



Fibonacci, pioneer in multidisciplinary mathematics education

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Abstract

Mathematics and (engineering) sciences match together and reinforce each other through modeling. The Fibonacci sequence where the next number is found by adding up the two numbers before it, can be considered as a role model in making mathematics more appealing, more interesting and more accessible due to its multidisciplinary character. Connections with architecture, art, structures in nature, music are opportunities for teachers to stimulate students' enthusiasm for Science, Technology, Engineering and Mathematics, in short STEM. Keeping in mind the unfulfilled demand for scientists and engineers, a priority area within education is to get young people involved in STEM. The subject of Fibonacci numbers and golden ratio fits well within the current tendency of inquiry-based and content-based mathematics education. We describe how the legendary Fibonacci can be a source of inspiration for science and mathematics teachers. We give some possible student activities and assignments for active involvement of the students concerning this subject. Moreover, projects targeting to make science education more interesting and relevant to student's lives are strongly encouraged by funding of the European Union.

Keywords: mathematics, motivation, STEM, multidisciplinary

1. STEM and multidisciplinary

The enrolment rates in STEM-based degree programs are decreasing [4], where STEM stands for Science, Technology, Engineering and Mathematics. This will lead to a workforce problem in the industrial sector as well as in research and development. Lewis [3] described the significance of the classroom climate in students' experience of disaffection towards mathematics. Content-based mathematics education can help to stimulate students' enthusiasm for mathematics. The inquiry-based activities should spark an interest in STEM which will motivate the student to endeavor future academic and scientific challenges. Mathematics educators should try to break the virtual walls between the scientific disciplines and offer a multidisciplinary approach that aims to enhance and broaden student's understanding while enabling them to gain a wider perspective of the physical world. When modeling (applied) scientific phenomena, mathematics appears naturally in describing the relationships between physical, biological, mechanical ... quantities. Generally, scientists use models to explain how the world works, whereas engineers use models to predict or estimate the behavior of a system given some input [2]. Whiteley [8] also stresses the importance of qualitative mathematics education to encounter the concept of uncertainty while using models to make predictions.

2. European projects

During recent years increasing attention was given to real world applications in mathematics education, encouraged by several European projects. The project Inspiring Science Education (ISE) (<http://www.inspiringscience.eu>) elaborates inspiring examples in mathematics and other sciences. ISE is a EU funded initiative that stimulates the use of inquiry-based learning in science by providing digital resources for teachers to help them make science education more interesting and relevant to student's lives. Another European project SciChallenge (<http://www.scichallenge.eu>) focuses on the development of novel concepts to get young people excited about science education. It uses a content-based approach towards self-produced digital education materials from young people for young people. The European Fibonacci project (<http://fibonacci.uni-bayreuth.de/home.html>) tried to disseminate inquiry-based teaching and learning methods in science and mathematics in primary and secondary schools. Stocker [5] and Verzosa [7] promote the use of real-world problems. Stocker



describes problems with a strong component of social justice in order to discuss the mathematics of problems that are central to student lives. The resulting content of these European projects does not always meet this requirement. This can be an area of attention for future projects.

3. Fibonacci

When promoting multidisciplinary mathematics, why not use the famous historical person Fibonacci himself as inspiring mentor of mathematics? Leonardo of Pisa (1170–1250), also known as Fibonacci, is often considered the most talented mathematician of the Middle Ages. In his book, *Liber Abaci*, he posed and solved a problem involving the growth of a hypothetical population of rabbits based on idealized assumptions. The solution was a sequence of numbers which came to be known as the Fibonacci sequence: the number of existing pairs of rabbits at a given month is the sum of the two previous numbers of pairs in the sequence: 1, 1, 2, 3, 5, 8, 13, 21... The biological topic about the breeding of rabbits is a wonderful example where the Fibonacci numbers are adopted in everyday life (Fig. 1).

Description of the problem:

Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits never die and that the female always produces one new pair (one male, one female) every month from the second month on. How many pairs will there be in one year?

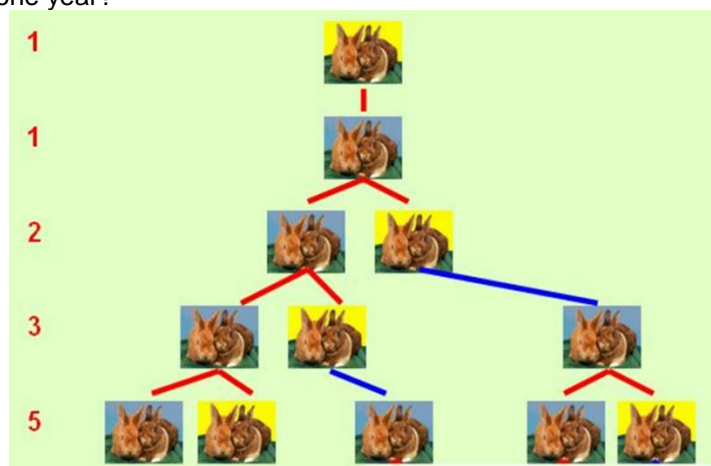


Fig. 1: Fibonacci numbers in the breeding process of rabbits.

Quadratic equations arise in a natural way when discovering that the ratio φ of two consecutive numbers in the Fibonacci sequence is a constant, named the golden mean [1], i.e. $(1 + \sqrt{5})/2$ as solution of $\varphi^2 - \varphi - 1 = 0$. The last is a consequence of the equation $1 + \frac{1}{\varphi} = \varphi$ as $\varphi = \frac{x_{n+2}}{x_{n+1}} = \frac{x_{n+1}}{x_n}$ and $x_{n+2} = x_{n+1} + x_n$.

In the mathematics class, this is the ideal opportunity to address the notion of limit, supported by a graph as in Fig. 2.

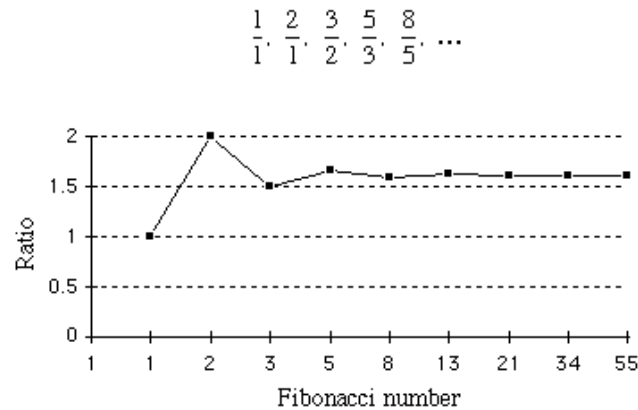


Fig. 2: Convergence of the ratio of Fibonacci numbers.

Moreover, when thinking in terms of natural growth patterns the connection between spirals and the Fibonacci sequence shows up. Spirals arise from a property of growth called self-similarity or scaling, i.e. the tendency to grow but to maintain the same shape. Not all organisms grow in this self-similar manner. Adult people, for example, are not just scaled up babies: babies have larger heads, shorter legs, and a longer torso relative to their size. But if we look at the shell of the chambered nautilus we see a different growth pattern. As the nautilus outgrows each chamber, it builds new chambers for itself, always the same shape. The shell of a very long-lived nautilus would spiral around and around, growing ever larger but always looking the same at every scale.

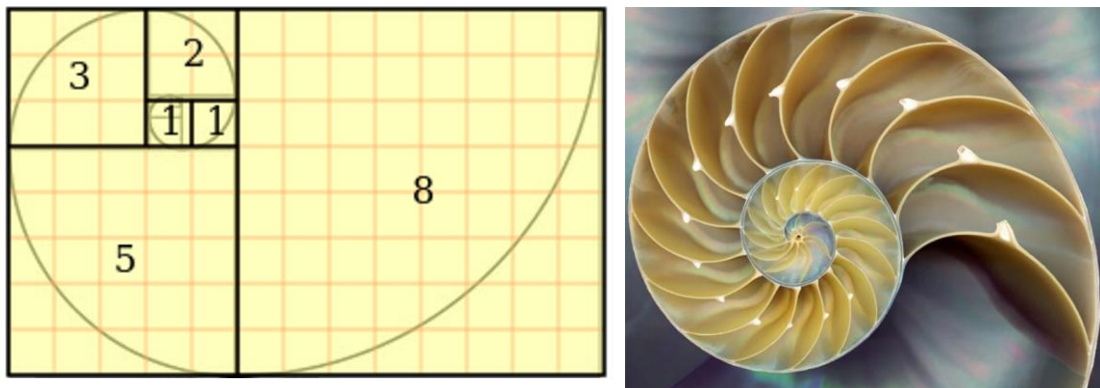


Fig. 3: Fibonacci numbers in the spirals of a nautilus.

Here it is where Fibonacci comes in: we can build a nautilus by starting from a square of size 1 and successively building on new rooms whose sizes correspond to the Fibonacci sequence (Fig.3).



Fig. 4: Fibonacci numbers in the seeds pattern of a sunflower.



Another biological application of the Fibonacci numbers is the pattern of the seeds of a sunflower (Fig.4). The Fibonacci spiral as for the nautilus is duplicated and rotated around the center. After the circular paths is mirrored, it results in the pattern of the seeds of a sunflower. The same phenomenon can be recognized in the pineapple (let the students count the number of spirals to the left and the number of spirals to the right) and the pinecone.

The link between the golden mean and architecture and art is well known. But music is an application field of Fibonacci's numbers as well. Van Gend [6] describes the presence of the Fibonacci sequence in the structure of the octave scale. He also notes the use of the golden ratio in instrument design and in certain musical works of composer Claude Debussy.

But even in every day habits as climbing a staircase, Fibonacci numbers are hidden. We assume that when climbing a staircase, you either take one step at a time or two steps at a time. When x_n is the number of different ways to climb a staircase with n stairs, we recognise a row of Fibonacci numbers. Fig. 5 shows the possibilities for $n = 1, n = 2, n = 3$.

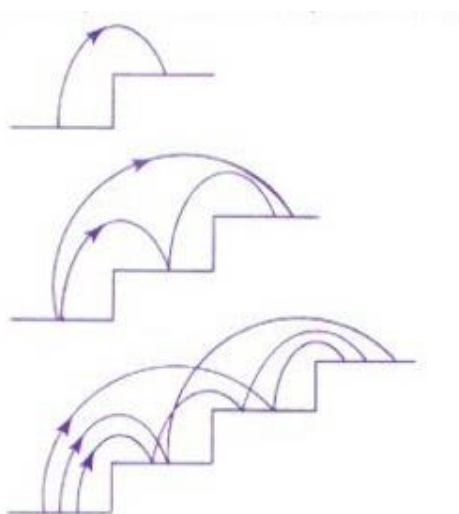


Fig. 5: Fibonacci numbers for counting ways to climb a staircase.

4. Student activities

Possible activities to involve student actively into this subject are:

- Make an exposition of pictures from real life containing the Fibonacci spiral.
- Write computer code that generates the Fibonacci numbers using recursion.
- Find a piece of art where the golden mean is used.
- Find a famous building where the golden mean is used in the architecture.
- Make a new sequence by dividing every term in the Fibonacci by 2. Does this sequence still act like a Fibonacci sequence? Why or why not?
- Make a new sequence by subtracting 4 to every term in the Fibonacci sequence. Does this sequence still act like a Fibonacci sequence? Why or why not?

5. Conclusion

Projects funded by the European Union attempt to stimulate content-based and inquiry-based learning where mathematics plays a prominent role in the multidisciplinary approach of studying science. The Fibonacci sequence is one example where biology and mathematics go together hand in hand. It can be used to appreciate and investigate a numerical pattern, to look for evidence of mathematical patterns in nature. Much of mathematics is done because of its intrinsic interest, without regard to its usefulness. Still, most mathematical disciplines do have applications, and much work in mathematics is stimulated by applied problems. Science and technology provide a large share of such applications



and stimulants. On the one hand, there have been some remarkable cases of finding new uses for centuries-old mathematics. On the other hand, the needs of natural science or technology have often led to the formulation of new mathematics.

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