THE MAXENT METHOD FOR PROBABILISTIC STRUCTURAL FIRE ENGINEERING – PERFORMANCE FOR MULTI-MODAL OUPUTS

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ABSTRACT

Probabilistic Risk Assessment (PRA) methodologies are gaining traction in fire engineering practice as a (necessary) means to demonstrate adequate safety for uncommon buildings. Further, an increasing number of applications of PRA based methodologies in structural fire engineering can be found in the contemporary literature. However, to date, the combination of probabilistic methods and advanced numerical fire engineering tools has been limited due to the absence of a methodology which is both efficient (i.e. requires a limited number of model evaluations) and unbiased (i.e. without prior assumptions regarding the output distribution type).

An uncertainty quantification methodology (termed herein as MaxEnt) has recently been presented targeted at an unbiased assessment of the model output probability density function (PDF), using only a limited number of model evaluations.

The MaxEnt method has been applied to structural fire engineering problems, with some applications benchmarked against Monte Carlo Simulations (MCS) which showed excellent agreement for single-modal distributions. However, the power of the method is in application for those cases where 'validation' is not computationally practical, e.g. uncertainty quantification for problems reliant upon complex modes (such as FEA or CFD).

A recent study by Gernay, et al., applied the MaxEnt method to determine the PDF of maximum permissible applied load supportable by a steel-composite slab panel undergoing tensile membrane action (TMA) when subject to realistic (parametric) fire exposures. The study incorporated uncertainties in both the manifestation of the fire and the mechanical material parameters. The output PDF of maximum permissible load was found to be bi-modal, highlighting different failure modes depending upon the combinations of stochastic parameters. Whilst this outcome highlighted the importance of an un-biased approximation of the output PDF, in the absence of a MCS benchmark the study concluded that some additional studies are warranted to give users confidence and guidelines in such situations when applying the MaxEnt method. This paper summarises one further study, building upon Case C as presented in Gernay, et al.

INTRODUCTION

Probabilistic methods in fire safety engineering are increasing in popularity and application. A recent revision to PD 7974-7:2019[1] provides a framework for probabilistic risk assessment (PRA), and further emphasises that in some cases PRA may be the only appropriate means through which adequate safety is demonstrated. The increased recognition of PRA has led to research by the structural fire safety community in the context of reliability, including a recent study by Gernay, et al.[2], which applies a novel method for un-biased probabilistic fire safety engineering, requiring a limited number of model realisations[3]. Within this paper the method, which will be referred to henceforth as MaxEnt, is applied in extension to the studies by Gernay, et al., with a specific emphasis on evaluating the ability of the MaxEnt method to produce accurate approximations of probability density functions (PDFs) which may be multi-modal. Multi-modal behaviour can manifest for numerous reasons, e.g. differences in failure modes or bifurcations in event and fault trees. This paper introduces the MaxEnt method, provides a summary of some of the studies undertaken to date, and introduces a further study targeted at evaluating the quality of MaxEnt estimations when encountering multi-modal responses.

UNCERTAINTY QUANTIFICATION & STRUCTURAL FIRE SAFETY

Performance based design (PBD) for structural fire safety has gained significant traction as a means of satisfying legislative fire safety requirements. Traditional performance based (structural) fire safety design is deterministic in nature, requiring the selection of design inputs, scenarios, and performance criteria that are deemed appropriately conservative by the engineer. In such a process, the safety level (or residual risk) associated with a given design is not evaluated as the full spectrum of consequences and their associated probabilities are not interrogated. Instead, it is assumed that an adequate, but unquantified, level of safety is attained based upon engineering judgement and on the pretence that: (a) real fire events have occurred, with performance observed; and (b) that society has not expressed dissatisfaction with the levels of performance witnessed. That is, the basis for acceptance of traditional performance-based design (or the safety foundation) is the experience of the fire safety profession (see left hand side of Figure 1) proposed in Hopkin, et al.[4] and Van Coile, et al[5]. This safety foundation can only be justified where there are sufficient real fire events to observe, guide design processes, and offer society opportunity to express views on their dissatisfaction (or otherwise) of the consequences witnessed. However, traditional (structural) fire safety design, and its associated safety foundation cannot be extrapolated to exceptional structures, those with atypical consequences of failure, nor those adopting innovative materials, as it is likely that insufficient instances exist where fires have occurred and performance witnessed. For such complex cases, there is a need to explicitly evaluate the residual risk (see right hand side of Figure 1).

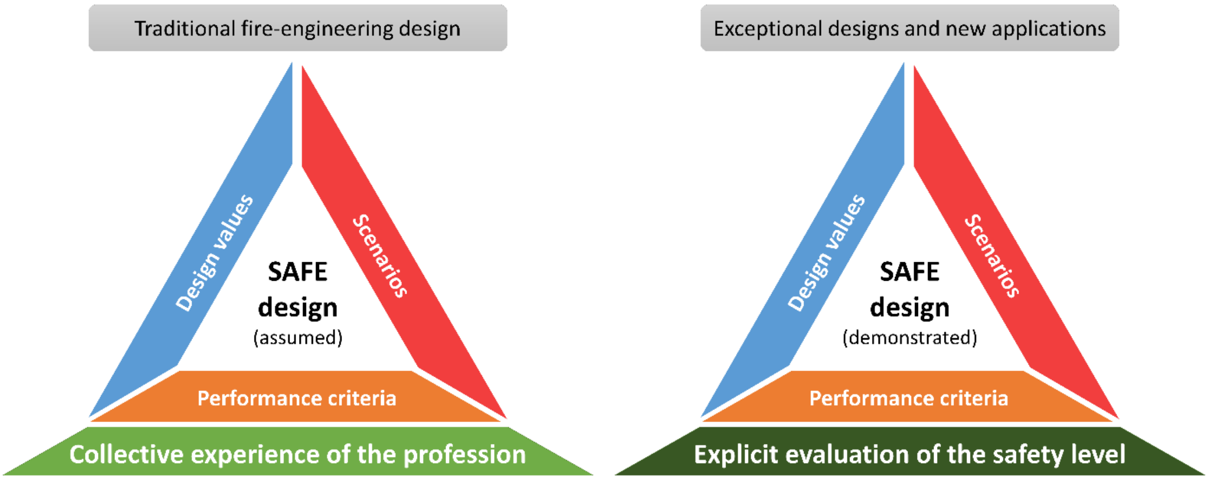

Figure 1. (left) assumed basis of safe design, (right) demonstrated basis of safe design where experience is not an adequate basis (Hopkin, et. al.[4])

Within the framework presented in PD 7974-7:2019, with background per Van Coile, et al.[5], there is an expectation that PRA methods be employed to demonstrate adequate safety for cases where the collective experience of the profession cannot be called upon to guide design approaches. In doing so, any design must be demonstrated to be tolerable to society, and the residual risk as low as is reasonably practicable (ALARP).

At a single element scale, the probabilistic response of a structural element exposed to fire can be evaluated using unbiased methods that require a high number of model realisations, e.g. Monte Carlo simulations. Studies are noted in the literature where such studies have been undertaken for developing fragility curves[6-8]. These approaches are viable where models are generally straightforward, and the associated ‘per realisation’ computational demand low. Increasingly, however, fire safety engineering calls upon complex tools to support design solutions. These complex tools are generally computationally expensive, meaning technological challenges exist with respect to their application in probabilistic frameworks, see Hopkin, et al[9]. In support of PRA applications in (structural) fire safety engineering, methods are being employed, such as Response Surface Modelling (see Van Weyenberge, et al.[10]) and the Maximum Entropy Multiplicative Dimensional Reduction Method (as per Van Coile, et al.[3], and referred to herein as ‘MaxEnt’). These seek to reduce computational demand by employing highly optimised numbers of model realisations in the process of computing an output probability density function (PDF). In the case of the MaxEnt, studies have been undertaken that verify the quality of optimised PDF estimate relative to ‘traditional’ unbiased MCS alternatives[2]. Generally, those results have shown good agreement between methods in the applications to date.

The MaxEnt Method

An uncertainty quantification methodology has recently been presented targeted at an unbiased assessment of the model output PDF, using only a limited number of model evaluations. The methodology has been modified from a calculation procedure proposed by Zhang and Pandey[11-12], which relies on the principle of maximum entropy. The principle of maximum entropy for stochastic output variables results in an analytical formulation for the estimated PDF and ensures that this estimate is the most unbiased estimate consistent with observed data when no prior knowledge on the shape of the PDF exists (e.g. no prior knowledge of a lognormal distribution[13]). Whereas a direct application of the maximum entropy principle can be used to estimate the PDF from a set of observed data, the concept presented by Zhang and Pandey takes advantage of the multiplicative dimensional reduction method and Gaussian interpolation to propose a very efficient calculation procedure (including sampling scheme) for estimating the PDF, f_Y, describing the output Y of a numerical model. It is this combination of methods which lends the calculation concept by Zhang and Pandey its name: the Maximum Entropy Multiplicative Dimensional Reduction Method, or ME-MDRM for short. Thus, the ME-MDRM returns an analytical formulation of an output PDF which is unbiased with respect to the fractional moments being assumed equal to the sample fractional moments. Opting for traditional distributions types such as the lognormal distribution may, however, still carry an advantage, both in communication and as a means towards standardization. The calculation procedure by Zhang and Pandey can be used to give a very efficient estimate of the parameters of such traditional (assumed) distribution types. Therefore, an aggregate step-wise calculation procedure has been proposed by Van Coile et al. for determining the PDF of a generic problem, opting for traditional distribution types such as the lognormal distribution when this is largely compatible with the maximum entropy result. This calculation procedure is further denoted as the MaxEnt method.

The MaxEnt Procedure

In Van Coile, et al.,[3] the ME-MDRM is introduced in detail starting from mathematical derivations. In the following (adapted from[14]) the same calculation procedure is directly introduced step-wise, focussing on procedural clarity in the calculation steps:

1. A deterministic model is developed describing the structural effect of interest. For example, a model capable of calculating the fire resistance time for a given set of input values;
2. Input variables with important uncertainty associated with their value, and which have (or are considered to have) a significant influence on the model output are identified and their stochastic distributions determined. The symbol n denotes the total number of stochastic variables;
3. For each stochastic variable X_l, 5 realizations $x_{l,j}$ are calculated through *Eq. (1)*, with the 5 ‘Gauss points’ z_j given in Table 1;
4. For each realization $x_{l,j}$, the model is evaluated using this realization for the variable X_l, and using the median value (i.e. $x_{k,3}$) for all other stochastic variables X_k, resulting in the model realization $y_{l,j}$. This implies 5 model realizations per stochastic variable, but as the model with all stochastic variables equal to their median value has to be evaluated only once (model realization y_0), the total number of model realizations is $4n+1$;

5. The minimization of *Eq. (2)* is performed, with $M_Y^{\alpha i}$ the α_i^{th} moment of Y evaluated from the $4n+1$ model evaluations through *Eq. (4)*, thus determining the parameters λ_i and α_i, with λ_0 a normalization constant calculated with *Eq. (5)*. The parameter m is the estimation order and can for practical purposes be set equal to 4. The evaluation of *Eq. (2)* is easily programmed, but the minimization result may depend on the starting solution. Therefore, the optimization is done in a step-wise approach:
   a. A large number of input values (i.e. Latin Hypercube samples) for α_i are generated. Without loss of generality, α_i can be chosen in the range [-2;2].
   b. For each set of α_i, the minimization of *Eq. (2)* is readily performed, resulting in corresponding values for λ_i.
   c. Across all realisations for α_i with associated minimizing values for λ_i, the set with the lowest function evaluation of *Eq. (2)* is maintained.
6. The estimated mathematical formulation for the PDF describing the model output Y is given by *Eq. (3)*. This mathematical formulation gives direct insight in the (estimated) shape of the PDF. *Eq. (3)* results from application of the Maximum Entropy principle (i.e. an acknowledgement of uncertainty with respect to the distribution shape).

$$x_{l,j} = F_{x_l}^{-1}\left(\Phi\left(z_j\right)\right) \quad \& \quad Minimize:\ \lambda_0 + \sum_{i=1}^{m} \lambda_i M_Y^{\alpha_i} \qquad (1 - 2)$$

where $F_{x_l}^{-1}(.)$ is the inverse cumulative distribution function for variable X_l

$\Phi(.)$ is the standard normal cumulative distribution function

$M_Y^{\alpha_i}$ is the a_i^{th} sample moment, calculated with *Eq. (4)*

λ_0 is a normalization constant calculated with *Eq. (5)*

$$f_Y(y) = \exp\left(-\lambda_0 - \sum_{i=1}^{m} \lambda_i y^{\alpha_i}\right) \quad \& \quad M_Y^{\alpha_i} \approx \left[y_0^{1-n}\right]^{\alpha_i} \prod_{l=1}^{n} \sum_{j=1}^{L} w_j y_{j,l}^{\alpha_i} \qquad (3 - 4)$$

where w_j is the Gauss weight associated with z_j (see *Table 1*)

$$\lambda_0 = \ln\left[\int_Y \exp\left(-\sum_{i=1}^{m} \lambda_i y^{\alpha_i}\right) dy\right] \qquad (5)$$

Table 1 - Gauss points z_j and associated Gauss weights w_j

	1	*2*	*3*	*4*	*5*
z_j	-2.857	-1.356	0	1.356	2.857
w_j	0.011257	0.222076	0.533333	0.222076	0.011257

Observed Bi-Modal MaxEnt Estimations

Gernay, et al., investigated the probabilistic response of a partially protected steel-composite slab panel undergoing tensile membrane action (TMA). The typology was influenced by a FEMA reference structure[15]. The ambient temperature design was according to ASCE 7-10[16] and the AISC Steel Construction Manual[17]. The panel was c. 9 m by 9 m on plan. The slab panel (Figure 2) was subject to both ISO 834 heating and Eurocode parametric fire heating, with the latter of primary interest herein.

The slab panel analysis was undertaken using SAFIR[18], with the output PDF sought for maximum permissible applied load. The stochastic variables comprised the fire load (input in the parametric fire model), the thermal conductivity of insulation (input in the thermal model), and the material strengths at ambient and elevated temperature (inputs in the structural model). For the 8 stochastic variables, the

MaxEnt procedure required 33 model realisations. Based upon these, the output PDF, CDF and cCDF are as indicated in Figure 3.

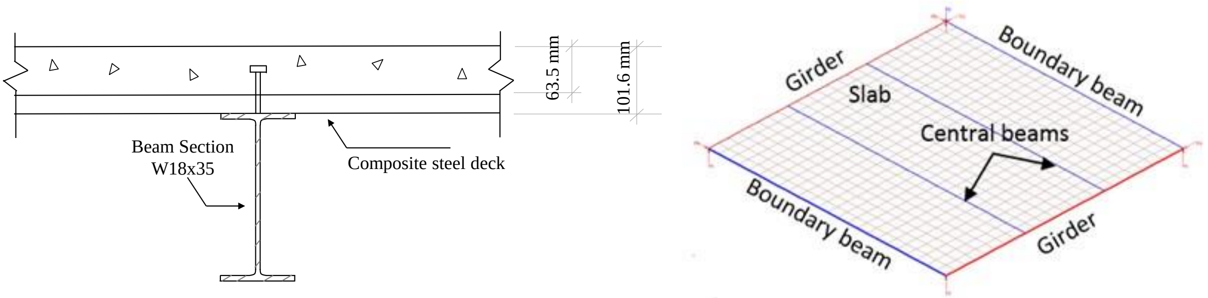

Figure 2. Steel-concrete floor: (a) cross section; (b) 3D view of the SAFIR numerical model[6]

As is shown in Figure 3, the MaxEnt procedure predicts a multi-modal (specifically bi-modal) distribution, which can be explained by the occurrence of different failure modes (slab failure vs. edge beam failure). In this case, assuming a log-normal distribution would not be adequate as is also indicated by a log-normal approximation; supporting that an unbiased method is needed. Critically, however, due to computational expense, the multi-modal response could not be separately verified by MCS or similar. Given the very premise and motivations for adopting the MaxEnt procedure, this will be a situation frequently encountered (computational constraints meaning independent MCS verification is impracticable). Therefore, it was concluded in Gernay, et al., that care should be taken when such behaviours might be anticipated in advance, and further analysis warranted when a multi-modal PDF is output by the MaxEnt method.

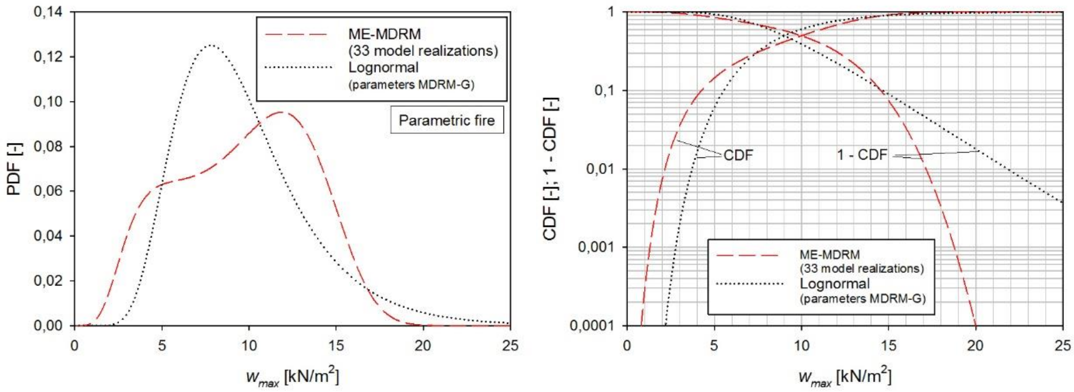

Figure 3. Minimum uniformly distributed load resulting in structural failure under Eurocode parametric fire. Comparison of ME-MDRM (33 simulations) and LN approximation (MDRM-G). (a) PDF; (b) CDF and complementary CDF.

FURTHER INVESTIGATIONS INTO MULTI-MODAL APPLICATIONS

Within Gernay, et al., a further application case was introduced (Case C), which conceives of a potential design problem where ‘a priori’ multi-modal behaviour is expected. The application was developed from the work presented in Hopkin, et al.[7], where an insulated structural steel element is subject to natural fires, idealised via either a traveling fire method[19] (TFM) or post-flashover (Eurocode parametric[20]) fire model. The transition between the two models is led by the stochastic variables influencing the ‘unit area burn-out time’ and fire spread rate. Geometry and inputs were chosen to as to lead to ‘severe’ heating when a parametric fire was instigated and less onerous heating when a travelling fire manifested. The clear bi-modal nature of the output is shown based upon MCS, considering 1000 realizations obtained through Latin Hypercube Sampling (LHS), as indicated in Figure 4. As is also apparent in Figure 4, the standard MaxEnt estimation based upon 17 model realizations (fire load density, spread rate, glazing failure percentage and near field temperature) was a relatively poor match.

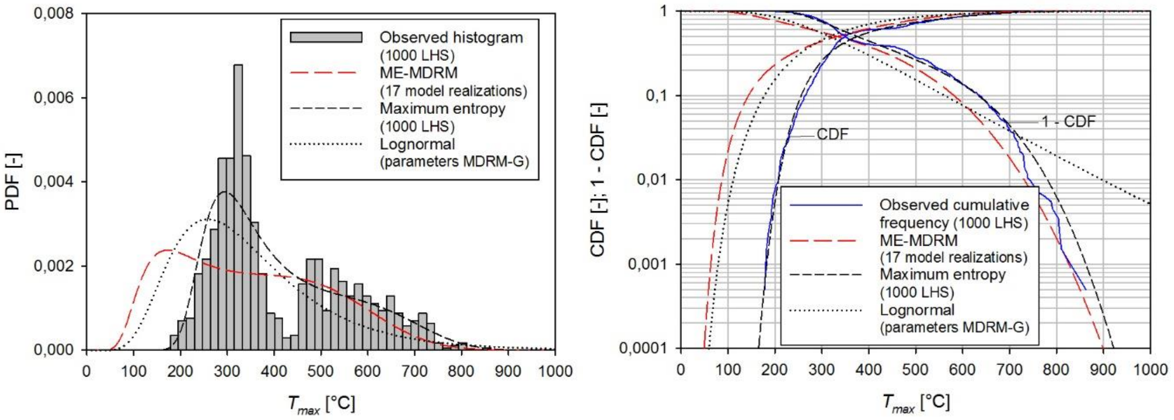

Figure 4. Maximum temperature in a protected steel element. Comparison of 1000 LHS with ME-MDRM (17 simulations), maximum entropy estimate using the 1000 LHS as input, and lognormal approximation. (a) PDF; (b) CDF and complementary CDF[2]

The ‘a priori’ expected performance of the standard MaxEnt procedure could be expected to be poorer given the stochastic variables and the relationship with the fire models. That is, for a parametric fire, near field temperature and spread rate have no influence. For a travelling fire, the ventilation condition has no bearing on the outcome, meaning only the fire load density variable was common across both fire modes. A benchmark assessment applying the optimization of Eq. (2) with all 1000 LHS realizations as input gave better results (dashed line), but still did not match the observed histogram.

Revised Bi-modal Heat Transfer Study – Overview and Inputs

In a revision to Case C presented in Gernay, et al., fire model modes have been modified to ensure utilisation of all stochastic variables irrespective of fire model choice. Per the previous evaluation, the maximum temperature of a protected steel element in a fire compartment is evaluated, when subject to a combination of travelling fires and post-flashover fire models, with the transition between fire models as previously defined in the literature[7]. This is conducted both based on MCS (adopting the tool defined in Fu, et al.[21]) and the MaxEnt procedure. In lieu of the Eurocode parametric fire, the proposals adopted in the NA to DIN EN 1991-1-2[22], as proposed by Zehfuss and Hosser[23], are adopted.

The time-temperature model of Zehfuss and Hosser requires a prior computation of the relationship between time and fire heat release rate (HRR), necessitating inputs for fire growth rate (α), heat release rate density (RHR_f) and fire load density (q_{fd}). These inputs are correspondingly present in the definition of a travelling fire time-temperature curve, albeit with spread rate adopted (s) in lieu of growth rate. In this case, the variables (α and s) are separately correlated as defined in Hopkin[19], assuming circular fire spread from a point origin.

Deterministic and stochastic variables adopted in the study are as defined in Table 2. Given the 3 stochastic variables in Table 2, the MaxEnt sampling regime requires 13 model realisations for the revised case, as defined in Table 3. For the MCS, the distributions are sampled adopting Latin Hypercube Sampling (LHS).

Monte Carlo Simulation Results

The PDF of maximum temperature attained by a protected steel element within an enclosure, with parameters as defined in Table 2, has been calculated based on 5,000 MCS. The resulting sample count vs. max attained temperature is shown in Figure 5, with separate plots indicated for travelling fires, DIN parametric fires and combined fires. Given the 5,000 samples, c. 53% are travelling fires vs. 47% DIN parametric fires. Relative to Figure 4, the bi-modal nature of the outcome is less pronounced.

Table 2. Stochastic and deterministic parameters

Input and units	**Distribution type**	**Mean**	**Standard dev.**	**Min.**	**Max.**
s - spread rate [m/s]	Uniform	N/A		0.00073	0.047
RHR_f – heat release rate density (MW/m^2)	Normal	0.4	0.1	N/A	
Q_{fd} – fire load density (MJ/m^2)	Gumbel	420	126	N/A	
Combustion efficiency [-]	Deterministic	1.0	N/A		
Max. near field temperature [°C]		1200			
Room breadth and depth [m]		12			
Room height [m]		4			
Total window width [m]		15			
Total window height [m]		3			
Room thermal inertia, b []		1500			
Section factor [m^{-1}]		150			
Protection thickness [mm]		18			
Protection conductivity [W/m.K]		0.2			
Protection density [kg/m^3]		800			
Protection specific heat [J/kg.K]		1700			
Beam location relative to ignition point [m]		9.6			

Table 3. MaxEnt sample combinations

Simulation	**s [m/s]**	**RHR_f [MW/m^2]**	**q_{fd} [MJ/m^2]**	**Simulation**	**s [m/s]**	**RHR_f [MW/m^2]**	**q_{fd} [MJ/m^2]**
1	0.01040	0.400	399.29	8	0.01040	0.535	399.29
2	0.00183	0.400	399.29	9	0.01040	0.680	399.29
3	0.00330	0.400	399.29	10	0.01040	0.400	184.78
4	0.01749	0.400	399.29	11	0.01040	0.400	275.80
5	0.01896	0.400	399.29	12	0.01040	0.400	598.10
6	0.01040	0.114	399.29	13	0.01040	0.400	967.37
7	0.01040	0.264	399.29				

Initial MaxEnt Estimation

For each realisation in Table 3, the maximum temperature attained by the steel element, as subject to MCS previously, is evaluated. The specific combination of inputs defines if the outcome is based upon a travelling fire vs. post-flashover fire model. Further, the samples in Table 3 are evaluated based upon all fires manifesting as travelling fires and all post-flashover fires. Results are as indicated in Table 4. For the combined case, 7 of the 13 samples manifest as travelling fires, with the remaining post-flashover fires. Adopting the MaxEnt optimisation procedure (with an estimation order of 4) and based upon the 13 combined fire model samples, Figure 6 compares the MCS and MaxEnt output PDFs, CDFs and cCDFs. Generally, there is good agreement in respect of the PDF range and peak. However, the MaxEnt procedure does not forecast a multi-modal response in this instance. Further, the MaxEnt procedure produces a log-normal outcome (indicating a reduced version of the procedure could be adopted, based upon predetermined bias).

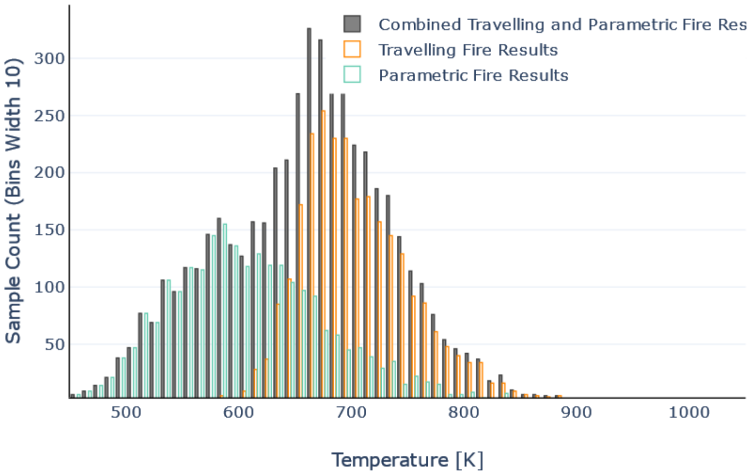

Figure 5. Maximum steel temperature vs. sample count for 5,000 LHS (showing travelling fire, DIN parametric fire and combined results)

Modified MaxEnt Estimation

Where an ‘a priori’ estimated of the likelihood of a given mode materialising can be made, a modified MaxEnt estimation can be achieved through separate evaluations of each mode. With reference to the MCS presented previously, there is a c. 50:50 likelihood of either a travelling or post-flashover fire materialising (the inputs were predetermined to deliver this). As such, the PDF for each fire mode can be estimated from the MaxEnt procedure, using samples corresponding to each fire model (as defined in Table 4; TF and PF, respectively), scaled and then combined.

Figure 7 indicates a comparison of the MCS and MaxEnt estimation of the CDF and cCDF for each fire mode. In this instance, the MCS instances corresponding to each fire mode have been isolated for direct comparison. Qualitatively, it can be said that the MaxEnt estimation is more closely aligned to the MCS for the post-flashover model, with a larger (albeit not significant) deviation noted for the travelling fire case.

Table 4. MaxEnt sample results – Max. steel temperature for combined, travelling fire (TF) and post-flashover (PF) fire in [°K]

Simulation	**Combined**	**TF**	**PF**	**Simulation**	**Combined**	**TF**	**PF**
1	667.9*	667.9	601.3	8	679.5*	679.5	551.9
2	710.1*	710.1	646.0	9	704.5*	704.5	495.8
3	718.5*	718.5	604.2	10	648.9*	648.9	463.6
4	600.2	621.5	600.2	11	648.9*	648.9	525.4
5	600.6	617.1	600.6	12	705.5	733.3	705.5
6	579.1	679.7	579.1	13	854.3	854.0	854.3
7	600.2	679.0	600.2	* Indicates travelling fire sampled, remaining are post-flashover			

Figure 8 compares the MCS results with the modified MaxEnt estimation, where each mode PDF is scaled and summed. In the case of the PDF, both the original and modified MaxEnt estimations are given. The modified procedure indicates a better (qualitative) fit with respect to the PDF shape, indicating an atypical distribution shape and suggestion of multi-modal behaviour (as is to be expected). The base MaxEnt estimation shows an improvement in estimation of the temperature with maximum likelihood. Relative to Figure 6, there is an improvement in correlation between the MCS and modified MaxEnt estimation at the distribution extremes (typically where fire performance is of most interest).

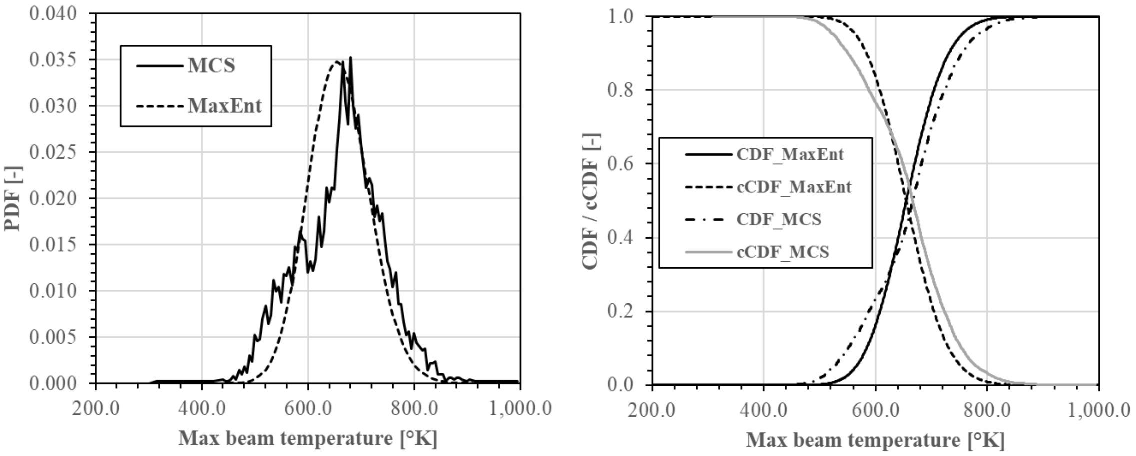

Figure 6. Maximum steel temperature PDF (left) and CDF (right) based upon 5,000 MCS vs. MaxEnt estimation (4th order)

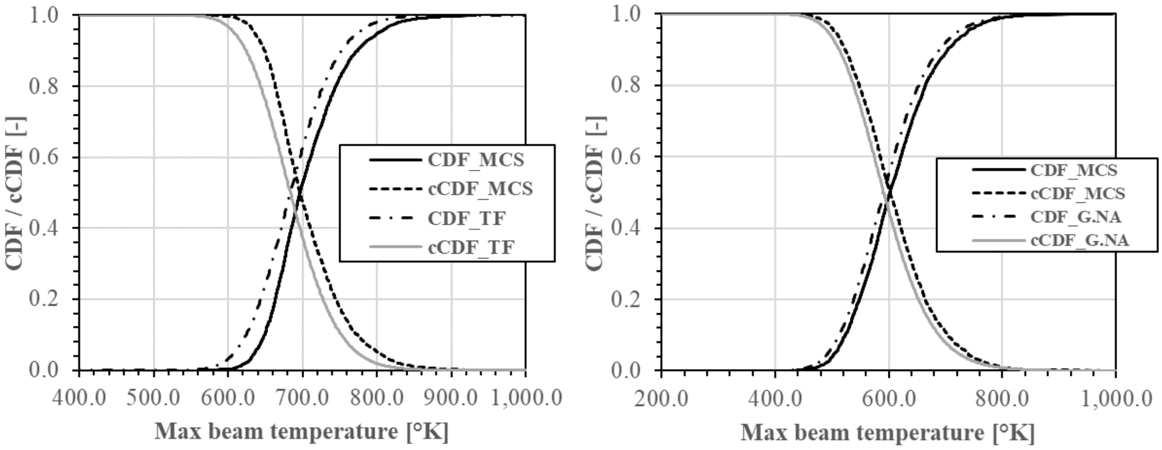

Figure 7. Maximum steel temperature CDF and cCDF (left) travelling fire only (right) post-flashover fire only – Comparison of MaxEnt (4th order) and MCS

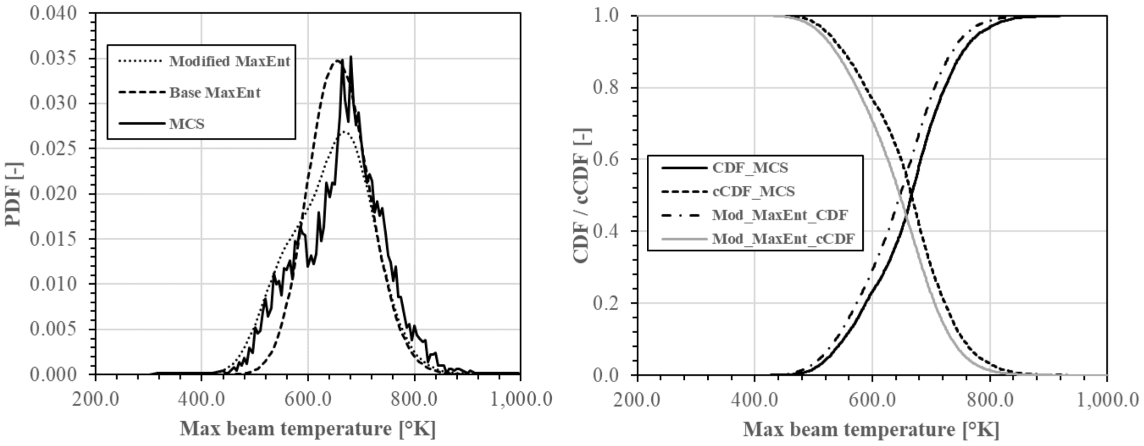

Figure 8. Maximum steel temperature (left) PDF (right) CDF / cCDF – Comparison of base MaxEnt (4th order), modified MaxEnt procedure and MCS

DISCUSSION

Previous studies[2] have shown the potential effectiveness of the MaxEnt method for uncertainty quantification in conjunction with advanced SFE modelling tools. Previous studies[2] have also illustrated the importance of applying an unbiased method in all situations where the shape of the response distribution is not known beforehand.

The standard MaxEnt method as proposed in Van Coile et al.[3], however, has limitations. The case study herein further examined the ability of the MaxEnt method to accurately predict a bi-modal output. In previous studies[2], it was confirmed that the method detects irregular PDF shapes (and hence the importance of the unbiased feature of this method), but the degree of correlation with MCS was poor (Figure 4). Building upon this previous study and noting the limitations of the case as reported in Gernay, et al., a further bi-modal study has been presented whereby it was ensured that stochastic variables have the potential to affect all modes of interest. Figure 6 indicates an improvement in the quality of MaxEnt estimation relative to that previously evaluated. The bi-modal output PDF from MCS suggests a less severe bi-modal distribution (i.e. only one clear peak in the PDF). As such, the MaxEnt approximation tends towards a lognormal distribution (i.e. also of singular peak in PDF). A MaxEnt evaluation with a larger number of Gauss integration points or alternative sampling scheme may lead to better results but would increase computational expense.

A modified procedure is presented on the pretence that the probability of a given mode can be forecast in advance. This requires separate MaxEnt estimations for each mode, leading to separate PDF estimates that must be scaled and amalgamated (Figure 8). This has resulted in (qualitatively) improved consistency in the PDF shape when assessed relative to that arising from MCS.

It should be noted that the MaxEnt method estimates the PDF describing the distribution of a positive scalar model output. Consequently, the method is not directly capable of assessing probabilities for binary or discrete output parameters. The nature of the application herein results in a bifurcation in the choice of fire models, each of which has a given probability. Given this, the base (combined) MaxEnt application results are better than might be anticipated and the modified approach is considered a more appropriate means of treating such bifurcations.

CONCLUSIONS

Based upon this study and others presented in the literature, i.e. Van Coile, et al.[14], and Gernay, et al.[2], the following conclusions are noted:

- PRA is an essential, and in some cases the only, means through which adequate safety in structural fire engineering can be demonstrated;
- Structural fire engineering analyses often necessitate the adoption of computational expensive models, such as FEA;
- Studies have illustrated the importance of applying an unbiased method in all situations where the shape of the response distribution is not known beforehand;
- An efficient un-biased (MaxEnt) method for probabilistic fire safety is presented in the literature and this has been applied to structural fire engineering computations, with good agreement when assessed relative to more typical MCS;
- The standard implementation of the MaxEnt method can forecast atypical PDF shapes and has in previous studies forecast that PDFs may be multi-modal;
- Simplified studies, the like of that herein, have sought to evaluate the ability of the MaxEnt to forecast multi-modal responses. The method, as is demonstrated, can forecast the presence of a multi-modal output, albeit the ‘goodness of fit’ could be improved;
- Herein, a more conscientious selection of stochastic inputs has been undertaken to ensure variables influence all modes and that each mode is evenly represented in the MaxEnt sampling

regime. Based upon this, a relatively good estimation of the PDF for the temperature of a protected beam is presented, albeit the shape is traditional (log-normal);

- A modified procedure is applied premised upon prior knowledge of the likelihood of a given fire mode manifesting. Based upon this, two MaxEnt estimations are made and the subsequent output variable PDFs combined. The outcome is an amalgamated PDF more closely reflecting the bi-modal results apparent in MCS;
- It is noted that the method is not directly capable of assessing probabilities for binary or discrete output parameters; and

  The results indicate that caution is advised when the method identifies a bi-modal distribution; the authors recommend investigation of the reasons underlying any irregular PDF shape and that the MaxEnt sampling scheme be carefully defined.

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