

Sparse MRI with a Markov Random Field Prior for the Subband Coefficients

Marko Panić*, Dejan Vukobratović†, Vladimir Crnojević* and Aleksandra Pižurica‡

*University of Novi Sad, BioSense Institute
mpanic@uns.ac.rs, crnjvc@gmail.

†University of Novi Sad, Department of Power, Electronics and Telecommunications
dejan.vukobratovic@gmail.com

‡Ghent University, Department Telecommunications and Information Processing, TELIN-IPI-iMinds
Aleksandra.Pizurica@ugent.be

Abstract—Recent work on compressed sensing in magnetic resonance imaging (CS-MRI) indicates benefits of modelling the *structure* of sparse coefficients. Comprehensive studies are available for tree-structured models. Much less work has been done on using statistical models for intra-scale (spatial) dependencies, like Markov Random Field (MRF) models in CS-MRI, although initial studies showed great potentials. We present here an efficient greedy algorithm with MRF priors and demonstrate encouraging performance in comparison to related methods, including those based on tree-structured sparsity.

I. INTRODUCTION

Compressed sensing (CS) for magnetic resonance imaging (MRI), dubbed CS-MRI, typically solves the problem

$$\min_{\mathbf{x}} \frac{1}{2} \|\mathbf{Ax} - \mathbf{y}\|_2^2 + \tau \phi(\mathbf{Px}) \quad (1)$$

where $\mathbf{x} \in \mathbb{C}^N$ is the ideal image and $\mathbf{y} \in \mathbb{C}^M$ are measurements obtained through partially observed Fourier transform $\mathbf{A} \in \mathbb{C}^{M \times N}$, $M \ll N$, with added noise $\mathbf{n} \in \mathbb{C}^M$ [1], [2]. $\mathbf{P} \in \mathbb{C}^{D \times N}$ denotes a sparsifying transform, $\tau > 0$ is a parameter and $\phi: \mathbb{C}^D \mapsto \mathbb{R} \cup \{-\infty, +\infty\}$ is a regularization function. When $\mathbf{P}$ is a wavelet-like transform, ϕ is typically the ℓ_1 norm: $\phi(\boldsymbol{\theta}) = \|\boldsymbol{\theta}\|_1$. Another common regularization is Total Variation (TV), where $\mathbf{P}$ is a discrete gradient operator. Compound regularization (a combination of ℓ_1 and TV) is often used as well [1]–[4]. Recent works incorporate modelling the *structured sparsity*, and in particular wavelet tree models have been proved beneficial in CS-MRI [5], [6]. An elegant algorithm LaMP (Lattice Matching Pursuit), which incorporates modelling of the spatial support of sparse images by a Markov Random Field (MRF), into a greedy solver was introduced in [7]. LaMP is not directly applicable to images that are not sparse in the canonical domain (and most MRI images are not). A related algorithm LaSB (Lattice Split Bregman) [8], which combines MRF modelling of the subband data with an augmented Lagrangian method showed promising results in MRI. It was unclear so far whether the success of LaSB could also be reached with a simpler, greedy type of methods, and it was also not clear how any of these methods would compare to alternative wavelet-tree sparsity methods [5], [6]. We address these questions and design a fast and simple MRF-based method for CS-MRI, demonstrating excellent performance.

II. A GREEDY CS-MRI ALGORITHM WITH MRF PRIORS

Let us first revisit briefly the original Lattice Matching Pursuit (LaMP) algorithm of [7], before analysing possible extensions to make it applicable to MRI. Our new algorithm, inspired by this analysis, will follow then.

The original LaMP, with the pseudocode (using our notation) in Alg. 1, assumes that the image is sparse in the canonical domain. Its main idea is to incorporate the estimation of the likely *support* $\mathbf{s}$ of the actual signal into the matching pursuit iterations. In particular, Step 4 in each iteration k of Alg. 1 assigns to $\mathbf{s}^{\{k\}}$ the *maximum a posteriori* (MAP) estimate of the support of the temporary signal estimate $\mathbf{x}_t^{\{k\}}$, assuming a MRF prior for the support. With a homogeneous Ising model, with labels $s_i \in \{-1, 1\}$, and using the common conditional independence assumption for the likelihood $p(\mathbf{x}_t | \mathbf{s}) = \prod_i p([\mathbf{x}_t]_i | s_i)$, the MAP estimate of the support of $\mathbf{x}_t^{\{k\}}$ (denoted as *MAP-support* $\{\mathbf{x}_t^{\{k\}}\}$ in Alg. 1) is:

$$\mathbf{s}_{MAP}^{\{k\}} = \max_{\mathbf{s} \in \{-1, 1\}^N} \sum_{\langle i, j \rangle} \beta s_i s_j + \sum_i [\alpha s_i + \log(p([\mathbf{x}_t^{\{k\}}]_i | s_i))]$$

where β and α are the parameters of the Ising model, controlling the strength of the pair-wise clique potentials and the preference of one type of labels over the other, respectively¹. The pseudo-inversion $\mathbf{A}^\dagger$ of the measurement matrix (Step 5) is then applied only for the columns of $\mathbf{A}$ selected by $\mathbf{s}^{\{k\}}$. Additional pruning to K largest signal components (Step 6) yields the signal estimate $\mathbf{x}^{\{k\}}$.

This algorithm is directly applicable to the problem (1), only with $\mathbf{P} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. We need to extend it such that it works in the case where $\mathbf{P}$ corresponds to a wavelet-like transform. A possible extension, which would allow applying LaMP to CS-MRI would be to replace steps 4–6 with:

$$\boldsymbol{\theta}_t^{\{k\}} = \mathbf{Px}_t^{\{k\}}; \quad \mathbf{s}^{\{k\}} = \text{MAP-support}\{\boldsymbol{\theta}_t^{\{k\}}\} \quad (2a)$$

$$\boldsymbol{\theta}_{t'}^{\{k\}} = \mathbf{PA}^\dagger \mathbf{y}; \quad \mathbf{t}[\mathbf{s}^{\{k\}} = 1] = \boldsymbol{\theta}_{t'}^{\{k\}}[\mathbf{s}^{\{k\}} = 1] \quad (2b)$$

$$\boldsymbol{\theta}^{\{k\}} = \text{Prune}(\mathbf{t}, K); \quad \mathbf{x}^{\{k\}} = \mathbf{P}^H \boldsymbol{\theta}^{\{k\}} \quad (2c)$$

¹In [7], a *non-homogeneous model* is allowed, with variable parameters $\beta_{i,j}$ and α_i depending on the spatial position, but this is not relevant here.

Algorithm 1 LaMP [7]

Input: $k = 1, \mathbf{y}, K, \mathbf{x}^{\{0\}}, \mathbf{t} = \mathbf{0}$

- 1: **repeat**{Matching Pursuit Iterations}
- 2: $\mathbf{r}^{\{k\}} = \mathbf{y} - \mathbf{A}\mathbf{x}^{\{k-1\}}$
- 3: $\mathbf{x}_t^{\{k\}} = \mathbf{A}^H \mathbf{r}^{\{k\}} + \mathbf{x}^{\{k-1\}}$
- 4: $\mathbf{s}^{\{k\}} = \text{MAP-support}\{\mathbf{x}_t^{\{k\}}\}$
- 5: $\mathbf{t} = \mathbf{0}; \quad \mathbf{t}[\mathbf{s}^{\{k\}} = 1] = \mathbf{A}^\dagger[\mathbf{s}^{\{k\}} = 1, :]\mathbf{y};$
- 6: $\mathbf{x}^{\{k\}} = \text{Prune}(\mathbf{t}, K)$
- 7: $k = k + 1$
- 8: **until** Maximum iterations or $\|\mathbf{r}^{\{k\}}\| \leq \text{threshold}$

Algorithm 2 The proposed algorithm: GreeLa

Input: $k = 1, \mathbf{y}, \mathbf{x}^{\{0\}}, \mathbf{t} = \mathbf{0}$

- 1: **repeat**
- 2: $\mathbf{r}^{\{k\}} = \mathbf{y} - \mathbf{A}\mathbf{x}^{\{k-1\}}$
- 3: $\mathbf{x}_t^{\{k\}} = \mathbf{A}^H \mathbf{r}^{\{k\}} + \mathbf{x}^{\{k-1\}}$
- 4: $\boldsymbol{\theta}_t^{\{k\}} = \mathbf{P}\mathbf{x}_t^{\{k\}}$
- 5: $\mathbf{s}^{\{k\}} = \text{MAP-support}\{\boldsymbol{\theta}_t^{\{k\}}\}$
- 6: $\mathbf{t} = \mathbf{0}; \quad \mathbf{t}[\mathbf{s}^{\{k\}} = 1] = \boldsymbol{\theta}_t^{\{k\}}[\mathbf{s}^{\{k\}} = 1]$
- 7: $\boldsymbol{\theta}^{\{k\}} = \mathbf{t}, \mathbf{x}^{\{k\}} = \mathbf{P}^H \boldsymbol{\theta}^{\{k\}}$
- 8: $k = k + 1$
- 9: **until** Maximum iterations or $\|\mathbf{r}^{\{k\}}\| \leq \text{threshold}$

Two important problems with this extension are: (i) the calculation of $\mathbf{P}\mathbf{A}^\dagger\mathbf{y}$ is costly, both in terms of the computation time and memory requirements and (ii) determining K in each subband is not trivial. Hence, we propose a simplified, greedy algorithm where the computation of the pseudo inverse is avoided by replacing $\boldsymbol{\theta}_t^{\{k\}}$ in (2b) by $\boldsymbol{\theta}_t^{\{k\}}$ and by excluding the additional pruning step (2c) (the sparseness is guaranteed already by the estimated support $\mathbf{s}^{\{k\}}$ using the right parameters of the prior MRF model). The proposed greedy algorithm named GreeLa (Greedy Lattice regularization) is summarized in Alg. 2. We employ the likelihood model from [8] and we also use the Metropolis sampler as in [8] for finding the MAP estimate of the support.

III. EXPERIMENTS AND DISCUSSION

In our experiments we used an MRI data set (brain scan) acquired on a Cartesian grid at the Ghent University hospital (UZ Ghent)², also used in [8], [9]. Here we show the results on 248 sagittal slices from this data set (each slice is a 256×256 image, and Fig. 1 shows some of them). We report the results for simulated *radial* undersampling trajectories in the k -space with different sampling rates (similar results – not shown here – were obtained with other trajectories). For the sparsifying transform we used the non-decimated shearlet transform, with the implementation from [10], with 3 scales and 8, 4, and 2 orientations per scale (fine-to-coarse). We compare the results to LaSB [8], FISTA [11] and the wavelet-tree sparsity (WaTMRI) method [5], [6] with the original implementation³.

²Data acquired thanks to Prof. Dr. Karel Deblaere at Radiology Department of UZ Ghent.

³<http://ranger.uta.edu/~huang/index.html>

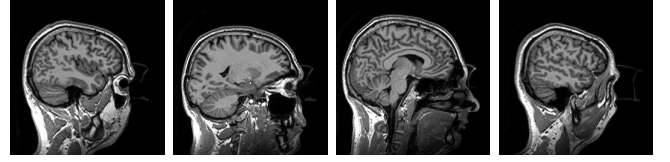


Fig. 1. Several sagittal slices from our MRI data set (256×256).

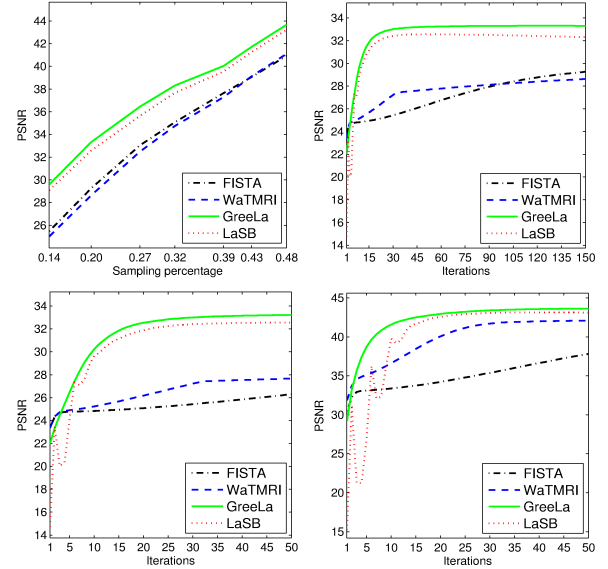


Fig. 2. **Top left:** PSNR for the reconstructions of one slice (second left in Fig. 1) for different sampling rates. **Top right and Bottom left:** Reconstruction performances on the same slice with 20% measurements in 150 and 50 iterations, respectively. **Bottom right:** Mean PSNR for 248 MRI slices with the sampling rate of 48%.

The MRF parameters were optimized separately for LaSB ($\alpha = .017, \beta = .07$) and for GreeLa ($\alpha = 1e - 4, \beta = .34$). Fig. 2 shows the Peak Signal to Noise Ratio (PSNR) for one slice, with sampling rate (SR) ranging from 14% to 48%, and the evolution of the PSNR per iteration for a particular SR (20%). The MRF-based methods GreeLa and LaSB achieve a consistent and significant improvement in PSNR (up to 3 dB) compared to FISTA and WaTMRI for all SR values, and they also approach convergence in fewer iterations. GreeLa yields slightly higher PSNR than LaSB. From the first 50 iterations (bottom left in Fig. 2), we can see a more stable behaviour of GreeLa (the PSNR of LaSB oscillates strongly in these first iterations). The average results on 248 MRI sagittal slices (bottom right in Fig. 2, SR=48%) lead to similar conclusions: although WaTMRI and FISTA increased their performances on average, GreeLa and LaSB yield a superior PSNR and converge in fewer iterations. A more stable behaviour of GreeLa compared to LaSB as well as slightly better PSNR are again observed. Given that the new algorithm is conceptually simpler, easier to implement and free of parameters of the Split-Bregman iterations, these results are highly encouraging.

REFERENCES

- [1] M. Lustig, D. Donoho, and J. M. Pauly, "Sparse MRI: The application of compressed sensing for rapid MR imaging," *Magnetic Resonance in Medicine*, vol. 58, no. 6, pp. 1182–1195, 2007.
- [2] M. Lustig, D. L. Donoho, J. M. Santos, and J. M. Pauly, "Compressed sensing MRI," *Signal Processing Magazine, IEEE*, vol. 25, no. 2, pp. 72–82, 2008.
- [3] J. Huang, S. Zhang, and D. Metaxas, "Efficient MR image reconstruction for compressed MR imaging," *Medical Image Analysis*, vol. 15, no. 5, pp. 670–679, 2011.
- [4] S. Ma, W. Yin, Y. Zhang, and A. Chakraborty, "An efficient algorithm for compressed MR imaging using total variation and wavelets," in *IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, 2008, pp. 1–8.
- [5] C. Chen and J. Huang, "Compressive sensing MRI with wavelet tree sparsity," in *Advances in neural information processing systems*, 2012, pp. 1115–1123.
- [6] C. Chen and J. Huang, "Exploiting the wavelet structure in compressed sensing MRI," *Magnetic Resonance Imaging*, vol. 32, no. 10, pp. 1377–1389, 2014.
- [7] V. Cevher, M. F. Duarte, C. Hegde, and R. Baraniuk, "Sparse signal recovery using Markov random fields," in *Advances in Neural Information Processing Systems*, 2009, pp. 257–264.
- [8] A. Pižurica, J. Aelterman, F. Bai, S. Vanlooce, Q. Luong, B. Goossens, and W. Philips, "On structured sparsity and selected applications in tomographic imaging," in *SPIE Conference on Wavelets and Sparsity XIV*, vol. 8138, 2011, pp. 81381D–1–12.
- [9] J. Aelterman, H. Q. Luong, B. Goossens, A. Pižurica, and W. Philips, "Augmented Lagrangian based reconstruction of non-uniformly sub-nyquist sampled MRI data," *Signal Processing*, vol. 91, no. 12, pp. 2731–2742, 2011.
- [10] B. Goossens, J. Aelterman, H. Luong, A. Pižurica, and W. Philips, "Efficient design of a low redundant discrete shearlet transform," in *IEEE International Workshop Local and Non-Local Approximation in Image Processing (LNLA)*, 2009, pp. 112–124.
- [11] A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," *SIAM Journal on Imaging Sciences*, vol. 2, no. 1, pp. 183–202, 2009.