Defining ALARP for fire safety engineering design via the Life Quality Index

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Fire, Probabilistic Risk Assessment, ALARP, cost-benefit analysis, LQI

Abstract

Fire safety engineering designs should comply with the ALARP requirement, meaning that fire safety measures should be implemented to ensure that fire risks are As Low As Reasonably Practicable (ALARP). However, previous approaches have often been contradictory or unclear, especially with respect to the concepts of risk aversion, the valuation of risk to life, the concept of disproportionality, and the need to take into account monetary benefits resulting from life safety investments. First, the As Low As Reasonably Practicable (ALARP) criterion is specified to a societal, risk-neutral, and scalar cost-benefit analysis in the field of fire safety engineering. This is done to overcome the shortcomings in current practice with respect to utilizing the ALARP criterion for Probabilistic Risk Assessment (PRA).

Second, the ethical dilemma of the valuation of risk to human lives is addressed by application of the Life Quality Index (LQI), as has been suggested in the field of structural engineering through the recent ISO 2394:2015 standard. Contrary to the existing literature, the proposed interpretation for the ALARP requirement corresponds to an application of the LQI maximum societal benefit criterion. This ensures that safety resources are allocated so as to maximize the number of lives saved, and implies that the ALARP criterion results in stronger minimum requirements for life safety investments, i.e. societal lower bound investment levels.

Third, the valuation of costs and benefits by the fire safety engineer is addressed by emphasizing the duty of care, and by specifying a benchmark criterion for the objective, diligent, and competent fire safety professional. This reinforces self-reflection amongst fire safety professionals, and diminishes the need for often unjustified or poorly specified disproportionality factors.

Finally, the relationship between fire safety design and private actor cost-optimisation considerations is discussed, and the presented concepts are illustrated using three examples. These clarify the application of the presented concepts and demonstrate how the societal ALARP requirement interacts with the (safety) preferences of private investors. The examples highlight the importance of the societal ALARP evaluation and show that this governs the safety investment when the private investor values damages low, applies a high discount rate, or takes into account indirect costs (e.g. aesthetic costs) related to the safety investment.

List of symbols

Symbols and abbreviations used throughout the paper are listed below. Symbols that are used only locally are explained at the appropriate point in the text. The utility of safety investments is expressed in non-descriptive units, below denoted as "[UNIT]". In most situations utilities are expressed in monetary terms, but other measures (e.g. life-years saved) are equally viable. Similarly, the risk is expressed by a risk indicator *RI* with units "[RI]". When considering investments in life safety, the risk indicator may relate e.g. to the expected number of fatalities.

a	[UNIT/RI]	(dis-)proportionality constant for the ALARP evaluation
B	[UNIT]	Benefit derived from the safety feature's existence; mathematically equal to the expected damage cost in absence of the safety feature
$C(\mathbf{p})$	[UNIT]	Cost of construction or implementation of the safety feature, including maintenance
C_{FM}	[years]	Demographic constant
$D(\mathbf{p})$	[UNIT]	Direct and indirect costs in case of failure
dX		Marginal change in X
dM	[1/years]	Mortality change
$E[.]$		Expected value operator
e	[years]	Life expectancy
g	[UNIT/year]	GDP per capita
N	[persons]	Exposed population
N_f	[persons]	Number of fatalities given failure
P_f	[-]	probability of failure
$\mathbf{p}$		Vector of design parameters p_i considered for optimization
q	[-]	LQI exponent considering work-leisure trade-off
$Z(\mathbf{p})$	[UNIT]	Total (net) utility derived from the safety feature's existence
ΔX		change in X
ΔC	[UNIT]	cost of the investigated safety feature in the ALARP evaluation
ΔRI	[RI]	change in the risk indicator for the ALARP evaluation
γ	[1/year]	Continuous discount rate
$\mu_{F,0}$	[UNIT]	expected costs corresponding with human losses in the reference situation, given fire occurrence
$\mu_{F,p}$	[UNIT]	expected costs corresponding with human losses when implementing the safety feature, given fire occurrence
μ_f	[persons]	expected number of fatalities, given fire occurrence
μ_H	[UNIT]	total expected costs in case of a failure event
$\mu_{M,0}$	[UNIT]	expected costs due to material losses in the reference situation, given fire occurrence
$\mu_{M,p}$	[UNIT]	expected costs due to material losses when implementing the safety feature, given fire occurrence
Φ		standard normal cumulative distribution function
ALARP		As Low As Reasonably Practicable
CBA		Cost Benefit Analysis
LCO		Lifetime Cost Optimization
LQI		Life Quality Index
PDF		probability density function

$SWTP_{1life}$	[UNIT/person]	Societal Willingness To Pay to save one statistical life; synonymous to Societal Capacity to Commit Resources (SCCR)

1 Introduction

Fire safety design aims to keep fire risk at acceptable levels. Specifically, fire risk levels should generally be As Low As Reasonably Practicable (ALARP) [1], [2]. The ALARP principle or criterion thus recognizes that a vision-zero strategy to reduce all risks as much as possible, irrespective of the associated costs, is not feasible [3] and that a balance must be sought between the costs of risk mitigation strategies and the benefits of these safety investments [4]. As societal safety resources are limited, significant overinvestment in reducing specific risks may result in a net loss of societal welfare, because more lives may be saved via other, more targeted risk reduction measures [5], [6]. Studies have indicated that an indirect loss of life in one area could result from costly safety investments in another (e.g. [7] and [8]).

When applying traditional, prescriptive fire safety solutions, the acceptability of the residual risk level for the (common) situation is considered to result directly from the application of the detailed design guidance [9]. In those situations, the ALARP requirement is (implicitly) assumed as being fulfilled. This is justified by invoking the 'collective experience of the profession' (i.e. common and longstanding application in design) [10], [11], and by recognizing that prescriptive guidance has, in many cases, been shaped through corrective actions in the aftermath of disasters or other demonstrated failures [9].

However, rapid changes in technology and innovative engineering applications can strain prescriptive guidance, particularly because attaining acceptable residual risk levels in prescriptive guidance requires failure events to learn from [9], [11]. Spinardi et al. have observed that the limitations of such prescriptive guidance has resulted in a trend towards deregulation [9], i.e. a shift towards Performance Based Design (PBD) approaches.

Common applications of PBD are deterministic in nature (i.e. probabilities of failure are neither directly nor indirectly considered), with the adequacy of the design assessed by comparison with pre-determined performance criteria in one or more potential fire scenarios [10]. The achievement of an ALARP residual risk level is not explicitly demonstrated in many cases, and is thus again implicitly assumed by relying on the 'collective experience of the profession' [11]. This reliance can be justified in common applications, and in such cases deterministic PBD can be a valuable addition to prescriptive guidance, justifying minor deviations from prescriptive rules.

A shortcoming of deterministic PBD is that it cannot provide an evidence-based answer in situations where the 'collective experience of the profession' is lacking, e.g. for exceptional structures, (very) low probability events, or innovative building designs and/or materials [12]. Neither does it provide a clear framework to take into account unknown deterioration of fire safety measures during the lifetime of the structure [13]. In these situations, the safety level of the design ought to be explicitly evaluated, taking into account the many uncertainties associated with the design and its performance. In other words, for innovative design applications a Probabilistic Risk Assessment (PRA) is recommended to determine whether the design complies with the ALARP requirement [11].

The explicit evaluation of ALARP through PRA is widely accepted as a possibility in design – see for example the UK guidance document PD7974-7 devoted to PRA for fire safety [4]. Despite this, examples of evaluation of ALARP through PRA are rare in practice.

To show the potential benefits of applying PRA in fire safety design and ensure that fire risk levels of designs are acceptable, this paper investigates the ALARP criterion within PRA. A practically feasible implementation of the ALARP criterion is proposed, with an emphasis on defining ALARP for life safety. Difficulties in the valuation of life safety investments are addressed using the Life Quality Index (LQI), thus attempting to ensure that safety investments are made in agreement with society's capacity to pay. Potential applications are shown for decisions related to sprinkler installation and egress width.

Many derivations given herein have been introduced and applied elsewhere (e.g. [5], [14], [15], [16]), and the presented ALARP interpretation builds upon seminal work by, for example, Fischer [3] and Rackwitz [17]. However, these concepts have not yet found common application in fire safety engineering practice, and fundamental discussion points remain unresolved within the fire safety community. Therefore, a review of the full framework, starting from its basic concepts, is given to elucidate the reasoning of the proposed evaluation of the ALARP requirement. Although the applicability of the presented concepts extends beyond fire safety engineering, the paper is targeted for a fire safety engineering audience.

2 PRA in fire safety design, and the ALARP requirement

2.1 The prerequisite of tolerability

For a design to have an acceptable residual risk level for a given fire safety objective, the design should be both tolerable and ALARP [11]; a design that is intolerable cannot be justified irrespective of the associated benefits [1]. Only once tolerability has been established, can the ALARP requirement be applied to determine the acceptability of the residual risk.

The above can be conceptually visualized through a Frequency-Consequence diagram (FC-diagram), as shown in Figure 1 [11]. For the fire safety objective of life safety, the consequence severity is generally specified by the number of fatalities N, resulting in FN-diagrams that are widely used by governmental bodies; see e.g. Frank and Farquharson [18] and the detailed discussion in Van Coile et al. [11]. These FN-diagrams make no reference to the size of the exposed population underlying the F-N evaluation, however, and thus can be established both on the level of a neighbourhood, a facility, or a sub-section of a facility depending on the situation at hand. When considering FN diagrams, the frequency of a single fatality may, however, be borne by a limited subgroup of the larger population at risk. Therefore, the tolerability limit in FN-diagrams is generally combined with an individual risk limit, e.g. a limit to *the frequency at which an individual who is permanently present at the location may be expected to sustain a given level of harm* [19].

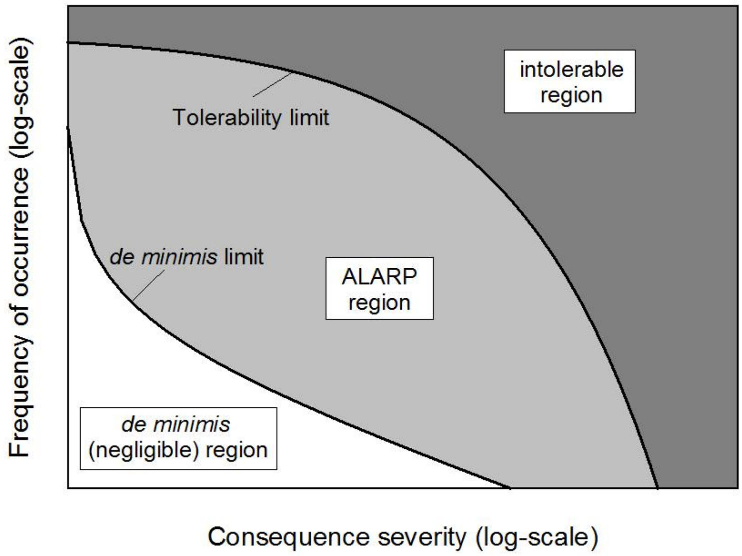

Figure 1: Generalized frequency-consequence (FC) diagram.

2.2 The ALARP requirement

Tolerable designs must also fulfil the ALARP requirement to correspond with an acceptable residual risk level [1], [11]. The ALARP requirement applies to both the ALARP and the *de minimis* regions in Figure 1 in which known cost-effective safety measures must still be implemented [1] but no active efforts must be made to identify further cost-effective safety investments.

2.2.1 Defining ALARP

The origin of ALARP is often traced back to the 1949 UK case Edwards vs. National Coal Board [20], in which Lord Asquith stated that mitigation measures need not be implemented [only] *"if it be shown that there is a gross disproportion between them – the risk being insignificant in relation to the sacrifice – the defendants discharge the onus on them"* [21]. The ALARP requirement is now commonly considered to require that safety measures be implemented up to the point where there is a 'gross disproportion' between the risks and the costs required to mitigate them [2].

The ALARP philosophy of Lord Asquith can symbolically be represented by Equation (1), where ΔC is the cost of the investigated safety feature, ΔRI is the associated change (<0) in the risk indicator RI, and a is the disproportionality threshold (valuation constant). This symbolic representation specifies a (societal) criterion for implementing safety features, i.e. the safety feature should be implemented when the cost-benefit ratio is below the threshold. Note that it is not the risk level that is directly deemed acceptable through Equation (1), but rather the efficiency of the safety measure.

$$\frac{\Delta C}{-\Delta RI} \leq a \tag{1}$$

As the ALARP criterion requires the implementation only of suitably 'efficient' safety measures, it is considered to be a cost-benefit analysis (CBA). This interpretation links the ALARP criterion to the 'maximum societal benefit criterion' proposed recently by Van Coile and Pandey [22]. Considering its conceptualization as a societal requirement aimed at achieving adequate safety, the CBA implied by the ALARP criterion should be evaluated from a societal perspective. As stated by Fischer [3], societal requirements for investments in safety function as lower bounds (minimum requirements) for considerations by private decision makers [11]. This implies that private decision makers are free to invest beyond societal minimum safety levels if they choose to, regardless of their motivation. Here, the notional private decision maker should generally be understood to be a private business or industry, but could apply to any private actor having direct decision authority on the implementation of (additional) fire safety features.

To define the set of safety features that constitute a societal minimum requirement necessitates a comparison of safety features; these may improve safety in very different ways. While some features may reduce the occurrence rate of severe fires overall, others may instead reduce the probability of high-consequence (multi-fatality) events. Comparing very different safety features implies a moral evaluation of different potential life-saving effects. However, once the tolerability of the design has been established, all possible consequences and associated occurrence rates are potentially acceptable to society. In those situations, only an equal ('risk-neutral') valuation of different risk-reduction effects is morally acceptable [5][6][11]. Because a risk-neutral evaluation implies that no further qualitative distinction is made between events, events with the same (aggregate) risk level are valued equally. Consequently, the ALARP evaluation should be made based on a scalar risk indicator (i.e. expected value) [11]. The above specifies the ALARP criterion to a 'societal, risk-neutral, and scalar cost-benefit analysis'.

2.2.2 The societal cost of the safety measure

Considering the ALARP criterion of Eq. (1), the (net) costs of the safety measure, ΔC, must be valued in common units to allow for the aggregation of, for example, monetary expenditures, time and effort, or loss of heritage value. The main cost of the safety measure will generally be monetary. Expressing the total cost in economic terms is the most practical approach. Reference to the 'net' costs of the safety measure suggests that indirect loss reductions (i.e. material benefits) resulting from the safety investment also ought to be considered.

A fundamental consideration relates to the perspective of the net cost valuation, i.e. as the ALARP requirement relates to the societal requirements for safety, the costs (and benefits) must be evaluated from a societal perspective. This introduces a number of practical difficulties. For instance, the designer's trustworthiness when evaluating the societal costs and benefits (damage reductions) is sometimes (indirectly) questioned, e.g. in the works by Fischer [3] and Jones-Lee and Aven [2]. Fischer proposed to determine societal minimum requirements for investments in life safety without considering monetary benefits due to cost or damage reductions [3]. Once the societal minimum investment level is determined, private decision makers can opt for additional safety investments considering their own cost-benefit considerations. Fischer's concept, where societal considerations result in a lower bound for private cost optimisation, greatly clarifies the ethical dilemma between societal and private considerations and is incorporated in the current proposal. However, neglecting the positive effect of societal reductions in material damage distorts the assessment in favour of not requiring the safety feature [22]. For example, the installation of a life safety sprinkler system will also reduce the likelihood of material damage. This reduction in material damage effectively reduces the net investment cost of the sprinkler installation and thus influences any ALARP assessment [23].

The concern with respect to the designer's evaluation of costs and benefits, however, does not seem fundamentally different from possible concern related to fire safety engineers using advanced computational models. As with the use of computational models, the designer has a duty of care and incurs professional liability when failing to execute their work according to the benchmark of an 'objective, diligent and competent fire safety professional' [11]. This benchmark pushes the fire safety professional towards self-reflection, as the professional's actions are assessed in relation to the current state-of-the-art within the fire safety profession. Considering the above, the issue of the designer's trustworthiness should be tackled through appropriate professional quality control, competence, and accreditation mechanisms, and not by systematically distorting the valuation of costs and benefits, e.g. by ignoring monetary benefits due to cost or damage reductions.

The valuation of societal costs and benefits can also be difficult because it is (generally) a private investor who bears the costs of a safety measure. However, it is reasonable to equate the societal cost of a safety feature with the investment cost for the private investor [11], implicitly assuming that the private investor will transfer the additional cost to the public when eventually charging for goods or services. Some material damage reductions resulting from fire safety investments will benefit private investors. For example, a reduction in expected losses in stocks of goods can be considered to benefit mostly the owner of the stock. If private investors are not inclined to transfer these benefits to society, but will only transfer the costs of the safety investment, then the societal benefit from the expected reduction in monetary losses effectively becomes zero. In other situations reductions in material losses may apply to society, which are beyond the normal scope of consideration of private decision makers. For example, reductions in the risk of fire spread between buildings and reductions in expected collateral damage can be considered a monetary benefit to society, even though a private investor may not value these risk reductions as a benefit.

With respect to this valuation, it is important to note that the costs and benefits associated with investments in structural safety for normal design conditions cannot be readily transposed to (structural) fire safety assessments. Consequently, the general relative safety investment cost assessments for normal design situations, as referenced e.g. by Van Coile et al. [24], cannot be directly applied to fire safety CBA. Neither should the reliability targets specified in e.g. the Eurocodes [25] and ISO2394:2015 [26] be adopted without further evaluation for fire safety applications.

2.2.3 The scalar and risk-neutral risk indicator

For a given safety objective, the risk incorporated within the design is determined by the full spectrum of possible consequences, *C*, and associated probabilities of occurrence, P_c, and can be represented by a risk curve [27] in the FC-diagram (see Figure 1). The full risk curve is necessary to determine the tolerability of a design. However, it can be reduced to a scalar risk indicator, *RI*, for the subsequent ALARP evaluation. This is because qualitative societal distinctions between events as a function of the consequence-size or occurrence frequency ought to be taken into account through the tolerability assessment only [11]. Once the tolerability of the design has been established, all possible consequences and associated occurrence frequencies are, in principle, potentially acceptable to society. Thus, the societally moral allocation of further safety resources is an allocation that maximizes the number of lives saved (i.e. maximizes societal welfare). To maximize the number of lives saved, all lives must be valued equally and, consequently, a societal decision-maker must make a risk-neutral assessment [5].

The equal valuation of consequences irrespective of the consequence magnitude or associated probability results in a scalar risk indicator *RI* equal to the expected value of the consequences in the considered time-frame of the probability evaluation. Generally, the evaluation is made on a yearly basis, resulting in scalar risk values with dimensions of e.g. 'fatalities per year' or 'annual monetary loss'.

2.2.4 The valuation constant *a*

The ALARP criterion of Equation (1) can be rewritten as Equation (2), where $-\Delta RI$ is a necessarily positive quantity for a risk indicator as defined in Section 2.2.3. For Equation (2) to have meaning, the right-hand side of the inequality must have the same dimension as the left-hand side, and thus the valuation constant a can be interpreted as a proportionality constant translating the change in the scalar risk indicator into an equivalent monetary value.

$$\Delta C \leq a \cdot \left(-\Delta RI\right) \tag{2}$$

The above terminology refers to the factor a as a proportionality constant and not a 'disproportionality constant'. This relates to the obligation of risk neutrality for the societal decision makers discussed above (Section 2.2.3), and is justified by considering the reference to *gross disproportion* in the original ALARP definition as an aspect of societal risk aversion. Societal risk aversion is included in the tolerability assessment (Section 2.1 and [11]). This results in an ALARP interpretation that is close to the precise Dutch interpretation as a pure cost-benefit assessment (CBA) [28].

Other justifications for applying *gross disproportion* in ALARP evaluations have been presented in the literature [2]. However, these (implicitly) relate to; (i) uncertainty and risk aversion with respect to high consequence events, (ii) possible incomplete assessments of direct and indirect costs, (iii) a wariness with respect to the designer's valuation of societal costs and benefits, and (iv) the issue of transferring risks to identifiable persons. All of these aspects are, however, in principle covered by the prerequisite of tolerability and the duty of care of the designer in the assessment of societal costs and benefits. Furthermore, all risk management procedures are complemented by a prohibition to transfer risks to identifiable persons [5], i.e. under no circumstances should it be permissible to use CBA to justify sacrificing an identifiable person 'for the greater good'.

To summarise, the valuation constant for the ALARP criterion is considered as a (risk-neutral) proportionality constant. It is acknowledged that an application of a disproportion factor reduces the burden on the engineer to make a detailed evaluation [28]. A disproportion factor is, however, not considered as an integral part of the ALARP criterion. By analogy to arguments from Sunstein [6], application of a disproportion factor cannot be considered to improve the overall societal welfare, since overinvestment with respect to reducing specific risks necessarily entails underinvestment with respect to reducing others. Or, as Sunstein puts it, *risk aversion does not exist* [6], and thus a 'risk-averse' disproportion factor cannot be justified.

2.2.5 Relationship with private decision analysis

Private considerations will often influence decisions around fire safety measures. The societal requirement of ALARP residual risk will impose a lower bound (minimum requirement) on any private cost-benefit considerations [29], [3]. The application of societal safety requirements as a lower bound to private CBA is in line with the recent international standard ISO2394:2015 on structural safety [26].

The distinction between societal and private cost-benefit assessments ensures that the private decision-maker has no constraints in the private valuation of costs and benefits and is free to invest in safety beyond the societal requirements and to perform the cost-benefit assessment using any discount rate. This freedom to overinvest is no different than the freedom of the private decision-maker to invest in specialized training, art, housing, or entertainment as part of the enjoyment of life and the fulfilment of specific private goals.

2.2.6 Cost-benefit analysis

The above discussions have specified the ALARP criterion as a risk-neutral cost-benefit analysis (CBA) with a societal valuation of costs and benefits. In general, CBA is conceptually represented by Equation (3), where $Z(\boldsymbol{p})$ is the total (net) utility, B is the benefit derived from the safety feature's existence, $C(\boldsymbol{p})$ is the cost of construction or implementation (including maintenance), $D(\boldsymbol{p})$ is the direct and indirect costs in case of failure, and $\boldsymbol{p}$ is the vector of design parameters p_i considered for optimization [30]. For simplicity, a single optimization parameter, p, will be considered further. The CBA evaluation then corresponds to maximizing the total net utility of Eq. (3). In this formulation, the benefit derived from the safety feature's existence is considered to correspond with the avoidance of the (expected) damage in the reference state absent the additional safety investment. Thus, B is independent of $\boldsymbol{p}$ in Eq. (3). This formulation is more practical for fire safety engineering applications in particular, where the need for additional safety features must generally be assessed. The damage term, D, then relates solely to the (expected) damages in the proposed design configuration.

$$Z(\boldsymbol{p}) = B - C(\boldsymbol{p}) - D(\boldsymbol{p}) \tag{3}$$

The societal optimization as part of the ALARP assessment is thus expressed through Eq. (4), which shows that the optimum investment level is independent of the initial (reference) safety situation incorporated in the benefit term B. The reference safety level incorporated in the benefit term is nevertheless important, since only designs with a positive lifetime expected utility $Z(p)$ are feasible (i.e. only investments that constitute net benefits can reasonably be required or implemented). This formulation of ALARP as a societal optimization is at odds with traditional net benefit assessments (e.g. [23] and [31]). In a net benefit assessment, the acceptability of the safety feature is determined by whether or not it increases the utility Z in Eq. (3). However, applying such a net benefit criterion as a societal minimum requirement for safety investments makes the societal requirement dependent on the base (reference) safety level, thus potentially resulting in a weak societal safety criterion [22]. Application of a net benefit may also result in significant (non-efficient) overinvestment. Highlighting the distinction with 'net benefit criteria', application of the optimization of Eq. (4) has been referred to as the 'maximum societal benefit criterion' [22].

$$\frac{dZ(p)}{dp} = -\frac{d}{dp}\left[C(p) + D(p)\right] = 0 \tag{4}$$

As the safety investment in fire safety engineering relates to the reduction of future risk, the CBA requires that future costs and benefits are discounted to present day values, or annualized, using a (continuous) discount rate γ [3]. For application of the ALARP criterion, a societal discount rate must be applied, as it relates to a societal valuation of uncertain future benefits resulting from current-day safety investments (when considering e.g. investing in an additional escape stair). A discount rate which is set very low will result in an increased preference for future life-saving relative to saving lives today, while a very high discount rate results in a focus on current-day life-saving operations and values future life-saving less. Fischer [3] and recently ISO2394:2015 [26] recommended a societal discount rate equal to the long-term economic growth rate. Note that the private decision maker is free in the choice of discount rate when performing a private CBA, as the only boundary condition for the private decision maker is not to invest below the societal ALARP design.

The CBA will indicate a preference for further investment up to the point where further investments are no longer balanced by a comparative reduction in risk [3], and thus CBA corresponds to the optimization of fire-safety costs over the lifetime of the structure (hence the alternative name of 'Lifetime Cost Optimisation' or LCO).

3 Valuation of reduction of risk to life: the Life Quality Index

3.1 Introduction

Lifetime Cost Optimisation with respect to investments that improve life safety necessitates a quantification of the value of reducing risk to human lives. As noted by Nathwani et al., the valuation of risk to life is often problematic in professional decision-making processes [5]. Valuation of risk to human life can be challenging because of a common view that human life has infinite value [8]. The tempting conclusion that investments in life safety must be made without regards for costs is very difficult to rationally justify however [3]. Public overspending in some areas has even been shown to result in a net *increase* of mortality in other areas, with disadvantaged population groups being particularly vulnerable to such circumstances [8].

A widely applied methodology for the valuation of the reduction of risk to life is derived from the Life Quality Index [14], [32]; this was recently incorporated into the ISO2394:2015 [26] standard and has been applied in (a limited number of) fire safety engineering studies (e.g. [33], [3], [23] and [16]).

3.2 The Life Quality Index (LQI) and the valuation of risk reduction

3.2.1 Introduction to the LQI

The Life Quality Index (LQI) was introduced by Nathwani et al. [5] to support decision making on investments affecting life safety. As stated by Lind et al. [34], the underlying goal of safety investments and risk management should be to cost-effectively improve overall societal welfare. As presented by Nathwani et al. [5], key indicators for societal welfare are the annual GDP per capita g and the life expectancy e. These parameters are considered together with an exponent q defining the trade-off between work and leisure to form the LQI index of Equation (5). While initially based on qualitative considerations, the LQI has later been derived from economic principles by Pandey et al. [32], maintaining Equation (5) but updating the assessment for the exponent q. According to Pandey et al., the exponent q is within the range of 0.16-0.20 for developed economies [32], while the recent standard ISO 2394:2015 states that q can in general be considered in the range 0.18-0.20.

$$LQI = g^q e \tag{5}$$

The LQI methodology has found widespread international application in the fields of earthquake engineering [35], structural safety [26], [17], and maritime safety [36], amongst others. In fire safety engineering, applications include egress width assessment [37] (see later), and tunnel fire safety [38]. Additional background on the origins of the LQI is given by Nathwani and Lind [39], [40].

3.2.2 The LQI net benefit criterion

In principle, the LQI can be used directly to assess whether or not a safety feature is beneficial to society. This can be understood by considering that Equation (5) defines indifference curves for societal welfare, as visualized in Figure 2. On each of these indifference curves, society is considered to be willing, in principle, to exchange a reduction of GDP per capita for a corresponding increase in life expectancy (and vice-versa).

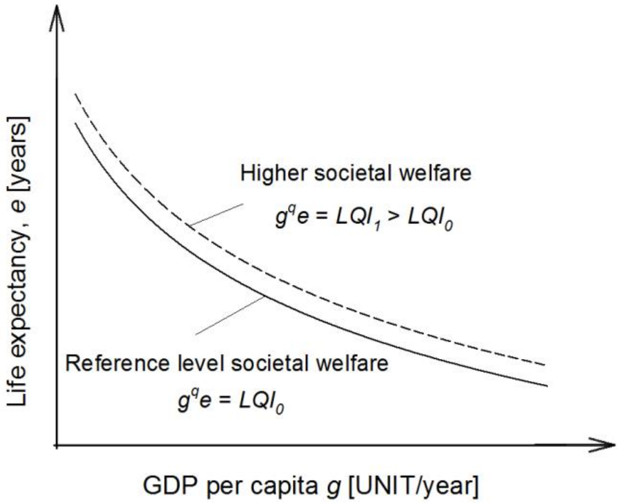

Figure 2: LQI indifference curves for societal welfare, based on [41].

A shift from one LQI indifference curve in Figure 2 to the other LQI indifference curve corresponds with a societal preference. Consider for example an investment in life safety. This investment reasonably corresponds to an expenditure, i.e. a change of GDP per capita $\Delta g < 0$, and an increase in life expectancy $\Delta e > 0$. This change in GDP and life expectancy can be set out from the LQI indifference curve corresponding with the initial situation prior to the proposed safety investment. As visualized in Figure 3, two scenarios for the safety feature can be discerned, where in Figure 3 both scenarios have the same life-saving benefit (i.e. the same increase in Δe) for illustrative purposes. In the left-hand graph, the cost Δg of the safety feature is small relative to the increase in life expectancy, and implementation of the feature results in an increase of the LQI: $LQI_{final} > LQI_{initial}$. The left-hand safety feature results in a move to a higher LQI indifference curve, is therefore beneficial for societal welfare and should be implemented. In the right-hand graph however, the costs of the safety feature are considerably larger and implementing the safety feature results in a reduction of the LQI: $LQI_{final} < LQI_{initial}$. Consequently, implementing the right-hand safety feature results in a reduction of societal welfare and thus (a societal requirement to implement) the safety feature cannot be justified on this basis.

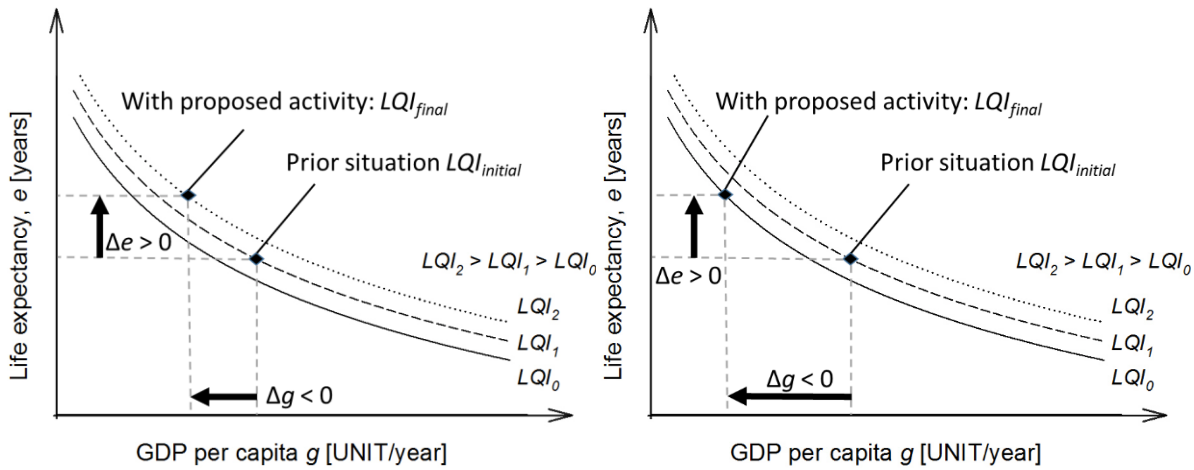

Figure 3: Conceptual scenarios for an investment in life safety. The left-hand scenario results in an increase of the LQI and is beneficial for societal welfare. The right-hand scenario results in a reduction of the LQI and correspondingly the life safety investment cannot be justified.

Mathematically, the above qualitative criterion for implementing a proposed safety investment can be written as Equation (6). Acknowledging that safety investments result in marginal changes *dg* and *de* of GDP and life expectancy, Equation (6) can be expressed as Equation (7).

$$\Delta LQI \geq 0 \tag{6}$$

$$dLQI = qg^{q-1}edg + g^q de \geq 0 \tag{7}$$

Dividing Equation (7) by the LQI definition of Equation (5) for convenience, the criterion is rewritten as a criterion on the relative change of the LQI, Equation (8). The criterion of Equation (8) is known as the LQI net benefit criterion and defines whether a proposed safety investment is beneficial for societal welfare.

$$\frac{dLQI}{LQI} = q\frac{dg}{g} + \frac{de}{e} \geq 0 \tag{8}$$

3.2.3 Valuation constant *a* for risk reduction to human life

The LQI net benefit criterion is applied to determine the valuation constant *a* necessary for the CBA in the ALARP evaluation (Section 2.2.4).

In the basic formulation of the LQI net benefit criterion, the relative change in life expectancy *de*/*e* represents the life safety benefit of a proposed safety feature. As discussed by Nathwani et al. [5] and by Rackwitz [17], this relative change in life expectancy can be approximated by a change in mortality *dM* through the demographic constant C_{FM}, resulting in Equation (9).

$$\frac{de}{e} \approx -C_{FM}dM \tag{9}$$

For an adverse event with an annual occurrence probability P_f, and N_f fatalities due to occurrence over a total exposed population *N*, the mortality change *dM* associated with a proposed safety feature can be evaluated by Equation (10), where the subscript *final* refers to the situation after the safety investment and the subscript *initial* refers to the initial situation, while $\Delta\mu_f$ is the change in expected annual number of casualties.

$$dM = \frac{1}{N}\left[\left(N_f P_f\right)_{final} - \left(N_f P_f\right)_{initial}\right] = \frac{\Delta\mu_f}{N} \tag{10}$$

By rewriting the LQI net benefit criterion and substituting Equations (9) and (10), Equation (11) is obtained, which indicates the maximum per capita reduction of GDP dg (<0) acceptable in accordance with the LQI net benefit criterion, for a reduction $\Delta\mu_f$ (<0) in expected annual number of casualties.

$$-dg \le \frac{g}{q}\frac{de}{e} = -\frac{g}{q}\frac{C_{FM}}{N}\Delta\mu_f \tag{11}$$

Setting $\Delta\mu_f$ to -1 casualty per year in Equation (11) gives an LQI net benefit threshold for the maximum acceptable per capita loss of GDP to save one statistical life per year, see Equation (12). Note that this constant values the reduction of risk to life, not life itself [5], i.e. the monetary valuation refers to a statistical risk to life and, importantly, not to an identifiable person.

The criterion of Equation (12) effectively means that more expensive safety investments are beyond the financial means of the society, and that other more cost-effective safety investments (possibly in other fields or applications) should be implemented first. Multiplying both sides of Equation (12) with the population size *N* gives the maximum total societal cost per year, *c*, to save one statistical life per year, see Equation (13). Since expenditures up to this maximum cost result in an increase of societal welfare (an increase in LQI), safety investments up to this maximum cost are of net benefit to society and should be implemented.

$$-dg_{\max} = \frac{g}{q}\frac{C_{FM}}{N}\left(\frac{1pers}{year}\right) \tag{12}$$

$$c = -dg_{\max}N = \frac{g}{q}C_{FM}\left(\frac{1pers}{year}\right) \tag{13}$$

Both sides of Equation (13) refer to an evaluation on a yearly basis. Cancelling the yearly aspect for both, results in Equation (14) as the LQI-based Societal Willingness To Pay for one statistical life, $SWTP_{1life}$. An equivalent terminology is to denote the obtained constant as the Societal Capacity to Commit Resources or SCCR.

$$\frac{g}{q}C_{FM} = SWTP_{1life} \tag{14}$$

Values below $SWTP_{1life}$ will not fully exhaust society's capacity to pay for life safety and will thus result in an unnecessary loss of life. Values above $SWTP_{1life}$ exceed society's capacity to pay, thus reducing its ability to pay for more cost-effective safety measures, and therefore result in unnecessary loss of life. In summary, based on the LQI methodology, risk to human life should be valued in accordance with the proportionality constant of $SWTP_{1life}$ for a societal risk-neutral cost-benefit evaluation of investments in life safety. This result is also directly obtained from application of the LQI maximum societal benefit criterion, as previously shown by Van Coile and Pandey [22].

4 Application examples

Illustrative examples of the ALARP evaluation in PRA fire engineering design and the application of the LQI $SWTP_{1life}$ are given in the following sections. For these examples, the tolerability of the reference (minimum) design is considered to be satisfied (both with respect to societal and individual risk). In other words, the reference design situations are all considered to be located in the ALARP region of Figure 1. Consequently, it must be assessed whether or not additional safety investments are required in accordance with the ALARP principle.

The first and third examples are derived from literature sources, respectively from Hasofer and Thomas [23] and De Sanctis and Fontana [37], limiting the required introduction and illustrating the compatibility of the presented concepts with LQI applications discussed elsewhere. The second example is conceptual, but clearly illustrates the trade-off between increased safety investment and cost, and the relationship with private considerations. Further examples that can be evaluated within the presented framework include the assessment of the benefit of smoke alarms presented by Fischer [3], and the concrete cover optimization presented in Van Coile et al. [42].

First, however, background is presented on the evaluation of uncertain costs and benefits as part of the CBA methodology. This background functions as a point of reference for the subsequent example applications. The derivations in Section 4.1 are specifically for situations where the safety investment affects the occurrence rate of damage-inducing fires, though more general formulations are possible. The concepts presented in Section 4.1 are well known in the field of structural reliability, but have so far found limited application in fire safety engineering with notable exceptions, e.g. [3] and De Sanctis [33].

4.1 Background: cost-benefit analysis for safety investments reducing P_f

Evaluating the lifetime utility and optimization of safety investments through Eqs. (3) and (4) requires an evaluation of the constituent terms. The initial investment cost (including maintenance), *C*, for a proposed safety feature may be well-determined or at least determinable. The fire-induced damages, *D*, on the other hand, relate to the uncertain occurrence of fire, the failure probabilities given fire exposure (e.g. failure to maintain tenable conditions in the escape routes), and the uncertain consequences associated with the failure definition. As societal demand for buildings does not stop at a single failure or fire event, the building is considered to be systematically restored after a fire, and the renewed structure is again exposed to possible damaging events. The utility of safety investments thus relates to the utility evaluation in a renewal process, see [43] and [44].

Describing the occurrence of failure by a homogeneous Poisson process with renewal rate λ given by the annual probability of failure P_f, and considering the costs H_i incurred at the i^{th} renewal (i.e. failure) to be independent and identically distributed with mean value μ_H, the expected present value of the damage cost *D* is given by Equation (15), see e.g. [42]. When the time horizon t_{max} approaches infinity, this reduces to the well-known result of Eq. (16). In most situations an infinite time horizon can reasonably be applied, since, for large finite time horizons, the present value contribution of costs incurred at t_{max} is small. Furthermore, Fischer et al. emphasize that the time horizon for societal requirements regarding life safety investments should be related to the 'societal need for the type of structure' (mostly infinite) and not to its conceptual (private investment) design life [29].

$$E\left[D\right] = \mu_H \int_0^{t_{\max}} P_f \exp\left(-\gamma t\right) dt = \mu_H \frac{P_f}{\gamma}\left(1-\exp(-\gamma t_{\max})\right) \tag{15}$$

$$\lim_{t_{\max}\to\infty} E\left[D\right] = \mu_H \frac{P_f}{\gamma} \tag{16}$$

As indicated by the equations above, a traditional CBA based on expected costs does not require knowledge of the variability of H_i, thus reducing the burden for assessing the input parameters in the assessment. As noted in [22], this reliance on the expected costs is rational and moral only because the tolerability of the design is a prerequisite for its acceptability [11]. In other words, the tolerability requirement has taken into account the damage variability and ensures that no 'possibility of ruin' (i.e. possibility of unbearable consequences) exists. In other words, the tolerability prerequisite functions as an implementation of the anti-catastrophe principle promoted by Sunstein as a workable (and more rational) implementation of the Precautionary Principle [6]. The benefit of safety investments is evaluated similarly to equations (15) and (16), but considering the failure occurrence rate and expected damages of the reference design situation.

For a more detailed elaboration of Eq. (3) and (4), consider a situation where an annual failure probability $P_{f,0}$ exists in absence of any (additional) safety investments and is associated with expected (direct and indirect) material losses $\mu_{M,0}$, and costs corresponding with human losses $\mu_{F,0}$. The benefits incurred by implementing the safety feature are, considering the above, defined by Equation (17). Similarly, the failure costs when implementing the safety feature are given by Equation (18). Here, the subscript p denotes the dependence on the safety investment parameter p. Together with the investment cost C these constituent equations specify the present value of the total lifetime expected utility (19), which is a particular evaluation of the general Eq. (3). The optimization criterion of Eq. (4) in this situation specifies to Eq. (20).

$$B(p) = \frac{\left(\mu_{M,0} + \mu_{F,0}\right)P_{f,0}}{\gamma} \tag{17}$$

$$D(p) = \frac{\left(\mu_{M,p} + \mu_{F,p}\right)P_{f,p}}{\gamma} \tag{18}$$

$$Z(p) = \frac{\left(\mu_{M,0} + \mu_{F,0}\right)P_{f,0}}{\gamma} - C(p) - \frac{\left(\mu_{M,p} + \mu_{F,p}\right)P_{f,p}}{\gamma} \tag{19}$$

$$\frac{dZ(p)}{dp} = \frac{d}{dp}\left[\frac{\left(\mu_{M,0} + \mu_{F,0}\right)P_{f,0}}{\gamma} - C(p) - \frac{\left(\mu_{M,p} + \mu_{F,p}\right)P_{f,p}}{\gamma}\right] = 0 \tag{20}$$

4.2 Example 1: Sprinklers in Australian single family residences

Hasofer and Thomas presented an application of the LQI to assess the societal benefit of sprinklers in single family dwellings in Australia [23]. A number of evaluations were made based on available statistical data, resulting in the input data reprinted in Table 1. Installation of sprinklers was considered to reduce the material losses and number of fatalities in case of fire. The exponent q was evaluated based on the earlier LQI definitions, giving a value of approximately 0.14 instead of the currently recommended value of 0.2. Considering the data in Table 1, Hasofer and Thomas concluded that sprinklers could not be justified in accordance with the LQI net benefit criterion. Details of the evaluation are given in [23]. Note that the formulated question on sprinkler installation corresponds with a binary decision problem: either sprinklers are installed or not. In such situations, the net benefit criterion provides the correct valuation in accordance with the proposed ALARP assessment. When the investment level relates to a continuous variable (as in the next sections), an optimization is required in accordance with Eq. (4), corresponding to an application of the maximum benefit criterion.

In the following discussion the example by Hasofer and Thomas is re-evaluated considering $SWTP_{1life}$ and the CBA concepts introduced above, illustrating the straightforward application of CBA concepts for assessing the need to implement a safety investment in accordance with the ALARP criterion.

Table 1: Fire statistics and assessed values, as given by Hasofer and Thomas [23].

Parameter	**Value**	**Unit**
avg persons household, N	2.99	persons
rate of fires, $\lambda_{ig,N}$	0.626	fires / 1000 persons / year
fatality rate without sprinkler, $\mu_{f,0}$	7	fatalities per 1000 fires
fatality rate with sprinklers, $\mu_{f,p}$	3.89	fatalities per 1000 fires
injury rate, i	15	per 1000 fires
cost per injury, μ_i	21100	AUD / injury
Property loss without sprinkler, $\mu_{M,0}$	24000	AUD / fire
property loss with sprinkler, $\mu_{M,p}$	3900	AUD / fire
sprinkler installation cost, c_0	1500	AUD
annual maintenance cost, m	400	AUD / year
nominal interest rate r	0.05	1 / year
useful life, L	20	years
CFM	20	years
G	39758	AUD / year
Q	0.143	
Derived parameter CBA	**Value**	**Dimension**
Societal Willingness To Pay, $SWTP_{1life}$	$5.57 \cdot 10^{6}$	AUD
Continuous discount rate, γ	0.049	1/year
Fire occurrence rate, $P_{f,0} = P_{f,p} = \lambda_{ig,N} \cdot N$ (fire as adverse event, not considered in dimension)	$1.87 \cdot 10^{-3}$	1/year
Reference expected equivalent human cost per fire, $\mu_{F,0}$	39279	AUD
Expected equivalent human cost per fire with sprinklers installed, $\mu_{F,p}$	21969	AUD
Installation cost including maintenance, C	6485	AUD

Lifetime benefit from sprinkler installation, *B*	1513	AUD
Lifetime damage when installing sprinklers, *D*	618	AUD

Based on the statistical data and assessment given by Hasofer and Thomas [23], the derived CBA parameters of Table 1 are obtained. The time horizon t_{max} has been set equal to the useful life of 20 years for consistency with the original paper by Hasofer and Thomas. This effectively limits the assessment to a 20-year window. An infinite time horizon can alternatively be considered if appropriate adjustments are made for renewed sprinkler installation costs every 20 years. The continuous discount rate is derived from the yearly discount rate as specified in Equation (21), where continuous and discrete interest valuations over a one-year time frame are equated. For fire-induced damage costs (and benefits), a continuous assessment is an improvement over a discrete calculation, considering the uncertain occurrence time of a fire. The discrete nominal interest rate of 5% has been maintained from the original paper, but a lower value is considered to more realistically capture long term growth rates. While generally, lower discount rates tend to favour safety investments, the conclusion presented further is in this case insensitive to the discount rate (considering the statistical data given in Table 1).

$$\exp\left(-\gamma\right) = \frac{1}{1+r} \Rightarrow \gamma = \ln(1+r) \quad (21)$$

The benefit from sprinkler installation is evaluated in accordance with Equation (17), with $\mu_{F,0}$ evaluated in accordance with Equation (22), taking into account parameters as specified in Table 1. Thus, as specified in Section 2.2.6, the benefit term *B* is evaluated as the fire-induced costs of the reference situation, which are 'avoided' by implementing the alternative safety scheme (i.e. sprinklers).

Equation (22) also applies to the valuation of $\mu_{F,\mathrm{p}}$ as part of the damage cost *D*. The present value of the investment cost *C* is evaluated by Equation (23) which includes the contribution of the discrete annual maintenance costs over the useful life of the sprinklers (excluding sprinkler maintenance at *t*=0, contrary to [23]). The total utility *Z* (monetary equivalent) of the sprinkler installation is given by Equation (24). As shown by Van Coile and Pandey [22], this formulation is equivalent to a present net value evaluation of the LQI net benefit criterion. The total expected lifetime utility is negative and thus sprinkler installation in a single family dwelling is not consistent with society's capacity for safety investments. The CBA evaluation is equivalent to the evaluation in [23] (with minor deviations as noted above).

$$\mu_F = i \cdot \mu_i + SWTP_{1life} \cdot \mu_f \quad (22)$$

$$C = c_0 + \sum_{t=1}^{L} \frac{m}{\left(1+r\right)^t} \quad (23)$$

$$\begin{aligned} Z &= \frac{\left(\mu_{M,0} + \mu_{F,0}\right) P_{f,0}}{\gamma}\left(1-\exp\left(-\gamma L\right)\right) - C - \frac{\left(\mu_{M,P} + \mu_{F,P}\right) P_{f,P}}{\gamma}\left(1-\exp\left(-\gamma L\right)\right) \\ &= B - C - D = -5591 AUD < 0 \end{aligned} \quad (24)$$

By looking at the valuation of *B*, *C* and *D* in Table 1, it is seen that sprinkler installation would not be required in accordance with the ALARP criterion even if sprinkler installation would result in certainty of no future fire losses (i.e. $D = 0$). The annual expected equivalent damage costs in the reference design situation (no sprinklers) are lower than the annual sprinkler maintenance costs, thus making it clear that sprinkler installation is not required in accordance with the ALARP criterion based on the input data assumed in this example.

The above valuation agrees with the original assessment by Hasofer and Thomas [23]. Where the original valuation applied the LQI net benefit criterion explicitly, the implementation as a CBA above is more transparent. Furthermore, the CBA allows making more definitive conclusions. Specifically, for the fire statistics in Table 1, even perfect sprinkler operation (i.e. guarantee of no future losses in cases of sprinkler installation) cannot justify the cost of implementing sprinklers. In other words, sprinkler installation is beyond the societal capacity to pay, and implementing sprinklers would result in a reduction of overall societal welfare as safety resources could be put to more beneficial use elsewhere.

4.3 Example 2: Generalized sprinkler requirement, considering different sprinkler classes

Sprinkler installation can provide reductions in risk to life and property [45]. The question of whether or not sprinklers are required to provide adequate safety applies not only to residential buildings, but is also an important focus of discussions regarding e.g. care homes [46], schools [47] and other institutional and commercial buildings.

The following is a conceptual example aiming to illustrate additional aspects of a potential ALARP assessment for sprinkler installations. For illustrative purposes, a continuous sprinkler classification system *S* is considered, ranging from Class 0 to Class 4, where the probability of successful suppression by the sprinkler system is conveniently given by Equation (25), where Φ is the cumulative standard normal distribution function. Readers who are familiar with the concept of reliability indices (as applied e.g. in the Eurocodes [25]) will note that the sprinkler Class *S* corresponds directly with the sprinkler class's reliability index β. The choice of a continuous sprinkler classification system is purely illustrative but allows for a clearer derivation below. Furthermore, a continuous sprinkler classification is considered conceptually reasonable considering the continuous range of possible levels of effort and quality control in technology, design, production, installation, and maintenance of sprinkler systems.

$$p_{\mathrm{sup}} = \Phi(S) \tag{25}$$

The left-hand graph in Figure 4 shows the probability of successful fire suppression p_{sup}, the sprinkler failure probability $P_{f,s} = 1 - p_{sup}$ and its derivative with respect to the Sprinkler Class *S*. The graphs show that the failure probability decreases rapidly with increasing sprinkler class for low *S*, but only limited additional benefits can be obtained for high sprinkler classes. The right-hand graph in Figure 4 shows the same values for p_{sup} and $P_{f,s}$, but emphasizes the order of magnitude changes of $P_{f,s}$ with increasing Sprinkler Class *S*. The lowest conceptual sprinkler installation, Class 0, corresponds to a failure rate of 50% (conceptually considered for low quality, low installation control, poor maintenance of systems).

The highest conceptual sprinkler class, Class 4, corresponds to a failure probability of only $3.17 \cdot 10^{-5}$, giving a very high level of effectiveness.

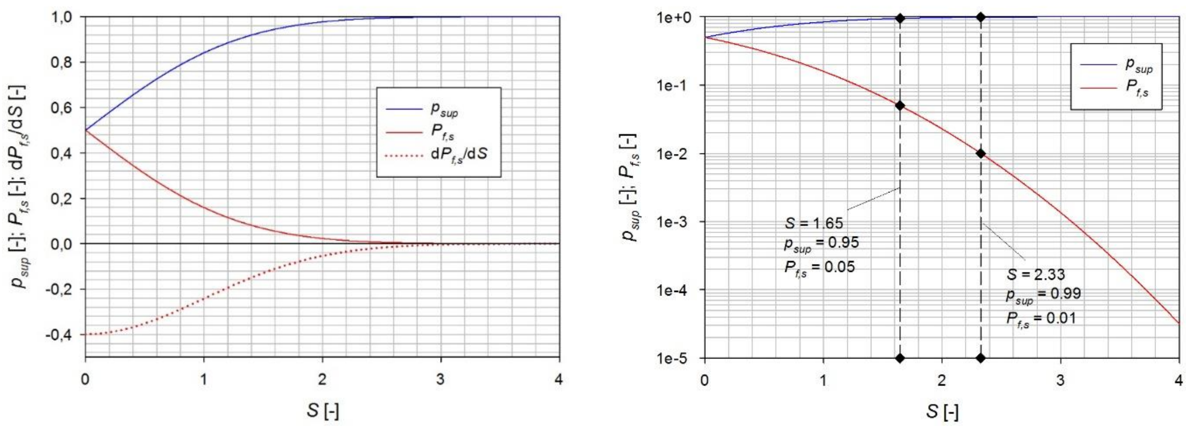

Figure 4: Sprinkler probability of successful early fire suppression p_{sup}, corresponding probability of failure to suppress the fire $P_{f,s}$, and derivative of $P_{f,s}$ with the sprinkler class.

Naturally, the improved performance of higher sprinkler classes comes at a higher (annualized) installation and maintenance cost. Considering an annualized cost has the advantage that different (hypothetical) sprinkler designs can be easily considered, irrespective of the design's useful service life. In this illustrative example, the cost model is linear with respect to *S* (and thus nonlinear with respect to the probability of suppression), as specified in Equation (26). In Equation (26), the parameters *a* and *b* denote, respectively, the *S*-independent annual cost of sprinkler installations and the additional annual *S*-dependent cost associated with increasing the Sprinkler Class *S*. The present net value evaluation of the installation and maintenance cost is given by Equation (27), with the last equality corresponding to an infinite time horizon.

$$c(S) = a + bS \tag{26}$$

$$C(S) = \int_0^{t_{max}} c(S)\exp(-\gamma t)\,dt = \frac{a+bS}{\gamma}\left(1-\exp(-\gamma t_{max})\right) \underset{t_{max}\to\infty}{=} \frac{a+bS}{\gamma} \tag{27}$$

Investment in a higher sprinkler class, or in sprinklers at all, ought to be justified by a reduction in expected fire damage. The flowchart of Figure 5 indicates that a fire which is not suppressed early on by users, suppression systems, or fire and rescue services results in consequences (losses) D_i. Figure 5 allows the PDF of D_i to include a non-zero probability of zero losses, corresponding for example with additional safety barriers which are not explicitly modelled. As CBA only requires knowledge of the mean loss μ_D (Section 4.1), a more detailed assessment is, in principle, not required for demonstrating adequate safety once tolerability has been established.

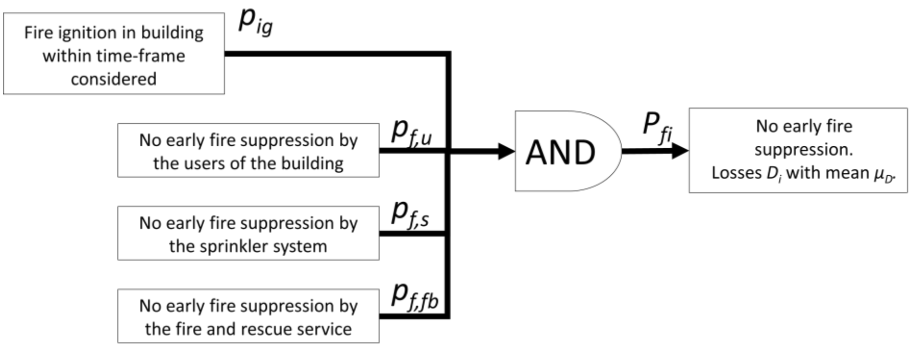

Figure 5: Conceptual flowchart for uncertain events leading to fire-induced losses D

As stated in Section 2.2.6, it is conceptually preferable to consider sprinkler installation as a change of regime, resulting in a benefit *B* of avoiding the expected damage of the (non-sprinklered) reference case. The damage costs *D* are then associated with the expected damage costs incurred despite the sprinkler installation. Considering the concepts introduced in Sections 2.2.6 and 4.1, and denoting with $\lambda_{fi} = p_{ig} \cdot p_{f,u} \cdot p_{f,fb}$ the reference annual frequency of fire ignition without early fire suppression, the benefit and damage terms are given by Eqs. (28) and (29). Note that P_{fi} in Figure 5 equals λ_{fi} for a non-sprinklered design, and $\lambda_{fi} \cdot P_{f,s}$ for sprinklered ones.

$$B(S) = \int_0^{t_{\max}} \mu_D \lambda_{fi} \exp(-\gamma t)\, dt \underset{t_{\max}\to\infty}{=} \frac{\mu_D \lambda_{fi}}{\gamma} \tag{28}$$

$$D(S) = \int_0^{t_{\max}} \mu_D \lambda_{fi} P_{f,s}(S) \exp(-\gamma t)\, dt \underset{t_{\max}\to\infty}{=} \frac{\mu_D \lambda_{fi}}{\gamma} P_{f,s}(S) \tag{29}$$

Combining Eqs. (27)-(29) results in an assessment for the lifetime utility. Multiplying by the discount rate γ gives Equation (30), which corresponds to an annualized total utility assessment Z_a.

Up to this point no distinction has been made between private and societal perspectives. The only limitation is the derivation of *B* and *D* from Figure 5, implying that only fire-induced damages are considered in the equations. Additional 'aesthetic' or other fixed costs can nevertheless be considered by the private investor as part of the annualized sprinkler investment cost.

$$Z_a(S) = \mu_D \lambda_{fi} - (a + bS) - \mu_D \lambda_{fi} P_{f,s}(S) \tag{30}$$

The annual cost of Equation (30) can be normalized by $\mu_D \lambda_{fi}$, resulting in the general formulation of Equation (31), with κ_1 and κ_2 two cost-factors as defined in Eqs. (32) and (33). For feasibility, only values of κ_1 between 0 and 1 can be considered. Just as the constituent parameters (*a, b* and μ_D), κ_1 and κ_2 will differ between different private decision makers depending on their valuation of costs. It is however only the societal valuation which determines compliance with the ALARP requirement.

$$Z_{a,norm}(S) = \frac{Z_a(S)}{\mu_D \lambda_{fi}} = 1 - \frac{a + bS}{\mu_D \lambda_{fi}} - P_{f,s}(S) = 1 - (\kappa_1 + \kappa_2 S) - P_{f,s}(S) \tag{31}$$

$$\kappa_1 = \frac{a}{\mu_D \lambda_{fi}} = \frac{\text{annual base sprinkler cost}}{\text{reference annual fire cost}} \quad (32)$$

$$\kappa_2 = \frac{b}{\mu_D \lambda_{fi}} = \frac{\text{annual extra } S\text{-cost}}{\text{reference annual fire cost}} \quad (33)$$

Considering the continuous Sprinkler Class *S*, the optimum sprinkler class is evaluated from Equation (31) by taking the derivative with respect to *S*, i.e. resulting in the optimum design criterion of Equation (34). Note that this optimum design formulation applies both to societal evaluations and to those by private decision makers, though the underlying parameter evaluations κ_1 and κ_2 differ.

$$\frac{dZ_{a,norm}(S)}{dS} = 0 = -\kappa_2 - \frac{dP_{f,s}(S)}{dS} \quad (34)$$

Although an optimum sprinkler class may be mathematically defined through Equation (34), it is uncertain that implementing the corresponding sprinkler class is feasible. This is illustrated in Figure 6 for a set of κ_1 and κ_2 combinations. Some optimum designs correspond to negative total utility $Z_{a,norm}$ and thus correspond to a reduction in utility compared to the reference situation (i.e. no sprinklers). This is because the optimum design formulation of Equation (34) is independent of the base sprinkler cost factor κ_1. In these situations of non-feasible optimum sprinkler class, the true optimum is to not install sprinklers, i.e. sprinkler installation cannot strictly be justified in accordance with the applied societal or private valuations.

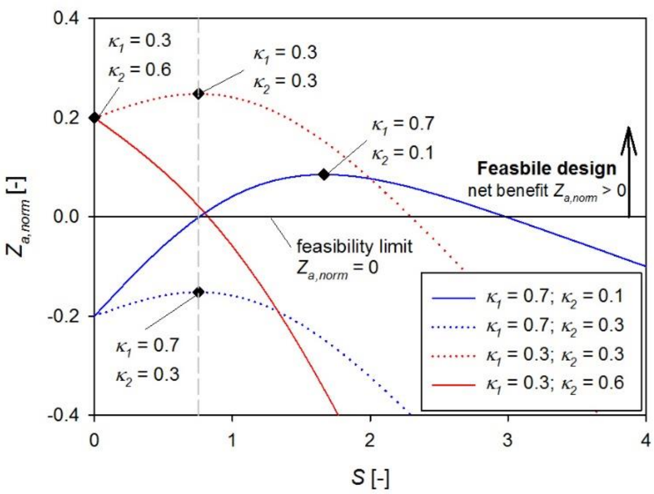

Figure 6: Net annualized utility $Z_{a,norm}$ as a function of implemented sprinkler class, for different cost-parameters κ_1 and κ_2.

To summarise, for the specified conceptual model the optimum sprinkler class is a function of κ_2 only, while κ_1 is determinative for the feasibility of this optimum design. This allows for the succinct representation of Figure 7. The bold (red) curve is the optimum sprinkler class solution as a function of κ_2, indicating that the optimum sprinkler class is the lowest Class *S* = 0 for κ_2 above 0.4. In other words, if the additional investment required for improved sprinkler reliability is too high compared to the annual expected damage costs without sprinklers, then the mathematical optimum design is to opt for the cheapest possible sprinkler protection.

However, the optimum is not necessarily feasible, and the different thinner (black) curves in Figure 7 denote different κ_1-dependent feasibility limits. The grey area corresponds in particular to the area of feasible designs for κ_1 = 0.7. As shown, the optimum design curve is intersected by the boundary of the indicated feasible area (i.e. intersected by the feasibility limit for κ_1 = 0.7), indicating that for κ_1 = 0.7 low sprinkler classes (S = 0 – 1.35) are never optimal. Indeed, the lowest sprinkler classes up to S = 0.5 are never feasible as they are outside the shaded feasible area.

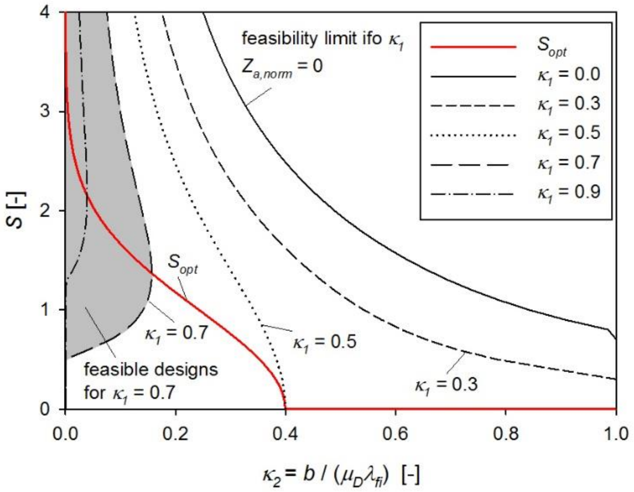

Figure 7: Optimum Sprinkler Class S as a function of κ_2 and feasibility limits as functions of κ_1. Shaded area represents area of feasible designs for κ_1 = 0.7.

The decision-lines in Figure 7 are derived from the dimensionless parameters applicable across expected fire damage costs μ_D and reference fire occurrence rates λ_{fi}, and apply, in principle, irrespective of the valuation perspective. Consider as an example the valuation parameters given in Table 2, all referring to exactly the same design but with different valuation perspectives, i.e. a societal perspective and three different private perspectives (SOC, PRIV1, PRIV2 and PRIV3).

Table 2: Sprinkler installation coefficient valuations: societal valuation (SOC) and three private valuations (PRIV1, PRIV2, PRIV3)

Parameter	**SOC**	**PRIV1**	**PRIV2**	**PRIV3**
Description	Valuation from societal perspective. Risk to life in accordance with *$SWTP_{1life}$*.	Nominal value in agreement with societal valuation, but higher private discount rate. $\gamma_{PRIV1} = \frac{9}{7}\gamma_{SOC}$	Same investment valuation and discount rate as PRIV1, augmented with higher fire-induced damages (e.g. reputational damage and other business concerns)	Same investment, damage valuation and discount rate as PRIV 2, augmented with additional cost of having a sprinkler installation (added to base sprinkler cost *a*).
Annual base sprinkler cost, *a* [UNIT/year]	70	90	90	270
Annual extra *S*-cost, *b* [UNIT/(year·*S*)]	10	12.86	12.86	12.86
Reference fire occurrence rate in	10^{-4}	10^{-4}	10^{-4}	10^{-4}

absence of sprinklers, λ_{fi} [1/year]				
Average damage cost in case of fire, μ_D [UNIT]	10^6	10^6	$3{\cdot}10^6$	$3{\cdot}10^6$
Relative base sprinkler cost, κ_1 [-]	0.7	0.9	0.3	0.9
Relative extra *S-class* sprinkler cost, κ_2 [-]	0.10	0.13	0.04	0.04
Optimum sprinkler class *S* [-]	1.66	1.50	2.11	2.11
Feasibility optimum sprinkler class ($Z_{a,norm} \geq 0$) [Boolean]	yes	no	yes	No
Preferred design based on CBA.	sprinkler installation, *S* = 1.66	no sprinkler installation	sprinkler installation, *S* = 2.11	no sprinkler installation
ALARP requirement	sprinkler installation, *S* = 1.66	sprinkler installation, *S* = 1.66	sprinkler installation, *S* = 1.66	sprinkler installation, *S* = 1.66
Conclusion				
Implemented design	sprinkler installation, *S* = 1.66	sprinkler installation, *S* = 1.66	sprinkler installation, *S* = 2.11	sprinkler installation, *S* = 1.66
Failure probability implemented sprinkler class	$4.8{\cdot}10^{-2}$	$4.8{\cdot}10^{-2}$	$1.7{\cdot}10^{-2}$	$4.8{\cdot}10^{-2}$

Note that the differences in sprinkler cost parameters *a* and *b* in Table 2 can refer both to an inclusion of additional private costs (e.g. aesthetic cost), as well as to different discount rates underlying the calculation of the annualized installation costs. When considering the same nominal upfront investment *C*, the effect of different discount rates can be understood from Equation (16) (infinite time horizon), as elaborated in Equation (35) denoting the difference in annualized cost *c*.

$$C = \frac{c_1}{\gamma_1} = \frac{c_2}{\gamma_2} \Rightarrow c_1 = c_2 \frac{\gamma_1}{\gamma_2} \tag{35}$$

In a specific design situation, the designer may be working directly for a societal institution, in which case the SOC optimum design, in accordance with Table 2, is evaluated and readily implemented. The design then fulfils the ALARP criterion precisely. In other situations, the designer will be working for a private client (private decision maker), e.g. PRIV1, PRIV2 or PRIV3. Furthermore, in those situations, the designer makes the SOC assessment. For a designer acting in accordance with the duty of care, this assessment must necessarily result in exactly the same SOC valuation as if the client was a societal institution. If the private client is interested in further cost-optimization (for example because the client is concerned with business interruption or reputational damage in the wake of a fire event), the designer makes a separate private valuation of costs and benefits in accordance with the private decision maker's valuation preferences, resulting in the PRIV-valuations in Table 2. If the client is not interested in a separate private valuation, the SOC design is readily implemented. The optimum designs for the valuations in Table 2 are shown in Figure 8 (note the reduced range of the horizontal axis) and listed in Table 2, with the SOC optimum functioning as the ALARP benchmark, resulting in the final implemented sprinkler classes specified in Table 2.

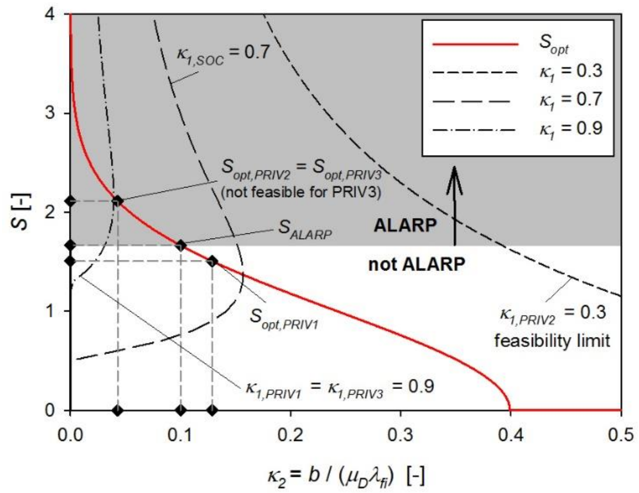

Figure 8: Optimum sprinkler class S as from a societal perspective and for three private investor CBA scenarios (Table 2). Indication of ALARP requirement.

In summary, this conceptual example illustrates the importance of the benefit term evaluation in the ALARP assessment in general, and for sprinkler installations specifically, and emphasizes the distinction between societal and private CBA evaluations. The benefit term evaluation as a 'change of regime' determines whether or not an optimum design solution is feasible. If the fire losses in the reference situation are small (relative to the cost of sprinkler installation), installing sprinklers cannot be considered a requirement in accordance with the ALARP criterion. Naturally, private investors are free to install sprinklers based on any other private considerations they may have (including private CBA). Most interestingly, the results illustrate the need for thorough evaluations when evaluating ALARP. As shown in Figure 8, low reliability sprinklers ($S \leq 0.5$) could never be justified for the considered sprinkler installation cost ($\kappa_1 = 0.7$), whereas sprinkler reliabilities below approximately 90% ($S = 1.35$) are never optimal on this basis. Furthermore, the optimum sprinkler reliability (sprinkler class) quickly increases when the relative cost of increasing sprinkler reliability is low (κ_2 tends to zero). This implies that for design situations with a very high reference annual fire cost in the absence of sprinklers, $\lambda_{fi} \cdot \mu_D$, thorough quality control and maintenance systems, in exceedance of common standards, may be required by the ALARP criterion.

4.4 Example 3: Exit width requirement for retail spaces

The third example is derived from a study by De Sanctis and Fontana [37] where the (single) egress width for retail premises is optimized using the $SWTP_{1life}$ valuation and CBA concepts presented above. In their introduction, De Sanctis and Fontana [37] state that the LQI-based criterion should be a lower bound for the private decision-maker, but this point is not further emphasized in the remainder of the original study; this aspect is therefore further clarified here. Reference is made to De Sanctis [33] and De Sanctis and Fontana [37] for background on the models for (amongst other issues) fire growth, evacuation, and risk to life.

In the following it is considered that the building meets the tolerability prerequisite when considering the reference free door width w of 0.8m (here subjectively considered to be a practical minimum for accessibility of the floor). This implies that, for the given compartment, a 0.8 m door width corresponds to a range of possible fatalities, which is potentially acceptable to society if further risk reduction measures (i.e. widening the door opening) are not 'reasonably practicable'. Thus, the question is whether or not the ALARP criterion requires a further increase of door width.

Consider the societal total (equivalent) utility evaluation of Equation (36). In its general formulation, Equation (36) can be evaluated either on an annual basis or on a lifetime basis. In accordance with the derivations by De Sanctis and Fontana [37], an annualized evaluation will be applied herein.

The investment cost C_{SOC} relates to the societal costs of increasing the door width itself and widening the staircase, and the associated increased construction and land-use costs. The costs are initially born by the private investor but can reasonably be considered to be transferred to society, i.e. $C_{SOC} = C_{PRIV} = C$, through increased pricing of goods and services (Section 2.2.2).

The failure costs D_{SOC} correspond with the $SWTP_{1life}$ valuation of risk to life (average annual number of fatalities) for a given door width. De Sanctis and Fontana evaluate the C and D_{SOC} through the approximations of Equation (37) and Equation (38), with c_i and r_i regression coefficients listed by De Sanctis [33] and w the single exit door width. No non-life saving societal benefits are considered to result from an increase of w. Consequently, B_{SOC} relates only to the change of regime and is evaluated as the costs avoided by the change of safety regime, using the equations for C_{SOC} and D_{SOC} for the reference door width w_{ref}. This results in the B_{SOC} valuation of Equation (39), as visualized further in Figure 9.

$$Z_{SOC} = B_{SOC} - C_{SOC} - D_{SOC} \tag{36}$$

$$C \approx c_1 w^2 + c_2 w + c_3 \tag{37}$$

$$D_{SOC} \approx SWTP_{1life}\left(r_1 \cdot \exp\left(-r_2 w\right) + r_3\right) \tag{38}$$

$$B_{SOC} \approx SWTP_{1life}\left(r_1 \cdot \exp\left(-r_2 w_{ref}\right) + r_3\right) + c_1 w_{ref}^2 + c_2 w_{ref} + c_3 \tag{39}$$

The societal CBA is applied to determine the societally optimal door width w by considering the optimum design criterion of Equation (40), i.e. the derivative with respect to w. The results are shown in Figure 9, using the coefficients specified in Table 3.

$$w_{opt} \Rightarrow \frac{dZ_{SOC}}{dw} = 0 \Rightarrow -r_1 \cdot r_2 \cdot \exp\left(-r_2 w_{opt}\right) \cdot SWTP_{1life} + 2c_1 w_{opt} + c_2 = 0 \tag{40}$$

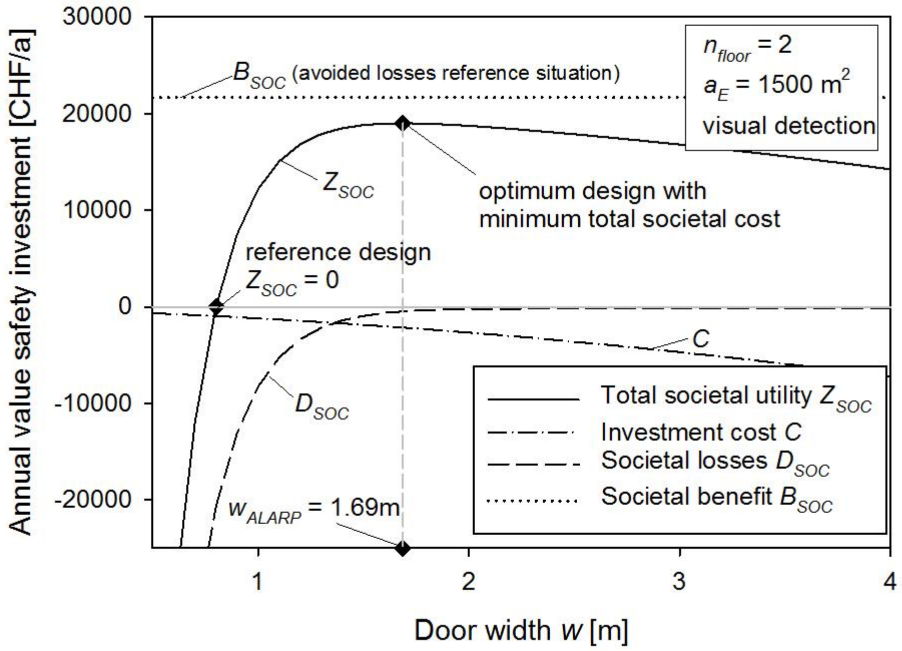

Figure 9: Annual benefits and costs from societal perspective; ALARP design as optimum design according to society's capacity to pay for risk reduction.

Table 3: Cost and risk to life coefficients listed in [33] (enclosure area 1500m², enclosure height 3m, visual detection possible, building with 2 floors). Cost parameter c_4 chosen for illustrative purposes.

Parameter	Value	Unit
Societal Willingness To Pay, $SWTP_{1life}$	$5 \cdot 10^6$	CHF
r_1	0.168	Fatalities/year
r_2	4.64	1/m
r_3	$3.29 \cdot 10^{-5}$	Fatalities/year
c_1	270.6	CHF/(m^2·year)
c_2	664.1	CHF/(m·year)
c_3	289.5	CHF/year
c_4	250	CHF/(m^3·year)

The societal optimum design, taking into account $SWTP_{1life}$ for the valuation of risk to life, is in accordance with Section 2.2 considered as the ALARP requirement and functions as a lower bound for (optional) further private cost-optimisation. Private decision makers are again free to apply their own valuation and to consider more (or less) costs and benefits associated with the safety investment. And again, the private discount rate can be chosen freely, as illustrated above in Example 2.

For illustrative purposes, three private valuation scenarios are considered corresponding to three different private investors (illustrated in Figure 10). The first investor $PRIV_1$ is motivated to avoid casualties in case of fire as these would result in loss of business. This investor values human casualties at (for illustrative purposes) a value numerically equal to 3 times the $SWTP_{1life}$. The second investor

$PRIV_2$ considers that fatalities are a societal concern governed through societal requirements, and neglects fatalities in their private optimisation. For this investor, the private optimum door width is trivially equal to the minimum practical door width of 0.8m. The third investor $PRIV_3$ on the other hand has similar concerns as $PRIV_1$ and values human casualties at a value numerically equal to 1.5 times the $SWTP_{1life}$, but also considers the door width to negatively influence the concept and aesthetics of their retail space. This additional private cost is a disadvantage/loss and is valued through Equation (41). Total cost-curves as functions of *w* and optimum designs are identified in Figure 10 for the three scenarios. For $PRIV_1$ the private optimum door width $w_{opt,PRIV1}$ exceeds w_{ALARP}. This investor will implement a door width beyond the societal requirement for adequate safety. Investors $PRIV_2$ and $PRIV_3$, on the other hand, both consider a private optimum door width below w_{ALARP}. For both investors a door width equal to w_{ALARP} (or exceeding) must be implemented in order to comply with the ALARP requirement, i.e. w_{ALARP} functions as a lower bound of safety for the private cost-optimisation.

Note that the above valuations of risk to life by private decision makers should not be considered as an application of risk aversion. Rather, private decision makers are free in their valuation of costs, benefits and discount rates (as indicated in Section 2.2.5). As such, the valuation of risk to life by private decision makers can have any underlying (and unknown) motivation, including a genuine motivation to improve life safety beyond societal requirements or a pure drive to avoid loss of revenue associated with negative publicity. However, at all times the societal lower bound investment level defined by the ALARP criterion applies, ensuring life safety investments in accordance with the societal capacity to commit resources for life safety risk reduction.

$$C_{PRIV3,AES} = c_4 \cdot w^4 \quad (41)$$

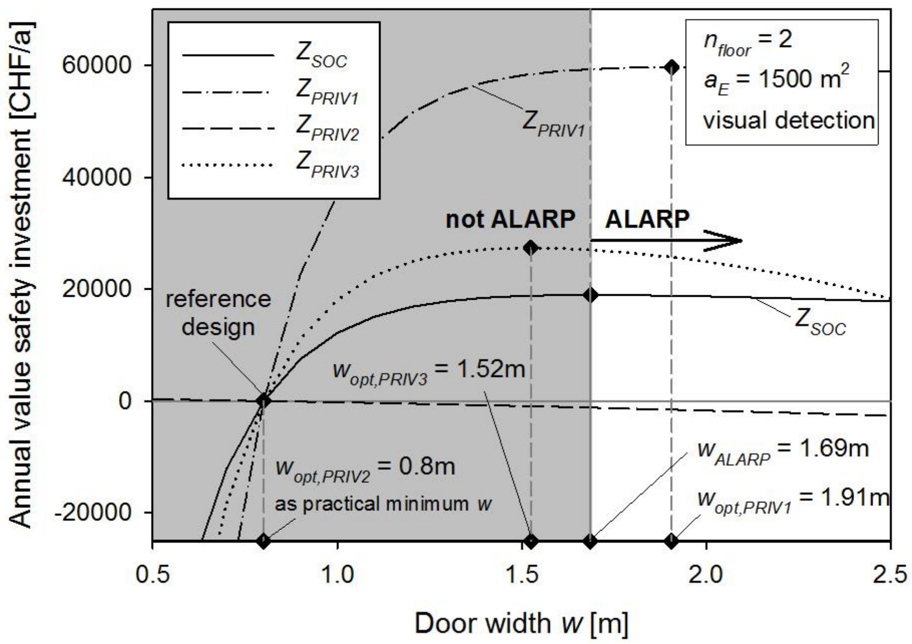

Figure 10: Annual costs and optimum design from societal perspective and for three private investor CBA scenarios. Indication of ALARP acceptance limit for adequate safety.

5 Conclusions

Current approaches to demonstrating that the residual risk levels for a fire safety design are As Low As Reasonably Practicable (ALARP) are often contradictory or unclear, especially with respect to the concepts of risk aversion, the valuation of risk to life, the concept of disproportionality, and the need to account for monetary benefits resulting from life safety investments. The ALARP requirement has therefore been clarified herein for application to a Probabilistic Risk Analysis-based fire safety design, as a societal, risk-neutral, and scalar cost-benefit analysis. Contrary to available literature that applies disproportionality factors, the proposed interpretation implies that qualitative societal differentiations as functions of consequence severity or occurrence frequency ought to be considered only in conjunction with a prerequisite of tolerability. Subsequently, the moral dilemma associated with the valuation of reduction in risk to human lives has been addressed through the compound social indicator of the Life Quality Index (LQI), in accordance with the recent ISO 2394:2015 standard in structural engineering. However, contrary to previous applications, the societal ALARP criterion is thus associated with an LQI maximum societal benefit criterion. This implies that societal monetary benefits from life safety investments ought to be included in the ALARP assessment and attempts to ensure that safety investments in life safety that are within society's means are implemented.

Furthermore, the relationship between the ALARP requirement and private cost-benefit considerations has been discussed, indicating that these evaluations are distinct. Private valuations are necessarily free to choose, both regarding the valuation of costs and benefits, and regarding the discount rate. Nonetheless, the societal evaluation of costs and benefits implied by the ALARP requirement functions as a lower bound (minimum safety investment) for private considerations. Safety measures that exceed society's current capacity to pay are therefore left to the discretion of private decision makers. Whilst this concept is consistent with recent literature in this area, the resulting societal minimum investment levels are more stringent in the proposed interpretation due to the application of the LQI maximum societal benefit criterion. This significantly enhances previous interpretations insofar as requiring increased safety investments. Finally, the wariness with respect to the valuation of costs and benefits by the fire safety engineer is addressed by emphasizing the duty of care, and by specifying the benchmark of the objective, diligent, and competent fire safety professional. This promotes self-reflection amongst designers and removes the need for often unjustified and poorly specified disproportionality factors.

Three example applications have been presented, illustrating application of the presented concepts to decisions on sprinkler installation, sprinkler efficiency, and egress width. While the first example demonstrates the compatibility of the presented concepts with earlier applications, the second example application illustrates the at times non-intuitive nature of ALARP evaluations. For the considered conceptual situation low reliability sprinkler classes were found to result in a net detriment to society, while more reliable sprinkler classes are at the same time required in accordance with the ALARP criterion. The third example demonstrates how private decision makers may or may not wish to invest beyond the societal minimum ALARP requirement. Together these examples highlight the importance of the societal ALARP evaluation by showing that it governs the safety investment when the private investor doesn't value damages, applies a high discount rate, or takes into account indirect costs (e.g. aesthetic costs) related to the safety investment.

Overall, the presented study provides a basis for evaluating ALARP in PRA fire safety design and shows how an ALARP evaluation procedure can be applied. It also provides a potential basis for deriving simplified design guidance towards reliability-based fire safety engineering.

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