Statistical physics of balance theory

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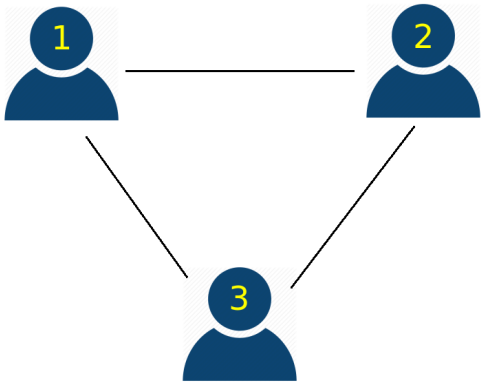
(Social) Balance Theory

- Proposed originally by Fritz Heider (1958).
- If two people have a positive (negative) relationship, their opinion about an object will be shared (differ)
- Friends tend to have similar preferences and common friends and enemies, and enemies tend to have the opposite.



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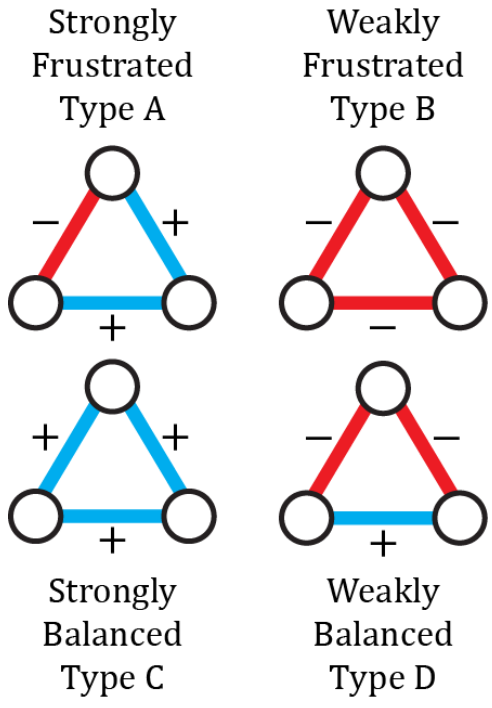
(Social) Balance Theory

- Balance Theory applied to 3-cycles : **Your friend's friend is your friend** and **your enemy's enemy is your friend**.
- Discrepancies have been observed - Not all frustrated triads behave equally. So we study the 4 cases.
- It has been measured that some unbalanced states are (sometimes) more common than balanced. What if we compare with a random network ?



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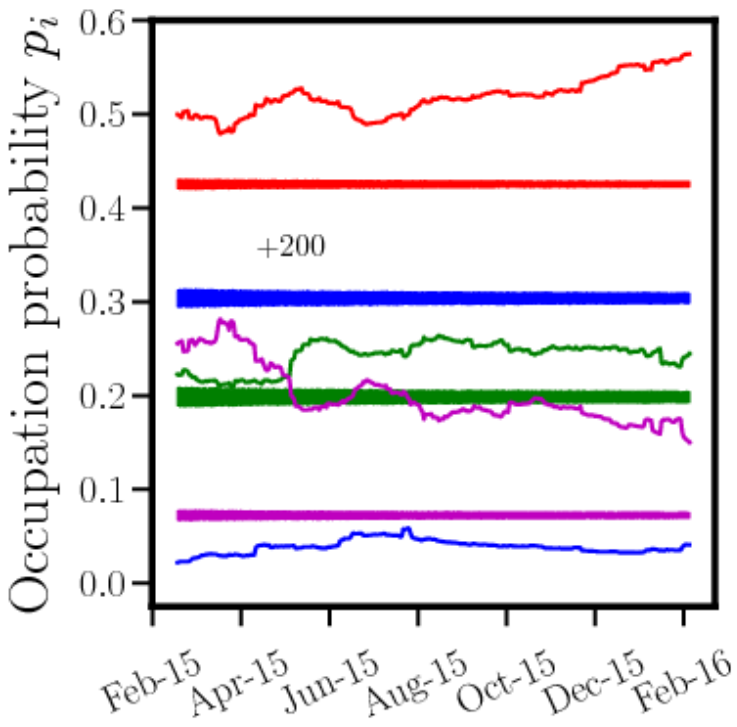
Comparative with a random network

Example : Timeseries of occupation probabilities in the political network of an Online World : Eve Online

- Balanced $[+--]$ (red) and $[+++]$ (magenta) are more common than random.
- Low Frustrated $[---]$ (green) is slightly more common than random.
- Strongly Frustrated $[++-]$ (blue) is much less common than random.



Energies

- Each probability (p_i) is related to an energy of state i (E_i) and a degeneracy $g(E_i)$.

$$p_i = \frac{g(E_i)e^{-E_i/T}}{\sum_j g(E_j)e^{-E_j/T}}$$

- If all energies are the same, the probability matches with the situation in a random network.

$$p_i = \frac{g(E_i)}{\sum_i g(E_i)}$$

- Lower (Higher) energies mean it would appear more often (less often) than in a random network :
  The systems' individual components "prefer" states with lower energies.

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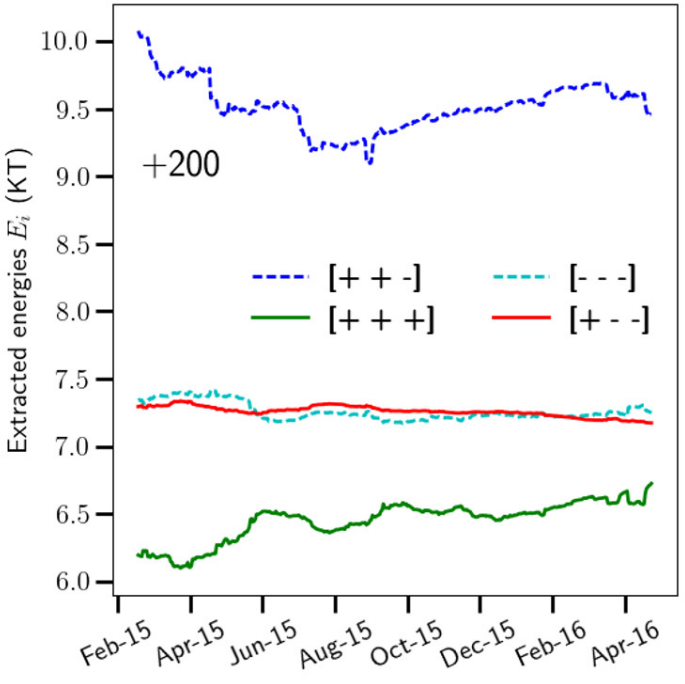
Political networks in Online World

$$E_i/T = -\ln(p_i/g(E_i)) + cte$$

- Strongly Balanced $[+++]$ (magenta) has the lowest energy.
- Low Frustrated $[---]$ (green) and Low Balance $[+--]$ (red) present similar energies
- Strongly Frustrated $[++-]$ (blue) has higher energy



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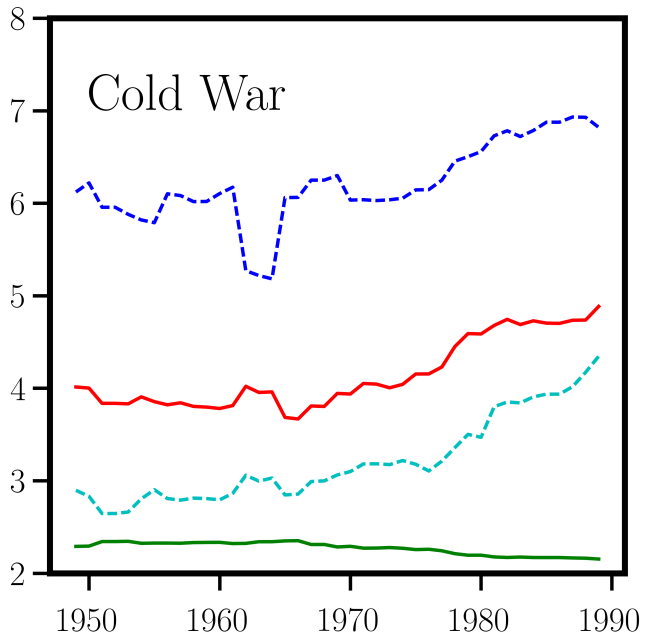
Cold War

$$E_i/T = -\ln(p_i/g(E_i)) + cte$$

- Strongly Balanced $[+ + +]$ (magenta) has the lower energy.
- Low Frustrated $[- - -]$ (green) and Low Balance $[+ - -]$ (red) present similar energies, but low frustrate is slightly more common
- Strongly Frustrated $[+ + -]$ (blue) has higher energy



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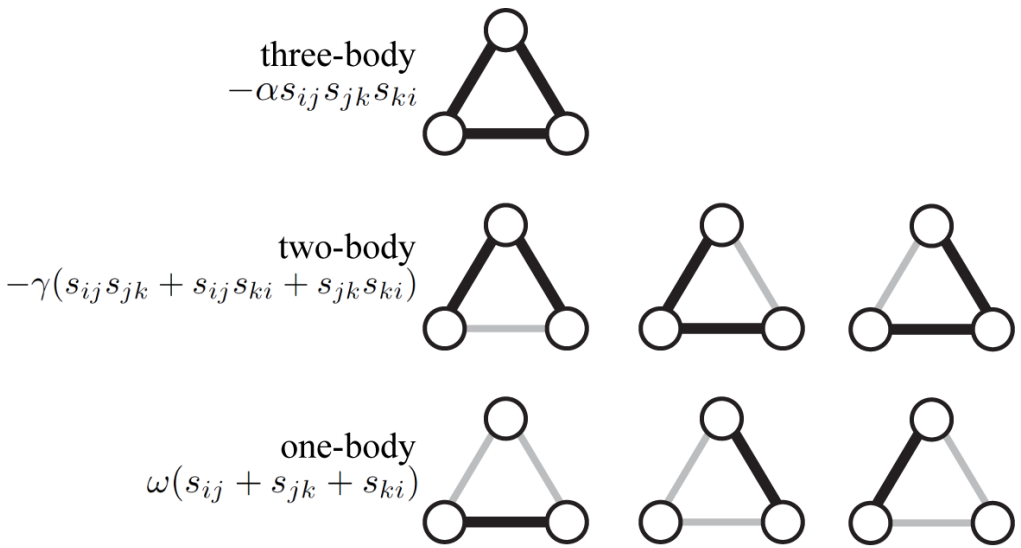
Hamiltonian

Can we describe the energies as a sum of energies related to the links ?

State	Energy
$[+\,+\,-]$	$\alpha + 2\gamma + \omega$
$[-\,-\,-]$	$\alpha - 3\gamma - 3\omega$
$[+\,-\,-]$	$-\alpha + 2\gamma - \omega$
$[+\,+\,+]$	$-\alpha - 3\gamma + 3\omega$



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Hamiltonian

Adjust of the Hamiltonian to the average of energies

Parameter	EVE (SOV)	EVE (+200)	Cold War	Middle East
α (T)	0.95 ± 0.03	1.02 ± 0.04	0.89 ± 0.07	1.09
γ (T)	0.38 ± 0.02	0.41 ± 0.02	0.61 ± 0.07	0.38
ω (T)	0.18 ± 0.02	0.14 ± 0.02	0.14 ± 0.07	0.22
Cte (T)	8.00 ± 0.03	8.04 ± 0.04	4.67 ± 0.07	2.70

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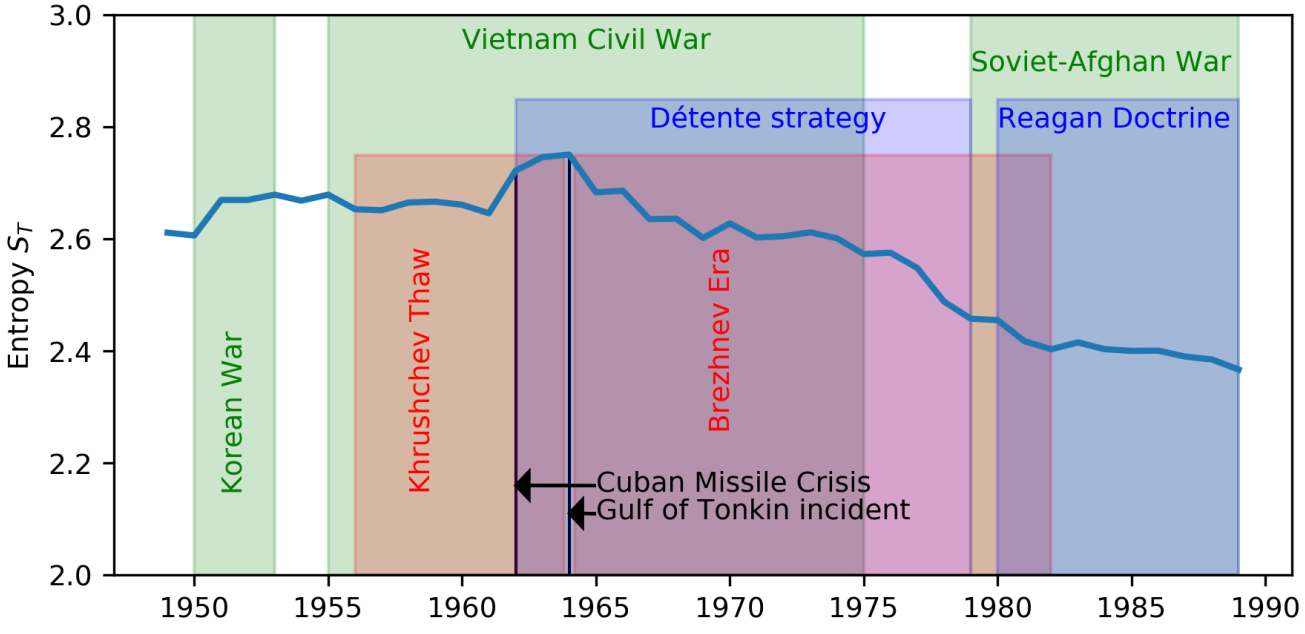
Entropy

Entropy associated with a system of discrete levels $\rightarrow$ Information on the Homogeneity in the system :



$$S = \sum_i p_i \ln(p_i)$$

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Overview

- The proposed model allows one to quantitatively study social balance.
- We can separate the dynamical mechanisms (regulated by the energies) and stochastic aspects (random network).
- We find a persistent hierarchy, where the $[+++]$ ($[++-]$) triad is the most (least) balanced.
- Shannon entropy $\rightarrow$ detect the change points in the time series.
- Importance SBT (three-body forces) ; however, strong corrections via the two-body force.

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Future Work : Extensions

- Transition Probabilities and their connection with the energy differences.
- Adding Neutral links : Introduce 6 new triads.
  $[0++]$, $[0+-]$, $[0--]$, $[00+]$, $[00-]$, $[000]$.

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End

Thanks for your attention.

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