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Parametric Uncertainty Quantification Using Polynomial Chaos Expansions Applied to a Wet Friction Clutch Model

Georges Tod, Wannes De Groote, Tom Lefebvre, Nele De Geeter, Bruno Depraetere, and Guillaume Crevecoeur

***Abstract*—The design of mechatronic systems relies more and more on models that are used to predict the performance in a virtual environment. The models involved are increasingly more complex multiphysical systems. Instead of spending more time modeling on increasingly detailed physical models, uncertainty can be explicitly considered to model the lack of knowledge. The mismatch between real life experiments and model simulations due to parametric uncertainty can be quantified by a likelihood estimation and Monte-Carlo sampling techniques for propagation. This paper proposes to significantly accelerate the process by using Polynomial Chaos Expansions for the propagation and a Genetic Algorithm to maximize likelihood. The soundness of the approach is demonstrated on a wet friction clutch system. The results show the method has a strong potential of scalability with respect to the number of uncertain parameters.**

***Index Terms*—Polynomial chaos expansions, system identification, uncertainty quantification, wet clutch.**

NOMENCLATURE

α	Constant for clutch plates' geometry and dynamic friction coefficient
β	Fraction of $z_{contact}$ at which $P(t)=x_1(t)$
γ	Constant for clutch plates' geometry and the fluid viscosity
$\Omega(t)$	Angular velocity of the input shaft
ω	Angular velocity of the output shaft
ω_m	Motor speed
a, k, τ	Current bias, gain and time delay
a_2, b_2, c_2, d_2	Oil pressure to piston position transfer function coefficients
aa	Oil pressure bias in piston position computation
$f_{reg}(t)$	Regularization function between torque transfer phases
f_{skTC}	Torque converter capacity factor function
f_{stTC}	Torque converter torque ratio function
g_r	Gear ratio between the output shaft of the clutch and the load shaft
J_i, T_{ic}, b_{ic}	Input shaft inertia, Coulomb friction torque and viscous friction coefficient
J_o, T_{oc}, b_{oc}	Output shaft inertia, Coulomb friction and viscous friction coefficients
k_T	Gain associated to oil temperature
$P(t)$	Clutch plates contact pressure
$u(t)$	Current
$x_1(t)$	Oil pressure inside the chamber
$x_2(t)$	Oil pressure inside the chamber time derivative
$x_3(t)$	State used to compute the piston position
$z(t)$	Piston position
$z_{contact}$	Maximum displacement of the piston

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I. INTRODUCTION

The design of products relies more and more on computational models that are used to predict the performance in a virtual environment. It allows the design process to be shorter resulting in faster design iterations that lead to more robust and reliable products at lower production costs. In addition, a trend in product design shows an interest from the industry to deliver smarter and more automated products to their customers. These trends translate into the need to deliver as fast as possible models of products that can predict design performance. In this context, the models involved are usually those of mechatronic systems which are increasingly more complex multiphysical systems. To validate such models, these are usually compared to experimental results. However, a mismatch will always remain between real life experiments and model simulations. If despite the mismatch the model can still predict trends acceptably enough, the model might still be considered to be useful [1]. If the model predictability is not high enough, there are two main approaches to improve its performance,

- spend more time to understand the underlying physics and build a more detailed physical model
- represent the lack of knowledge explicitly to compensate for model structure and parameters that cannot be easily determined

The first approach is the most comprehensive one but cannot be achieved in reasonable time when the system is too complex. The second approach gives a chance to find a balance between the modeling effort and the model usefulness. However, the representation of uncertainty in the model structure or parameters will result in uncertain predictions: the challenge is therefore to quantify the sufficient uncertainty that allows to predict experimental observations.

In this paper, the strong assumption is made that the model structure is known and that the uncertainty is only of parametric nature and probabilistic. The uncertainty that is assumed to remain in a given model is due to the difficulty or impossibility to measure some physical parameters such as for example friction coefficients. Additionally, it is assumed that the uncertainty structure is fixed, and only its parameters, such as mean and standard deviation (for a normal distribution), need to be identified. Sampling techniques such

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as Monte-Carlo methods [2] have then been extensively used for the parametric uncertainty propagation. Such an approach to propagation involves sampling values for each uncertain parameter from their uncertainty range, then evaluating a model using this parameter set and repeating the process many times to obtain a probability distribution of the model output. The accuracy of the result depends on the number of runs that are performed and in general a large number of them is usually necessary. As a result, this approach is computationally expensive. More structured sampling techniques such as Latin hypercube sampling [3], adaptive sampling [3] and importance sampling [4] have been proposed with modest computational speed improvements for complex systems. More recent works have shown that functional expansions such as Polynomial Chaos Expansions (PCEs) [5]-[7] can lead to significant gains in term of computational cost [6]. When dealing with complex models, the use of PCEs can be facilitated by non-intrusive PCEs [7], by allowing to perform the propagation without the need to modify the model itself. In this work, the model parametric uncertainty is represented using normal distributions. In addition, a term (zero-mean normally distributed) is added at the output of the model. Other distributions could have been employed without any loss of generality of the approach. The originality of this investigation lies in the use of PCEs for the propagation of parametric uncertainty inside a Maximum Likelihood Estimation (MLE) [8], [9] loop, see Fig. 1. The goal of the approach is to find the sufficient parametric uncertainty a model needs to include, to explain different experiments [10]. To maximize the likelihood of an experiment being predicted by the model a Genetic Algorithm (GA) is used to select the parameters of the chosen uncertainty structure (mean and standard deviation). As GAs are based on heuristic rules, a global maximum cannot be guaranteed to be found. However, the method is particularly useful for complex problems in which developing a more detailed physical would be too time consuming and for simulation models that are computationally expensive.

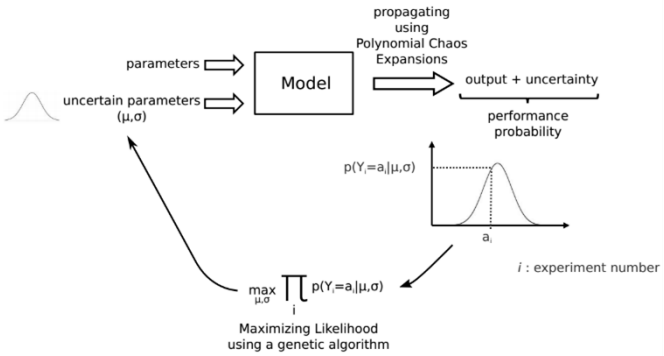

Fig. 1. Proposed approach for uncertainty quantification.

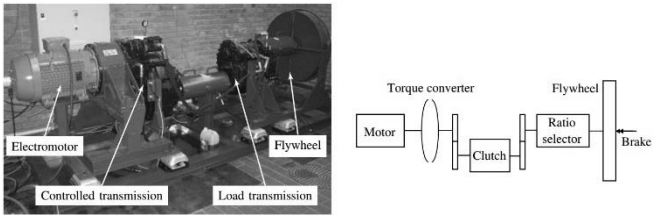

Fig. 2. Test bench of a wet friction clutch system.

As a proof of concept, the approach is applied to a wet friction clutch system, see Fig. 2. Wet friction clutches are hydraulic-mechanical devices used to transmit torque from their input shaft to their output shaft by means of friction. They are used in various types of automatic transmissions to selectively engage gear elements. They are used for off-road vehicles and agricultural machines where high torques are transmitted. For our investigation, a test bench was used, see Fig. 2.

To illustrate the proposed approach for uncertainty quantification, the paper starts with a section about the experiments that were performed and how an initial deterministic model of the physical system was derived and how it performs. The subsequent section describes the theoretical foundations of the approach and numerical results in application to the wet friction clutch.

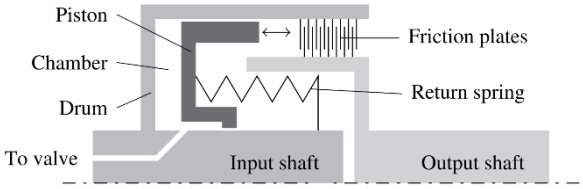

Fig. 3. Wet friction clutch system.

II. Wet-Clutch Experiments and Model

The following section describes the experiments that were performed on the test bench Fig. 2 and how a physical model of the system was derived.

A schematic cross-section of a wet clutch is shown in Fig. 3. As explained in [11], its input shaft is connected to a hollow cylinder with internal grooves, called the drum. A first set of friction plates (clutch plates) with external toothing can slide in those grooves, while a second set of friction plates (clutch discs) with internal toothing can slide over a grooved bus connected to the output shaft. Torque is transferred between the shafts by pressing both sets together with a hydraulic piston, realized by sending a control signal to the servo-valve in the hydraulic line to the clutch. When this is done, the clutch chamber first fills up with oil and the pressure builds up until it is high enough to compress the return spring and move the piston towards the friction plates, see Fig. 4. This is called the filling phase, and it ends once the piston advances far enough and presses the plates together such that torque transfer commences. At this moment the slip phase begins and the system dynamics change considerably, yielding strongly non-linear system behavior [12], [13]. The difference in rotation speeds between the in- and output shafts, denoted the slip, then decreases until both shafts rotate synchronously. A good engagement is obtained when torque transfer starts as soon as possible without introducing torque peaks, which can be realized by a short filling phase and a smooth transition into the slip phase. The pressure in the line of the clutch, the rotational speeds of the in- and output shafts as well as the torque of the output shaft torque are measured.

A. Experiments

In total 216 experiments are performed, and the oil pressure, angular velocities and output shaft torque are recorded. Between experiments, variations of the control signal, the motor speed, inertia and friction of the load are

performed, see Fig. 2. A classic key performance indicator (KPI) of interest in industry is the shifting time from neutral to first gear of the automatic transmission, see Fig. 4. The KPIs obtained experimentally are reported on Fig. 5 and the difference between experimental and simulation results Δt is the one on Fig. 5. Even though 4 parameters could change during the experiments, some important quantities remain incontrollable and not measurable on the test bench such as the oil temperature dynamics and the piston position. The piston position is generally not measured on industrial machines which makes the case closer to an industrial application.

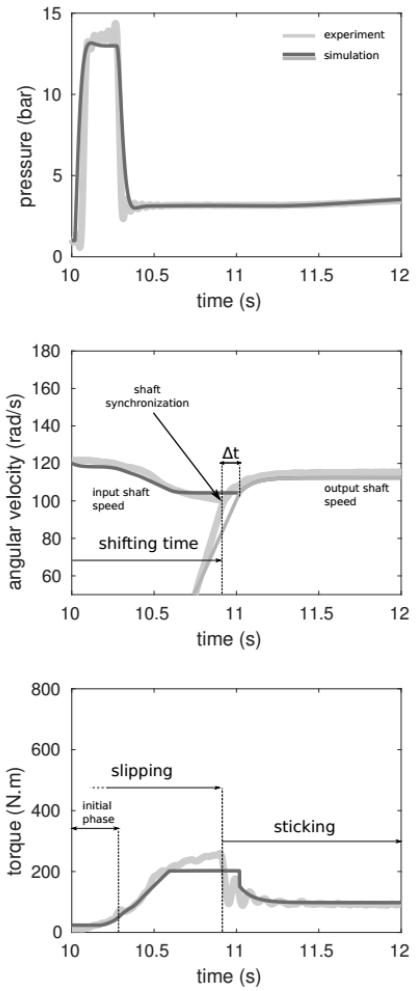

Fig. 4. Wet friction clutch system dynamics.

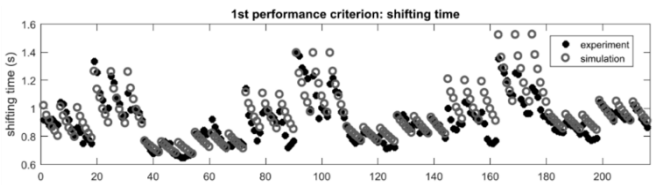

Fig. 5. Experimental versus deterministic simulation results

B. Physical Modeling

To predict the performance using a physical model, the test bench system is virtually decomposed into subsystems. The motor is idealized by assuming it always delivers a predefined speed and required torque. The torque converter is modeled using a torque ratio function $\mathrm{f_{stTC}}\left(\frac{\omega_{\mathrm{input}}}{\omega_{\mathrm{output}}}\right) = \frac{\mathrm{T_{output}}}{\mathrm{T_{input}}}$ and a capacity factor function $f_{skTC}\left(\frac{\omega_{output}}{\omega_{input}}\right) = \frac{\omega_{input}}{\sqrt{T_{input}}}$ provided by the manufacturer. The load of the system is composed of the ratio selector, the flywheel and the brake, see Fig. 2. The ratio selector is a gearbox that allows to vary the inertia, while the brake allows to vary the friction load seen by the clutch.

In addition, a state-space model identified based on previous work [14] was used to represent the dynamics from the current input signal to the piston position in the chamber,

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = -\frac{6}{\tau^2}.x_1 - \frac{4}{\tau}.x_2 - \frac{6.a.k}{\tau^2} + \frac{6.k}{\tau^2}.u - \frac{2.k}{\tau}.\dot{u} \\ \dot{x_3} = \frac{b_2}{c_2}.x_1 + \frac{a_2}{c_2}.x_2 - \frac{d_2}{c_2}.x_3 - \frac{aa.b_2}{c_2} \\ z = min\left(z_{contact}, max(0, k_T.x_3)\right) \\ P = max\left(0, \frac{\frac{z}{z_{contact}} - \beta}{1-\beta}\right).10^5.x_1 \end{cases} \quad (1)$$

The wet friction clutch model is decomposed according to the three torque phases of Fig. 4. The initial torque phase is the phase where the friction plates are not yet in contact but as there is oil between them, some torque is transferred between input and output shafts. This phase is modeled based on planar Couette flow assumption and leads to the equation below,

$$J_o.\frac{d\omega}{dt} = \frac{\gamma}{z}.(\omega - \Omega) \quad (2)$$

where J_o and ω are the inertia and speed of the output shaft, z is the distance between the friction plates, Ω is the speed of input shaft and γ is a constant that captures the geometry of the plates and the fluid viscosity. The slipping phase is the one where the plates enter in contact. This is modeled as it would be assumed for a dry friction clutch,

$$\begin{cases} J_i.\dot{\Omega} = -b_{iv}.\Omega - T_{ic} - f_{reg}.\alpha.P \\ +\omega_m^2.\frac{f_{stTC}\left(\frac{\Omega}{\omega_m}\right)}{f_{skTC}\left(\frac{\Omega}{\omega_m}\right)^2} - (1-f_{reg}).\frac{\gamma}{z_{contact}-z}.\left(\omega - \frac{1}{g_r}\Omega\right) \\ J_o.\dot{\omega} = -b_{ov}.\omega - T_{oc} \\ +f_{reg}.g_r.\alpha.P + (1-f_{reg}).\frac{g_r.\gamma}{z_{contact}-z}.\left(\omega - \frac{1}{g_r}\Omega\right) \end{cases} \quad (3)$$

In the sticking phase, plates are in contact, input and output shaft speeds become equal and as soon as the required torque is not too high, the transferred torque becomes oil pressure independent,

$$\begin{aligned} &J_i + \frac{J_o}{g_r^2}.\dot{\Omega} = -b_{iv}.\Omega - T_{ic} \\ &+\omega_m^2.\frac{f_{stTC}\left(\frac{\Omega}{\omega_m}\right)}{f_{skTC}\left(\frac{\Omega}{\omega_m}\right)^2} - \frac{1}{g_r^2}.b_{ov}.\Omega - \frac{1}{g_r}.T_{oc} \end{aligned} \quad (4)$$

The equations are implemented in MATLAB and solved using a solver for nonstiff differential equations (ode45).

C. Experimental Vs. Deterministic Simulation Results

The model contains parameters such as viscous friction coefficients, Coulomb friction coefficients that were identified using specific experiments in steady state. Inertias were estimated based on known masses, materials and geometry approximations. Other parameters, such as current bias, gain & delay, oil temperature, pressure bias for position computation, clutch friction coefficient and initial piston position cannot be measured and are considered as uncertain. As a first approximation, these parameters are fitted by nonlinear least-squares using the experimental results. A comparison between the experimental and the simulation results can be found on Fig. 5. The difference between experimental and simulation results Δt is the one on Fig. 5. As mentioned in the previous paragraphs, the oil temperature can hardly be measured and there is no piston position sensor on our setup. The lack of knowledge of these dynamics is a source of uncertainty that the method proposes to quantify. To reduce the influence of oil temperature dynamics, all experiments were performed after a warm-up of the machine that guarantees some identical initial conditions in terms of temperature. It is an assumption that the uncertainty of the model is of only parametric nature. Future work will investigate how to include model structure uncertainty as well.

III. Parametric Uncertainty Quantification

The parametric uncertainty quantification is done accordingly to the approach of Fig.1. To verify the approach, the results will be compared to a densely sampled quasi Monte-Carlo (for the propagation) and exhaustive grid search (that maximizes likelihood) that we call "classic" throughout the rest of the paper. This classic approach allows to brute force the solution and can be very effective when the number of uncertain parameters is not too high or the computational cost of evaluating the model is very low. The proposed approach in this paper allows to overcome those two limitations. The main originality being that for such class of problems PCEs are applied for the propagation of parametric probabilistic uncertainty. This section presents some theory about PCEs and the validation of the proposed approach on the wet friction clutch model described in the previous section.

A. Propagation Using Polynomial Chaos Expansions

As determined by the architecture in Fig. 1, we require an efficient means to quantify the probability that performance, y_i, of experiment i, as obtained through simulation, takes the same value as the experimentally measured performance, a_i, and that for a specified input uncertainty. Mathematically, we desire to evaluate the conditional output probability density function, $p_{Y_i}(a_i|\beta), where$ is a parameter set that fully determines the input uncertainty.

To that end we will engage the Polynomial Chaos Expansion (PCE) framework [15]. The PCE framework offers an efficient and accurate tool to quantify and propagate uncertainty in nonlinear context and is found computationally superior over brute force Monte Carlo (MC) approaches. The propagation of input uncertainty to the output space is realized by modelling the explicit forward deterministic model through a polynomial series expansion in function of the random parameters. When the polynomial basis functions are chosen to be orthonormal w.r.t. the input probability distribution, optimal approximation conditions are established to propagate uncertainty to the output space for that specific input distribution [15]. Numerical details are given next.

According to the framework, any sufficiently smooth forward model $y(\cdot)$ can be represented as an infinite polynomial series expansion [16].

$$\mathrm{y}(x) = \sum_{i=0}^{\infty} c_i \phi_i(x) \longrightarrow y_{(d)}(x) = \sum_{i=0}^{n_p} c_i \phi_i(x) \qquad (5)$$

Here c_i represent the polynomial coefficients and $\phi_i(\cdot)$ a range of polynomial basis functions that can be used to write any polynomial. For practical purposes we consider the n_d-th order approximation, $y_{(d)}(\cdot)$, that is obtained by omitting any higher order polynomials from the series expansion and thus omitting any polynomial basis function $\phi_i(\cdot)$ of order $> n_d$. When the basis is ordered properly this corresponds with truncating the series after the n_p-th term. The latter value is determined by the polynomial order, n_d, and the input dimension, n_x, as $n_p = \frac{(n_d+n_x)!}{n_d!n_x!}$. The polynomial order required to represent a given forward model accurately, is determined by the model's nonlinearity and input dimension. Exact error bounds in function of the polynomial order are given in [17]. The error vanishes for $d \to \infty$.

Now consider that the input parameter x corresponds with a stochastic variable X. We are interested in quantifying the stochastic behaviour of the output variable $Y = y(X)$. As mentioned, $y_{(d)}(\cdot)$ is suited optimally to quantify the uncertainty propagation to the output space when the polynomials are orthonormal w.r.t. the probability distribution of X. A polynomial basis is orthonormal with respect to the arbitrary probability distribution $p_X(\cdot)$ when the following condition holds, here $\mathcal{X}$ represents the value set of the input uncertainty.

$$\langle \phi_i, \phi_j \rangle = \int_{\mathcal{X}} \phi_i(x)\phi_j(x)p_X(x)\,dx$$

Several standard probability distributions are associated to known univariate orthonormal polynomial bases by the Wiener-Askey scheme [15], Table I. A multivariate orthonormal basis is easily constructed from a set of univariate orthonormal bases under the sole assumption that the stochastic input variables are independent. How to construct such a basis is detailed in appendix.

In this work two random input variables are considered, and it is assumed that both are distributed normally. Hence, we can express these two input variables as a linear transformation of the standard normal random variables, θ_1 and θ_2. According to the Wiener-Askey scheme, the multivariate Hermite polynomials, H_i, are optimal to construct the polynomial series (see appendix). Remark that the forward model then becomes a function of the standard

normal random variable, θ, invoking the transformation, $x(\theta|\beta)$, that is fully determined by the parameter, $\beta = (\mu_1, \mu_2, \sigma_{11}, \sigma_{22}, \sigma_{12})$.[1]

$$\mathrm{x}(\theta|\beta) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}; \theta_1, \theta_2 \sim \mathcal{N}(0,1)$$

The coefficients, $c_i(\beta)$, in function of the transformation parameters are calculated by projection of the forward model on the polynomial space exploiting the inner product definition. Remark that indeed all other terms vanish due to the orthonormality of the multivariate Hermite polynomials, $H_i(\cdot)$, w.r.t. the multivariate normal distribution, $\varphi(\cdot)$ (consult appendix for details).

$$\begin{aligned} c_i(\beta) = \langle y(x(\cdot|\beta)), H_i \rangle &= \int_{\Theta} y(x(\theta|\beta)) H_i(\theta) \varphi(\theta) \mathrm{d}\theta \\ &\approx \sum_{j=1}^{n_q} y\left(x(\theta_j|\beta)\right) H_i(\theta_j) w_j \end{aligned} \tag{6}$$

In this work quadrature rules were used to approximate the resulting integral. A quadrature rule is determined by a number of collocation points θ_j and collocation weights w_j. A unique optimal quadrature rule is associated to each polynomial basis covered by the Wiener-Askey scheme [17]. For one dimensional input dimension a quadrature set of n_q points is exact for polynomials up to degree $2n_q - 1$. Hence the quadrature order determines the number of coefficients that can be retrieved correctly and therefor the accuracy of the polynomial approximation. For multivariate input, full tensor products can be used. We wish to emphasize that characterization of the expansion in (5) still requires to perform a total of n_q forward model evaluations. In general, however, the number of collocation points required to optimally approximate the forward model is far less than the number of MC samples that is required to accurately quantify the output uncertainty, justifying the computational efficiency of PCE over MC.

In conclusion, recall that we were interested in the conditional output density function with respect to the transformation parameters β. In order to evaluate the probability density function of the output variable, a number of MC simulations can be performed on the polynomial in (5) using the coefficients obtained by (6), therefor omitting any further direct evaluations of the forward model.

TABLE I: WIENER-ASKEY POLYNOMIAL CHAOS

distribution	polynomials	support
Gaussian	Hermite	$[-\infty, \infty]$
Gamma	Laguerre	$[0, \infty]$
Beta	Jacobi	$[-1,1]$
Uniform	Legendre	$[-1,1]$

B. Application and Numerical Results

To validate the proposed approach, see Fig. 1, it is applied to the wet friction clutch model presented in Section 2. As previously stated, the model contains 7 uncertain parameters but as a first step, only 2 uncertain parameters are considered: pressure bias and initial piston position. To vary those two uncertain parameters, nominal pressure bias is multiplied by x_1 and nominal initial piston position by x_2. To avoid the cost of repeating computations of the model several times, the model performance (shifting time from neutral to 1st gear) is stored in a look-up table for 50×50 variations of $x_1 \times x_2$ between 0.5 and 1.5 for the 216 experimental conditions. The 540k simulations are ran on a supercomputer and stored in a lookup table that replaces the model block in the approach Fig. 1. Future work will also investigate the use of surrogate models to replace, when relevant, this step of the approach.
It is assumed that the two uncertain parameters are normally distributed, but other distributions could have been chosen without any loss of generality; the impact on the PCE would be on the kind of polynomials to be used [5].

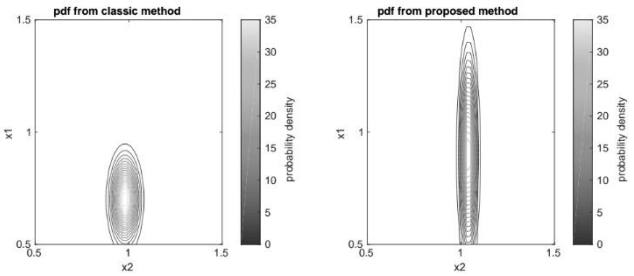

Fig. 6. Quantified uncertainty on inputs.

For a given input multivariate normal distribution of x_i, the propagation through the model and the addition of an additive output uncertainty term (zero mean Gaussian) allows to obtain a performance probability distribution. Covariance terms were removed as they appeared to have no significant effect in this case. The likelihood of the given uncertainty $(\mu_{x_1}, \mu_{x_2}, \sigma_{x_1}, \sigma_{x_2}, \sigma_{output})$ is then computed as the product of the probability of the simulation model to be the experimentally measured performance (Δt_i from data) for the i^{th} experimental scenario,

$$L(\mu, \sigma) = \prod_{i}^{216} p(Y_i = \Delta t_i | \mu, \sigma) \tag{7}$$

The higher $L(\mu, \sigma)$ is, the more the simulation results are likely to represent the data obtained experimentally. In the classic approach, the maximization of L is found by evaluating the likelihood of a very large number of combinations of means and variances. The maximum decimal log-likelihood is -269 and is plotted on Fig. 6. For a given uncertainty structure, this is the best result that can be found, and it is used as a benchmark of the proposed approach. The figure 6 shows the region of x_i's that best fits the data. Since x_2's mean is close to 1, the nominal parameter appears to be an acceptable approximation. The probability density functions for all experiments are presented on Fig. 7 including the experimental results (black dots).

In comparison, the proposed approach gives an approximation of the previous solution. The maximum decimal log-likelihood is found to be -273. An important downside of the approach is it cannot achieve the exact same performance as the brute force approach. This is due to the fact PCEs introduce a model approximation and that a GA is used to replace the grid search which cannot guarantee the solution is a global optimum. However, the so-called classic

[1] When neglecting input correlation, we can disregard σ_{12} by definition.

brute force approach is not scalable with the number of uncertain parameters due to its computational cost.

For the proposed approach, the grid search of uncertain inputs is replaced by a genetic algorithm that dramatically reduces the number of necessary model evaluations. In addition, the PCEs accelerate the propagation of uncertain parameters and leads to an interesting solution, see Fig. 6.

TABLE II: COMPUTATIONAL COST OF EACH METHOD PERFORMED ON A DESKTOP PC (8XCORE I7-6900K - 128 GB RAM)

Method	Computational cost
Classic: quasi Monte-Carlo + grid search	≈ 277 hours
Proposed: PCEs + GA	≈ 1.65 hours

Due to the low cost of the computation, see Table II, scalability is a great potential of the method. In future work, the accuracy of the result Fig. 6 could be improved by refining the search method after this initial solution.

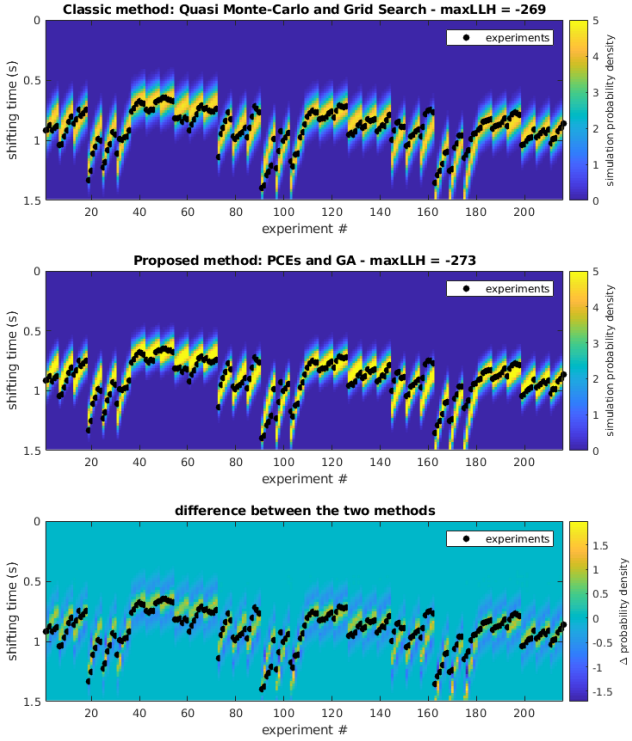

Fig. 7. Simulation performance probability versus measured performance

IV. CONCLUSIONS AND FUTURE WORK

An approach using Polynomial Chaos Expansions for the propagation, and a Genetic Algorithm for the selection of the uncertainty structure parameters, is presented to quantify the sufficient parametric uncertainty a given model should include in order to explain experimental results. It is illustrated by applying it to the complex problem of predicting the shifting time of an automatic wet friction clutch system. The method is validated by comparing the obtained solution to the one obtained by brute forcing the problem. The results show the method gives an approximation of the solution at a very interesting computational cost. The proposed approach has therefore a great potential of scalability with respect to the number of uncertain parameters.

Future work will investigate the inclusion of not only parametric but also model structure uncertainty.

APPENDIX

Hermite Polynomials and Multivariate Bases

The univariate Hermite polynomials, $H_i(\cdot)$, are *orthogonal* w.r.t. to the univariate standard normal distribution, $\mathcal{N}(0,1)$, wich probability density function is given by $\varphi(\theta) = \sqrt{2\pi^{-1}}e^{\frac{-\theta^2}{2}}$. In the main text we mentioned that the Hermite polynomials are *orthonormal* meaning that the inner product not only vanishes when applied on different Hermite polynomials but also normalizes when applied on the same Hermite polynomials. Note that within a normalization factor both definitions are equal and hence we opted merely to mention the normalized version so to ease the readability of the section. It follows that

$$\langle H_i, H_j \rangle = \int_{-\infty}^{\infty} H_i(\theta) H_j(\theta) \frac{1}{\sqrt{2\pi}} e^{\frac{-\theta^2}{2}} \mathrm{d}\theta = \delta_{ij}$$

In the univariate case a basis, $H_1^{n_d}$, for the univariate polynomial space of at most degree n_d is spanned by the first $n_d + 1$ Hermite polynomials, $\{H_0, \ldots, H_{n_d}\}$. By construction this basis is orthogonal w.r.t. the univariate standard normal distribution. A basis, $H_{n_\theta}^{n_d}$, for the n_θ-variate polynomial space of at most degree n_d can be generated from the n_θ univariate bases, $H_1^{n_d}$. The basis vectors are defined as $H_{|i| \leq n_d} = \prod_{k=1}^{n_\theta} H_{i_k}(\theta_k)$, with multi-index $i = (i_1, \cdots, i_{n_\theta})$ and where $|i| = \sum_{k=1}^{n_\theta} i_k$. For notational convenience we further exploit the bijection that exists between the multi-indices i that satisfy $|i| \leq n_d$ and the integer index i taking values in $\left(1, \ldots, \frac{(n_\theta + n_d)!}{n_\theta! n_d!}\right)$. Remark that this basis is indeed orthogonal w.r.t. the multivariate standard normal distribution, defined as $\varphi(\theta) = \prod_{k=1}^{n_\theta} \varphi(\theta_k)$.

$$\langle H_i, H_j \rangle = \prod_{k=1}^{n_\theta} \int_{-\infty}^{\infty} H_{i_k}(\theta) H_{j_k}(\theta) \varphi(\theta_k) \mathrm{d}\theta_k = \prod_{k=1}^{n_\theta} \delta_{i_k j_k} = \delta_{ij}$$

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