional Padé approximants [3].

II. MODIFIED PADÉ PROPAGATORS

On the past decade, Hadley [1] proposed the WA-BPM algorithm based on the standard Padé approximation using the following recurrence relation with initial value of $\partial/\partial z\big|_0 = 0$:

$$\left.\frac{\partial}{\partial z}\right|_{n+1} = i\,\frac{\dfrac{P}{2k}}{1-\dfrac{i}{2k}\left.\dfrac{\partial}{\partial z}\right|_n} \qquad (1)$$

where $P = \nabla_\perp^2 + k_0^2(n^2 - n_{ref}^2)$, $k = k_0 n_{ref}$, n_{ref} the reference refractive index, k_0 the vacuum wavevector.

For $\partial/\partial z\big|_2$, this gives us the well-known Padé(1,1) approximant-based WA beam propagation formula as follows:

$$\frac{\partial H}{\partial z} \approx i\,\frac{\dfrac{P}{2k}}{1+\dfrac{P}{4k^2}}\,H \qquad (2)$$

If Eq. (2) is compared with a formal solution of Eq. (1) in [1] written in the well-known form

$$\frac{\partial H}{\partial z} = i(\sqrt{P+k^2} - k)H = ik(\sqrt{1+X} - 1)H \qquad (3)$$

where $X = P/k^2$, we obtain the approximation

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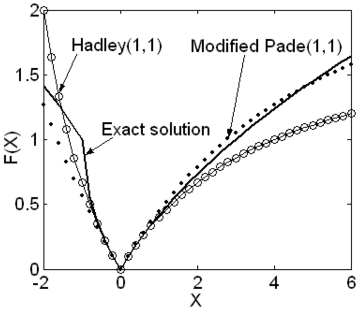
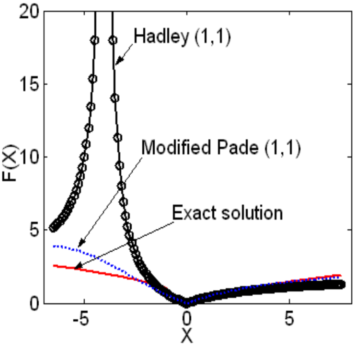
formula

$$\sqrt{1+X}-1 \approx \frac{\frac{P}{2k^2}}{1+\frac{P}{4k^2}} = \frac{\frac{X}{2}}{1+\frac{X}{4}} \qquad (4)$$

Since the operator X has a real spectrum, it is useful to consider the approximation of $\sqrt{1+X}-1$by the Padé approximant operator. Figure 1(a) shows the absolute value of $\sqrt{1+X}-1$ and its first-order Padé approximant called Hadley(1,1) as a function of X.



(a)



(b)

Figure 1. The absolute values of $\sqrt{1+X}-1$ (solid line), its first-order standard Padé approximant (solid line with circles) and modified Padé approximant (dot line).

However, as the denominator of Hadley (1,1) gradually approaches zero, its absolute value approaches ∞ as can clearly be seen in Figure 1(b). Physically this means that the conventional Padé approximant incorrectly propagates evanescent modes.

To circumvent this problem we proposed a modified Padé approximant operator by introducing a different initial value of $\partial/\partial z|_0 = -k\beta$ (β is a damping parameter) [4]. For $\partial/\partial z|_2$ the first-order modified Padé(1,1) approximant operator is given as follows:

$$\frac{\partial H}{\partial z} \approx i \frac{\frac{P}{2k}}{1+\frac{P}{4k^2(1+\frac{i\beta}{2})}} H \qquad (5)$$

The absolute value of the modified Padé (1,1) approximant of $\sqrt{1+X}-1$ is also depicted in Figure 1(a). It is seen that our modified Padé approximant operator (with β=2) allows more accurate approximations to the true Helmholtz equation than the conventional Padé approximant operator. Furthermore, the conventional rational Padé approximant incorrectly propagates the evanescent mode as their denominator gradually approaches zero while the modified Padé approximant gives the evanescent mode the desired damping as clearly seen in Figure 1(b).

Acknowledgements

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References

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