Labour Market Imperfections, Expectations, and Fiscal Policy in Dynamic Macro Models

BRECHT BOONE

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“Learning never exhausts the mind.”

Leonardo da Vinci

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Nederlandstalige Samenvatting

De macro-economische wetenschap en haar theoretische modellen zijn sterk geëvolueerd in de laatste decennia. Het nooit aflatende streven naar almaar realistischere macro-economische modellen zorgde ervoor dat meerdere assumpties die ooit de kern van deze modellen vormde, versoepeld werden. Het gevolg is dat de modellen die heden ten dage gebruikt worden niet meer vergelijkbaar zijn met de modellen die macro-economen destijds gebruikten. Maar hoewel de theoretische funderingen van deze modellen fundamenteel gewijzigd zijn, blijft hun doel nog altijd hetzelfde. Hun doel is namelijk het verklaren van werkelijke fenomenen. Bovendien zijn het belangrijke instrumenten om het effect van beleid op het gedrag van de verschillende actoren in de economie en hun interactie te bestuderen.

Deze verhandeling, bestaande uit vier papers, bouwt verder op deze evolutie langs twee belangrijke dimensies. In hoofdstuk 2 en 3 ligt de focus van deze verhandeling op een gedetailleerde modellering van de arbeidsmarkt van zowel hoog- als laaggeschoolden. Hoofdstukken 4 en 5 focussen dan weer op de verwachtingen van gezinnen en de impact van deze verwachtingen op de macro-economie. Het resultaat is dat elk model dat geconstrueerd wordt in deze verhandeling bijdraagt aan ons begrip van de macro-economie en het effect van het begrotingsbeleid hierop.

Het vertrekpunt in hoofdstukken 2 en 3 is de observatie dat OESO-landen onderling sterk verschillen inzake de arbeidsmarktprestatie van de laaggeschoolden, veel meer dan in termen van arbeidsmarktprestatie van de hooggeschoolden. Bovendien is er een sterke correlatie tussen de geaggregeerde werkgelegenheidsgraad in een bepaald land en de relatieve prestatie van laaggeschoolden op de arbeidsmarkt in dat land. Anders gezegd, in landen waar de geaggregeerde werkgelegenheidsgraad hoog is, is de achterstand van laaggeschoolden ten opzichte van hooggeschoolden op de arbeidsmarkt typisch klein.

Deze vaststellingen onderstrepen het belang van theoretische macro-economische modellen die niet alleen kunnen verklaren waarom meer of minder laaggeschoolden aan het werk zijn, maar ook hoe het beleid de arbeidsmarktprestatie van deze individuen kan verbeteren. Dit is hetgeen hoofdstukken 2 en 3 ambiëren. In hoofdstuk 2 richten we ons op de determinanten van de arbeidsmarktprestatie van de laaggeschoolden. Waarom is het percentage laaggeschoolden dat werkt hoger dan wel lager in een bepaald land? Welke rol spelen het begrotingsbeleid en de arbeidsmarktinstituties? Of gaat het hier vooral om individuele preferenties? Hoe belangrijk is elke component? Dit zijn de vragen die we proberen te beantwoorden in het tweede hoofdstuk. Hoofdstuk 3 bouwt hierop verder, maar de focus ligt nu veeleer op het simuleren van een reeks beleidsmaatregelen die mogelijk de arbeidsmarktprestatie van laaggeschoolden bevorderen. Welke middelen heeft de overheid om het percentage tewerkgestelde laaggeschoolden te verhogen? Zijn deze middelen even effectief? Hoe kunnen vakbonden hiertoe bijdragen? Gaat hogere werkgelegenheid ook gepaard met hogere welvaart of is dit afhankelijk van de genomen maatregelen?

Om een antwoord op al deze vragen te kunnen geven, maken we gebruik van een macro-economisch algemeen evenwichtsmodel met overlappende generaties. De grootste innovatie in ons model ten opzichte van de bestaande literatuur is de zeer realistische modellering van de verschillende componenten van de arbeidsmarkt van de laaggeschoolden. In het model wordt het loon voor de laaggeschoolden door de vakbonden bepaald. De bedrijven kiezen daarna het aantal laaggeschoolden die zij willen tewerkstellen en tegelijk bepalen de gezinnen de hoeveelheid uren die zij willen werken wanneer zij tewerkgesteld zijn.

De resultaten in hoofdstuk 2 tonen aan dat het model een groot deel van de geobserveerde variatie in arbeidsmarktprestatie van de laaggeschoolden tussen landen kan verklaren. Dit is een eerste indicatie van hoe waardevol het model en haar voorspellingen zijn. Wanneer we kijken naar wat die variatie bepaalt, dan stellen we vast dat verschillen inzake begrotingsbeleid en verschillen in arbeidsmarktinstituties elk ongeveer 50% van de verklaarde variatie bepalen. Als we dus willen verklaren waarom een land beter dan wel slechter scoort op het vlak van de arbeidsmarktprestatie van de laaggeschoolden, moeten we zowel naar het gevoerde begrotingsbeleid als naar de arbeidsmarktinstituties kijken. Echter, niet elke beleidsparameter heeft dezelfde informatieve waarde. De belangrijkste beleidsparameter is de hoogte van de werkloosheidsuitkering of de netto-vervangingsratio, d.w.z. de werkloosheidsuitkering na belastingen als percentage van het vroegere loon na belastingen. De rol van deze parameter is zeer sterk in zowel de continentaal-Europese en de Scandinavische landen. Al het overige gelijk, hoe hoger de netto-vervangingsratio in een land, hoe slechter de arbeidsmarktprestatie van de laaggeschoolden.

Na het lezen van hoofdstuk 2 weten we waarom landen verschillen inzake werkgelegenheid onder de laaggeschoolden. Maar wat kan de overheid nu concreet doen om de werkloosheid onder laaggeschoolden te verlagen? Een aantrekkelijke optie is de combinatie van lagere belastingen op arbeid en gematigde looneisen vanwege de vakbonden. Een belangrijke bevinding in hoofdstuk 3 is dus dat het zowel voor de overheid en de vakbonden aantrekkelijk is om de krachten te bundelen. De gematigde looneisen zorgen voor meer werkgelegenheid, terwijl de lagere belastingen op arbeid ervoor zorgen dat het nettoloon van de laaggeschoolden op peil blijft of zelfs kan stijgen, ondanks het lagere brutoloon. Een alternatieve optie om de arbeidsmarktpositie van laaggeschoolden te verbeteren, is het verlagen van de netto-vervangingsratio. Ook deze zal leiden tot een toename in de werkgelegenheid van laaggeschoolden, maar de impact op de welvaart van de werklozen is naar alle waarschijnlijkheid sterk negatief. Daardoor is deze optie minder interessant. Verder toont de analyse in hoofdstuk 3 aan dat de impact van werkgeversbijdragen en werknemersbijdragen niet equivalent zijn. De impact van de laatste op het gedrag van individuen en dus de macro-economie is veel sterker waardoor het een interessanter instrument is voor de overheid.

Hoofdstukken 4 en 5 dragen op hun beurt dan weer bij tot een groeiende literatuur die alternatieven zoekt voor het paradigma van de rationele verwachtingen. Ondanks de dominantie van dit paradigma groeit de kritiek erop. Immers, de veronderstelling van rationele verwachtingen impliceert dat economische agenten extreme cognitieve capaciteiten hebben en dat ze een perfect inzicht hebben in de werking van de economie. In hoofdstukken 4 en 5 veronderstellen we als alternatief dat het inzicht van de economische agenten beperkt is. Zij observeren maar een klein deel van de economie en baseren hun beslissingen hierop. Echter, hun inzicht mag dan wel beperkt zijn, toch zijn deze economische agenten bereid tot en capabel om hun verwachtingen aan te passen wanneer nieuwe informatie beschikbaar is. Het loon kan bijvoorbeeld afwijken van het eerder verwachte loon. In zo'n situatie zullen de economische agenten hun verwachtingen aanpassen.

Dit beperkt inzicht en de leerprocessen vormen een belangrijk onderdeel van de economische modellen in hoofdstuk 4 en 5. Hoe evolueren deze verwachtingen over tijd? Is de dynamiek van het model met alternatieve verwachtingen anders dan in het model met rationele verwachtingen? En in welke mate wordt de effectiviteit van het begrotingsbeleid beïnvloed door de verwachtingen van de economische agenten?

In hoofdstuk 4 richten we onze focus op de korte termijn. In de literatuur is het gebruikelijk om de conjuncturele schommelingen te analyseren in een lineare benadering van het macro-economisch model rond het evenwicht. In tegenstelling tot het leeuwendeel van de literatuur, echter, analyseren wij het leergedrag van de economische agenten in het originele niet-lineaire model. Doorgaans is het lineaire model een goede benadering, zeker als de afwijkingen van het evenwicht klein zijn. Maar bij grote afwijkingen – een diepe recessie bijvoorbeeld – wordt een lineaire benadering snel minder nauwkeurig. In zo'n situaties vormt een niet-lineaire analyse een zeer waardevol alternatief.

Gebruikmakend van dit model tonen we aan dat de effectiviteit van extra overheidsuitgaven binnen het niet-lineaire Real Business Cycle (RBC) model varieert over de tijd, een bevinding die in lijn is met heel wat empirische studies. Deze tijdsvariatie vindt zijn oorsprong in de wijzigende verwachtingen van de economische agenten. Dit resultaat staat in schril contrast met de uitkomst van het standaard RBC model met rationele verwachtingen, namelijk dat de effectiviteit van extra overheidsuitgaven constant is over tijd. Het model in hoofdstuk 4 kan dus een instrument zijn voor de overheid om de effectiviteit van extra overheidsuitgaven in te schatten wanneer de economie zich dicht én ver van haar evenwicht bevindt.

De aanpak in hoofdstuk 5 verschilt van deze in hoofdstuk 4 op twee manieren. Ten eerste verlaten we de korte termijn en bestuderen we de evolutie van de economie op lange termijn. Ten tweede veronderstellen we nu dat de economische agenten gebruikmaken van heuristieken oftewel vuistregels om verwachtingen te vormen. Wat we doen in hoofdstuk 5 is de economische dynamiek simuleren die ontstaat na het permanent verlagen van de belasting op arbeid, zowel onder rationele verwachtingen alsook onder de veronderstelling dat individuen vuistregels gebruiken om verwachtingen te vormen. Dat individuen dergelijke vuistregels gebruiken werd reeds aangetoond in empirische studies. De simulaties onthullen dat het traject van de economie met vuistregels vooral in de eerste jaren na de beleidsingreep sterk kan verschillen van het traject onder rationele verwachtingen. Rationele verwachtingen is dus niet altijd een goede benadering. Verder stellen we vast dat dit traject sterk afhankelijk is van de (hoeveelheid) vuistregels die beschikbaar zijn en hoe snel economische agenten overschakelen op beter presterende vuistregels. Wanneer zij bijvoorbeeld maar één vuistregel kunnen gebruiken, zal het exacte traject (en dus de impact van begrotingsbeleid) sterk afhankelijk zijn van de heuristiek die op dat moment gehanteerd wordt alsook van de mate waarin zij hun verwachtingen aanpassen wanneer nieuwe informatie beschikbaar is. Wanneer individuen bijvoorbeeld hun verwachtingen altijd zeer sterk aanpassen aan de nieuwe economische realiteit, zal de economie zelf ook onstabieler worden. Echter, wanneer zij de mogelijkheid hebben om over te schakelen op beter presterende vuistregels, zal de economie niet alleen stabieler worden, maar zal het traject van de economie ook veel minder afhankelijk zijn van de verschillende vuistregels. Het gevolg is dat de variatie inzake mogelijke trajecten veel kleiner wordt. Economische agenten maken dan ook kleinere voorspellingsfouten. Dit is een belangrijke les voor economen en onderzoekers. Indien het model heuristieken bevat, is het zeer sterk aan te raden om agenten de mogelijkheid te geven te wisselen tussen de verschillende heuristieken.

Chapter 1

Introduction

The macroeconomic profession and its theoretical models have evolved strongly over the last decades. In a never-ending attempt to build increasingly realistic macroeconomic models, several assumptions that once formed the core of theoretical models have been relaxed. As a consequence, the structural models used these days are not comparable to the ones used a few decades ago. But while the theoretical foundations of these models have changed significantly, their aim has not. They still serve to explain real-world phenomena. Furthermore, they are important instruments to study the effects of public policy on the behaviour of different economic actors and their interactions.

This dissertation, consisting of four papers, makes progress in this ongoing evolution along two important dimensions. In Chapters 2 and 3, the focus of this dissertation lies on a detailed modelling of the labour markets of respectively individuals who have and individuals who have not enjoyed tertiary education[1]. Furthermore, special attention is devoted to introducing frictions on the labour market of the latter. In Chapters 4 and 5, on the other hand, this dissertation zooms in on the formation of expectations of economic agents. As a result, the models constructed in these four chapters significantly enhance our understanding of the macroeconomy and the effect of fiscal policy thereon.

The starting point in Chapters 2 and 3 is Figure 1.1[2]. This figure shows first of all that the labour market position of low-skilled individuals is highly indicative of the aggregate employment rate within a country. Countries with lower aggregate employment rates are typically countries with higher unemployment rates (panel a) and relatively lower

[1] In what follows, individuals who have enjoyed tertiary education are denoted as high-skilled, while those who have not are denoted as low-skilled.

[2] The focus of the first two chapters of this dissertation is on 2001-2007 as this was the last period of relative stability on the labour market before the financial crisis and the euro crisis. Considering that the analysis in this dissertation aims at studying equilibrium unemployment, it is clearly more appropriate to use data for a relatively stable period.

employment rates (panel b) among the low-skilled. As a second important result, Figure 1.1. (panel c) reveals a much larger cross-country variance in the employment rate among the low- than among the high-skilled[3].

These observations indicate that a specific focus on understanding and improving the labour market performance of low-skilled individuals should be an important (policy) objective. The lion's share of the literature studying the effects of public policy on employment focuses on aggregate employment, however. Furthermore, they raise the question whether restricting the focus on aggregate employment sufficiently captures the potential impact on the low-skilled, especially given the distinct and precarious labour market status of the latter. They also highlight the importance of modelling the labour markets of high- and low-skilled individuals separately, with a specific focus on the latter. Ultimately, to address these concerns one needs theoretical models capable of capturing these observations and use them to simulate a wide range of possible policy reforms and study their impact on the labour market performance of the low-skilled. All these concerns and observations lie at the core of Chapters 2 and 3.

In **Chapter 2**, co-authored with Freddy Heylen, we address these concerns by developing a five generations Overlapping Generations model for a small open economy with two key assumptions. The first one – given the importance of skills and education – is the assumption that individuals are heterogeneous by ability. They enter our model with different human capital stocks and have different capacity to build more human capital. Our second assumption and key novelty is the assumption of a unionised labour market for the low-skilled[4].

Having established its empirical reliability, the model is used to find out which policy or preference parameters account for the cross-country differences in the aggregate unemployment rate. We extend the literature by explicitly testing the potential explanatory power of labour market imperfections, different union preferences in particular, and different tastes for leisure of the households. Performing a Shapley decomposition, we find an almost equal role for differences in fiscal policy variables and in union preferences.

[3]For this sample of countries, the variance of the employment rate among the high-skilled 25- to 64-year-olds is 4.75, whereas it is 20.35 among the low-skilled.

[4]A unionised labour market is not the only manner to introduce labour market frictions, though. Our choice is motivated by the set-up of the model, the targeted skill group, and the observation that unions are still an active actor in most OECD countries. Regarding the set-up of the model, given the length of each generation, a search and matching framework might lead to difficulties when calibrating the model. Indeed, within a span of several years, the probabilities that an individual finds a job, that a vacancy gets filled, and that a job is terminated are very high. Introducing an efficiency wage framework is another way. This framework is especially relevant when the effort of a worker is not easily measurable, when replacing an employee is very costly and/or when firms want to attract high-potential workers. These characteristics apply, however, more to high- than to low-skilled labour. Combined with the role unions still play in most OECD countries, these arguments motivate our choice for a unionised labour market.

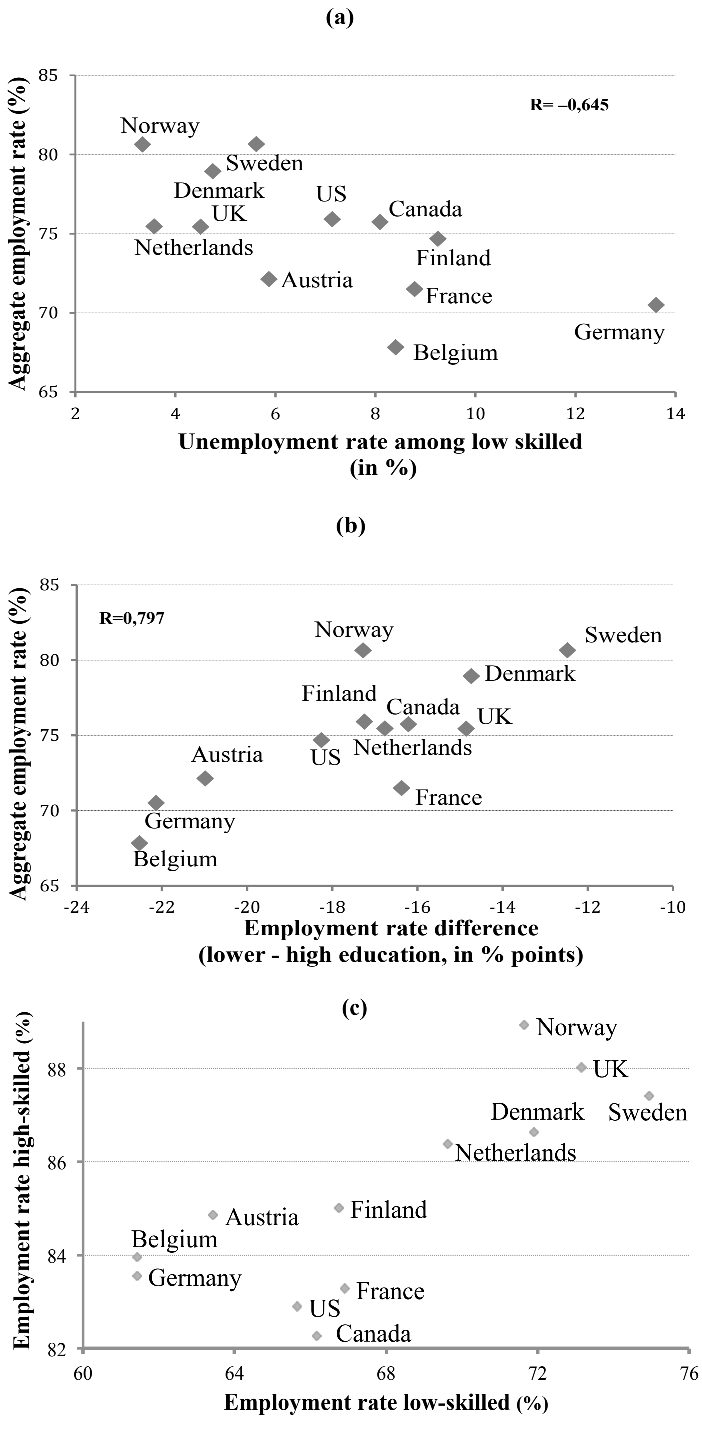

Figure 1.1: Employment and unemployment in OECD countries, 2001-2007.

Note: The (un)employment rate among lower skilled individuals is computed as the average of the (un)employment rates among individuals with less than upper secondary education and among individuals with upper secondary, but no tertiary degree. The (un)employment rate among individuals with higher education relates to those with a tertiary degree. Unless defined differently, all reported employment and unemployment rates concern the age group 25-64. The employment rate indicates the fraction of individuals who have a job. Data sources: Eurostat (LFS series: lfsa_ergaed, lfsa_urgaed) and OECD Labour Force Statistics (Total Employment).

Each account for about half of the explained variation in unemployment rates across countries. By contrast, any differences in the households' taste for leisure play virtually no role. We argue in the paper that the above market-clearing wage chosen by the unions is the source of unemployment, while the fiscal policy variables explain a significant part of the magnitude of unemployment. Going into greater detail on the fiscal side, we find that the key variable driving cross-country differences in unemployment of lower ability individuals is the unemployment benefit replacement rate. In the Nordic countries and (even more) the continental European countries, this has a significant impact on the reference wage of the union. We find no contribution, however, from differences in labour taxes to account for cross-country unemployment variation on an interregional level.

The aim in **Chapter 3** is to extend and generalise the model used in Chapter 2 and use it to quantitatively investigate the impact of fiscal and union policies on the labour market outcome for low-skilled individuals. The key innovation in this chapter is that the unionised labour market now leads to endogenously generated idiosyncratic employment risk. The simulations reveal that modelling the different components of aggregate employment is important to explore the effectiveness of fiscal and union policies. While most fiscal policy and union instruments have a highly similar impact on aggregate output, capital, and labour, these instruments all have a different impact on unemployment, labour force participation, hours worked, and workers' welfare. The simulations thus highlight that the choice for a given policy instrument is dependent on the target variable.

That being said, several policies lead to desirable results in terms of unemployment. Overall, however, the simulations show that fiscal governments and unions have a mutual interest in joining forces to improve the labour market performance of the low-skilled. For example, our most preferred policy is a decrease in labour taxes levied on low-skilled employees combined with wage moderation accepted by the unions. Not only does this lead to a decrease in the unemployment rate of the low-skilled, it also has a positive impact on the number of hours worked per employed individual and it is very likely to increase welfare of employed low-skilled individuals. Furthermore, it does not directly harm unemployed individuals.

The last conclusion one can draw from the simulations is that, unlike in perfectly competitive labour markets, labour taxes levied on respectively employers and employees are not equivalent within the model. Even more, the behavioural effects of a decrease in the labour tax rate paid by employers are very small. Within the current set-up, the fiscal government can thus achieve more by lowering the tax rates paid by employees.

In Chapters 4 and 5, this dissertation focuses on the formation of household expectations in the short and the long run. Indeed, by now it is well understood that the macroeconomic impact of (fiscal) policy crucially depends on the behavioural response of households to these policies. An important determinant of this behavioural response is the approach households apply to form expectations regarding the evolution of different endogenous macroeconomic variables. The dominant paradigm used to model expectations in macroeconomics is the rational expectations (RE) hypothesis. According to this hypothesis, households have perfect knowledge about the structure of the model and understand the full complexities of the macroeconomy. Notwithstanding its dominance, the rational expectations paradigm has been under attack as it endows economic agents with unrealistic cognitive capabilities. An alternative to the rational expectations hypothesis is provided by the learning literature. In this literature, agents form expectations using a perceived law of motion. Over time, as new information becomes available, agents update the coefficients of their perceived law of motion.

Chapter 4, co-authored with Ewoud Quaghebeur, studies learning behaviour and the effects of government spending in the original non-linear model, unlike the vast majority of the learning literature in which the model is often first linearised around the steady state. We show that our set-up leads to time variation in the transmission of government spending shocks in the model economy. Furthermore, the time variation in the government spending multiplier is endogenously determined within our set-up. As the economic agents update their beliefs, their response to a change in government spending changes as well, leading to a different impact of government spending shocks on the economy. This result stands in sharp contrast with the rational expectations solution of the model.

While Chapter 4 focuses on the role of expectations in the short run, **Chapter 5** shifts the focus to the long run. Agents now use simple rules, heuristics, to forecast the future course of the interest rate and the real wage within an Overlapping Generations setting with idiosyncratic risk. Compared to the literature focusing on least-squares learning, these agents using heuristics do not act as econometricians, though. Instead, they have a certain number of different heuristics at their disposal and on a regular basis, they assess the predictive power of the heuristic they are currently applying. If it performs well, the probability that agents will use the same heuristic in the next period will be higher. If it does not perform well, there is a higher probability that they switch to another rule.

This chapter shows that in a context in which agents only have one heuristic at their disposal to form expectations, the transitional dynamics can be substantially different from the rational expectations solution. This finding indicates that rational expectations is not always a good approximation. Furthermore, there is a lot of variation in the dynamics over different values for the preference parameters and heuristics. Moreover, for some parameter configurations, the model becomes unstable and the resulting dynamics oscillate or even diverge.

Second, after activating the heuristic switching regime, the variation in the transitional dynamics decreases significantly, meaning that the transitional effects of a decrease in labour taxes become more predictable and monotonic over all different configurations of the parameters. Thus, allowing for alternative expectations does no longer lead to a wide range of possible transitional paths.

Third and last, the heuristic switching mechanism has a stabilising effect on the transitional dynamics. For certain configurations of the parameter values for which the dynamics were very unstable in the absence of heuristic switching, the dynamics now converge to the steady state in most cases.

Chapter 2

Cross-country Differences in Unemployment: Fiscal Policy, Unions, and Household Preferences in General Equilibrium[1]

Abstract

We develop a five period overlapping generations model with individuals who differ by ability and an imperfect labour market (union wage setting) for the individuals of lower ability. The model explains human capital formation, hours worked and unemployment within one coherent framework. Its predictions match the differences in the unemployment rate across 12 OECD countries remarkably well. A Shapley decomposition of these differences reveals an almost equal contribution of fiscal policy variables and union preferences. As to fiscal policy, differences in unemployment benefits play a much more important role than tax differences. Differences in households' taste for leisure are unimportant.

Keywords: Low-skilled unemployment, Heterogeneous agents, Union wage setting, Fiscal policy, Shapley decomposition
JEL classification codes: E24, E62, J51, J64

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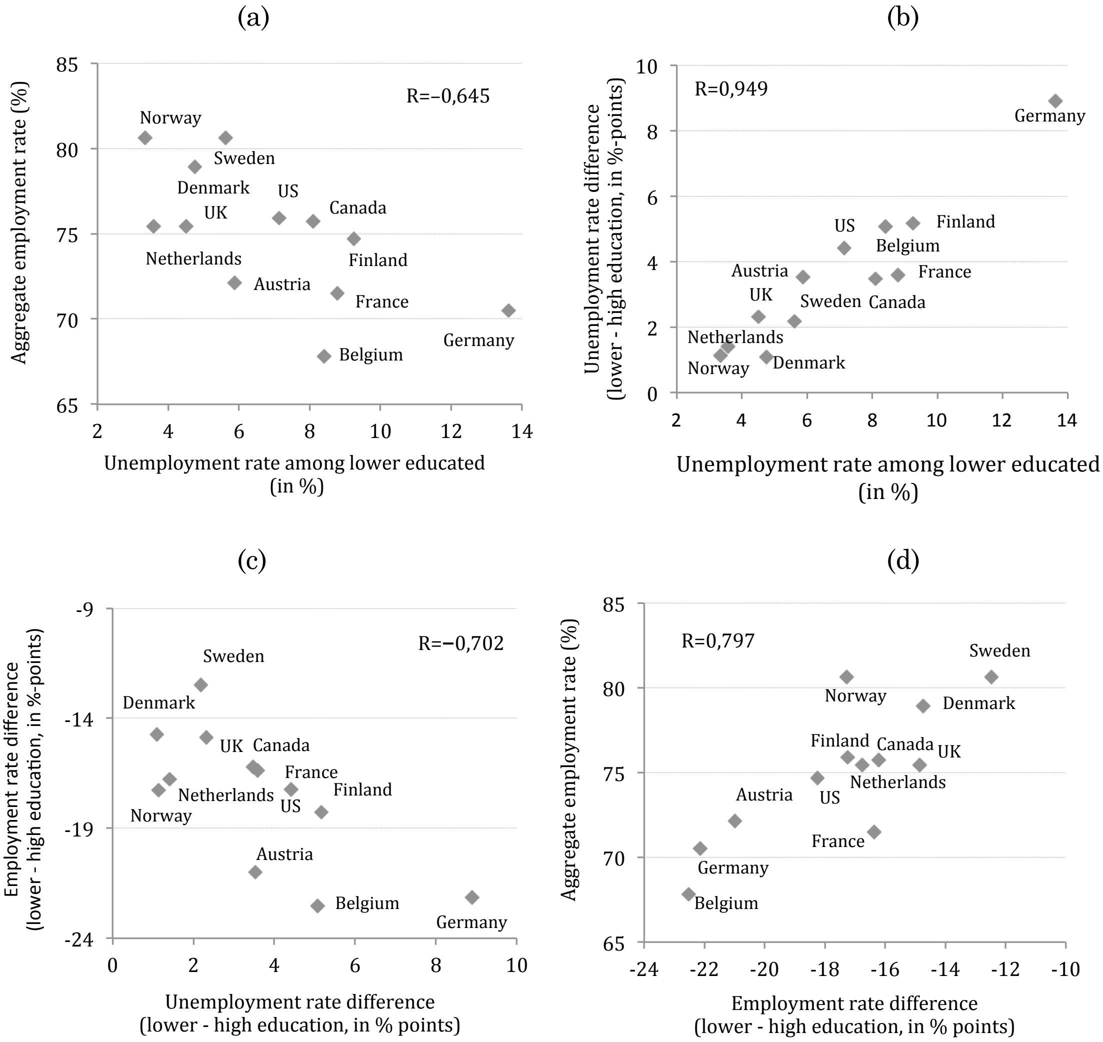
2.1 Introduction

Labour market performance differs widely across OECD countries. Since about a decade many researchers have built gradually richer general equilibrium models to account for these differences. Initial contributions by Prescott (2004), Rogerson (2007), Dhont and Heylen (2008) and Ohanian et al. (2008) tried to explain differences in aggregate per capita hours worked. Later work introduced a life-cycle dimension in labour supply and employment in order to explain also the huge cross-country differences in employment among persons older than 50 (see Rogerson and Wallenius (2009), Erosa et al. (2012), and Alonso-Ortiz (2014)). Another advantage of introducing a life-cycle dimension is that it became possible to model the time allocation of young people between labour and education, and to explain human capital formation as an endogenous variable (see e.g. Ludwig et al. (2012); Heylen and Van de Kerckhove (2013); Wallenius (2013)).

Despite the enormous progress that has been made in this literature, one clear weakness has not been dealt with. A striking observation in all the aforementioned models is their assumption of a perfectly competitive labour market. They cannot explain equilibrium unemployment, let alone the huge and persistent differences in unemployment between for example high and lower educated individuals. Yet, as demonstrated in Figure 2.1 for 12 OECD countries in 2001-2007, cross-country differences in aggregate employment are strongly related to differences in unemployment, in particular unemployment among lower educated individuals. In panel (a), we observe the highest aggregate employment rates in countries (e.g. Denmark, Norway, and Sweden) that are relatively successful in avoiding unemployment among lower educated individuals. By contrast, countries that fail in fighting unemployment among the lower educated, like Belgium and Germany, also show relatively bad aggregate employment performance. The other panels in Figure 2.1 reveal a number of interesting other regularities, which will guide us later in this paper. Panel (b) establishes the fact that almost all cross-country variation in the gap between the unemployment rates of lower and high educated individuals is due to variation in the unemployment rate among the lower educated. Correlation in this panel is almost 0.95. Countries vary much less when it comes to the labour market situation of the high educated. (Correlation between the unemployment rate among individuals with a tertiary degree and the unemployment gap between the lower and the high educated is only 0.14). Panel (c) shows a strong inverse relationship between the unemployment gap and the employment gap between lower and high educated individuals. Finally, panel (d) reveals that the aggregate employment rate is strongly related to this employment gap. We conclude that if it is the objective of countries to raise aggregate employment, an important challenge will be to fight unemployment among lower educated individuals. The

Figure 2.1: Employment and unemployment in OECD countries, 2001-07.



Notes: We compute the (un)employment rate among lower educated individuals as the average of the (un)employment rates among individuals with less than upper secondary education and among individuals with upper secondary, but no tertiary degree. The (un)employment rate among individuals with higher education relates to those with a tertiary degree. Unless defined differently, all reported employment and unemployment rates concern the age group 25-64. The employment rate indicates the fraction of individuals who have a job. Data sources: Eurostat (LFS series: lfsa_ergaed, lfsa_urgaed) and OECD Labour Force Statistics (Total Employment).

existing (dynamic) general equilibrium models for labour market analysis in the tradition of Prescott (2004) and Rogerson (2007) have no clear answer to deal with this challenge.

Next to excluding a potential role for labour market imperfections, the above mentioned general equilibrium literature also leaves little room for differences in individual preferences across countries to show up. Blanchard (2004) and Alesina et al. (2005) have argued that a key factor behind the lower employment in many European countries compared to the US is a higher taste for leisure. Yet, the general equilibrium literature generally imposes the same preferences upon individuals.

Our contribution in this paper is to extend the dynamic general equilibrium literature studying employment with a labour market imperfection and to use our extended model to quantitatively explore which variables drive cross-country differences in unemployment, in particular unemployment among lower educated individuals. More precisely, we develop a five generations OLG model for a small open economy with two key assumptions. The first one - given the importance of skills and education - is the assumption that individuals are heterogeneous by ability. They enter our model with different human capital stocks and have different capacity to build more human capital. This approach may offer the best match to findings by Huggett et al. (2006), Huggett et al. (2011) and Keane and Wolpin (1997) that heterogeneity in human capital endowment at young age and in learning abilities, rather than shocks to human capital, account for most of the variation in lifetime utility. Our second assumption and key novelty compared to previous work in this tradition is the assumption of a unionised labour market for lower ability (lower educated) individuals[2]. Like Faia and Rossi (2013), we introduce a monopolistic firm-specific trade union that determines the real pre-tax wage for these workers while taking aggregate variables and fiscal policy parameters (e.g. tax rates, unemployment benefits) as given. We specify a Stone-Geary utility function for the union with both wages and employment as arguments, albeit with a different weight. As to wages, the firm-specific union only derives utility from the difference between the after-tax wage and a reference wage. The monopoly union chooses the wage in a first stage. In the next stage, the firm will choose employment (number of workers), while the households of lower ability individuals decide on the supply of hours per employed. Both the firm and the households take the wage set by the union as given.

The union wage setting framework in our model is motivated by the observation that in Europe union wage bargaining is still the most common way of wage determination. While union membership rates have decreased over time, the coverage of collective bargaining is still at least 80% in most continental European countries and Nordic countries. Also, despite the fact that unions are not that powerful in the US, there exists a form of minimum wage in the US. As such, a union pushing the wage above its perfectly competitive counterpart might be a valid assumption for all countries to introduce unemployment.

Firms in our model act competitively on the goods market. Furthermore, we introduce a government with a rich set of fiscal policy instruments. Government spending on goods and unemployment benefits are financed by taxes on labour, capital and consumption. As

[2]For higher ability workers we assume that wages and employment are determined in a perfectly competitive way. Several authors have provided empirical evidence that the effects of the presence of unions are much stronger for low skilled individuals than for the high skilled, e.g. through a higher union-non union wage premium among the low skilled workers. See e.g. Card et al. (2004), and Checci and García-Peñalosa (2008).

to labour taxes, we distinguish between taxes paid by the employer and the employees[3]. Another novelty is the modelling of progressive income taxes paid by the households. We follow the approach used by Guo and Lansing (1998) and Koyuncu (2011). Lump sum transfers balance the budget.

We then use our model to investigate the main drivers of the differences in unemployment across OECD countries. A large range of variables play a role in the model. To find out which of these matter most, our procedure is as follows. First, we calibrate our model and show its empirical relevance for twelve countries belonging to three groups (five continental European countries, four Nordic countries and three Anglo-Saxon countries). More precisely, we simulate our calibrated model for each country imposing common technology on all countries, but country-specific fiscal policy parameters and country group-specific household and union preferences. We find that the predictions of our model match the main facts in most countries. These facts concern hours worked per employed person and the unemployment rate. Having established its empirical reliability, we then use the model to find out what policy or preference parameters account for the cross-country differences in the aggregate unemployment rate. Our objective is similar to the one of Dhont and Heylen (2008), Wallenius (2013) and Alonso-Ortiz (2014) in earlier work. We make progress by also explicitly testing the potential explanatory power of labour market imperfections, different union preferences in particular, and different tastes for leisure of the households. Performing a Shapley decomposition, we find an almost equal role for differences in fiscal policy variables and in union preferences. Each account for about half of the explained variation in unemployment rates across countries. By contrast, any differences in the households' taste for leisure play virtually no role. Our story will then be that the above market-clearing wage chosen by the unions is the source of unemployment, while the fiscal policy variables explain a significant part of the magnitude of unemployment. Going into greater detail on the fiscal side, we find that the key variable driving cross-country differences in unemployment of lower ability individuals is the unemployment benefit replacement rate. In the Nordic countries and (even more) the continental European countries, this has a significant impact on the reference wage of the union. We find no contribution, however, from differences in labour taxes to account for cross-country unemployment variation on an interregional level.

Our finding that both policies and institutions should be taken into account for a good

[3]In a perfect labour market situation, whether labour taxes are levied on workers or firms does not matter for the cost of labour, nor for after-tax wages and employment. However, as Heijdra and Ligthart (2009) argue, it is not immediately clear whether the same result holds in imperfectly competitive labour markets. We therefore choose to distinguish explicitly between the two.

explanation of differences in unemployment across OECD countries matches well with the results of many econometric studies, like those of Nickell et al. (2005), Bassanini and Duval (2009), and Nymoen and Sparrman (2015). The generosity of the unemployment benefit system and the characteristics of wage setting are often found among the main drivers of unemployment in these studies. Our focus on unemployment among low educated workers, however, is missing in most econometric studies (despite its importance for the aggregate employment situation that we highlighted in Figure 2.1). As we have emphasised before, introducing a labour market imperfection and explaining unemployment is also our main contribution to the dynamic general equilibrium analysis of labour market performance in the tradition of Prescott (2004), Rogerson (2007), Rogerson and Wallenius (2009), and Heylen and Van de Kerckhove (2013), among others.

Several earlier contributions have made an attempt to introduce unemployment in dynamic macro models. Daveri and Tabellini (2000), Corneo and Marquardt (2000), and Ono (2010) among others developed OLG models with a unionised labour market, while Ravn and Sørensen (1999), Cahuc and Michel (1996) and Sommacal (2006) introduced minimum wages. Gali et al. (2011) extended the Smets and Wouters New Keynesian DSGE model to allow for involuntary unemployment. In their model, unemployment also results from market power in labor markets, reflected in positive wage markups. Other authors embed a search and matching setup in a life-cycle model, e.g. de la Croix et al. (2013). We also make progress compared to this literature. First, to the best of our knowledge, all the existing OLG models where unions are present are populated by only two generations, which means that they lack a life-cycle dimension of labour supply. The fact that we model the labour market outcome of different generations is clearly different from the DSGE literature on unemployment as well. Second, most of the models incorporating unions in a life-cycle model leave the intensive margin of employment (i.e. hours worked) unexplored. However, hours of work per employed person are substantially lower in most European countries than in the Anglo-Saxon countries. Third, most of these models do not allow for individuals with different ability. The consequence being that these are not suited to explore the labour market situation of lower educated individuals separately. And last, given the rich specification of the model, both in terms of fiscal and institutional variables, we are able to explore the drivers of unemployment in much more detail.

The structure of this paper is as follows. In Section 2.2, we describe the basic setup of our model. Section 2.3 discusses optimal behaviour of unions and firms, and how this drives hours worked, unemployment and real output. Section 2.4 presents our calibration procedure. In Section 2.5 we test and show the empirical validity of our model for 12 OECD countries as described above. Finally, in Section 2.6 we investigate the relative importance of institutional and (household and union) preference related variables versus several fiscal policy variables to explain differences across countries in the unemployment rate. Section 2.7 concludes.

2.2 The model: setup, preferences and constraints

Time is discrete and runs from 0 to infinity. We assume a small open economy populated by five overlapping generations of households, firms, unions and a fiscal government. Individual members of the household enter the model at the age of 18 and live for five periods j of 12 years. Individuals have either high or low innate ability. Households have only higher or lower ability members, but not both. Both the goods market and the labour market for higher ability individuals are competitive, whereas the labour market for lower ability individuals is unionised. In every period t, wages for lower ability workers are set by a monopoly union at the firm level. The government in our model disposes of a rich set of fiscal policy instruments. Government spending on goods and unemployment benefits are financed by taxes on capital, labour and consumption. As to labour taxes, we distinguish between taxes paid by the employer (linear) and by the employees (non-linear). There is no uncertainty.

2.2.1 Households

In the spirit of Merz (1995) and Andolfatto (1996), we assume a number of households each consisting of a continuum of members of the same age and the same ability. Each household has unitary mass. We normalise the number of households of a given age and ability type to one. Therefore, the economy consists of 10 households in total[4]. All members at working age participate on the labour market and pool their income, meaning that consumption across household members is the same. As such there is perfect insurance within the household against the risk of unemployment. A household that enters the model in period t (a household of generation t) is denoted with a superscript t. Subscripts are reserved for the age $j \in \{1, 2, 3, 4, 5\}$ and the ability type $a \in \{H, L\}$. Hence, n_{2H}^{t+1} denotes the fraction of time devoted to labour services by a member of a higher ability family who is in the second period of life and who started active life in period $t+1$. Tables 2.1 and 2.2 present a brief overview of the model structure with respect to the households:

[4] With our assumption of five periods, we are able to isolate the young (18-29) and the elderly (54-65) and look at whether the model correctly predicts the labour market outcome for these generations. Including more generations would not lead to additional insights with respect to the aggregate unemployment rate. Having fewer generations might lead to generations that are too big (especially the young and the older workers).

Table 2.1: Life-cycle of a member of a higher ability household of generation t

Time	t	$t+1$	$t+2$	$t+3$	$t+4$
Hours worked	n^t_{1H}	n^t_{2H}	n^t_{3H}	n^t_{4H}	0
Education	e^t_{1H}	0	0	0	0
Participation rate	1	1	1	1	0
Unemployment rate	0	0	0	0	0
Employment rate	1	1	1	1	0
Leisure time	$1-n^t_{1H}-e^t_{1H}$	$1-n^t_{2H}$	$1-n^t_{3H}$	$1-n^t_{4H}$	1

Table 2.2: Life-cycle of a member of a lower ability household of generation t

Time	t	$t+1$	$t+2$	$t+3$	$t+4$
Hours worked when employed	n^t_{1L}	n^t_{2L}	n^t_{3L}	n^t_{4L}	0
Education	0	0	0	0	0
Participation rate	1	1	1	1	0
Unemployment rate	$u_{1,t}$	$u_{2,t+1}$	$u_{3,t+2}$	$u_{4,t+3}$	0
Employment rate	$1-u_{1,t}$	$1-u_{2,t+1}$	$1-u_{3,t+2}$	$1-u_{4,t+3}$	0
Leisure time when employed	$1-n^t_{1L}$	$1-n^t_{2L}$	$1-n^t_{3L}$	$1-n^t_{4L}$	1

– Members of the higher ability household enter the model with a human capital stock h^t_{1H}. They have a time endowment of one in each period which they can devote to work, education when young, or leisure. During four active periods (age $j = 1, 2, 3, 4$), all these individuals are employed on a perfectly competitive labour market. During the fifth period ($j = 5$), they are retired. The household chooses an optimal consumption path, the optimal amount of non-human wealth, the time each individual devotes to education when young and the amount of hours each member supplies labour.

– Members of the lower ability household enter with a human capital stock $h^t_{1L} < h^t_{1H}$. Just like their higher ability counterparts, they have a time endowment of one. Lower ability individuals do not pursue tertiary education. A fraction $1 - u_{j,t+j-1}$ of all lower ability individuals of generation t at age j will be employed in period $t+j-1$, the others are (involuntarily) unemployed. Employed members devote time to either work or leisure, unemployed members only have leisure. The household chooses an optimal consumption path, the optimal amount of non-human wealth and the amount of time the employed members supply labour.

2.2.1.1 Higher ability households

Lifetime utility of the higher ability household of generation t is given by

$$u^t_H = \sum_{j=1}^{5} \beta^{j-1} \left(\ln c^t_{jH} + \gamma_j \frac{\left(1 - e^t_{jH} - n^t_{jH}\right)^{1-\theta}}{1-\theta} \right) \tag{2.1}$$

with $0 < \beta < 1$, $\gamma_j > 0$, $\theta > 0$ ($\theta \neq 1$), and where $e^t_{2H} = e^t_{3H} = e^t_{4H} = e^t_{5H} = n^t_{5H} = 0$. In this equation, β represents the discount factor, γ_j is an age-specific parameter determining the value of leisure relative to consumption and $\frac{1}{\theta}$ is the intertemporal elasticity of substitution in leisure[5]. The household's budget constraints are given by (2.2). Income is derived from labour, non-human wealth and lump sum transfers from the government. It is allocated to either consumption or savings.

$$(1+\tau_c)c^t_{jH}+\Omega^t_{jH} = w_{H,t+j-1}\varepsilon_j h^t_{jH} g(n^t_{jH})(1-\tau_{jH})+(1+r_{t+j-1})\Omega^t_{j-1,H}+z_{t+j-1} \quad (2.2)$$

for $j \in \{1,2,3,4,5\}$ and with $\Omega^t_{0H} = \Omega^t_{5H} = n^t_{5H} = 0$. As in Rogerson and Wallenius (2009), $g(n^t_{jH}) = \max\{n^t_{jH} - \bar{n}_H, 0\}$, with n^t_{jH} the chosen fraction of their time endowment that individuals allocate to labour services and $n^t_{jH} - \bar{n}_H$ the effective labour time that individuals supply. We assume that if an individual with human capital stock h^t_{jH} devotes a fraction n^t_{jH} of his/her time to the labour market, this will yield $(n^t_{jH} - \bar{n}_H)h^t_{jH}$ units of effective labour market services if $n^t_{jH} \geq \bar{n}_H$, and 0 otherwise. A fraction $\bar{n}_H$ is not productive due to e.g. commuting and getting setup in a job. Individuals earn an after-tax wage of $w_{H,t+j-1}\varepsilon_j\ h^t_{jH}g(n^t_{jH})(1-\tau_{jH})$ during their four active periods, where $w_{H,t+j-1}$ is the pre-tax real wage per unit of effective labour, ε_j is an age-specific productivity parameter, and τ_{jH} is the average tax rate on labour. Due to our modelling of a progressive labour income tax system, tax rates depend on individuals' ability and age. We specify the tax system in greater detail below. As to other variables, we denote the lump sum transfer from the government at time t by z_t. The consumption tax rate is τ_c. The households' accumulated non-human wealth at the end of their j-th period of life is Ω^t_{jH}. Households enter the model without wealth and leave no bequests.

2.2.1.2 Lower ability households

In the spirit of the previous subsection, we again consider a large household, consisting of a continuum of lower ability members. Again, the decision unit is the household. The key difference is that in this household only a fraction $1 - u_{j,t+j-1}$ of the individuals is employed. A fraction $u_{j,t+j-1}$ is unemployed. Hence, $u_{j,t+j-1}$ represents the aggregate unemployment rate among the lower ability individuals of generation t at age j and time $t + j - 1$. The household derives utility from consumption, while it only enjoys utility from the leisure of each employed member[6]. Thus, lifetime utility of the household of

[5] In the empirical part of our paper (i.e. Sections 4 and 5), we allow γ_j to differ between regions (continental European countries, Nordic countries, and Anglo-Saxon countries). The region-specific values will be calibrated.

[6] A similar utility function is also used in both business cycle and overlapping generations models with search and matching frictions, e.g. Tomas (2008) and de la Croix et al. (2013). Similar to these contributions, we assume that the leisure of the unemployed members is neutral in terms of utility.

lower ability individuals of generation t is given by

$$u_L^t = \sum_{j=1}^{5} \beta^{j-1} \left(\ln c_{jL}^t + \gamma_j (1 - u_{j,t+j-1}) \left[\frac{(1 - n_{jL}^t)^{1-\theta}}{1-\theta} \right] \right) \tag{2.3}$$

where $n_{5L}^t = 0$. The household's budget constraints are given by (2.4). Again, we assume that all members of the household pool their income:

$$(1+\tau_c)c_{jL}^t + \Omega_{jL}^t = w_{L,t+j-1}\varepsilon_j h_{jL}^t (1 - u_{j,t+j-1}) g(n_{jL}^t)(1 - \tau_{jL}) + \widetilde{B}_j + (1 + r_{t+j-1})\Omega_{j-1,L}^t + z_{t+j-1} \tag{2.4}$$

for $j \in \{1,2,3,4,5\}$ and with $\Omega_{0L}^t = \Omega_{5L}^t = n_{5L}^t = 0$. For the fraction $u_{j,t+j-1}$ of its members who are unemployed, the household receives an unemployment benefit, equal to a fraction $\widetilde{b}_j$ of the after-tax labour income that these individuals would receive if they were employed. Formally, $\widetilde{B}_j = \widetilde{b}_j w_{L,t+j-1}\varepsilon_j h_{jL}^t g(n_{jL}^t)(1 - \tau_{jL})u_{j,t+j-1}$, where $j \in \{1,2,3,4\}$. The household takes both the unemployment benefit and the unemployment rate as given, when choosing consumption, savings and the supply of working hours. Note that, due to the progressivity of labour income taxes, lower ability households will face a different tax rate than higher ability households ($\tau_{jL} < \tau_{jH}$).

2.2.1.3 Human capital

Individuals enter our model at the age of 18 with a predetermined level of human capital. This level is generation-invariant, but higher for individuals with high innate ability. The latter reflects for example higher intelligence and greater capacity to learn and accumulate knowledge at primary and secondary school. We normalise the human capital of a young individual with high ability to h_0. A young individual with low ability enters the model with only a fraction $\epsilon_L h_0$, where $\epsilon_L < 1$. The parameter ϵ_L will be calibrated.

During youth, individuals with high ability will invest a fraction of their time in tertiary education. They accumulate more human capital, making them more productive in later periods. Formally,

$$h_{2H}^t = h_{1H}^t (1 + \phi (e_{1H}^t)^\sigma) \text{ with } 0 < \sigma \leq 1, \phi > 0 \tag{2.5}$$

We adopt in Equation (2.5) a human capital production function similar to Lucas (1990) and Bouzahzah et al. (2002). The production of new human capital by these individuals rises in the amount of time they allocate to education (e_{1H}^t) and in their initial human capital (h_{1H}^t). The parameter σ indicates the elasticity of time input, ϕ is an efficiency parameter. Individuals with low innate ability do not study. Their human capital remains constant: $h_{2L}^t = h_{1L}^t$. Finally, we assume that the human capital of all individuals remains unchanged after the second period. A rationale for this assumption is that learning-by-doing in work may counteract depreciation. This means that $h^t_{4a} = h^t_{3a} = h^t_{2a}$, for $a \in \{H, L\}$.

2.2.2 Firms

Both the goods market and the labour market for higher ability individuals are perfectly competitive, whereas the labour market for lower ability individuals is unionised. All firms are identical. They maximise profits, pay taxes on capital income and social security contributions when hiring labour. Total domestic output is given by the production function (2.6). Production exhibits constant returns to scale in aggregate physical capital (K_t) and labor in efficiency units ($A_t H_t$). Given our assumption of perfect competition on the goods market, profits are zero in equilibrium.

$$Y_t = K_t^{\alpha}(A_t H_t)^{1-\alpha} \tag{2.6}$$

Technology A_t is growing at an exogenous and constant rate x: $A_{t+1} = (1 + x)A_t$. As to total effective labour H_t, we assume that higher and lower ability individuals are imperfectly substitutable in production. This framework was pioneered by Katz and Murphy (1992). So,

$$H_t = \left[\eta_H H_{H,t}^{\frac{\iota-1}{\iota}} + \eta_L H_{L,t}^{\frac{\iota-1}{\iota}}\right]^{\frac{\iota}{\iota-1}}, \text{ with } \eta_H + \eta_L = 1 \tag{2.7}$$

with η_a being a share parameter and ι the elasticity of substitution between higher and lower ability labour. Furthermore, workers of the same ability type but different age are assumed to be perfect substitutes. Formally, this gives respectively for higher and lower ability labour

$$H_{H,t} = \sum_{j=1}^{4}(n_{jH}^{t-j+1} - \bar{n}_H)\varepsilon_j h_{jH}^{t-j+1}, \quad H_{L,t} = \sum_{j=1}^{4}(1 - u_{j,t})(n_{jL}^{t-j+1} - \bar{n}_L)\varepsilon_j h_{jL}^{t-j+1} \tag{2.8}$$

2.2.3 Unions

The economy is populated by decentralised trade unions, operating at the firm level. Every single union represents all the lower ability workers in a firm. As such, unions are large compared to the workers. The union will determine the lower ability workers' wage while taking aggregate variables and fiscal policy parameters as given. Just like in e.g. Pencavel (1984) and de la Croix et al. (1996), the objective function of the union follows the Stone-Geary specification,

$$V_t = \sum_{j=1}^{4}\left[\frac{1}{\chi_j}(w_{L,t}(1 - \tau_{jL}) - \bar{w}_{j,t})^{\chi_j}(1 - u_{j,t})\right] \tag{2.9}$$

with $\chi_j > 0$. The union derives utility from both wages and employment, albeit to a different degree[7]. As to wages, what matters is the difference between the after-tax wage $w_{L,t}(1-\tau_{jL})$ and a reference wage, $\bar{w}_{j,t}$. The age-specific parameter χ_j measures the concavity with respect to the excess wage gap. The higher χ_j, the higher the preference of the union for wages versus employment for the age group j. Every union has the same reference wage. We define this as a weighted average or combination of the after-tax wage that would prevail if the lower ability labour market were competitive, the after-tax wage of higher ability workers and the unemployment benefit. The respective weights are ρ_1, ρ_2 and ρ_3. They sum up to 1. Formally, the reference wage is given by $\bar{w}_{j,t} = \rho_1 w^c_{L,t}(1-\tau^c_{jL}) + \rho_2 w_{H,t}(1-\tau_{jH}) + \rho_3 \widetilde{b}_j w_{L,t}(1-\tau_{jL})$[8].

2.2.4 Government

Unemployment benefits and government spending on goods are financed by taxes on capital, labour (both on employers and employees) and consumption. Lump sum transfers ensure a balanced budget. Formally,

$$G_t + B_{L,t} + Z_t = T_{nH,t} + T_{nL,t} + T_{k,t} + T_{c,t} \tag{2.10}$$

and

$$G_t = gY_t, \;\; Z_t = 10z_t \tag{2.11}$$

$$T_{k,t} = \tau_k \alpha Y_t \tag{2.12}$$

$$T_{c,t} = \tau_c \sum_{j=1}^{5} (c_{jH}^{t+1-j} + c_{jL}^{t+1-j}) \tag{2.13}$$

$$B_{L,t} = \sum_{j=1}^{4} \widetilde{b}_j w_{L,t} \varepsilon_j h_{jL}^{t+1-j} g(n_{jL}^{t+1-j})(1-\tau_{jL}) u_{j,t} \tag{2.14}$$

$$T_{nH,t} = \sum_{j=1}^{4} w_{H,t} g(n_{jH}^{t+1-j}) \varepsilon_j h_{j,H}^{t+1-j} (\tau_{jH} + \tau^p) \tag{2.15}$$

$$T_{nL,t} = \sum_{j=1}^{4} w_{L,t}(1-u_{j,t}) g(n_{jL}^{t+1-j}) \varepsilon_j h_{jL}^{t+1-j} (\tau_{jL} + \tau^p) \tag{2.16}$$

The government spends a fraction g of output on goods. In Equations (2.15) and (2.16), τ_{ja} is the average tax rate that applies to the labour income of an individual of age j and

[7] In our companion paper (Boone and Heylen, 2015), we elaborate more in detail on the Stone-Geary functional form.

[8] Like for the household taste for leisure parameters (γ_j), we also assume for χ_j and ρ_k ($j \in \{1,2,3,4\}$ and $k \in \{1,2,3\}$) that their empirical values may differ across country groups (continental European countries, Nordic countries, and Anglo-Saxon countries).

ability a and τ^p is the tax rate paid by the employer. What remains is the specification of progressive income taxes. The tax rates appearing in the budget constraints of the households are average tax rates and given by $\tau_{ja} = \Gamma\left(\frac{y^{lab}_{ja,t}}{\bar{y}^{lab}_t}\right)^{\xi}$, where $\xi \geq 0$ and $0 < \Gamma \leq 1$. Here, $y^{lab}_{ja,t}$ is total pre-tax labour income of the household at time t and $\bar{y}^{lab}_t$ is the average labour income in the economy. Just like in Guo and Lansing (1998) and Koyuncu (2011), the parameters ξ and Γ govern the slope and level of the tax schedule. The average tax rate τ_{ja} increases with the total taxable labour income of the household when $\xi > 0$. Households are aware of the progressive structure of the tax system when making decisions, but take this as given. The marginal tax rate τ^m_{ja} is then simply the rate applied to the last euro earned: $\tau^m_{ja} = (1+\xi)\Gamma\left(\frac{y^{lab}_{ja,t}}{\bar{y}^{lab}_t}\right)^{\xi}$. Rewriting this yields $\frac{\tau^m_{ja}}{\tau_{ja}} = 1+\xi$. The marginal tax rate is higher than the average tax rate when $\xi > 0$, i.e. the tax schedule is said to be progressive. When $\xi = 0$, the average and marginal tax rates coincide.

2.3 Optimisation and Equilibrium

All households will optimally choose consumption in each period of life and hours worked when employed. Households of higher ability will also choose the fraction of time allocated to education when young. The first order conditions are standard. We report them in Appendix 2.A. Our focus in this section is on the maximisation problem of the firms and the unions. Moreover, we also present a detailed description of how we solve for the subgame perfect equilibrium of the game played between a triplet of a union, a firm and a household. For a description of the general equilibrium of our model, we refer to Appendix 2.B.

2.3.1 Firms

The representative firm in our model operates in a small open economy with perfect mobility of physical capital. It chooses the optimal capital stock and the amount of effective labour. In terms of production, the firm prefers a combination of few people working more hours over a combination of many people working few hours, as each individual causes a time cost for commuting and getting setup in a job. More precisely, firms maximise profits with respect to the vector $[K_t, H_{1H,t}, H_{2H,t}, H_{3H,t}, H_{4H,t}, (1-u_{1,t}), (1-u_{2,t}), (1-u_{3,t}), (1-u_{4,t})]$, leading to the following first order conditions,

$$\alpha\left[\frac{A_t H_t}{K_t}\right]^{(1-\alpha)}(1-\tau_k) = r_t \tag{2.17}$$

$$(1-\alpha)A_t^{1-\alpha}\left[\frac{K_t}{H_t}\right]^{\alpha}\eta_a\left[\frac{H_t}{H_{a,t}}\right]^{\frac{1}{\iota}}=w_{a,t}(1+\tau^p),\;\; a\in\{H,L\} \tag{2.18}$$

where $H_{H,t}$ and $H_{L,t}$ are defined in Equation (2.8) and $\eta_H = 1-\eta_L$. Due to the perfect mobility of capital, the firm in Equation (2.17) will hire capital until its after-tax marginal product equals the exogenous world interest rate, r_t. There is no depreciation of capital. Whenever the net return to investment exceeds the world interest rate, capital will flow into the country until optimality is restored. According to Equation (2.18), firms hire higher and lower ability labour up to the point where their marginal product equals their real wage cost. On the labour market for higher ability workers, wages are determined competitively. Wage flexibility on this market implies that they will all work. The wage will be such that the total supply of higher ability labour, $H_{H,t}$, in Equation (2.8) equals the firms' demand in (2.18). For lower ability workers, however, the wage is controlled by the union. Since hours of work are chosen by the households, in order to satisfy Equation (2.18) the firm can only choose the fraction of persons it wants to employ, or the unemployment rate.

Our assumptions of constant population and of individuals entering the model with a predetermined and generation-invariant level of human capital imply that in steady state effective labor will be constant. Physical capital, output and real wages by contrast will all grow at the exogenous technology growth rate x.

2.3.2 Union

The union chooses the wage $w_{L,t}$ to maximise Equation (2.9) subject to the firm's and the lower ability households' optimal choice of (un)employment and hours of work, i.e.

$$\text{s.t.}\begin{cases} F(n_{jL}^{t-j+1},1-u_{j,t},w_{L,t}) = (1-\alpha)A_t^{1-\alpha}\left[\frac{K_t}{H_t}\right]^{\alpha}\eta_L\left[\frac{H_t}{H_{L,t}}\right]^{\frac{1}{\iota}} - w_{L,t}(1+\tau^p)=0 \\ G(n_{jL}^{t-j+1},1-u_{j,t},w_{L,t}) = \frac{\gamma_j}{(1-n_{jL}^{t-j+1})^{\theta}} - \frac{w_{L,t}\varepsilon_j h_{jL}^{t-j+1}(1-\tau_{jL}^m)}{(1+\tau_c)c_{jL}^{t-j+1}} = 0 \end{cases}$$

with $j\in\{1,2,3,4\}$. To derive the first order condition, one has to know how the optimal unemployment rate resulting from the second stage of the game changes when the chosen wage changes. From the second stage, we derive a system of two implicit equations in the supply of hours worked, the unemployment rate, and the wage rate.

Using matrix notation, evaluating at the equilibrium values of the supply of hours worked, the unemployment rate, and the wage rate, and taking the total differential yields:

$$\begin{bmatrix}\frac{\partial F}{\partial n_{jL}^{t-j+1}} & \frac{\partial F}{\partial(1-u_{j,t})}\\ \frac{\partial G}{\partial n_{jL}^{t-j+1}} & \frac{\partial G}{\partial(1-u_{j,t})}\end{bmatrix}\begin{bmatrix}dn_{jL}^{t-j+1}\\ d(1-u_{j,t})\end{bmatrix}=\begin{bmatrix}-\frac{\partial F}{\partial w_{L,t}}dw_{L,t}\\ -\frac{\partial G}{\partial w_{L,t}}dw_{L,t}\end{bmatrix} \tag{2.19}$$

Under normal parameter values the (2x2)-matrix is non-singular. We can then take the inverse to calculate $\frac{d(1-u_{j,t})}{dw_{L,t}}$, the change in the equilibrium unemployment rate resulting from the second stage of the game, following an increase in the chosen wage rate by the union. Using the assumption that the union treats every generation equally when it comes to generating union utility, the first order condition for the union is given by[9]:

$$w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t} = -\frac{\chi_j(1-u_{j,t})}{\frac{\partial(1-u_{j,t})}{\partial w_{L,t}}}(1-\tau_{jL}^m) \tag{2.20}$$

As the right-hand side of this equation is positive, the left-hand side has to be positive as well. This implies that the after-tax wage determined by the union will exceed the reference wage. The higher χ_j (i.e. the relative weight on wages as opposed to employment for age group j), the higher the ex-ante union wage premium. Ex-post, a rise in χ_j will also be reflected in higher unemployment u_j, though. The reason is that if the union has a higher preference for wages for a particular age group, the firm will in the end replace the workers of this age group by low ability workers of other age. Unemployment among these other age groups might fall. Due to more expensive low ability labour, however, aggregate unemployment among low ability workers will rise.

2.3.3 Solving for the Subgame Perfect Equilibrium

Within every period t, a dynamic two-stage game is played between a triplet of a firm, a union and a lower ability household. In the first stage, the unions choose the wage for the lower ability workers, whereas in the second stage firms choose the fraction of people they want to employ, and lower ability households choose their labour supply in hours. As such, the second stage is a static game, played between the firm and the lower ability household. We use backward induction to solve for the subgame perfect equilibrium of our game. In the second stage, the firm and the households play simultaneously, taking the union's wage, the fiscal policy variables, and the action of the other player as given. In the previous sections, we already solved for the best responses of the household and the firm. In Graph 1, we show the second stage of the game. The flatter curves are the 'best response'-functions of the households given the real wage rate, the unemployment rate chosen by the firm, tax rates and the unemployment benefit. If the unemployment rate increases, the income and consumption of the household will decrease, implying that the marginal benefit of working increases. Household members will then supply more hours. This argument explains the negative slope of the households' best-response curves. If the wage chosen by the union increases, the best-response curves shift upwards, leading to a

[9] For details on the derivation of this equation, we refer the reader to Appendix 2.D.

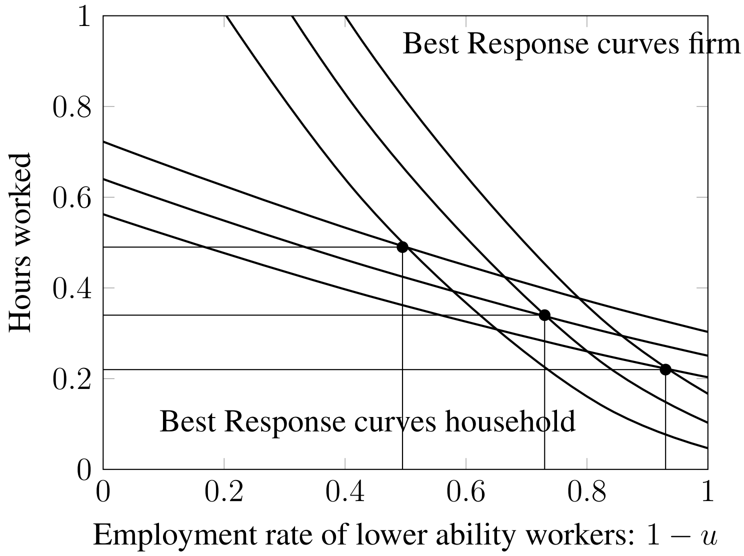
higher supply of hours for a given unemployment rate. Intuitively, the substitution effect of a higher wage dominates the income effect. The best-response curves of the firm are calculated using the first-order conditions of the firm. If the households decide to supply more hours for a given wage rate, the firm will employ fewer people. If the wage chosen by the union increases and households maintain their supply of hours, the firm will also employ fewer people, implying that the best-response curves will shift to the left. The intersection of the 'best response'-functions for different wage rates represent the Nash equilibria of the second stage of our game. In Graph 2, an indifference curve of the union has been drawn. The other curve is the collection of Nash equilibria for different wage rates resulting from the second stage of the game. This curve indicates the employment rates which are Nash Equilibria in the second stage given different levels of the real wage rate.

The optimal combination of the wage and the employment rate is found where the indifference curve of the union is tangent to the Nash-function of the second stage. In Graph 2, one optimal point is drawn. From this value for the wage, we can calculate the exact Nash equilibrium in the second stage. The firm chooses the different employment rates such that Equation (2.18) holds for every age group. The exact composition of employment follows from the second stage of the game. First, notice that Graph 1 only represents one age group. To solve the full game, one needs a similar graph for all age groups (four in total). Then, for a given wage rate chosen by the union, the best-response curves of the firm and the household will be different over the age groups. The workers of an old household react differently to an increase in the wage compared to the members of the youngest household, for example. The same goes for the firm. Given the same wage increase, as there are differences in productivity and the amount of hours worked, the decrease in the employment rate for the oldest workers which is required to restore optimality (Equation 2.18) will differ from the required decrease for the youngest workers. Thus, the shift in the best-response curves of the household and the firm will be different over all the ages and therefore, for the same wage, one will obtain different employment rates. Finally, as every game is symmetric, the wage will be the same at every firm in the economy.

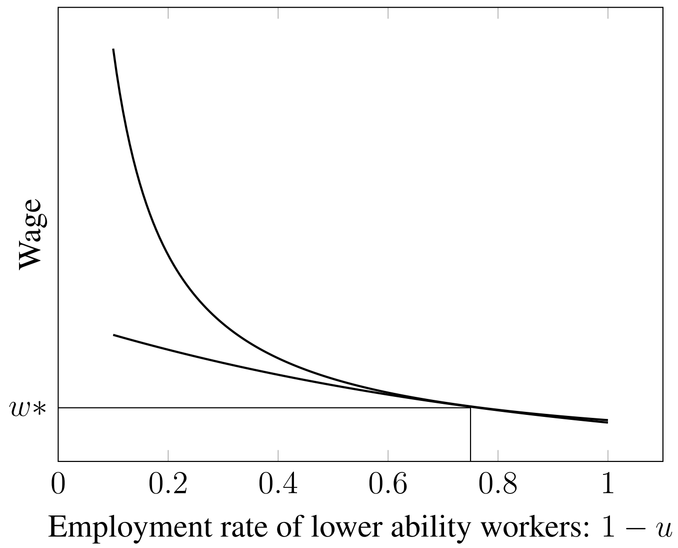
2.4 Calibration and empirical relevance of the model

In a first step, we calibrate our model and compare its predictions regarding the main labour market variables to the data for three groups of countries: Anglo-Saxon countries (the US, the UK and Canada), continental European countries (Belgium, France, Germany, the Netherlands, and Austria) and Nordic countries (Denmark, Finland, Norway

Graph 1: Second stage of the game



Graph 2: First stage of the game



and Sweden). To make our predictions, we impose common technology and productivity parameters on all countries. Most household and all union preference parameters are assumed to be common for the countries within the same group, but they may differ across groups. The parameters involved are the relative utility of leisure versus consumption for the households (γ_j), the relative weight that the unions assign to wages as opposed to employment (χ_j), and the relative weight of each of the three determinants of the unions' reference wage (ρ_k). To highlight their country group or region-specific character we add a superscript R to these parameters from now on, which gives γ_j^R, χ_j^R and ρ_1^R, ρ_2^R and ρ_3^R

respectively, with $j \in \{1,2,3,4\}$ and $R \in \{Eur, Ang, Nor\}$. Last but not least, fiscal policy parameters are all country-specific.

All common and all country group-specific parameters are reported in Table 2.3 below. So are the country-group averages of the unemployment rate by age, the country-group averages of hours worked per employed person by age, and the overall average education rate and per capita growth rate. These performance indicators play an important role in our calibration, as we will describe in Section 2.4.2. First, however, we say more about the construction of our main data in Section 2.4.1. For country-specific data on labour market performance and for the country-specific fiscal policy parameters we refer to Appendix 2.C.

2.4.1 Data

In our model we have assumed that all individuals (except the retired) of both higher and lower ability participate in the labour market. Those of higher ability will all work. Among those of lower ability, only a fraction $1 - u_{j,t}$ will work. The difference between both employment rates corresponds to the *rate of unemployment* $u_{j,t}$. The data for unemployment that we report in Table 2.3 reflect this setup. They are the difference in percentage points between the actual employment rate (in persons) among those within a particular age group who enjoyed tertiary education and those who did not. Although consistent with the setup of our model, this proxy for unemployment among the lower educated differs from official unemployment series. Our unemployment rate also captures differences in the labour market participation rate between high and low educated individuals in the data. Lower participation among low educated individuals implies higher unemployment in our data. When we account for cross-country differences in unemployment in Section 2.5, our results will then also capture any impact of household preferences, union preferences, and fiscal policy on participation. This should not affect our main conclusions, though[10]. The (negative) correlation in actual data between the difference in employment rates and the difference in unemployment rates between higher and lower educated individuals is strong (see also Figure 2.1.c). Depending on the age group considered, it varies between -0.60 and -0.85. Our data for *hours worked per employed n* indicate the fraction of time that employed individuals devote to work relative to their total time endowment

[10]Balleer et al. (2009) have investigated the determinants of labour force participation in the euro area. They find that labour taxes, union density and unemployment benefits have an impact, but this impact is not robust in sign nor statistical significance across age and gender groups, and countries. Moreover, since the deeper objective of this paper is to contribute to an explanation of labour market performance, and employment in particular (see the introduction to this paper), our approach to compute unemployment rates is even to be considered an advantage.

on an annual basis. We follow Wallenius (2013) in our computation[11]. The *education rate* e is the fraction of time that higher ability individuals devote to tertiary education. The data reflect the number of students in tertiary education in full-time equivalents divided by (the assumed higher ability) half of the population of age 18-29.

2.4.2 Calibrated parameters

How well does our model match reality in individual countries? When we impose each country's fiscal policy parameters, how close are the model's predictions to the data? To find out, we first parameterise the model. We discuss our procedure in this section. Table 2.3 contains an overview of all parameters. Many have been set in line with, or taken from, the existing literature. Others have been calibrated to match key data.

We set the rate of time preference at 2% per year and the (exogenous and constant) world real interest rate at 4.5% per year. Considering that periods in our model last 12 years, this choice implies a discount factor $\beta = 0.788$ and interest rate $r = 0.696$. In the production function for goods, we assume a capital share coefficient α equal to 1/3. Following Caselli and Coleman (2006), who state that the empirical labour literature consistently estimates values between 1 and 2, we set the elasticity of substitution ι between the two ability types in effective labour equal to 1.5. We set $\theta = 4$. Rogerson (2007) considers a value for the inverse of the intertemporal elasticity of substitution in leisure θ between 1 and 3 as reasonable. We do not have an endogenous participation decision, however. Therefore, we adopt a value for θ that is somewhat higher. Two other sets of parameters that we took directly from the literature are the age-specific productivity parameters ε_j and the time cost of commuting and being set-up in a job $\bar{n}$. For the former we follow the hump-shaped pattern imposed by Miles (1999). For the latter we impose a value of 0.05, in line with Wallenius (2013). We impose the same ϵ_j and $\bar{n}$ for both ability types.

Three parameters relate to the production of human capital and to the level of human capital with which individuals enter the model. For the elasticity with respect to education time (σ) we choose a conservative value of 0.3. This value is within the range considered by Bouzahzah et al. (2002), but much lower than the value imposed by Lucas (1990). To determine the relative initial human capital of lower ability individuals (relative to the initial human capital of high ability individuals, ϵ_L) we follow Buyse et al. (2017) who rely on PISA science scores. We use the average of the test scores of students at the 17th and the 50th percentile as representative for lower ability individuals in our model, and the test

[11] We assume that the total time endowment of each individual is 14 hours a day, 7 days a week and 52 weeks per year. This time can be allocated to work, leisure or (for young higher ability individuals) education.

Table 2.3: Basic parameterisation of the model

Parameters imposed on all countries - Values			
Discount factor in utility		β	0.788
World real interest rate		r	0.696
Capital share parameter in goods production		α	0.33
Elasticity of substitution between different ability types of labour		ι	1.5
Inverse of the intertemporal elasticity of substitution in leisure		θ	4
Age-specific productivity parameters		ε_1,ε_2,ε_3,ε_4	2.325, 2.770, 2.776, 2.341
Time cost for e.g. commuting		$\bar{n}$	0.05
Relative initial human capital of lower ability individuals (to h_0)		ϵ_L	0.755
Elasticity of human capital with respect to education time		σ	0.3
Efficiency parameter in human capital production		ϕ	3.53
Share parameter of higher ability individuals in effective labour		η_H	0.594
Exogenous technology growth		x	0.255
Calibrated region-specific parameters			
	cont. European	Nordic	Anglo-Saxon
γ_1^R	0.195	0.216	0.106
γ_2^R	0.481	0.391	0.355
γ_3^R	0.356	0.289	0.262
γ_4^R	0.239	0.204	0.187
χ_1^R	2.513	1.740	1.792
χ_2^R	4.013	2.355	2.913
χ_3^R	4.017	2.366	2.888
χ_4^R	4.280	2.598	2.960
ρ_1^R	0.8	0.9	0.9
ρ_2^R	0.05	0	0.1
ρ_3^R	0.15	0.1	0
Target values for the calibrated parameters ϕ, x, and η_H (2001-07)			
Education rate (e) (average over 12 countries)			23.4%
Per capita annual growth (average over 12 countries)			1.91%
Relative gross wage of young low versus high educated workers US			0.53
Region-specific targets: unemployment and hours worked by age (2001-07)			
	cont. European	Nordic	Anglo-Saxon
u_1	14.0%	8.7%	16.9%
u_2	14.9%	11.8%	14.8%
u_3	15.5%	12.7%	14.7%
u_4	20.3%	18.7%	17.2%
n_1	0.295	0.293	0.331
n_2	0.313	0.343	0.371
n_3	0.313	0.343	0.371
n_4	0.306	0.331	0.355

score of students at the 83th percentile as representative for high ability individuals. The data are remarkably robust across countries. The science test scores of students at the 17th percentile and students at the 50th percentile are always very close to 67% and 85% of the test score of students at the 83th percentile[12]. The differences across countries being so small, we take these relative scores as objective indicators of the relative cognitive capacity of lower and high ability individuals, and will correspondingly set ϵ_L equal to 0.755 (= the average of 0.67 and 0.85). Last but not least, the efficiency parameter ϕ in the human capital production function of the individuals with high ability has been determined by a calibration procedure that we discuss now.

We determined 36 parameters by calibration. Next to the efficiency parameter in human capital production (ϕ), these are the exogenous technology growth rate (x), the share parameter in aggregate effective labor (η_H), the four household taste for leisure parameters ($\gamma_1^R, \gamma_2^R, \gamma_3^R, \gamma_4^R$), the four union 'preference for wage' parameters ($\chi_1^R, \chi_2^R, \chi_3^R, \chi_4^R$), and the three weights in the unions' reference wage ($\rho_1^R, \rho_2^R, \rho_3^R$). The former three parameters (ϕ, η_H, x) are assumed to be the same for all countries. The 11 household and union preference parameters may differ by country group. The efficiency parameter in human capital production (ϕ) is determined to correctly predict the average fraction of time allocated to tertiary education (e) by individuals of age 18-29 in all twelve countries in our sample. The parameter turns out to be 3.53. The exogenous growth rate of technology (x) is calibrated to match actual per capita growth. The underlying target for the annual growth rate is 1.91%, being the average annual per capita growth in our sample of twelve countries in 2001-2007. Following Buyse et al. (2017), we calibrate the share parameter in aggregate effective labor (η_H) to match the relative wage of young workers without a tertiary degree versus young workers with tertiary degree in the US. This relative wage is 0.53[13]. As shown by Equation (2.18), the share parameter η_H is an important determinant of the relative productivity of labour and relative wages. Actual wages are informative if a close link can be assumed between wages and productivity. This condition is much more likely fulfilled in the US than in Europe, which explains the introduction here of US relative wages. The value for η_H that emerges is 0.59.

Finally, we calibrated the four taste for leisure parameters of the households (γ_j^R), the four union preference for wage parameters (χ_j^R), and the weights in the reference wage of the union for the hypothetical competitive wage of lower ability workers (ρ_1^R), the wage of higher ability workers (ρ_2^R), and the unemployment benefit ($\rho_3^R = 1 - \rho_1^R - \rho_2^R$) for

[12] The data that we report are averages of the PISA results for the years 2000, 2003 and 2006. The available data concern students aged 15.

[13] OECD data (Education at a Glance, 2009, table A7.1a) show a relative wage of 0.43 for workers of age 25-34 without upper secondary education versus workers of this age with a tertiary degree. The relative wage of workers of age 25-34 with upper secondary degree is 0.63. On average this is 0.53.

each of the three country groups. That is a total of 33 parameters. To do this, we used a procedure in line with Heylen and Van de Kerckhove (2013). Basically, this procedure comes down to (i) exactly matching the 24 country-group averages of the unemployment rates and hours worked by age reported in Table 2.3, and (ii) to minimise the deviation of our model's predictions for the aggregate unemployment rate (over all age and both ability groups) from the data in the twelve individual countries in our sample. More precisely, in a first step we imposed 9 values for ρ_1^R, ρ_2^R and ρ_3^R ($R \in \{Eur, Nor, Ang\}$). With these imposed values we calibrated the 12 parameters χ_j^R and the 12 parameters γ_j^R with $j = \{1, 2, 3, 4\}$ and $R \in \{Eur, Nor, Ang\}$ to exactly match the country-group averages of actual unemployment rates u in four generations and the country-group averages of actual hours worked n per generation (over both ability types). The obtained set of household taste for leisure and union preference parameters for each of the three country groups - together with all other calibrated parameters - would then allow us to compute the predictions of our model for all unemployment rates and all hours worked in all generations in each of the twelve countries in our sample separately. We repeated this procedures many times, each time starting from different values for ρ_1^R, ρ_2^R and ρ_3^R. Our guideline to pin down our final values for these parameters and the corresponding values for γ_j^R and χ_j^R was to minimise the deviation of our model's predictions from the actual data for the aggregate unemployment rate (over all age and ability groups)[14].

The results in Table 2.3 reveal by far the highest values for χ_j^{EUR} in continental Europe, which implies that in these countries union indifference curves are flatter, wages are more rigid and the union wage mark-up is higher. Given the dominance of sectoral wage bargaining in these countries, this result matches well with the famous Calmfors and Driffill (1988) hypothesis. Furthermore, we observe that in each country group χ_j^R rises in the age of the workers involved. Wages are therefore the most (least) rigid and the highest (lowest) for the oldest (youngest) workers. Seniority pay systems and the insider-outsider theory may provide an explanation for this result. As to the specific weights in the unions' reference wage, we notice in each country group a major role for the competitive wage of lower ability individuals ($\rho_1^R \geq 0.8$). In the Anglo-Saxon countries the only other variable that matters in the unions' reference wage is the wage of the high skilled. In the Nordic countries, it is the unemployment benefit. In continental Europe, both these other variables have an impact on union wage setting, with unemployment benefits being more important. For the household taste for leisure parameters γ_j^R, we observe the lowest values in the Anglo-Saxon countries, which would confirm Blanchard (2004). Except among

[14]From the predictions of our model and the data for 12 countries we computed each time for the aggregate unemployment rate the root mean squared error normalised to the mean. We minimised the average normalised RMSE.

the youngest households, however, cross-country differences in γ_j^R are fairly small.

2.4.3 Evaluation and empirical relevance of the model

In Figures 2.2-2.4, we evaluate the empirical relevance of the model regarding the cross-country variation in unemployment and hours worked. The interrupted line in each figure is the 45°-line. In each figure, we also report the slope of the regression line (not shown). Plugging each country's policy parameters into our calibrated model, it matches the facts for the aggregate unemployment rate very well. Correlation between the predictions of the model and the facts in Figure 2.2.a is over 80%, with a slope of the regression line very close to 1. Our model also captures the cross-country differences in the unemployment rates of different age groups quite well. Figure 2.3 shows this for the youngest and the oldest age groups. Last but not least, in Figures 2.2.b and 2.4 our model explains an important fraction of cross-country differences in hours worked per employed. The reported slope of the regression line in these figures remains below one, though, suggesting that our model somewhat exaggerates the effect of policy differences on hours worked.

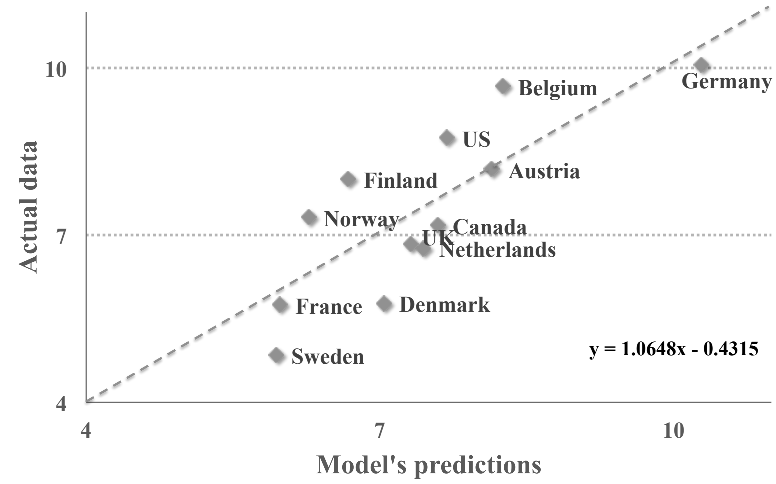
2.5 Accounting for cross-country variation in unemployment

Figure 2.2.a showed large differences across OECD countries in the unemployment rate. From Figure 2.1 we know that these differences explain an important fraction of cross-country differences in aggregate employment and labour market performance. The final step in this research is to account for these unemployment differences. What exactly causes higher unemployment in some countries compared to other countries?

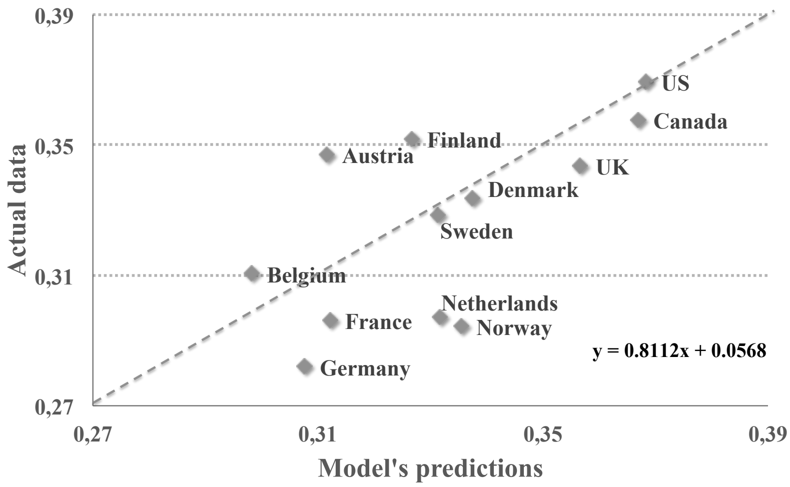
2.5.1 Description of the experiment

We find a correlation of 0.804 between the predictions of our model for the aggregate unemployment rate and the unemployment data. From this, we derive a R^2 of 0.646. Following Israeli (2006), we perform a Shapley decomposition of the R^2-coefficient in order to evaluate the relative importance of the different fiscal policy variables, union wage setting and household preferences in generating cross-country unemployment differences. More specifically, according to a Shapley decomposition, the contribution of each of our variables equals its marginal effect measured by the change in the R^2-coefficient

Figure 2.2: Aggregate labour market performance in individual countries (2001-07)



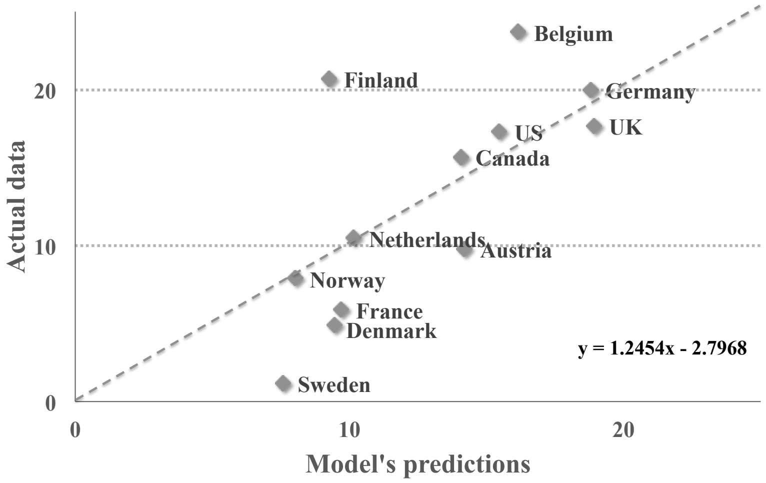
(a) Unemployment rate (in %) - Correlation: 0.804



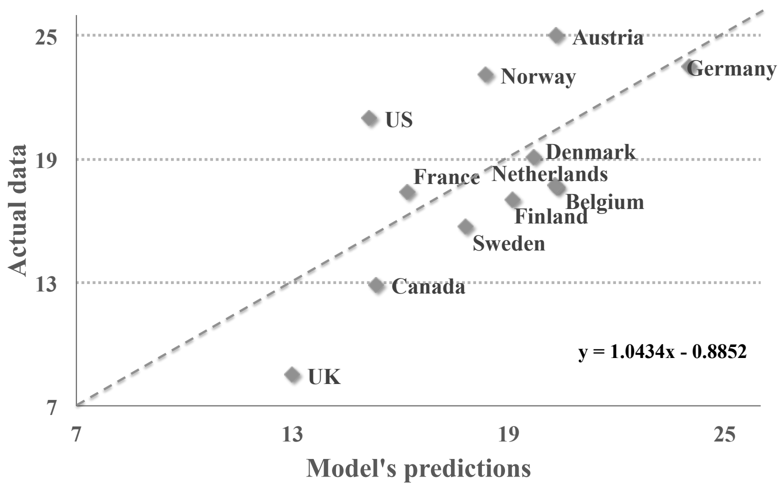
(b) Hours worked per employed - Correlation: 0.64

after eliminating the cross-country differences in this variable. This change in R^2 is computed for every subset S of other explanatory variables. For example, if we had four explanatory variables, x_1, x_2, x_3, and x_4, the marginal effect of x_1 on R^2 would be $M_1 = R^2[x_1, S \subseteq \{x_2, x_3, x_4\}] - R^2[S \subseteq \{x_2, x_3, x_4\}]$ for every subset S. Next, we take a weighted average over all these marginal effects where the weight is respectively $\frac{s!(n-s-1)!}{n!}$, with s the number of elements in the subset and n the total number of explanatory variables. Hence, for each of our variables in the Shapley decomposition, we successively replace their country-specific values by the average value over all countries in our sample, implying that we shut down the influence of these specific variables in generating cross-country differences in the unemployment rate. These variables are (i) the labour tax imposed on employers; (ii) the labour tax imposed on employees; (iii) the

Figure 2.3: Unemployment rate (%) among lower ability individuals



(a) Young (age 18-29) - Correlation: 0.713

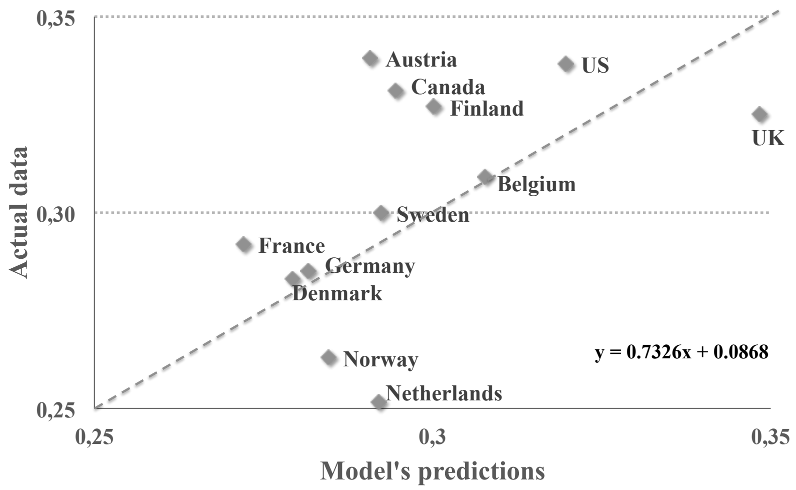


(b) Older (age 54-65) - Correlation: 0.674

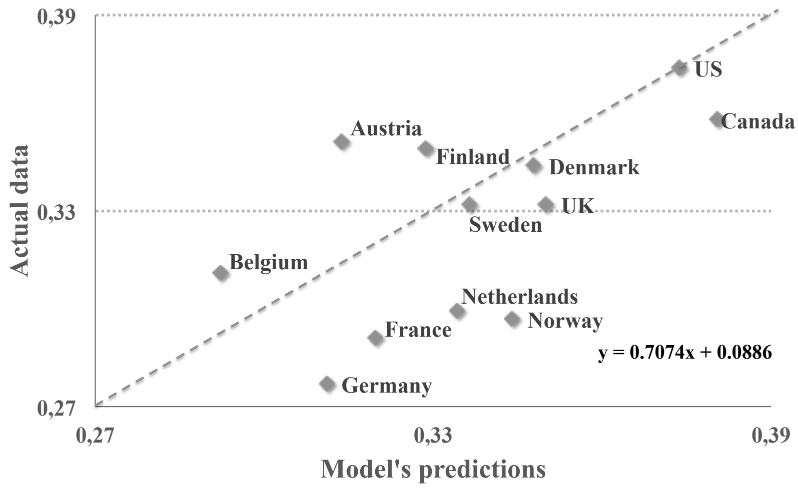
replacement rate in the unemployment benefit formula; (iv) the tax rate on capital; (v) the tax rate on consumption; (vi) government spending on goods; (vii) the union preference parameters, and (viii) the taste for leisure of the households.

Our model generates cross-country variation in aggregate unemployment due to these country-specific and region-specific variables. The Shapley decomposition makes it possible to formally explore the contribution of the variation in each of those variables to the predicted variation in the aggregate unemployment rate. While the decomposition can be applied to any model independent of the underlying theory, we believe that there are a few features of our model and our results that make the Shapley decomposition highly relevant. First, the richer the model, the more explanatory variables contribute to the R^2-coefficient and the more relevant is a decomposition of R^2 into its contributing variables.

Figure 2.4: Hours worked by age over both ability types



(a) Young (age 18-29) - Correlation: 0.515



(b) Older (age 54-65) - Correlation: 0.590

Second, the R^2-coefficient between the model's predictions and the actual data for the aggregate unemployment rate is quite high. Thus, the variation in the exogenous variables and parameters is highly informative about the cross-country variation in the aggregate unemployment rate.

2.5.2 Fiscal policy, union preferences, or households' taste for leisure?

Is the cross-country variation in unemployment rates due to differences in union behaviour, household preferences or fiscal policy variables. Blanchard (2004) and Alesina et al. (2005) emphasise the role of union behaviour and differences in the taste for leis-

Table 2.4: Shapley decomposition of u_t

Variables		**Contribution to R^2**	
Preferences/Institutions		**Absolute**	**Relative**
Wage preference of union	χ^R	0.217	33.6%
Weights in union reference wage	$\rho_1^R, \rho_2^R, \rho_3^R$	0.121	18.7%
Household taste for leisure	γ_j^R	-0.009	-1.4%
Total preferences/institutions		0.329	50.9%
Fiscal policy		**Absolute**	**Relative**
Tax on employers, employees, consumption	$\tau^p, \xi, \Gamma, \tau_c$	-0.057	-8.8%
Net unemployment benefit replacement rate	$\tilde{b}_j$	0.198	30.6%
Tax on capital income	τ_k	0.049	7.6%
Fraction of output spent on goods	g	0.128	19.7%
Total fiscal policy		0.318	49.1%
Total		0.647	100%

ure of households. Other authors such as Prescott (2004), Ohanian et al. (2008) and Dhont and Heylen (2008) conclude that differences in fiscal policy are superior. Looking at Table 2.4, the conclusion is that both fiscal policy variables and union preferences and wage setting matter. They account each for about half of the unemployment variation across countries. A correct diagnosis of the unemployment problem and analysis of cross-country differences clearly seems to require both components. On the other hand, any differences in households' taste for leisure can safely be ignored. Integrating these findings, our interpretation is that while the above market-clearing wage chosen by the unions is the source of unemployment, the fiscal policy variables explain a large part of the magnitude of the unemployment rate.

If we explore the impact of the union parameters into more detail, we notice that the contribution of the variation in χ_j^R (i.e. the relative weight on wages as opposed to employment for age group j) is superior to that of the variation in ρ_1^R and ρ_2^R (i.e. the weights in the specification for the reference wage of the unions).

Looking at the different components of fiscal policy, a surprising result is that - despite huge cross-country variation in ξ, Γ, τ^p, and τ_c - these tax rates and parameters have no role to play when it comes to explaining unemployment differences across countries. Countries with higher average tax rates and a higher degree of progressivity in labour taxes are not necessarily the countries with the highest aggregate unemployment rate. Ambiguous effects from higher taxes may explain this. A rise in τ^p for example will imply higher unemployment because it raises the cost of low skilled labour for the firms. On the other hand, it also generates effects that may lead to lower unemployment. One is the negative effect of a rise in τ^p on competitive gross wages, which will imply more moderate wage claims from the unions. Another is that higher taxes may feed through into higher lump sum transfers in our model. The negative effect of higher transfers on the supply of hours per worker will induce firms to hire more people. Similar ambiguity follows after a rise in Γ. On the one hand, this negatively affects labour supply, pushing wages up and making low skilled workers more expensive. Firms will then hire fewer workers, and unemployment rises. On the other hand, the fact that individuals supply fewer hours because of higher taxes (and an expected increase of lump sum transfers), will induce firms to hire more people.

The major role of the replacement rate $\widetilde{b}_j$ is not a surprising result, however. From the results of the calibration in Table 2.3, it is clear that unions in both continental Europe and Nordic countries attach a positive weight ρ_3^R to the level of these benefits. This weight is the largest in continental Europe. Benefit changes will therefore affect the cost of low educated labour most in these countries, and therefore firms' willingness to hire. Important differences in benefits as exist for example between France and Germany can then be expected to explain a significant fraction of unemployment differences in the period studied. The contribution of $\widetilde{b}_j$ might even be an underestimation, as it is the combined impact of the net replacement rate and the region-specific union preferences that drive the Shapley results for the union preferences.

The variation in the capital tax rate has a small positive contribution to the R^2. Its influence runs via the first order condition of the firm with respect to capital, and has an impact on the variation in labour demand over countries. This effect dominates the indirect effect on the lump sum transfers. Thus, a decrease in τ_k leads to a decrease in u due to the higher labour demand. Government spending affects u via the lump sum transfers. An increase of g leads to a decrease in lump sum transfers. Households will consume less and supply more hours. Therefore, the firm will hire fewer individuals (u increases).

2.6 Conclusion

Huge differences in labour market performance across OECD countries have attracted the attention of many researchers during the last decade. One strand of the literature has emphasised the major role of the composition of fiscal policy, i.e. the level and structure of taxes and government expenditures (e.g. Prescott (2004), Rogerson (2007), Dhont and Heylen (2008), Wallenius (2013), Alonso-Ortiz (2014)). The focus of these studies is mainly on explaining employment (hours worked). All assume perfect competition and as such disregard any role for labour market imperfections. Unemployment is not an issue in these studies. A second tradition in the literature also recognises the role of labour taxes and unemployment benefits, but this tradition has put much more emphasis on the role of unions (e.g. union power and wage bargaining) and labour and product market institutions (e.g. Daveri and Tabellini (2000), Nickell et al. (2005), Alesina et al. (2005)). Last but not least, some other authors (e.g. Blanchard (2004)) have pointed to the key role of household preferences. In their view, a major element behind the weaker employment performance in many European countries compared to the US is a higher taste for leisure in Europe. Alesina et al. (2005) explain that stronger unions may have contributed to this higher taste for leisure in Europe.

This paper is complementary to the first strand of the literature. We also develop a general equilibrium model (OLG model) to study cross-country differences in labour market performance. While we somewhat simplify the approach by assuming exogenous participation, our main contribution is to extend this literature so that it can also explain equilibrium unemployment among lower educated individuals. This extension is important given that differences in employment rates among OECD countries are strongly related to countries' success or failure in avoiding unemployment among the lower educated. Two assumptions are key in our model. The first one is the assumption that individuals are heterogeneous by ability. They enter the model with different human capital stocks and have different capacity to build more human capital. A second assumption and key novelty compared to previous work in this tradition is the assumption of a unionised labour market and union wage setting for lower ability (lower educated) individuals. For higher ability individuals we assume that wages and employment are determined in a perfectly competitive way.

Calibrating and simulating this richer model for twelve OECD countries, we are able to assess the relative importance of a whole range of explanatory variables for cross-country differences in unemployment. What is the contribution of (progressive) tax rates on labour, tax rates on consumption, unemployment benefits, etc.? What is the contribution of union preferences and wage setting? What is the contribution of differences in households' taste for leisure? Performing a Shapley decomposition we find an almost equal role for differences in fiscal policy variables and in union preferences. Each account for about half of the cross-country variation in unemployment rates explained by our model. By contrast, any differences in the households' taste for leisure play no role. Integrating our findings, our interpretation will then be that the above market-clearing wage chosen by the unions is the source of unemployment, while the fiscal policy variables explain the major share of its magnitude. Going into greater detail on the fiscal side, we find that differences in unemployment benefit generosity play a much more important role than tax differences. In the Nordic countries and (even more) the continental

European countries, the unemployment benefit replacement rate has a significant impact on union wage setting.

Our results highlight the relevance of integrating heterogeneity in individuals' ability and labour market imperfections into dynamic general equilibrium analyses of labour market performance. Imposing perfect competition seems to imply that an important fraction of reality is unfortunately ignored. By contrast, cross-country differences in households' taste for leisure seem insignificant for unemployment, and can safely be disregarded.

Appendices

Appendix 2.A First-order conditions of the household

2.A.1 Higher ability households

The maximisation problem of the higher ability households boils down to:

$$\max_{\Upsilon_H} \quad u_H^t = \sum_{j=1}^{5} \beta^{j-1} \left[\ln c_{jH}^t + \gamma_j \frac{\left(1 - e_{jH}^t - n_{jH}^t\right)^{1-\theta}}{1-\theta} \right]$$

s.t. the household budget constraints and the human capital accumulation process, while taking fiscal policy variables, the wage and the interest rate as given. The vector Υ_H of decision variables is $[\Omega_{1H}^t, \Omega_{2H}^t, \Omega_{3H}^t, \Omega_{4H}^t, n_{1H}^t, n_{2H}^t, n_{3H}^t, n_{4H}^t, e_{1H}^t]$. Optimisation yields the following first order conditions guiding the optimal consumption path (2.21), the labour-leisure choice (2.22, 2.23) and the optimal time allocation to education (2.24):

$$\frac{c_{j+1,H}^t}{c_{jH}^t} = \beta(1 + r_{t+j}), \;\; j \in \{1,2,3,4\} \tag{2.21}$$

$$\frac{\gamma_1}{(1 - n_{1H}^t - e_{1H}^t)^\theta} = \frac{w_{H,t}\varepsilon_1 h_{1H}^t (1 - \tau_{1H}^m)}{(1+\tau_c) c_{1H}^t} \tag{2.22}$$

$$\frac{\gamma_j}{(1 - n_{jH}^t)^\theta} = \frac{w_{H,t+j-1}\varepsilon_j (1 + \phi(e_{1H}^t)^\sigma) h_{1H}^t (1 - \tau_{jH}^m)}{(1+\tau_c) c_{jH}^t}, \;\; j \in \{2,3,4\} \tag{2.23}$$

$$\frac{\gamma_1}{(1 - n_{1H}^t - e_{1H}^t)^\theta} = \beta \frac{1}{c_{2H}^t} \frac{\partial c_{2H}^t}{\partial e_{1H}^t} + \beta^2 \frac{1}{c_{3H}^t} \frac{\partial c_{3H}^t}{\partial e_{1H}^t} + \beta^3 \frac{1}{c_{4H}^t} \frac{\partial c_{4H}^t}{\partial e_{1H}^t} \tag{2.24}$$

$$\text{with: } \frac{\partial c_{jH}^t}{\partial e_{1H}^t} = \sigma \phi (e_{1H}^t)^{\sigma - 1} \frac{w_{H,t+j-1}\varepsilon_j h_{1H}^t (1 - \tau_{jH}^m) \left(n_{jH}^t - \bar{n}_H\right)}{(1+\tau_c)}, \;\; j \in \{2,3,4\} \tag{2.25}$$

2.A.2 Lower ability households

For the lower ability individuals, the objective is to

$$\max_{\Upsilon_L} \quad u_L^t = \sum_{j=1}^{5} \beta^{j-1} \left[\ln c_{jL}^t + (1 - u_{j,t+j-1})\gamma_j \frac{(1 - n_{jH}^t)^{1-\theta}}{1-\theta} \right]$$

s.t. the household budget constraints and the human capital accumulation process, while taking the unemployment rate, wages, the interest rate, taxes and the unemployment benefit as given. The vector Υ_L of decision variables is $[\Omega_{1L}^t, \Omega_{2L}^t, \Omega_{3L}^t, \Omega_{4L}^t, n_{1L}^t, n_{2L}^t, n_{3L}^t, n_{4L}^t]$. Optimisation yields the following first order conditions:

$$\frac{c_{j+1,L}^t}{c_{jL}^t} = \beta(1 + r_{t+j}), \;\; j \in \{1, 2, 3, 4\} \tag{2.26}$$

$$\frac{\gamma_j}{(1 - n_{jL}^t)^\theta} = \frac{w_{L,t+j-1}\varepsilon_j h_{jL}^t (1 - \tau_{jL}^m)}{(1 + \tau_c)c_{jL}^t}, \; j \in \{1, 2, 3, 4\} \tag{2.27}$$

Appendix 2.B Description of the equilibrium of the model

Definition 1 *Given an initial value for the technology stock A_0 and a value for the predetermined human capital stock of young higher ability individuals h_0, a vector of exogenous fiscal policy variables $\{\tau_c, \tau_k, \tau^p, \Gamma, \xi, g, \widetilde{b}_j\}_{j=1}^4$ and the exogenous world interest rate, an intertemporal equilibrium consists of sequences of household decision rules $\{c_{1a}^t, c_{2a}^t, c_{3a}^t, c_{4a}^t, c_{5a}^t, \Omega_{1a}^t, \Omega_{2a}^t, \Omega_{3a}^t, \Omega_{4a}^t, e_{1H}^t, n_{1a}^t, n_{2a}^t, n_{3a}^t, n_{4a}^t\}_{t=0}^\infty$, sequences of prices $\{w_{a,t}\}_{t=0}^\infty$, human capital stocks $\{h_{1a}^t, h_{2a}^t, h_{3a}^t, h_{4a}^t\}_{t=1}^\infty$, lump sum transfers $\{Z_t\}_{t=0}^\infty$, tax rates $\{\{\tau_{ja}, \tau_{ja}^m\}_{j=0}^4\}_{t=0}^\infty$, unemployment rates $\{u_{1,t}, u_{2,t}, u_{3,t}, u_{4,t}\}_{t=0}^\infty$ and aggregate variables $\{Y_t, K_t, H_t, A_t\}_{t=0}^\infty$ for $a \in \{H, L\}$ such that*

1. *Decision rules of the higher ability households maximise the intertemporal utility function (2.1) subject to the budget constraints (2.2) and the human capital accumulation process (2.5), whereas decision rules of the lower ability households maximise the intertemporal utility function (2.3) subject to the budget constraints (2.4) and the human capital accumulation process described in Section 2.2.1.3..*

2. *Firms' choices $\{K_t, H_{1H,t}, H_{2H,t}, H_{3H,t}, H_{4H,t}, (1-u_{1,t}), (1-u_{2,t}), (1-u_{3,t}), (1-u_{4,t})\}$ maximise profits. These choices are determined via the optimality conditions of the firm in (2.17) and (2.18).*

3. *The wage $\{w_{H,t}\}$ is determined via (2.18) and is such that the labour market for higher ability individuals clears.*

4. *Given the wages chosen by the union using (2.20), each couple $\{n^t_{jL}, 1 - u_{j,t}\}$ forms a Nash equilibrium in the second stage of the dynamic game played between the household, the firm and the union. The union chooses $\{w_{L,t}\}$ to maximise its utility (2.9) subject to the optimal responses of the household and the firm (2.19). These actions form a subgame perfect equilibrium.*

5. *Human capital of the individuals evolves according to the human capital accumulation process described in Section 2.2.1.3..*

6. *Lump sum transfers ensure that the government budget (2.10) is balanced.*

7. *Government aggregates are determined via the equations described in (2.11-2.16). Average tax rates follow $\tau_{ja} = \Gamma\left(\frac{y^{lab}_{ja,t}}{\bar{y}^{lab}_t}\right)^{\xi}$, whereas marginal tax rates are determined via $\tau^m_{ja} = (1+\xi)\Gamma\left(\frac{y^{lab}_{ja,t}}{\bar{y}^{lab}_t}\right)^{\xi}$.*

8. *Y_t follows from the production function (2.6) and the values for K_t and A_tH_t (2.7-2.8).*

Appendix 2.C Details on data construction and sources

In this Appendix, we provide details on the construction and sources of our data.

2.C.1 Individuals of high and low ability

Individuals of high ability pursue tertiary education when they enter our model at the age of 18. Individuals of lower ability do not: they achieve at most an upper secondary degree, but no tertiary degree. These assumptions explain why we use existing data for individuals with a tertiary degree as proxy for variables (e.g. wages, employment) relating to higher ability individuals in our model, and the average of existing data for individuals with a lower secondary degree and individuals with an upper secondary degree (but no tertiary degree) as proxy for variables relating to lower ability individuals. According to ISCED classification, data for high ability individuals therefore relate to education levels 5-6. Data for lower ability individuals are constructed as the average for education levels 0-2 and 3-4.

Table 2.5: Unemployment rates among four generations of lower educated individuals (2001-2007), in %

Unemployment rates	18-29	$30-41$	$42-53$	$54-65$
Belgium	23.7	17.1	18.9	17.6
France	5.9	12.5	10.2	17.4
Germany	20.0	18.5	18.5	23.5
Austria	9.8	14.1	16.4	25.0
Netherlands	10.5	12.4	13.5	17.7
Denmark	4.9	10.2	11.4	19.1
Finland	20.7	12.5	13.8	17.0
Norway	7.9	13.5	14.2	23.1
Sweden	1.2	11.1	10.9	15.7
United States	17.3	15.7	15.8	21.0
United Kingdom	17.7	14.5	13.9	8.5
Canada	15.7	14.3	14.5	12.9
Average	13.5	14.0	14.3	18.2

2.C.2 Unemployment

In our model, all individuals participate in the labour market during four periods of working age (18-29, 30-41, 42-53, 54-65). A fraction of lower ability individuals becomes unemployed. High ability individuals are always employed. Since we do not model participation as an endogenous variable, our setup implies that the unemployment rate among lower ability individuals in a particular age group is the same as the gap in percentage points between the employment rate among higher ability individuals and the employment rate among lower ability individuals in that age group. In line with our explanation in C.1, as a proxy for the former we use data for individuals with a tertiary degree. As a proxy for the latter, we compute the average of data for individuals with a below upper secondary degree and individuals with an upper secondary degree, but no tertiary degree.

Data sources: Eurostat (Employment rates by sex, age and highest level of education attained (%) [lfsa_ergaed]) provides employment rates in persons by level of educational attainment and by age for all EU15 countries and Norway since 1995 at the latest. Data are available for the age groups 20-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59 and 60-64, among others. We compute the data for our four larger age groups as weighted averages of the Eurostat data. The data that we report and employ are for 2001-2007. For the United States and Canada we use data provided by OECD Education at a Glance. Data by educational attainment are available for the age groups 25-34, 35-44, 45-54, 55-64.

2.C.3 Hours worked

For hours worked, we proceed as follows. First, we gather available OECD (Labour Force Statistics) data on the usual weekly number of hours that individuals work. These data are available for the age groups 15-24, 25-54 and 55-64. We compute the data for our four age groups as weighted averages of the OECD data. Second, we compute how many weeks individuals work per year. We divide OECD data on total annual hours worked per employee by the average number of hours worked per week. Data on total annual hours are only available at the level of the aggregate labour force. As such, we obtain a proxy for the number of weeks individuals work per year, also at the level of the aggregate labour force. Multiplying the first variable (usual hours per week) by the second (weeks per year), we obtain our proxy for total annual hours worked per employee in each generation. We express this number as a fraction of the total time endowment. Like Wallenius (2013), we assume that the total time endowment of each individual is 14 hours a day, 7 days a week and 52 weeks per year. This time can be allocated to work, leisure or (for young higher ability individuals) education.

2.C.4 Education rate of the young higher ability individuals

The education rate indicates the fraction of their total time endowment that high ability individuals allocate to schooling. Considering that in countries like Canada and Norway almost (or even more than) 50% of the 25-34 year-olds succeed in obtaining a tertiary degree, it will be our assumption that 50% of the population in each country has high ability, and therefore the potential to succeed at high level. The extent to which this potential is fully exploited may however differ across countries. Differences may show up in the fraction of individuals that effectively succeed in tertiary education and in the level of the tertiary degree that these individuals eventually achieve. We expect that the latter will be reflected in the number of years that is studied. Building on this assumption and these considerations, our empirical proxy for the education rate is the number of students in tertiary education in full-time equivalents divided by 50% of size of the population of age 18-29. Data on the number of students is obtained from Eurostat (Students by ISCED level, study intensity (full-time, part-time) and sex [educ_enrl1ad]). Population data are from the OECD database (Total Population by sex and age). The education rates that we report are averages for 2001-2007.

Table 2.6: Hours worked per person employed (fraction of time) - average for tertiary and non-tertiary educated individuals (2001-2007)

Hours worked	18-29	$30-41$	$42-53$	$54-65$
Belgium	0.309	0.311	0.311	0.311
France	0.292	0.301	0.301	0.291
Germany	0.285	0.283	0.283	0.277
Austria	0.339	0.349	0.349	0.351
Netherlands	0.252	0.319	0.319	0.299
Denmark	0.283	0.354	0.354	0.344
Finland	0.327	0.366	0.366	0.349
Norway	0.263	0.309	0.309	0.297
Sweden	0.300	0.341	0.341	0.332
United States	0.338	0.383	0.383	0.374
United Kingdom	0.325	0.359	0.359	0.332
Canada	0.331	0.371	0.371	0.358
Average	0.305	0.338	0.338	0.323

Table 2.7: Fraction of time allocated to education by young high ability individuals (2001-2007)

	Education
Belgium	0.450
France	0.450
Germany	0.386
Austria	0.434
Netherlands	0.442
Denmark	0.522
Finland	0.564
Norway	0.512
Sweden	0.444
United States	0.574
United Kingdom	0.366
Canada	0.474
Average	0.468

2.C.5 Growth rate of real per capita output

To compute the growth rate of real per capita output, which we need for the calibration of the exogenous rate of technical progress (x), we use data on real potential GDP and on population at working age (15-64). The former are available from OECD Economic Outlook (supply block), the latter from OECD Labour Force Statistics. In line with all other data we compute average growth rates over 2001-2007.

2.C.6 Fiscal policy variables

The government in our model finances spending on goods and unemployment benefits from taxes on labour (on both employers and employees), consumption, and capital. Lump sum transfers ensure a balanced budget. For the tax rates τ_k and τ_c, we use the same data as Heylen and Van de Kerckhove (2013). Regarding labour tax rates, we distinguish between social security contributions paid by employers and taxes and social security contributions on labour income paid by employees. Our data source is OECD (Taxation, Tax Database, Tables I.4, I.5, I.6). More specifically, we use the OECD's average rate of employer social security contributions for τ^p (Table I.5). We calibrate the level parameter Γ in the workers' income tax rate using the OECD data for all-in average personal income tax rates at average wage. The all-in tax rate is calculated as the combined central and sub-central government income tax plus employee social security contribution, as a percentage of gross wage earnings. The OECD provides these tax rates for four family types (Table I.6). We computed the average over these types. A novelty compared to previous work is the inclusion of progressive income taxation. Just like Koyuncu (2011), we calibrate the country-specific degree of progressivity ξ according to our description in Section 2.2.4. as the ratio (minus 1) of the marginal tax rate on workers' gross wage to the average tax rate, both including social security contributions. The OECD provides these marginal and average tax rates for a single person without dependent at four different income levels (Tables I.4 and I.5). Our proxy for ξ reflects the average of the results over these four income levels. All computed and reported tax data in our Table 8 are averages over the period 2000-2007. For government spending on goods in percent of GDP (g) we compute the sum of their data for government consumption and productive government spending. For details on the construction of these fiscal policy variables, we refer to Heylen & Van de Kerckhove (2013, their Appendix 1).

For the unemployment benefit replacement rate ($\widetilde{b}$) we make use of data provided by the OECD (Tax-Benefit Models). Since in our model unemployment is a structural or equilibrium phenomenon, the data that we use concern net transfers received by structurally or long-term unemployed people. They include social assistance, family benefits and

Table 2.8: Labour tax rates on employees and employers (2000-2007) and unemployment benefit replacement rates (2001-2004), in %

	ξ	Γ	τ^p	$\widetilde{b}_j$
Belgium	0.343	33.4	29.1	59.6
France	0.296	21.7	38.7	46.0
Germany	0.245	30.2	19.9	64.7
Austria	0.311	27.3	28.6	56.3
Netherlands	0.391	25.2	11.2	55.0
Denmark	0.233	35.3	0	61.9
Finland	0.424	28.9	24.1	61.3
Norway	0.392	25.1	12.7	56.9
Sweden	0.376	28.9	32.2	55.4
United States	0.330	17.1	7.7	30.5
United Kingdom	0.383	24.0	9.8	51.1
Canada	0.331	17.9	11.1	44.4
Average	0.337	26.3	18.8	53.4

Table 2.9: Tax rates on consumption and capital (in %), and government spending on goods (in % of GDP), (1995-2001)

	τ_c	τ_k	g
Belgium	13.4	27.1	24.8
France	17.1	21.7	28.5
Germany	11.1	34.4	23.2
Austria	13.2	17.3	23.4
Netherlands	12.2	24.3	27.3
Denmark	18.9	22.5	29.8
Finland	15.2	17.2	26.8
Norway	16.4	22.1	26.3
Sweden	17.9	16.1	32.6
United States	7.2	23.6	19.5
United Kingdom	14.5	21.2	21.4
Canada	14.5	24.8	23.6
Average	14.3	22.1	25.7

housing benefits in the 60th month of benefit receipt. They also include unemployment insurance or unemployment assistance benefits if these benefits are still paid, i.e. if workers can be structurally unemployed for more than five years without losing benefit eligibility. The OECD provides net replacement rates for six family situations and three earnings levels. Our data in Table 8 are the averages of these 18 cases. Data are for 2001-2004.

Appendix 2.D Derivation of the first-order condition of the union

The union maximises Equation (2.9) with respect to $w_{L,t}$. In this derivation, we use the structure of average and marginal tax rates as discussed in Section 2.2.4. So,

$$\frac{\partial V_t}{\partial w_{L,t}} = 0 \Longleftrightarrow \sum_j [\frac{1}{\chi_j} \left(w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t}\right)^{\chi_j} \frac{\partial(1-u_{j,t})}{\partial w_{L,t}} + \\ (1-u_{j,t}) \left(w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t}\right)^{\chi_j - 1} \left[(1-\tau_{jL}) + w_{L,t}\left(-\frac{\partial \tau_{j,L}}{\partial w_{L,t}}\right)\right]] = 0$$

$$\Longleftrightarrow \sum_j [\frac{1}{\chi_j} \left(w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t}\right)^{\chi_j} \frac{\partial(1-u_{j,t})}{\partial w_{L,t}} + \\ (1-u_{j,t}) \left(w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t}\right)^{\chi_j - 1} * \\ \left[(1-\tau_{jL}) + w_{L,t}\left(-\left[\xi\Gamma\left(\frac{y^{lab}_{jL,t}}{\bar{y}^{lab}_t}\right)^{\xi-1}\left(\frac{g(n^{t-j+1}_{jL})h^{t-j+1}_{jL}\varepsilon_j(1-u_{j,t})}{\bar{y}^{lab}_t}\right)\right]\right)\right]] = 0$$

$$\Longleftrightarrow \sum_j [\frac{1}{\chi_j} \left(w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t}\right)^{\chi_j} \frac{\partial(1-u_{j,t})}{\partial w_{L,t}} + \\ (1-u_{j,t}) \left(w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t}\right)^{\chi_j - 1} \left[1 - (\tau_{jL} + \xi\Gamma\left(\frac{y^{lab}_{jL,t}}{\bar{y}^{lab}_t}\right)^{\xi})\right]] = 0$$

$$\Longleftrightarrow \sum_j [\frac{1}{\chi_j} \left(w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t}\right)^{\chi_j} \frac{\partial(1-u_{j,t})}{\partial w_{L,t}} + \\ (1-u_{j,t}) \left(w_{L,t}(1-\tau_{jL}) - \bar{w}_{j,t}\right)^{\chi_j - 1} \left[1 - \tau^m_{jL}\right]] = 0$$

We impose an extra assumption, namely that the union treats every generation equally when it comes to generating union utility. Due to this assumption, each part of this sum is put equal to zero. The result is the first-order condition that appears in the text (Equation 2.20), which applies for each generation. For each generation, the excess wage gap is now related to the corresponding response of the household within that generation and the firm.

Chapter 3

Fiscal Policy, Union Preferences, and the Labour Market Position of Low-skilled Individuals[1]

Abstract

I explore the potential of fiscal and union policies to improve the labour market position of the low-skilled in an OLG model with heterogeneous abilities. The key innovation is that the probabilities in the Markov transition matrix governing the stochastic employment process for the low-skilled are endogenous. The results indicate that alternative fiscal policy and union instruments may have a very different impact on respectively hours worked, unemployment, labour market participation, and workers' welfare. Most preferable to improve the labour market position of the low-skilled is a decrease in labour taxes on employees combined with wage moderation accepted by the unions. Finally, unlike in perfectly competitive labour markets, labour taxes levied on respectively employers and employees are no longer equivalent.

Keywords: Low-skilled unemployment, Endogenous Markov matrix, Heterogeneous agents, Union wage setting, Fiscal policy
JEL classification codes: E24, E62, J51, J64

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3.1 Introduction

This paper explores the potential of fiscal and union policies to improve the labour market performance of low-skilled individuals[2]. While there is considerable evidence on the impact of public policy on aggregate employment[3], there is much less evidence on the impact thereof on employment across different skill groups, notably among the low-skilled individuals. Figure 3.1 indicates, however, that focusing on the latter is of particular interest[4]. This figure shows first of all that the labour market position of low-skilled individuals is highly indicative of the aggregate employment rate within a country. Countries with lower aggregate employment rates are typically countries with higher unemployment rates (panel a) and relatively lower employment rates (panel b) among the low-skilled. As a second important result, Figure 3.1. (panel c) reveals a much larger cross-country variance in the employment rate among the low- than among the high-skilled[5].

These observations indicate that a specific focus on improving the labour market performance of low-skilled individuals should be an important policy objective. Furthermore, they raise the question whether restricting the focus on aggregate employment sufficiently captures the potential impact on the low-skilled, especially given the distinct and precarious labour market status of the latter. They also highlight the importance of modelling the labour markets of high- and low-skilled individuals separately, with a specific focus on the latter. Ultimately, to address all these concerns one needs a theoretical model (i) with labour market frictions on the low-skilled labour market, (ii) that is capable of capturing the cross-country variation in low-skilled employment, and (iii) that can be applied to simulate a wide range of possible policy reforms to explore their impact on the different components that determine the labour market performance of the low-skilled. These concerns and findings lie at the core of this paper.

The aim of this paper is twofold. The first objective is to develop a large-scale computable OLG model that contains all the different components of aggregate employment. The key innovation compared to most of the literature is that low-skilled individuals face idiosyncratic unemployment risk. As the source of unemployment, I introduce a union-

[2]In this paper, the term low-skilled is given to individuals who have not enjoyed tertiary education, whereas the term high-skilled refers to individuals who have enjoyed tertiary education.

[3]Examples using a deterministic life-cycle model include Prescott et al. (2009); Erosa et al. (2012); Wallenius (2013); Heylen and Van de Kerckhove (2013), and Alonso-Ortiz (2014) while Heathcote (2005); Nishiyama and Smetters (2005, 2007), and Conesa et al. (2009) use heterogeneous agent life-cycle models with idiosyncratic productivity risk.

[4]The focus of this paper is on 2001-2007 as this was the last period of relative stability on the labour market before the financial crisis and the euro crisis. Considering that the analysis in this paper aims at studying equilibrium unemployment, it is clearly more appropriate to use data for a relatively stable period.

[5]For this sample of countries, the variance of the employment rate among the high-skilled 25- to 64-year-olds is 4.75, whereas it is 20.35 among the high-skilled.

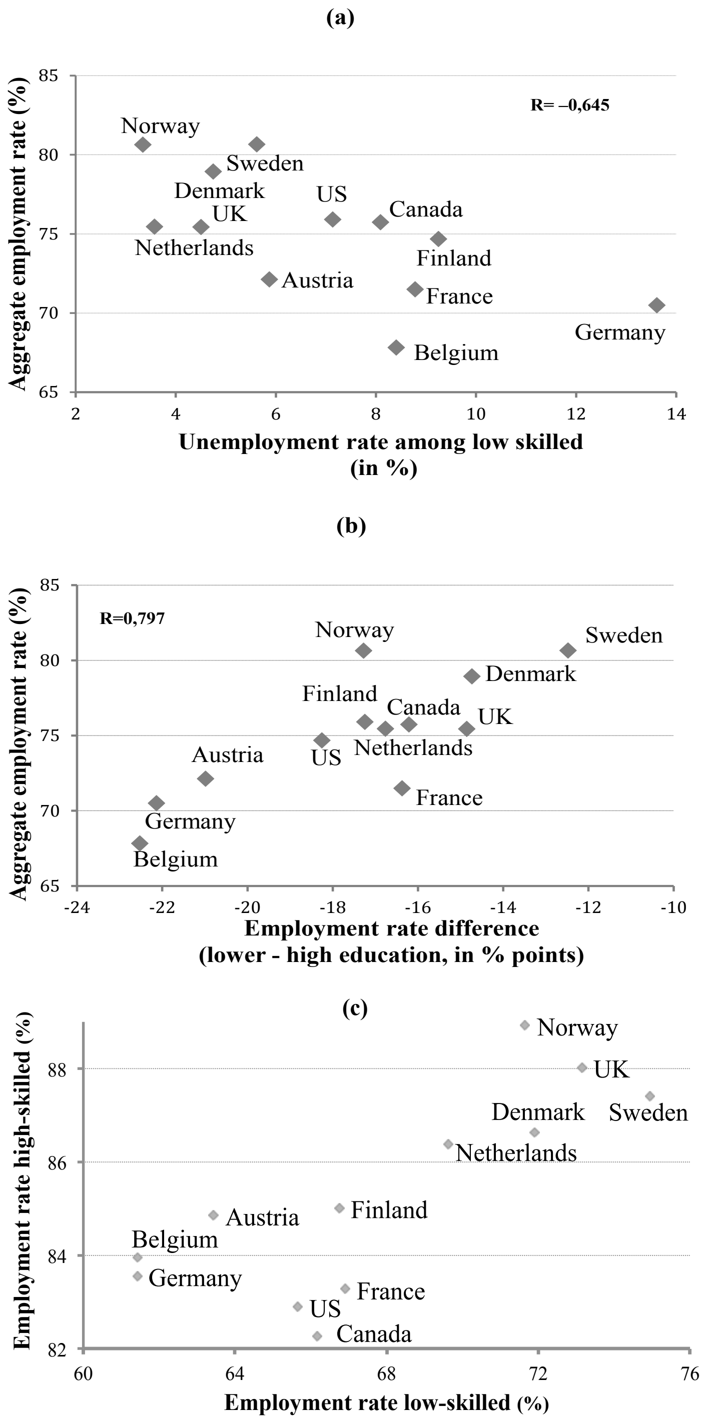

Figure 3.1: Employment and unemployment in OECD countries, 2001-2007.

Note: The (un)employment rate among lower skilled individuals is computed as the average of the (un)employment rates among individuals with less than upper secondary education and among individuals with upper secondary, but no tertiary degree. The (un)employment rate among individuals with higher education relates to those with a tertiary degree. Unless defined differently, all reported employment and unemployment rates concern the age group 25-64. The employment rate indicates the fraction of individuals who have a job. Data sources: Eurostat (LFS series: lfsa_ergaed, lfsa_urgaed) and OECD Labour Force Statistics (Total Employment).

ised labour market for the low-skilled with a rich specification of union preferences. Due to this set-up, the probabilities in the age-specific Markov matrix governing the stochastic employment process are endogenously determined. By attending college or not, households choose the employment risk they will face. The model contains two additional ingredients that will determine the choices that different individuals make when it comes to attending college or not. These ingredients are heterogeneity in individuals' innate ability and a college wage premium. Furthermore, both skilled and unskilled individuals face idiosyncratic productivity risk. Last but not least, given that early retirement determines the employment rate for the elderly to a large extent, the participation decision of 54- to 65-year-old individuals will also be endogenous in the model.

The second objective is to simulate a number of different public policies that are expected to improve the labour market outcome for low-skilled individuals. Due to the rich set-up of the model, I can study the effects of both fiscal and union policies on aggregate and skill-specific employment in great detail. For example, a change in fiscal policy might have several effects: (i) the union might adopt a different wage rate, leading to a possible change in employment rates and different Markov matrices, (ii) 54- to 65-year-olds might change their decision to participate or not on the labour market, (iii) the number of 18-year-olds choosing to attend college might change, leading to fewer low-skilled individuals in the economy, and (iv) employed individuals might choose to supply a different number of hours. Furthermore, the model allows to investigate the impact on the welfare of both employed and unemployed low-skilled individuals. The life-cycle structure, finally, allows the government to target specific groups of individuals: e.g. low-skilled individuals, 54- to 65-year-olds and 54- to 65-year-old low-skilled individuals. Most of the aforementioned papers are not able to identify all these effects.

The simulations reveal that modelling the different components of aggregate employment is important to explore the effectiveness of fiscal and union policies. While most fiscal policy and union instruments have a highly similar impact on aggregate output, capital, and labour, these instruments all have a different impact on unemployment, labour force participation, and hours worked. The simulations thus highlight that the choice for a given policy instrument is dependent on the target variable. Decreasing only labour taxes, for example, has no use when the fiscal government aims to decrease low-skilled unemployment. Decreasing the net unemployment benefit replacement rate and less aggressive wage policies by the union, on the other hand, have a strong impact on low-skilled unemployment. But they have no use if the target variable is the amount of hours worked per employed individual.

That being said, several policies lead to desirable results in terms of unemployment. Overall, however, the simulations show that fiscal governments and unions have a mutual interest in joining forces to improve the labour market performance of the low-skilled. For example, a particularly interesting policy is a decrease in labour taxes levied on low-skilled employees combined with wage moderation accepted by the unions. Not only does this lead to a decrease in the unemployment rate of the low-skilled, it also has a positive impact on the number of hours worked per employed individual and it is very likely to increase welfare of employed low-skilled individuals. Furthermore, it does not harm unemployed individuals directly.

The last conclusion one can draw from the simulations is that, unlike in perfectly competitive labour markets, the effects of labour taxes levied on respectively employers and employees are not equivalent within the model. Even more, the behavioural effects of a decrease in the labour tax rate paid by employers are very small. Within the current set-up, the fiscal government can thus achieve more by lowering the tax rates paid by employees.

This paper is organised as follows. Section 3.2 describes the model equations and the recursive formulation of the household, while Section 3.3 describes the definition of the equilibrium. The data and the calibration are outlined in Section 3.4. Section 3.5 reports the simulation results. Section 3.6, finally, concludes.

3.2 The model

The model is an OLG model for a closed economy. Time is discrete and runs from 0 to ∞. The economy is populated by heterogeneous individuals, firms, unions, and a fiscal government. Individuals enter the model at the age of 18 and live at most J periods. Both the goods market and the labour market for high-skilled individuals are competitive, whereas the labour market for low-skilled individuals is unionised. In every period t, the pre-tax real wage for low-skilled workers is set by a monopoly union operating at the firm level. The government in the model disposes of a rich set of fiscal policy instruments.

3.2.1 Demographics

At each moment in time, the economy is populated by J overlapping generations. At the beginning of each period, a continuum of new agents with measure 1 enters the model. Individuals have an uncertain lifespan, since all individuals face an age-specific survival probability φ_j between the age of j and $j + 1$. The demographics of the model are exogenous and given by:

$$N_{j+1,t+1} = \varphi_j N_{j,t} \tag{3.1}$$

where $N_{j,t}$ represents the number of individuals of age j at time t. Every individual who survives J periods will die with certainty after the J-th period. For the purpose of clarification, I immediately introduce a few additional notations which are frequently used below: $N_{js,t}$, $P_{j,t}$, $P_{js,t}$, $L_{j,t}$ and $L_{js,t}$. These variables denote respectively the number of individuals of age j and skill s at time t, the number of individuals of age j who participate on the labour market at period t, the number of individuals of age j and skill s who participate at time t, the number of individuals of age j employed at period t and the number of individuals of age j and skill s employed at period t.

3.2.2 Individuals

Individuals enter the model without wealth and have no intentions to leave bequests. Any remaining resources at the time of death are collected by the government and are equally distributed among the living population in the next period. Before any decision is made, individuals draw their innate ability $\pi \in \{\pi_1, \pi_2, ..., \pi_N\}$. This value remains unchanged for the remainder of their life. Each agent decides on whether to attend college or not. Going to college requires an ability-specific fraction of time $0 < \Pi(\pi) < 1$, which individuals spend during their first period of life (Krueger and Ludwig, 2013). More able people require less time to learn, meaning that the time cost decreases when the innate ability is higher. Individuals who choose to attend and complete college are referred to as high-skilled, individuals who have not enjoyed tertiary education are denoted as low-skilled. In line with the literature, both high- and low-skilled individuals face labour productivity shocks during their active life[6].

[6] Several authors argue that the introduction of idiosyncratic productivity risk has a few advantages - see e.g. Heathcote et al. (2009) and Fehr (2009). For example, they add an additional precautionary motive for savings. This in turn will influence the levels of the aggregate equilibrium quantities and prices. These authors also argue that the welfare implications can differ substantially over heterogeneous households. As such, they can be much larger than the cost for a representative agent. Heckman (2001) argues that in order to properly evaluate large-scale government programs, researchers need models that take into account both general equilibrium effects and the impact of these policies on heterogeneous agents. Nishiyama

Going to college has two different benefits. First, wages for high-skilled individuals are higher. Second, low-skilled individuals face extra idiosyncratic uncertainty as only a fraction l_t of the participating low-skilled individuals is employed. The probability that a participating low-skilled agent of any age is employed ($l = e$) or is unemployed ($l = u$) is endogenously determined within the model due to the monopoly union framework (cf. infra).

An individual will decide to attend college if his/her maximised expected lifetime utility of being high-skilled ($s = \bar{s}$) is bigger than the maximised expected lifetime utility of staying low-skilled ($s = \underline{s}$) :

$$I_\pi = \begin{cases} 1 \text{ if } V(0, \pi, s = \bar{s}) \geq V(0, \pi, s = \underline{s}) \\ 0 \text{ otherwise} \end{cases} \tag{3.2}$$

where $V(0, \pi, s)$ gives the maximised expected lifetime utility at age $j = 0$ (i.e. before individuals enter the model and start making decisions), given the ability level π and where the expectations are taken over the stochastic processes governing the labor productivity and employment shocks.

All agents are endowed with one unit of time, which they devote to labour supply, leisure, and education if they decide to attend college. Individuals reaching the age of J_R retire. However, from the age of J_{R-3} onwards, they do have the opportunity to withdraw from the labour market and enjoy early retirement[7].

At any given point in time, individuals are characterised by a state vector (a, l, π, η, s), where a denotes the accumulated non-human wealth at the beginning of the period, l is the employment status, π is the ability level, s is the skill level, and η is the productivity shock. Let $\Phi_{j,t}(a, l, \pi, \eta, s)$ denote the share of agents aged j of type (a, l, π, η, s) at date t. For each t and j we have $\int \Phi_{t,j}(da \times dl \times d\pi \times d\eta \times ds) = 1$.

The expected lifetime utility of an individual entering the model at period t and age $j = 1$ is given by:

$$U^t = E\left\{\sum_{j=1}^{J} \beta^{j-1}\left[\frac{\left(c_j{}^{(1-\mu)}(1 - I_\pi \Pi(\pi) - n_j - (1 - I_P)\rho n_j^*)^\mu\right)^{1-\theta}}{1-\theta} - I_P \Theta_s\right]\right\} \tag{3.3}$$

with $1 \geq \mu \geq 0$, $\theta > 0$, $1 \geq \rho \geq 0$ and where the expectations are taken over the stochastic processes governing the mortality risk, and the labor productivity and employ-

and Smetters (2007) simulate the privatisation of social security in a setup respectively with and without idiosyncratic wage shocks. They find that the effects on both aggregate equilibrium quantities and prices on the one hand and welfare on the other hand are indeed different in both setups.

[7] In the empirical part, the length of one period is four years. The generation of age J_{R-3} represents the age group 54-57. So from the age of 54 onwards, individuals have the opportunity to withdraw from the labour market. Furthermore, J_R coincides with the age group 66-69. Thus, the official retirement age is 66.

ment risk. Note that the individuals endogenously choose the fraction of time endowment n_j they allocate to labour services. Agents derive utility from consumption (c_j) and leisure $\left(1 - I_\pi \Pi(\pi) - n_j - (1 - I_P)\rho n_j^*\right)$. Two indicator functions are present: I_π takes the value 1 in the first period of active life when an individual decides to attend college and I_P takes the value 1 when the individual has reached age J_{R-3} (and is entitled to early retirement) but continues to participate on the labour market. Θ_s is a fixed utility cost of participating[8]. Furthermore, it is my assumption that the early retired individuals do not fully enjoy their leisure time yet. The leisure they enjoy as an early retiree is equal to $1 - \rho n_j^*$ where n_j^* is the fraction of time an individual would work if he or she still participated, while the actual fraction of time he or she devotes to labour market services n_j is equal to 0[9].

An employed individual with state (j, a, e, π, η, s) faces the following recursive problem:

$$V(j, a, e, \pi, \eta, s) = \max_{n \in [0, 1 - I_\pi \Pi(\pi)], c, a' \geq 0}$$
$$\left\{ u(c, 1 - I_\pi \Pi(\pi) - n) + \beta \varphi_j \sum_{\eta'} \sum_{l'} \Delta(\eta'|\eta) \Delta_s(l'|l) V(j+1, a', l', \pi, \eta', s) \right\} \quad (3.4)$$

subject to the budget constraint

$$(1 + \tau_c)c_j + a'_j = w_{s,t+j-1} \varepsilon_j \eta \varpi_s(\pi) n_j (1 - \tau_{js}) + (1 + r_{t+j-1})(a_j + Tr_{t+j-1}). \quad (3.5)$$

Individuals aged j who participate on the labour market earn an after-tax wage of $w_{s,t+j-1}$ $\varepsilon_j \eta \varpi_s(\pi) n_j (1 - \tau_{js})$, where $w_{s,t+j-1}$ is the real wage per unit of effective labour for an individual with skill s at time $t + j - 1$, ε_j is an age-specific productivity parameter, η is the labour productivity shock, $\varpi_s(\pi)$ denotes the component making wages depend on ability, and τ_{js} is the average tax rate on labour, which is specified more completely below. The interest rate is given by r_{t+j-1}. The consumption tax rate is τ_c. The accumulated

[8] This assumption is based on Kitao and Imrohoroglu (2012) and Kitao (2014). These authors include a fixed utility cost of participating as well. In the latter paper, the author includes a fixed cost of participation for each generation, where the cost itself is a function of the age of the individuals. The calibrated utility cost only slightly increases between the ages of 20 and 50. Between the ages of 50 and 85, the cost of participating increases strongly. Based on this profile, I choose to include a utility cost only for the age group 54-65. I do not, however, assume that the cost of participation changes as the individual ages.

[9] A study conducted by Age Wave and Merrill Lynch (2016) found that retirees may have difficulties to adapt to their new life, especially early retirees. They do not yet organise their leisure activities efficiently. Age Wave and Merrill Lynch also conclude that it matters more who retirees engage with in their leisure activities than the actual leisure activity itself. Furthermore, Chang et al. (2014) conclude that relationships and the social network are an important determinant of the leisure enjoyed as a retiree. As an early retiree, however, friends and relatives might still be in the labour force.

non-human wealth at the end of the period is denoted by a'_j. Furthermore, I impose that individuals are not able to borrow: $a'_j \geq 0$. Due to accidental bequests, individuals receive a transfer Tr_{t+j-1} from the government. Finally, $\Delta(\eta'|\eta)$ and $\Delta_s(l'|l)$ denote respectively the productivity process and the skill-specific employment process. Note that for high-skilled individuals, the probability that they will be employed in the next period is always equal to 1 if they participate.

A low-skilled unemployed individual with current state $(j, a, u, \pi, \eta, \underline{s})$ faces the following recursive problem:

$$V(j, a, u, \pi, \eta, \underline{s}) = \max_{c, a' \geq 0} \left\{ u(c) + \beta \varphi_j \sum_{\eta'} \sum_{l'} \Delta(\eta'|\eta) \Delta_{\underline{s}}(l'|l) V(j+1, a', l', \pi, \eta', \underline{s}) \right\} \tag{3.6}$$

subject to the budget constraint

$$(1+\tau_c)c_j + a'_j = B_{t+j-1} + (1+r_{t+j-1})(a_j + Tr_{t+j-1}). \tag{3.7}$$

The only difference between Equation (3.5) and (3.7) is the presence of an unemployment benefit instead of labour income. I assume that the unemployment benefit is given by $B_{t+j-1} = bw_{\underline{s},t+j-1}\varepsilon_j \eta \varpi_{\underline{s}}(\pi) n_j (1-\tau_{j\underline{s}})$, i.e. a fraction b of the income of an individual with the same state, except for the employment status.

From the age of J_{R-3} onwards, individuals have the opportunity to withdraw from the labour market. If they do so, they receive an early retirement benefit $ep_{j,t+j-1}$ which is a fraction b_p of the income they would have earned if they had participated on the labour market. Formally, the budget constraint is then

$$(1+\tau_c)c_j + a'_j = (1+r_{t+j-1})(a_j + Tr_{t+j-1}) + ep_{j,t+j-1} \tag{3.8}$$

At the age of J_R, individuals have to retire and receive a public pay-as-you-go pension. The budget constraint for the ages $j \geq J_R$ is given by

$$(1+\tau_c)c_j + a'_j = (1+r_{t+j-1})(a_j + Tr_{t+j-1}) + pp_{s,t+j-1} \tag{3.9}$$

For simplicity, it is assumed that all individuals of the same skill group receive the same pension. This pension is a fraction b_p of the skill-specific average wage in the economy. Algebraically,

$$pp_{s,t} = b_p \left(\frac{\sum_{j=1}^{J_{R-1}} L_{js,t} \int w_{s,t} n_j(a,e,\pi,\eta,s)\eta\varepsilon_j \varpi_s(\pi)(1-\tau_{js})\Phi_{j,t}(da \times d\pi \times d\eta)}{\sum_{j=1}^{J_{R-1}} L_{js,t}} \right) \tag{3.10}$$

3.2.3 Firms

Both the goods market and the labour market for high-skilled individuals are perfectly competitive, whereas the labour market for low-skilled individuals is unionised. Firms maximise profits and pay social security contributions when hiring labour. The production function of the representative firm is given by

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha},\ \alpha \in [0,1] \tag{3.11}$$

The level of technology is represented by A and is assumed to stay constant over time. K_t is the amount of capital used by the firm. Inspired by Katz and Murphy (1992) and Borjas (2003), it is my assumption that high- and low-skilled individuals are imperfectly substitutable in production:

$$H_t = \left[\nu_1 H_{\bar{s},t}^{\frac{\iota-1}{\iota}} + (1-\nu_1) H_{\underline{s},t}^{\frac{\iota-1}{\iota}} \right]^{\frac{\iota}{\iota-1}} \tag{3.12}$$

where ν_1 is a share parameter and ι the elasticity of substitution between high-skilled labour $H_{\bar{s},t}$ and low-skilled labour $H_{\underline{s},t}$. Furthermore,

$$H_{\bar{s},t} = \sum_{j=1}^{J_R-1} L_{j\bar{s},t} \int n_j(a,e,\pi,\eta,\bar{s}) \varpi_{\bar{s}}(\pi)\eta\varepsilon_j \Phi_{t,j}(da \times d\pi \times d\eta) \tag{3.13}$$

$$H_{\underline{s},t} = \sum_{j=1}^{J_R-1} P_{j\underline{s},t}(1-u_t) \int n_j(a,e,\pi,\eta,\underline{s}) \varpi_{\underline{s}}(\pi)\eta\varepsilon_j \Phi_{t,j}(da \times d\pi \times d\eta) \tag{3.14}$$

Note that for the low-skilled a fraction $1 - u_t = l_t$ of the number of participating individuals of each generation $P_{j\underline{s},t}$ is employed at period t.

3.2.4 Unions

The economy is populated by decentralised trade unions, operating at the firm level. Every firm negotiates with a single union, representing all the low-skilled individuals employed at the firm. As such, unions are large compared to the workers, while taking aggregate variables and fiscal policy parameters as given. Just like in e.g. Pencavel (1984), de la Croix et al. (1996), Corneo and Marquardt (2000) and Ono (2007), the objective function of the unions follows the Stone-Geary specification:

$$W_t = \left[\chi(w_{\underline{s},t}(1-\bar{\tau}_{\underline{s}}) - \bar{w}_{\underline{s},t})^{1-\frac{1}{\kappa}} + (1-\chi)l_t^{1-\frac{1}{\kappa}}\right]^{\frac{\kappa}{\kappa-1}} \tag{3.15}$$

The union takes both wages and employment into account, albeit to a different degree. As Equation (3.15) indicates, the firm-specific trade union only derives utility from the difference between the negotiated net wage $w_{\underline{s},t}(1-\bar{\tau}_{\underline{s}})$ and a reservation wage, $\bar{w}_{\underline{s},t}$. The elasticity of substitution $\kappa > 0$ between the wage gap and the employment rate takes different values than 1. The share parameter χ measures the relative utility value of a higher wage versus higher employment for the union. I assume that the union derives utility from the fraction of participating low-skilled individuals employed at the firm. Every union has the same reservation wage, i.e. a weighted average or combination of the hypothetical net wage that would prevail if the labour market for the low-skilled were perfectly competitive, the net wage of the high-skilled and the net unemployment benefit, respectively with weights ϱ_1, ϱ_2 and ϱ_3. These sum up to 1. The different tax rates $\bar{\tau}^c_{\underline{s}}$, $\bar{\tau}_{\underline{s}}$ and $\bar{\tau}_{\bar{s}}$ are the average of all average tax rates faced by respectively the low- and the high-skilled.

The specification for the reservation wage is the following:

$$\bar{w}_{\underline{s},t} = \varrho_1 w^c_{\underline{s},t}(1-\bar{\tau}^c_{\underline{s}}) + \varrho_2 w_{\bar{s},t}(1-\bar{\tau}_{\bar{s}}) + \varrho_3 b w_{\underline{s},t}(1-\bar{\tau}_{\underline{s}})$$

At the beginning of each period t, in the first stage the monopoly union decides on the pre-tax wage, knowing that in the next stage the firm and the members decide on the extensive and intensive margin of employment. Both individuals and the firms take the wage set by the monopoly union as given. Moreover, as they move simultaneously, they take the action of the other player as given. Once the wage has been determined, the firm decides on the extensive margin (fraction of available low-skilled individuals to employ) and the individuals who participate on the intensive margin (hours worked per employed).

3.2.5 Government

Government expenditures on unemployment benefits, government consumption, early retirement pensions and public PAYG pensions are financed by taxes on labour (both on employers and employees) and consumption. The fraction of output that is devoted to government consumption is given by g_c. Formally, the government budget constraint is given by

$$G_{c,t} + B_t + EP_t + PP_t = T_{n,t} + T_{c,t} \tag{3.16}$$

$$\text{with: } \begin{cases} G_{c,t} = g_c Y_t \\ PP_t = \sum_{j=j_R}^{J} N_{j,t} \int pp_{s,t} \Phi_{j,t}(da \times d\pi \times ds) \\ EP_t = \sum_{j=j_{R-3}}^{J_{R-1}} (N_{j,t} - P_{j,t}) \int ep_{j,t}(a,e,\pi,\eta,s) \Phi_{j,t}(da \times d\pi \times d\eta \times ds) \\ B_t = B_{\underline{s},t} \\ T_{n,t} = T_{n\bar{s},t} + T_{n\underline{s},t} \\ T_{c,t} = \tau_c \sum_{j=1}^{J} N_{j,t} \int c_j(a,l,\pi,\eta,s) \Phi_{j,t}(da \times dl \times d\pi \times d\eta \times ds) \end{cases}$$

And

$$B_{\underline{s},t} = \sum_{j=1}^{J_R-1} P_{j\underline{s},t} u_t \int bw_{\underline{s},t} \eta \varepsilon_j \varpi_{\underline{s}}(\pi) n_j(a,e,\pi,\eta,\underline{s})(1-\tau_{j\underline{s}}) \Phi_{j,t}(da \times d\pi \times d\eta) \tag{3.17}$$

$$T_{n\bar{s},t} = \sum_{j=1}^{J_R-1} L_{j\bar{s},t} \int w_{\bar{s},t} n_j(a,e,\pi,\eta,\bar{s}) \eta \varepsilon_j \varpi_{\bar{s}}(\pi)(\tau_{j\bar{s}} + \tau^p) \Phi_{j,t}(da \times d\pi \times d\eta) \tag{3.18}$$

$$T_{n\underline{s},t} = \sum_{j=1}^{J_R-1} L_{j\underline{s},t} \int w_{\underline{s},t} n_j(a,e,\pi,\eta,\underline{s}) \eta \varepsilon_j \varpi_{\underline{s}}(\pi)(\tau_{j\underline{s}} + \tau^p) \Phi_{j,t}(da \times d\pi \times d\eta) \tag{3.19}$$

What remains is the specification of the progressive income taxes. The tax rates appearing in the budget constraints of the households are average tax rates and are given by

$$\tau_{js} = \Gamma \left(\frac{y_{j,t}^{lab}(a,l,\pi,\eta,s)}{\bar{y}_t} \right)^{\xi} \tag{3.20}$$

where $y_{j,t}^{lab}$ is the total pre-tax labour income of an individual of age j and state (a,l,π,η,s) at time t and $\bar{y}$ is the average labour income in the economy. Just like in Guo and Lansing (1998) and Koyuncu (2011), the parameters ξ and Γ govern the level and slope of the tax schedule. The average tax rate τ_{js} increases with the total taxable labour income of the household when $\xi > 0$. Households are aware of the progressive structure of the tax system when making decisions. The marginal tax rate τ_{js}^m is then simply the rate applied to the last euro earned:

$$\tau_{js}^m = (1+\xi)\Gamma \left(\frac{y_{js,t}^{lab}(a,l,\pi,\eta,s)}{\bar{y}_t} \right)^{\xi} \tag{3.21}$$

Rewriting this yields

$$\frac{\tau^m_{js}}{\tau_{js}} = 1 + \xi \tag{3.22}$$

This means that the marginal tax rate is higher than the average tax rate when $\xi > 0$, i.e. the tax schedule is said to be progressive. When $\xi = 0$, the average and marginal tax rates coincide.

3.3 Definition of Equilibrium

Let $\Phi_{j,t}(a, l, \pi, \eta, s)$ denote the share of agents aged j at time t with state (a, l, π, η, s). For each j and t we have $\int \Phi_{j,t}(da \times dl \times d\pi \times d\eta \times ds) = 1$.

Definition 1 Given an initial capital stock K_0, initial measures $\{\Phi_{j,0}\}_{j=1}^{J}$, and a given vector of exogenous fiscal policy variables $\{\tau_{c,t}, \tau^p_t, b_t, b_{p,t}, \xi_t, \Gamma_t\}_{t=0}^{\infty}$, an intertemporal equilibrium consists of sequences of household value and policy functions $\{V_{j,t}, a'_{j,t}, c_{j,t}, n_{j,t}, I_\pi, I_p\}_{t=0}^{\infty}$, sequences of transfers $\{Tr_t\}_{t=0}^{\infty}$, sequences of prices $\{w_{\bar{s},t}, w_{\underline{s},t}, r_t\}_{t=0}^{\infty}$, sequences of taxes, social security policies and government spending $\{G_c, \tau_{js,t}, \tau^m_{js,t}, pp_{s,t}, ep_{j,t}\}_{t=0}^{\infty}$, unemployment rates $\{u_t\}_{t=0}^{\infty}$ and aggregate variables $\{Y_t, K_t, H_t, H_{\bar{s},t}, H_{\underline{s},t}\}_{t=0}^{\infty}$ and sequences of measures $\{\Phi_{j,t}\}_{t=0}^{\infty}$ such that:

1. Given prices and government policies, $V_{j,t}$ satisfy the Bellman equations as given in subsection 3.2.2 for all ages j and all states (a, l, π, η, s), where $\{a'_{j,t}, c_{j,t}, n_{j,t}, I_\pi, I_p\}$ are the related policy functions.

2. Interest rates and wages for high-skilled individuals are given by:

$$r_t = \alpha A \left(\frac{H_t}{K_t}\right)^{(1-\alpha)} - \delta \tag{3.23}$$

   and

$$w_{\bar{s},t}(1 + \tau^p) = (1 - \alpha) A \left(\frac{K_t}{H_t}\right)^{\alpha} \nu_1 \left(\frac{H_t}{H_{\bar{s},t}}\right)^{\left(\frac{1}{\iota}\right)} \tag{3.24}$$

3. The pension pp_t is determined by:

$$pp_{s,t} = b_p \left(\frac{\sum_{j=1}^{J_{R-1}} L_{js,t} \int w_{s,t} n_j(a, e, \pi, \eta, s) \eta \varepsilon_j \varpi_s(\pi)(1 - \tau_{js}) \Phi_{j,t}(da \times d\pi \times d\eta)}{\sum_{j=1}^{J_{R-1}} L_{js,t}} \right) \tag{3.25}$$

for $s \in \{\bar{s}, \underline{s}\}$ and where $b_{p,t}$ is the net replacement rate for the individuals. The early retirement benefit is formally given by $ep_{j,t} = b_p w_{s,t} n_j^* \eta \varepsilon_j \varpi_s(\pi)(1 - \tau_{js})$.

4. The transfer Tr_{t+1} is determined by:

$$Tr_{t+1} = \left(\frac{\sum_{j=1}^{J} (1-\varphi_j) N_{j,t} \int a'_j(a,l,\pi,\eta,s) \Phi_{j,t}(da \times dl \times d\pi \times d\eta \times ds)}{\sum_{j=1}^{J} N_{j,t+1}} \right) \tag{3.26}$$

5. Given the wage $w_{\underline{s},t}$ chosen by the union, the optimal response of the low-skilled individuals $n_j(a,e,\pi,\eta,\underline{s})$ and the optimal response of the firm $u_{j,t}$ form a Nash equilibrium in the second stage of the dynamic game played between the individuals, the firm and the union. These actions form a Subgame Perfect Equilibrium.

6. The capital market, the labour market for high-skilled individuals and the goods market clear every period t:

$$K_{t+1} = \sum_{j=1}^{J} N_{j,t} \int a'_j(a,l,\pi,\eta,s) \Phi_{j,t}(da \times dl \times d\pi \times d\eta \times ds) \tag{3.27}$$

$$H_{\bar{s},t} = \sum_{j=1}^{J_{R-1}} L_{j\bar{s},t} \int n_j(a,e,\pi,\eta,\bar{s}) \varepsilon_j \eta \varpi_{\bar{s}}(\pi) \Phi_{j,t}(da \times d\pi \times d\eta) \tag{3.28}$$

$$Y_t = \sum_{j=1}^{J} N_{j,t} \int c_j(a,l,\pi,\eta,s) \Phi_{j,t}(da \times dl \times d\pi \times d\eta \times ds) + G_{c,t} + (K_{t+1} - (1-\delta)K_t) \tag{3.29}$$

7. $H_{\underline{s},t}$ is given by:

$$H_{\underline{s},t} = \sum_{j=1}^{J_{R-1}} P_{j\underline{s},t}(1-u_t) \int n_j(a,e,\pi,\eta,\underline{s}) \varepsilon_j \eta \varpi_{\underline{s}}(\pi) \Phi_{j,t}(da \times d\pi \times d\eta) \tag{3.30}$$

8. Government consumption as fraction of Y_t (g_c) is endogenously determined such that the government budget is balanced each period:

$$g_c = \frac{T_{n,t} + T_{c,t} - EP_t - PP_t - B_t}{Y_t} \tag{3.31}$$

9. Average and marginal tax rates are determined via Equations (3.20) and (3.21).

10. Y_t is determined by Equation (3.11).

11. $\Phi_{j+1,t+1} = Z_{j,t}(\Phi_{j,t})$ where $Z_{j,t}$ is the law of motion induced by the exogenous mortality rates, the exogenous Markov process for labour productivity, the endogenous Markov process for employment shocks and the endogenous asset accumulation and education choice.

3.4 Data and calibration

In this section, the parameterisation and calibration of the model is outlined in detail. The lion's share of the calibration is in line with the literature on quantitative OLG models with idiosyncratic risk. I make a distinction between shared and country-specific variables. Shared parameters are those that I consider to hold for all OECD countries. In line with most of the literature these include many household preference and technology parameters. Country-specific parameters include the survival probabilities, the probability of being employed in the next period given unemployment in this period, the union preference parameters and the fiscal policy parameters[10]. The shared parameters are either taken from the literature or calibrated on Belgium. Afterwards, I explore the performance of the model in its capacity to replicate the data for Denmark and the United Kingdom[11]. The values for the shared parameters and the corresponding moments can be found in Table 3.1. The values for the country-specific variables can be found in Table 3.2.

3.4.1 Demographics

Individuals enter the model at the age of 18 and live at most 19 periods of 4 years until the age of 94. The conditional survival probabilities $\{\varphi_j\}$ are country-specific and are taken from the Human Mortality Database and are for 2000.

3.4.2 Participation and employment rates

In the model, all high- and low-skilled individuals aged 18-53 participate, those aged 54-65 choose whether they participate or not, and all individuals aged 66 and over have no choice but to retire. Furthermore, all the participating high-skilled individuals are employed, whereas only a fraction $l_t = (1 - u_t)$ among the participating low-skilled

[10]Note that in the spirit of Boone and Heylen (2017), the union parameters χ^R, ϱ_1^R and ϱ_2^R are in fact region-specific. Hence, the superscript R is added from now on. Boone and Heylen (2017) distinguish three regions: continental European countries (including Belgium, Germany, France, Austria, and the Netherlands), the Nordic countries (including Denmark, Sweden, Finland, and Norway), and the Anglo-Saxon countries (including the US, the UK and Canada). However, as I pick one country from each region, region-specific coincides with country-specific in this set-up.

[11]The choice for these two countries is arbitrary. The conclusions do not depend on this choice.

individuals works. To construct the different variables of interest I use data from Eurostat on both the participation and employment rate by age and by educational attainment (respectively lfsa_argaed and lfsa_ergaed). Table 3.3 contains the main labour market performance data for Belgium, Denmark, and the UK. A comparison later with the model's predictions will provide a first and simple test of the quality of the model. A detailed description of these data is given in Appendix 3.B.

3.4.3 Employment shocks

The set-up of the model leads to idiosyncratic employment risk among the low-skilled workers. The Markov matrix governing the stochastic process is given by:

$$\Omega_{\underline{s}} = \begin{bmatrix} \rho_{ll} & 1-\rho_{ll} \\ \rho_{ul} & 1-\rho_{ul} \end{bmatrix} \tag{3.32}$$

where ρ_{ll} and ρ_{ul} denote the probability of being employed in the next period for respectively an employed and an unemployed individual today. To calibrate the age-specific Markov matrix governing the stochastic employment process, I use job-finding rate estimates provided by Hobijn and Sahin (2009). Using their estimated hazard function, I computed 48 consecutive monthly job-finding rates for Belgium, Denmark and the UK. I use the average of these job finding rates for ρ_{ul}. The data at the bottom of Table 3.2 reveal much lower job finding rates in Belgium compared to Denmark and the UK. Finally, ρ_{ll} is adjusted endogenously such that the stochastic process leads to the correct macroeconomic employment rates.

3.4.4 Technology and employment

The parameters regarding technology are $\{\alpha, A, \delta, \iota, \nu_1\}$. The capital share in production α equals 0.36. The level of technology A is normalised such that the equilibrium real wage rate for high-skilled individuals $w_{\bar{s},t}$ is equal to 1. The depreciation rate δ is calibrated using a target for the equilibrium annual real interest rate of $r = 4.5\%$. Following Caselli and Coleman (2006), who state that the empirical labour literature consistently estimates values between 1 and 2 for the elasticity of substitution between the two skill groups, ι is set to 1.4, the value reported by Katz and Murphy (1992). The share parameter for high-skilled labour in composite labour H_t, ν_1, is calibrated such that the equilibrium real wage rate of the low-skilled individuals delivers an unemployment rate as observed in the data (cf. Section 3.4.2).

3.4.5 Labour productivity shocks and parameters

In the model, an employed individual of age j with innate ability π, skill s and idiosyncratic shock η who works n hours will earn a pre-tax wage of

$$w_{s,t} n \eta \varepsilon_j \varpi_s(\pi) \tag{3.33}$$

I use the specification reported by Cournède and Gonand (2006) to calibrate the age-specific productivity profile ε_j. Like Krueger and Ludwig (2013) and Kindermann and Krueger (2014), I assume that the ability-dependent component of wages takes the following log-linear form:

$$\ln \varpi_s(\pi) = \vartheta_{1s} \pi \tag{3.34}$$

Following this specification, two parameters $\{\vartheta_{1\bar{s}}, \vartheta_{1\underline{s}}\}$ have to be calibrated. A priori, $\vartheta_{1\underline{s}}$ is normalised to 1, so only one parameter is left to be calibrated. $\vartheta_{1\bar{s}}$ is chosen such that in equilibrium the benchmark model generates an average college wage premium equal to 1.8. Heathcote et al. (2010) report this value for US wages.

The stochastic process governing the labour productivity shocks is the same for both skill groups. The productivity shock η can take three values η_j with $j \in \{1, 2, 3\}$. The Markov transition matrix is a (3x3)-matrix:

$$\Upsilon = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{bmatrix} \tag{3.35}$$

where ρ_{ij} is the probability $Pr(j|i)$ to end up in state j in the next period given state i in the current period. Taking all this information together, the states of the Markov chain $\{\eta_1, \eta_2, \eta_3\}$ and the Markov transition matrix Υ still have to be determined. For the labor productivity states $\{\eta_1, \eta_2, \eta_3\}$ in the labor earnings process, I use a discretised Markov chain for a continuous AR(1)-process with persistence ζ_s and variance σ^2_η. These parameters are the same for both skill groups. The persistence is chosen to be 0.969 and the variance 0.01 (Krueger and Ludwig, 2013). The latter authors estimate these values using an econometric model for log wages on PSID data for the United States.

The markov transition matrix for the idiosyncratic productivity risk is then given by

$$\Omega = \begin{bmatrix} 0.8851 & 0.1113 & 0.0035 \\ 0.0557 & 0.8887 & 0.0557 \\ 0.0035 & 0.1113 & 0.8851 \end{bmatrix}$$

while the values for η_1, η_2 and η_3 are respectively 0.6029, 1, and 1.6587.

3.4.6 Preferences

The instantaneous utility function of the individuals is given by[12]:

$$u(c_j, n_j) = \frac{\left(c_j{}^{(1-\mu)}(1 - I_\pi \Pi(\pi) - n_j - (1 - I_p)\rho n_j^*)^\mu\right)^{1-\theta}}{1-\theta} - I_P \Theta_s. \tag{3.36}$$

The parameters to be calibrated are $\{\beta, \mu, \theta, \Theta_{\bar{s}}, \Theta_{\underline{s}}, \rho\}$[13]. As in Krueger and Ludwig (2013), θ is chosen to be 4, in line with the literature using this functional form for the utility function. The relative weight on leisure μ is determined such that hours worked per Belgian employed individual is close to the value found in the data (see Table 3.3) and β is chosen such that a capital-output ratio of approximately 2.8 is obtained for Belgium[14]. $\Theta_{\bar{s}}$ and $\Theta_{\underline{s}}$ are calibrated on the aggregate participation rate of respectively the high- and the low-skilled individuals aged 54-65. Finally, ρ is calibrated such that given the values for $\Theta_{\bar{s}}$ and $\Theta_{\underline{s}}$, the deviation between the model's predictions and the data for the different participation rates of the individuals aged 54-57, 58-61 and 62-65 over both skill groups is minimised for Belgium.

3.4.7 Innate ability and time spent in college

Before any decisions are made, individuals draw their innate ability from a time-invariant distribution $F(\pi)$. As IQ is often mentioned as an adequate measure for innate ability, it is assumed that the distribution $F(\pi)$ follows a normal distribution $\mathcal{N}(\mu_\pi, \sigma_\pi^2)$ with $\mu_\pi = 1$ and $\sigma_\pi = 0.15$, which are standard values for the distribution of IQ. The chosen normal distribution $\mathcal{N}$ is then discretised to 10 equidistant values: $\pi \in \{\pi_1, \pi_2 ..., \pi_{10}\}$. The probabilities follow from the probability density function. In the model these probabilities coincide with the actual fraction of individuals with ability π_i in the economy.

After having observed their innate ability, individuals decide whether to attend college or not. The actual fraction of time required for completing tertiary education conditional on the innate ability is exogenous and given by the following function which is linear in ability:

$$\Pi(\pi) = \gamma_0 + \gamma_1 \pi, \ \gamma_0 > 0, \ \gamma_1 < 0 \tag{3.37}$$

[12]This functional form is often used in the quantitative OLG literature with idiosyncratic risk: see e.g. Conesa and Krueger (2006) and Krueger and Ludwig (2013).

[13]Remember that $I(\pi)$ takes the value 1 in the first period of active life when an individual decides to attend college, I_P takes the value 1 when the individual is aged 54-65 and participates on the labour market, Θ_s is a fixed utility cost of participating, and I_{ER} takes the value 1 when an 54-65 year-old individual decides to withdraw from the labour market. The leisure these early retired individuals enjoy is equal to $1 - \varrho n_j^*$ where n_j^* is the fraction of time this individuals would work if they would participate.

[14]For more details on the construction of hours worked per employed individual, I refer the reader to Boone and Heylen (2017).

In line with Krueger and Ludwig (2013), for individuals with the lowest innate ability π_1, the fraction of time they have to devote to college education in order to graduate is 1. For individuals with the highest innate ability π_{10}, the required time cost is calibrated such that the number of college graduates in the economy is as close as possible to the number of college graduates in the data. In OECD (2014), values for the percentage of people who obtained tertiary degree in 2000 and 2005 can be found. Using these two points, the parameters γ_0 and γ_1 are easily determined.

3.4.8 Union preferences

The parametrisation of the union framework is in line with Boone and Heylen (2017). Remember that in order to highlight their region-specific character, the notation of the union preference parameters has been augmented with a superscript R. In Boone and Heylen (2017), the authors calibrate the union's preference for wage parameter (χ^R) and the weights in the reference wage of the union (ϱ_1^R, ϱ_2^R and $\varrho_3^R = 1 - \varrho_1^R - \varrho_2^R$) for a sample of Anglo-Saxon, continental European and Nordic countries using a sensitivity analysis à la Heylen and Van de Kerckhove (2013)[15]. In this paper, I use these calibrated values for χ_j^R, ϱ_1^R and ϱ_2^R. The values for the two latter parameters are directly applicable to the model in this paper. However, the specification of the utility function of the union is slightly different from the functional form used in Boone and Heylen (2017). I thus have to transform the calibrated values for χ_j^R from Boone and Heylen (2017) such that the relative weight on wages as opposed to employment is the same in this model. The new value is obtained as follows:

$$\chi^{N,R} = \frac{\frac{\sum_{j=1}^{4} \chi_j^{O,R}}{4}}{1 + \frac{\sum_{j=1}^{4} \chi_j^{O,R}}{4}}$$

where $\chi^{N,R}$ is the new χ, used in the remainder of this paper. $\chi_j^{O,R}$ are the ones obtained in Boone and Heylen (2017). Finally, κ is determined such that given all these parameters, the resulting unemployment rates among the low-skilled are consistent with the data for respectively Belgium, the UK, and Denmark[16].

[15] In a first step - executed separately for each country group - the authors imposed values for ϱ_1^R, ϱ_2^R and ϱ_3^R ($R \in \{Eur, Nor, Ang\}$). With these imposed values the parameters χ_j^R with $R \in \{Eur, Nor, Ang\}$ were calibrated to match the average of actual unemployment rates in four generations over all countries in the country group. The obtained set of union preference parameters for each of the three country groups - together with all other calibrated parameters - allowed them to compute predictions for all unemployment rates in all generations in each of the twelve countries in their sample. This procedure was repeated many times, each time starting from different values for ϱ_1^R, ϱ_2^R and ϱ_3^R. The ultimate aim was to minimise the average normalised RMSE for the aggregate unemployment rate over all ability and age groups.

[16] Although the size of the effects of the different scenarios reported in Section 3.5.2 might differ somewhat if different values for κ are used, the main findings of this paper do not change.

3.4.9 Fiscal policy variables

The government in the model finances government spending on goods, PAYG pension and benefits to the unemployed with taxes on consumption and labour. Data on b, ξ, Γ, τ^p and τ_c are the same as Boone and Heylen (2017).

For the tax rate τ_c, I use the same data as Heylen and Van de Kerckhove (2013). For details on the construction of this fiscal policy variable, I refer to Heylen & Van de Kerckhove (2013, their Appendix 1). For the unemployment benefit replacement rate b data provided by the OECD (Tax-Benefit Models) is used. Since in the model unemployment is a structural or equilibrium phenomenon, the data that are used concern net transfers received by structurally or long-term unemployed people. They include social assistance, family benefits and housing benefits in the 60th month of benefit receipt. They also include unemployment insurance or unemployment assistance benefits if these benefits are still paid, i.e. if workers can be structurally unemployed for more than five years without losing benefit eligibility. The OECD provides net replacement rates for six family situations and three earnings levels. The data in Table 3.2 are the averages of these 18 cases. Data are for 2001-2004.

Regarding labour tax rates, I distinguish between social security contributions paid by employers and taxes and social security contributions on labour income paid by employees. The data source is the OECD (Taxation, Tax Database, Tables I.4, I.5, I.6). More specifically, I use the OECD's average rate of employer social security contributions for τ^p (Table I.5). The level parameter Γ in the workers' income tax rate is calibrated using the OECD data for all-in average personal income tax rates at average wage. The all-in tax rate is calculated as the combined central and sub-central government income tax plus employee social security contribution, as a percentage of gross wage earnings. The OECD provides these tax rates for four family types (Table I.6). The average over these types is computed. A novelty compared to previous work is the inclusion of progressive income taxation. Just like Koyuncu (2011), I calibrate the country-specific degree of progressivity ξ according to Equation (3.21) as the ratio (minus 1) of the marginal tax rate on workers' gross wage to the average tax rate, both including social security contributions. The OECD provides these marginal and average tax rates for a single person without dependent at four different income levels (Tables I.4 and I.5). Our proxy for ξ reflects the average of the results over these four income levels. All computed and reported tax data in Table 3.2 are averages over the period 2000-2007.

For the basic PAYG-pension received by the (early) retired households in the model, I use data on the net replacement rate for the average level of earnings obtained from the OECD (Pensions at a Glance, 2005, Table 4.2.).

Table 3.1: Calibration summary: parameter values imposed on all countries

Parameters taken from the literature: definition, parameter, value, and source			
Elasticity of substitution between different types of labour	ι	1.4	Katz and Murphy (1992)
Coefficient of risk aversion of the individuals	θ	4	Conesa and Krueger (2006); Krueger and Ludwig (2013)
Age-specific component of wages	ε_j		Cournède and Gonand (2006)
Stochastic component of wages	ζ, σ_η^2	see Section 3.4.5	Krueger and Ludwig (2013)
Capital share in production	α	0.36	
Parameters calibrated to Belgium: definition, parameter, value, and target			
Weight on leisure in the individuals' utility function	μ	0.6164	Average fraction of time spent working = 0.31 (Boone and Heylen, 2017)
Discount factor in the individuals' utility function	β	0.97	Capital-output ratio of 2.8 (Penn World Table)
Utility cost of participation for high-skilled individuals	$\Theta_{\bar{s}}$	0.01	Employment rate for high-skilled individuals (54-65) equal to 53%
Utility cost of participation for low-skilled individuals	$\Theta_{\underline{s}}$	0.029	Participation rate for low-skilled individuals (54-65) equal to 34.5%
Increase in leisure time as early retiree*	ρ	0.65	Age profile of participation rates 54- to 65-year-olds in Belgium (see Table 3.3)
Fraction of time required to complete tertiary education	γ_0, γ_1	$1.827, -1.272$	% of high-skilled individuals equal to 31% ((OECD, 2014)
Level of technology	A	2.836	$w_{\bar{s}} = 1$
Depreciation rate	δ	0.378	$r = 4.5\%$
Share parameter of high-skilled labour	ν_1	0.528	Wage rate for low-skilled individuals
Parameters calibrated to other targets: definition, parameter, value, and target			
Parameter determining the ability dependent component of wages	$\vartheta_{1\bar{s}}$	1.322	College wage premium of 80% in the US (Heathcote et al., 2010)
Elasticity of substitution utility function union	κ	1.4	Unemployment rates for Belgium, UK, and Denmark (see Table 3.3)

* This increase in leisure is expressed as a percentage of the fraction of time this individual would work if he/she still participated.

Table 3.2: Calibration summary: country-specific parameters

Country-specific parameters				
		Belgium	**UK**	**Denmark**
Weight of union on wages relative to employment	χ^R	0.787	0.725	0.694
Weight in union reference wage for competitive wage	ϱ_1^R	0.8	0.90	0.9
Weight in union reference wage for high-skilled wage	ϱ_2^R	0.05	0.1	0
Weight in union reference wage for unemployment benefit	ϱ_3^R	0.15	0	0.1
Degree of progressivity of tax code	ξ	0.34	0.383	0.233
Labour tax rate for the average wage	Γ	33.4	24.0	35.3
Labour tax rate levied on employers	τ^p	29.1	9.8	0.0
Consumption tax rate	τ_c	13.4	14.5	18.9
Net replacement rate for unemployment benefit	b	59.6	51.1	61.9
Net replacement rate PAYG pension for (early) retired	b_p	63.1	47.6	54.1
Probability to move from unemployment to employment	ρ_{ul}	0.035	0.092	0.082

3.4.10 Model performance

Table 3.3 gives an overview of the performance of the model. Given that the model is calibrated to Belgium, it is no surprise that the model's predictions are very close to the actual numbers in the data. However, using the calibrated parameters together with the country-specific parameter values for both the UK and Denmark and exploring the model's predictions provides an additional test of the performance of the model. Looking at Table 3.3, one can conclude that the model goes a long way in explaining the differences in labour market outcome between Belgium, the UK and Denmark. For both Denmark and the UK, the model correctly predicts that high- and low-skilled individuals aged 54-65 participate far more on the labour market. Within age groups, the model's predictions do deviate somewhat from the actual data. The lower aggregate unemployment rates and

Table 3.3: Labour market data and model performance for Belgium, the UK and Denmark

	Belgium		**UK**		**Denmark**	
	Data	Model	Data	Model	Data	Model
Average participation rate high-skilled (54-65)	53.0%	54.0%	82.3%	87.1%	81.4%	76.0%
Participation rate low-skilled (54-57)	55.5%	53.7%	78.3%	86.5%	89.0%	78.0%
Participation rate low-skilled (58-61)	32.1%	33.9%	73.8%	71.5%	62.3%	59.6%
Participation rate low-skilled (62-65)	15.9%	17.4%	72.0%	47.1%	37.5%	34.0%
Aggregate unemployment rate[(a)]	11.0%	13.3%	8.4%	8.9%	5.2%	5.6%
Fraction of time devoted to work per employed[(a)]	0.310	0.312	0.343	0.345	0.334	0.342
Government spending as % of GDP	27.0		19.7		23.4	
% of skilled individuals	31.1		31.1		31.1	

(a) averaged over both skill groups and all generations.

Note: For details on the construction of the data, I refer the reader to Appendix 3.B.

Table 3.4: Simulation outline - Different scenarios

	Ages 18-53	**Ages 54-65**	**Ages 18-65**		
	Labour tax level (Γ)	Labour tax level (Γ)	Employer tax (τ^p)	Unemployment benefit (b)	Union wage intensity (χ^R)
(1)	30.45	30.45	29.1	59.6	χ^{EUR}
(2)	33.4	33.4	24.92	59.6	χ^{EUR}
(3)	33.4	33.4	29.1	35.2	χ^{EUR}
(4)	30.45	30.45	29.1	35.2	χ^{EUR}
(5)	33.4	33.4	24.92	35.2	χ^{EUR}
(6)	33.4	12.2	29.1	59.6	χ^{EUR}
(7)	33.4	12.2	29.1	35.2	χ^{EUR}
(8)	33.4	33.4	29.1	59.6	χ^{ANG}
(9)	30.45	30.45	29.1	59.6	χ^{ANG}
(10)	33.4	33.4	24.92	59.6	χ^{ANG}
(11)	33.4	12.2	29.1	59.6	χ^{ANG}
(12)	30.45	30.45	29.1	35.2	χ^{ANG}
(13)	33.4	33.4	24.92	35.2	χ^{ANG}

Note: The ex-ante effect of all changes in fiscal policy parameters is equal to 1% of output. Policy changes only apply to low-skilled individuals. Benchmark values (in %) are: $\Gamma = 33.4$, $\tau^p = 29.1$, $b = 59.6$.

the higher fraction of time devoted to work per employed individual are captured as well.

3.5 Simulations: outline and results

3.5.1 Outline

Table 3.4 contains a detailed outline of the different simulated scenarios. The ex-ante effect of all these policy changes is equal to 1% of output, i.e. if no one changed behaviour, they would all cost or save 1% of output. Every scenario described in Table 3.4 specifically targets low-skilled individuals and is designed such that an improvement in the labour market position of these individuals can be expected. The fiscal variable used to balance the budget ex-post is τ_c.

In (1) the level parameter Γ determining the labour taxes levied on low-skilled employees is decreased for all generations from 33.4% in the benchmark model to 30.45%, while (2) applies the same logic to τ^p, the tax rate levied on employers for employing low-skilled individuals. In (3), the net unemployment benefit replacement rate b is reduced. Scenarios (4) and (5) combine the respective scenarios (1) and (2) with the decrease in b of scenario (3). Scenarios (6) and (7) focus on the 54- to 65-year-old low-skilled agents. In

(6) and (7) Γ is reduced for these individuals. In (7) this is complemented by a reduction of the net unemployment benefit replacement rate b for all generations.

In scenarios (8-13), the fiscal policy component is augmented with a change in the union preference for wage parameter χ. More specifically, its value is reduced from 0.787 (χ^{EUR}) to 0.725 (χ^{ANG}) (see Table 3.2).

3.5.2 Results

The results are split into three main categories of variables and are displayed in Table 3.5. The first category groups the % change compared to the old steady state in respectively (a) aggregate output Y_N, (b) capital K_N and (c) total effective labour H_N. The subscript N denotes the new steady-state. The second category looks into the different components of aggregate labour: (d) % change in the average fraction of time spent working by a low-skilled, employed individual relative to the same indicator in the old steady state, (e) the %-point change in the aggregate unemployment rate, (f) and the %-point change in the average participation rate of the low-skilled individuals aged 54-65.

The third category looks at the welfare effects (i) for a newborn who will stay low-skilled and (ii) for a newborn who will become high-skilled. To calculate the welfare effects of the different tax reforms, I use the ex-ante expected utility of a newborn to compute the Consumption Equivalence Variation. This indicator measures the uniform percentage change in consumption that would make the expected lifetime utility of a newborn before the reform equivalent to the expected lifetime of a newborn after the reform. Formally, this metric is given by:

$$\text{CEV} = \left[\left(\frac{V_a^t(0, \pi, a = 0)}{V_b^t(0, \pi, a = 0)} \right)^{\frac{1}{(1-\mu)(1-\theta)}} - 1 \right] * 100 \tag{3.38}$$

where $V_a^t(0, \pi, j = 0)$ and $V_b^t(0, \pi, j = 0)$ are the expected lifetime utility of a newborn respectively after the reform and before the reform. I compute this metric for newborns who remain low-skilled and newborns who attend college and become high-skilled.

3.5.2.1 Discussion

Before diving into the details of the different situations, let me start by describing the main conclusions. First, the simulations reveal that, while most fiscal policy and union instruments have a similar impact on aggregate output, capital, and labour, these instruments may have a very different impact on unemployment, labour force participation, hours worked, and worker's welfare. This result definitely highlights the added value of modelling these different components. As a result, the fiscal government has to choose its desired policy instrument carefully and dependent on the variable it wants to target. Second, fiscal governments and unions have a mutual interest in joining forces to improve the labour market performance of the low-skilled. For example, the most preferable policy is a decrease in labour taxes levied on low-skilled employees combined with wage moderation accepted by the unions. The third and final conclusion one can draw from the simulations is that, unlike in perfectly competitive labour markets, in this set-up labour taxes levied on respectively employers and employees are not equivalent. Even more, the behavioural effects of a decrease in the labour tax rate paid by employers are very small.

Let us explore the impact of the different policies on the different categories separately now. Looking at the impact on output (Y), capital (K) and labour (H) in Table 3.5, the different policies seem more or less similar at first glance. A decrease in labour taxes levied on all low-skilled employees (Γ), a decrease in the net unemployment benefit replacement rate (b), and a decrease in the wage preference of the unions (χ), these policies lead to a strong increase in the different aggregate variables. These effects are even stronger when two of these instruments are adjusted simultaneously. For example, a joint decrease in Γ and b leads to a 5% increase in Y. Adding a decrease in χ even leads to a 10% increase in steady state output. On the other hand, the effect on Y, K, and H are much smaller (or even negative) following a decrease in τ^p and a decrease in Γ_{54-65}.

Zooming in on the different components of aggregate labour, however, the model really proves its usefulness. Indeed, the simulations reveal that the impact of Γ, b, and χ on respectively unemployment, labour force participation, and hours worked differs substantially relative to each other. Decreasing only labour taxes (Γ), for example, have no use when the fiscal government aims to decrease low-skilled unemployment. They are, however, very useful to increase the amount of hours worked by employed low-skilled individuals. Decreasing the net unemployment benefit replacement rate and less aggressive wage policies by the union, on the other hand, have a strong impact on low-skilled unemployment. But they have no use if the target variable is the amount of hours worked per employed individual. A decrease in Γ_{54-65} has a strong impact on the participation decision of the low-skilled aged 54-65, but a small impact on the other components of aggregate labour.

Both a decrease in Γ for all low-skilled individuals and a decrease in b have a negative impact on the labour force participation of the individuals aged 54-65. When faced with lower labour taxes, employed low-skilled individuals work, earn and save more. As such, they are able to accumulate more non-human wealth before the age of 54 and by the time they reach the age of 54, they are less dependent on labour income. More specifically, the higher level of non-human wealth allows them to retire earlier.

Table 3.5: Simulations - Results

	% change				%-point change		Welfare	
Policy change	Y	K	H	n	u	p	$CEV_{\underline{s}}$	$CEV_{\bar{s}}$
(1) $\triangle\Gamma$ (all ages)<0, $\triangle\tau_c=+1.7\%$	2.3	3.5	1.6	2.5	-0.3	-1.8	2.462	-0.209
(2)$\triangle\tau^p$ (all ages)<0, $\triangle\tau_c=+1.7\%$	0.3	0.5	0.2	0.1	0.0	-0.1	1.637	-1.070
(3)$\triangle b$ (all ages)<0, $\triangle\tau_c=-3\%$	3.1	2.8	3.3	1.4	-5.5	-12.9	-4.099	5.722
(4)$\triangle\Gamma$ (all ages)<0, $\triangle b$ (all ages)<0, $\triangle\tau_c=-1.4\%$	4.8	5.3	4.5	3.7	-5.2	-13.8	-1.882	4.986
(5)$\triangle\tau^p$ (all ages)<0, $\triangle b$ (all ages)<0, $\triangle\tau_c=-1.4\%$	3.3	3.0	3.4	1.5	-5.2	-12.8	-2.620	4.386
(6)$\triangle\Gamma$ (54-65)<0, $\triangle\tau_c=+2.8\%$	0.2	-1.1	0.9	0.4	1.4	20.1	-1.252	-0.981
(7)$\triangle\Gamma$ (54-65)<0, $\triangle b$ (all ages)<0, $\triangle\tau_c=-0.6\%$	3.3	1.4	4.5	1.1	-6.2	-4.2	-4.993	5.118
(8)$\triangle\chi<0$, $\triangle\tau_c=-1.6\%$	2.9	3.0	2.9	0.5	-4.0	-5.1	-0.443	3.901
(9)$\triangle\Gamma$ (all ages)<0, $\triangle\chi<0$, $\triangle\tau_c=-0.2\%$	5.2	6.4	4.6	2.9	-4.4	-7.4	1.915	3.891
(10)$\triangle\tau^p$ (all ages)<0, $\triangle\chi<0$, $\triangle\tau_c=0.0$	3.3	3.6	3.1	0.6	-3.9	-5.6	1.159	2.676
(11)$\triangle\Gamma$ (54-65)<0, $\triangle\chi<0$, $\triangle\tau_c=+0.6$	3.5	2.1	4.3	0.6	-3.4	13.1	-1.639	3.749
(12)$\triangle\Gamma$ (all ages)<0, $\triangle b<0$, $\triangle\chi<0$, $\triangle\tau_c=-2.8$	9.0	9.9	8.6	3.6	-10.5	-16.1	0.044	9.777
(13)$\triangle\tau^p$ (all ages)<0, $\triangle b<0$, $\triangle\chi<0$, $\triangle\tau_c=-2.6$	7.4	7.8	7.2	1.5	-10.1	-15.1	-0.601	8.756

Notes: both the percentage change in aggregate output (Y), aggregate capital (K), aggregate labour (H), and average amount of hours worked per employed low-skilled (n) as well as the percentage point change in the aggregate unemployment rate (u) and the participation rate (p) are relative to the steady state values displayed in Table 3.3.

Following a decrease in b, it is not due to higher savings per employed individual, but due to the fact that more individuals are employed. More individuals are employed, which means that they earn a labour income and are able to accumulate more non-human wealth.

Finally, Table 3.5 also contains a welfare comparison between the two steady states for each policy. As mentioned, the welfare is looked upon from an ex-ante perspective[17]. Note that fiscal policy variables have both a direct and an indirect impact on ex-ante welfare, while union instruments only have an indirect effect. That explains why the impact on welfare is on average stronger if a fiscal policy instrument is changed. Unemployed low-skilled individuals facing a decrease in b are worse off in the new steady state. This result is driving the negative effect on ex-ante welfare for the low-skilled. The welfare effects of the other policies are less negative or even positive. For example, a decrease in Γ leads to a positive effect on ex-ante welfare. The results for high-skilled individuals are primarily determined by the change in the consumption tax rate.

3.5.3 Application: Germany 2010-2016

Figure 3.1 depicts Germany as the country with the biggest unemployment rate among the low-skilled. Since 2001-2007, the benchmark period considered in the previous sections, Germany has imposed drastic fiscal policy reforms, though. As a result, Germany thus forms a highly interesting case to explore the explanatory power of the model.

In this section, I simulate the model for Germany respectively for the periods 2001-2007 and 2010-2016 and compare its predictions with the data. More specifically, I impose the values for the fiscal policy variables as observed in these two time periods together with the calibrated values as outlined in Section 3.4. Table 3.6 displays these values. Note that all these variables are determined using the data sources in Section 3.4.9. The fiscal reform imposed by the German government includes a decrease in labour taxes (both on employers and employees), a decrease in the net replacement rate for the PAYG pension, and a decrease in the net unemployment benefit replacement rate.

The simulations are displayed in Table 3.7. Indeed, the performance of the German labour market has increased significantly following the reform. Both high- and low-skilled 54- to 65-year-old individuals participate far more compared to 2001-2007. The aggregate unemployment rate has decreased as well: from 11.2% to 9.7%. Looking at the model's predictions, one conclusion is that the model has difficulties in capturing the level of the participation rates. The predictions of the model for both the high- and the low-skilled are lower than the levels observed in the data. The aggregate unemployment rate, on the

[17] Note that this is only a preliminary welfare analysis and that these results are only informative. A thorough welfare analysis requires the computation of the full transition path. The transitional path is, however, beyond the scope of this paper.

Table 3.6: Calibration for Germany (2001-2007 and 2010-2016): fiscal policy and union variables

Fiscal Policy and Union variables		2001-2007	2010-2016
Weight of union on wages relative to employment	χ^R	0.787	0.787
Weight in union reference wage for competitive wage	ϱ_1^R	0.8	0.8
Weight in union reference wage for high-skilled wage	ϱ_2^R	0.05	0.05
Weight in union reference wage for unemployment benefit	ϱ_3^R	0.15	0.15
Degree of progressivity of tax code	ξ	0.25	0.22
Labour tax rate for the average wage	Γ	30.2	28.6
Labour tax rate levied on employers	τ^p	19.9	18.9
Consumption tax rate	τ_c	11.1	12.0
Net replacement rate for unemployment benefit	b	59.6	50.7
Net replacement rate PAYG pension for (early) retired	b_p	58.0	55.0
Probability to move from unemployment to employment	ρ_{ul}	0.065	0.065

other hand, is somewhat higher. Notwithstanding these observations, an upside is that the predicted impact of the fiscal policy reform goes in the right direction. It does not fully capture the evolution, though. Let me start with the 18- to 53-year-old low-skilled individuals. Following the fiscal policy reform, they face a higher probability of being employed. A lot more low-skilled individuals will be employed at each age. Furthermore, the activity rate among the elderly also increases, albeit not so strong for the low-skilled. As discussed in the previous section, this is due to the lower unemployment rate. More low-skilled individuals are employed and accumulate more non-human wealth. As such, they are less dependent on labour income at the age of 54-65.

There is a caveat, however. So far, the model is only able to compare steady states. Given that only few years have passed since the fiscal policy reform, the German economy might not have reached its steady state yet. The predictions of the model, on the other hand, are the values for the steady state of the model. This might, of course, influence the results to some extent.

Table 3.7: Labour market performance in Germany in 2001-2007 and 2010-2016: data versus model's predictions

2001-2007

	Data	Model's predictions
Participation rate 54-65 high-skilled	71.1%	57.5%
Participation rate 54-65 low-skilled	51.5%	45.1%
Aggregate unemployment rate	11.2%	13.0%

2010-2016

	Data	Model's predictions
Participation rate 54-65 high-skilled	87.0%	65.0%
Participation rate 54-65 low-skilled	66.5%	49.1%
Aggregate unemployment rate	9.7%	10.0%

3.6 Conclusion

This paper explores the potential of fiscal and union policies to improve the labour market performance of low-skilled individuals. While there is considerable evidence on the impact of fiscal policy and labour market institutions on aggregate employment , there is much less evidence on their impact on employment among low-skilled individuals.

My aim in this paper is twofold. The first objective is to develop a large-scale computable OLG model with idiosyncratic productivity risk in which households choose the tax and wage profile they will face during their active period of life by attending college or not. The key innovation here is that equilibrium unemployment among unskilled individuals is endogenous and leads to idiosyncratic employment risk. More specifically, I introduce a unionised labour market for low-skilled individuals. Due to this set-up, the probabilities in the age-specific Markov matrix governing the stochastic employment process are endogenously determined. Therefore, by attending college or not, households also choose the employment risk they will face.

The second objective is to use the model to quantitatively investigate the impact of fiscal and union policies on the labour market position of low-skilled individuals. The simulations reveal that modelling the different components of aggregate employment is important to explore the effectiveness of fiscal and union policies in great detail. While most fiscal policy and union instruments have a highly similar impact on aggregate output, capital, and labour, these instruments all have a different impact on unemployment, labour force participation, and hours worked. The simulations thus highlight that the choice for a given policy instrument is dependent on the target variable. Only decreasing labour taxes, for example, has no use when the fiscal government aims to decrease low-skilled unemployment. Decreasing the net unemployment benefit replacement rate and less aggressive wage policies by the union, on the other hand, have a strong impact on low-skilled unemployment. But they have no use if the target variable is the amount of hours worked per employed individual.

Several policies lead to desirable results in terms of unemployment. Overall, however, the simulations show that fiscal governments and unions have a mutual interest in joining forces to improve the labour market performance of the low-skilled. For example, the most preferred policy is a decrease in labour taxes levied on low-skilled employees combined with wage moderation accepted by the unions. Not only does it lead to a decrease in the unemployment rate of the low-skilled, it also has a positive impact on the number of hours worked per employed individual and it is very likely to increase welfare of employed low-skilled individuals. Furthermore, it does not harm unemployed individuals directly.

The last conclusion one can draw from the simulations is that, unlike in perfectly competitive labour markets, labour taxes levied on respectively employers and employees are not equivalent within the model. Even more, the behavioural effects of a decrease in the labour tax rate paid by employers are very small. Within the current set-up, the fiscal government can thus achieve more by lowering the tax rates paid by employees.

Finally, given the remarkable evolution of the German labour market since 2007, the above analysis is complemented with a simulation of the German fiscal policy reform in the period 2010-2016. Notwithstanding the difficulties of the model to grasp the level of the high- and low-skilled participation rates, it does a fairly good job at capturing the evolutions that are characterising the German labour market since 2007: both the lower aggregate unemployment rate and the increase in participation.

Appendices

Appendix 3.A Details on the computational approach

This Appendix describes the computational approach used to solve the model in the paper. The model is solved using three algorithms: one algorithm to solve for the different policy and value functions, one algorithm to solve for the equilibrium quantities and one algorithm to solve the union problem. First, I solve the competitive model in which the unions are excluded, meaning that there is no involuntary unemployment. Using the output from this model (e.g. the competitive wage for unskilled workers), I use a minimisation routine to compute the solution for the union problem. Once a solution for the union problem is obtained, I solve for the equilibrium of the model with unemployment. Due to the stochastic unemployment process, there are some minor differences between the household problem in the competitive model and the imperfect competitive model. In this Appendix, I will describe the algorithm for the imperfect competitive model, as this is the most complex one.

3.A.1 Computation of the policy and value functions

The algorithm that is used to solve for the policy and value functions is standard in the literature. In this subsection, the implementation of this algorithm is outlined in detail.

First, as the asset dimension of the state space is continuous, I have to discretise this dimension in order to calculate the value and policy functions on a computer. Therefore, I construct a set of discrete points $\{a^1, a^2, a^3, ..., a^n\}$. As the computing time required to solve the model increases with the amount of discrete grid points for the state space for asset holdings and the results do not change if I choose higher values, I set $n = 25$. The discretised asset dimension of the state space is then used to solve the maximisation problem of the household. The algorithm iterates on the following steps:

- Compute the decisions of the individuals at the maximum age J for any (a^i, π, s). As the surviving agents die with certainty in the next period and since they leave no bequests, they consume all remaining resources. Therefore, the policy functions for consumption $c_J(a^i, \pi, s)$ are computed for every (a^i, π, s) with the associated value functions.

- As the policy and value functions in the last period of life are calculated in the previous step, these are used to iterate backwards to solve for the household decisions for all ages. Focusing on employed individuals of working age, $a'_j(a^i, e, \pi, \eta, s)$, $c_j(a^i, e, \pi, \eta, s)$ and $n_j(a^i, e, \pi, \eta, s)$ are computed. I use the minimisation routine of Chris Sims[18] to solve for the optimal a'_j, where the optimal level of consumption and labour supply follow from this value for a'. Note that as this is a minimisation routine, I minimise $-V_j$, i.e. the negative of the value function. As the routine requires a continuous function, I have to interpolate the value function V_{j+1}. The unskilled individuals that are not employed in the current period only choose next period asset holdings $a'_j(a^i, u, \pi, \eta, n)$ and consumption $c_j(a^i, u, \pi, \eta, n)$.

3.A.2 Solving for the macroeconomic equilibrium

I use the Gauss-Seidel-Quasi-Newton algorithm proposed by Ludwig (2007) to solve for the macroeconomic prices. Starting from an initial guess for the interest rate and the wages for high-skilled individuals (remember that the wage for the unskilled individuals is already determined by the union problem), I compute household decisions and the distribution of households on the state space and macroeconomic quantities. Using these quantities, I compute new values for the interest rate and the wage for high-skilled individuals. Using these, the initial values are updated. These steps are executed until the distance between the initial guesses and the resulting prices is sufficiently small.

3.A.3 Solving the union maximisation problem

Using the output of the competitive model and the minimisation routine of Chris Sims, the union maximisation problem is solved. Similar to the household decision with respect to next period asset holdings, I minimise the negative of the utility function of the union. While the minimisation routine executes, for every new update of the wage for unskilled individuals, I solve for the policy functions of the unskilled individuals and the unemployment rates determined by the firm while taking the decisions of the high-skilled individuals, the wage for high-skilled individuals and the interest rate resulting from the competitive model and the fiscal policy variables as given.

[18]This routine can be found on http://sims.princeton.edu/yftp/optimise/.

Appendix 3.B Details on the construction of the data for labour force participation rates and the aggregate unemployment rate

This Appendix gives a detailed description of the construction of the data for the labour force participation rates, employment rates, and the aggregate unemployment rate. The details for Belgium are given in Table 3.8 below. The data for Denmark and the UK are constructed in a similar way.

In a first step, the employment rate for the high-skilled 18- to 53-year-olds is constructed. This value serves as the benchmark to construct the other data. Looking at Table 3.8, the benchmark is 85.9%. More specifically, all other variables that follow are normalised with respect to this benchmark of 85.9%. The choice of this employment rate as benchmark follows from the assumption in the model that all high-skilled individuals aged 18-53 participate on the labour market and have a job. For this group, there is full employment.

Let me start with the employment rates among the age groups 54-57, 58-61, and 62-65 of the high-skilled. The data are 67.4% for the age group 54-57, 43.5% for the age group 58-61, and 25.7% for the age group 62-65. On average, this gives an employment rate of 45.5%. The next step is normalising the aforementioned values by the average participation and employment rate for the high-skilled individuals aged 18-53. On average, this implies an employment rate of 53% (see also Table 3.3). Given our assumption of zero unemployment among the high-skilled, this number is also the average participation rate among the older high-skilled. The latter values are the data used in the paper for the calibration of $\Theta_{\bar{s}}$, $\Theta_{\underline{s}}$, and ρ.

I then perform the same analysis for the low-skilled individuals. The average employment rate of low-skilled individuals aged 18-53 in the data is 65.7%. The %-point difference between the employment rate of the high- and low-skilled individuals aged 18-53 equals 20.2. This value is used to construct the aggregate unemployment rate (u_t) assuming that for each generation of low-skilled 20.2% of the participating individuals is unemployed and given that all participating high-skilled work. Next, I use the actual values for the participation rates of the low-skilled individuals aged 54-57, 58-61, and 62-65. These are respectively 47.7%, 27.6%, and 13.7%. Just like for the high-skilled, I normalise these values using the value for the employment rate of high-skilled individuals aged 18-53. These can be found in the column to the right.

Table 3.8: Construction of the data for labour force participation in Belgium

Labour market indicator	**Data**	**Normalised on employment rate high-skilled aged 18-53**
(a) Employment rate high-skilled individuals aged 18-53	85.9%	100%
(b) Employment rate low-skilled individuals aged 18-53	65.7%	
Employment rate difference in %-points between high- and low-skilled individuals aged 18-53	20.2%	
Employment rate high-skilled 54-65	45.5%	53.0%
Participation rate low-skilled 54-57	47.7%	55.5%
Participation rate low-skilled 58-61	27.6%	32.1%
Participation rate low-skilled 62-65	13.7%	15.9%

Chapter 4

Real-time Parameterised Expectations and the Effects of Government Spending[1]

Abstract

In this paper, we explore the effects of government spending in the real business cycle model where agents use a learning mechanism to form expectations. In contrast to most of the learning literature, we study learning behaviour in the original non-linear model. Following the learning interpretation of the parameterised expectations method, agents' forecast rules are approximations of the conditional expectations appearing in the Euler equation. We show that variation in agents' beliefs about the coefficients of these rules, generates time variation in the transmission of government spending shocks to the economy. Hence, our modelling approach provides an endogenous mechanism for time-varying government spending multipliers in the standard real business cycle model.

Keywords: Non-linear learning, Parameterized expectations, Fiscal Policy, Time-varying multipliers
JEL classification codes: E62, D83, D84, E32

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4.1 Introduction

The macroeconomic impact of fiscal policy depends crucially on the behavioural response of households to these policies. An important determinant of this behavioural response is the approach households apply to form expectations regarding the evolution of different endogenous, macroeconomic variables. The dominant paradigm used to model expectations in macroeconomics is the rational expectations (RE) hypothesis. According to this hypothesis, households have perfect knowledge about the structure of the model and understand the full complexities of the macro-economy. An alternative to the rational expectations hypothesis is provided by the learning literature (see e.g. Evans and Honkapohja, 2001).[2] In this literature, agents form expectations using a perceived law of motion. Over time, as new information becomes available, agents update the coefficients of their perceived law of motion.

This observation raises the question of whether the effects of government spending and the transmission thereof in the macro-economy are different in the basic Real Business Cycle (RBC) model using respectively rational expectations and a learning set-up. Indeed, it is well known that government spending multipliers generated by standard RBC and Dynamic Stochastic General Equilibrium (DSGE) models using rational expectations are typically constant. This theoretical finding is, however, not in accordance with the findings of several empirical studies. Auerbach and Gorodnichenko (2012) and Owyang et al. (2013), among others, show that the government spending multiplier is time-varying. In this paper, we start from these empirical findings and show that the introduction of a learning set-up in the standard RBC model can generate substantial time variation in the government spending multiplier.

Several papers have already explored the effects of fiscal policy using a learning framework within an RBC or DSGE model. Evans et al. (2009), for example, study the effects of anticipated fiscal policy changes both within an endowment economy and the Ramsey model. Their assumption is that agents fully understand and anticipate the evolution of taxes but have to forecast future factor prices using a linear learning mechanism. Building on this framework, Mitra et al. (2013) generalise the analysis of Evans et al. (2009) to a stochastic environment with elastic labour supply. Gasteiger and Zhang (2014) extend the model even further by introducing distortionary taxes. Following a similar learning approach, Benhabib et al. (2014) investigate the effects of fiscal stimulus in a new Keynesian model with a zero lower bound on the nominal interest rate.

[2] In this paper we consider *Euler equation* learning as put forward by Evans and Honkapohja (2001). In this approach, agents make one-step ahead forecasts. By contrast, the *infinite horizon* approach of Preston (2005) assumes that at each date agents make forecasts about variables into the infinite future. For a discussion of these two approaches see Honkapohja et al. (2013).

All the aforementioned papers have enriched our knowledge on the macroeconomic effects of fiscal policy. They show that these effects can be substantially different when using the learning approach instead of the rational expectations approach. However, none of these studies explicitly focuses on the evolution and level of the government spending multiplier. Furthermore, they all study learning in the linearised counterpart of the non-linear model they are using. Indeed, it is common in the learning literature that non-linear models are first linearised around the rational expectations solution before studying their dynamics under learning. Exploring learning and the link with fiscal policy in the original non-linear model has several advantages, though. First, there is no longer the need to linearise the RE model around its steady state. Furthermore, contrary to linearised models, which necessarily lead to a local stability analysis, the context of the original non-linear system allows one to provide a more global stability analysis. Last, due to the non-linearity of the system, it is more natural to use non-linear forecasting rules compared to linearised models. This way, the usefulness of non-linear forecasting rules can be studied as well. As such, one can allow for the possibility of non-linear responses from households.

In this paper, we explore the transitional effects of government spending and the behaviour of the government spending multiplier by introducing learning in the in the original non-linear RBC model. More specifically, we adopt the learning interpretation of the parameterised expectations algorithm (PEA). This algorithm was initially developed as a solution method for non-linear, stochastic models with rational expectations (see for example den Haan and Marcet, 1990; Marcet and Lorenzoni, 1999; Marcet and Marshall, 1994). The idea behind the PEA is to replace the conditional expectations in the equilibrium conditions of the model with flexible functional forms with a finite number of arguments, e.g. polynomials. However, in Marcet and Marshall (1994), the authors also give an alternative, learning interpretation to the solution of the PEA.

We find that learning in the non-linear model leads to substantial time variation in the transmission of structural shocks in the model economy. As such, this result stands in sharp contrast with the RE and PEA solutions of the model. Our set-up thus leads to time-varying government spending multipliers. Furthermore, the time variation in our set-up itself is endogenously determined. As the economic agents update their beliefs, their response to a change in government spending changes as well, leading to a different impact on the economy.

The remainder of this paper is structured as follows. Section 4.2 describes the model. In Section 4.3, the learning mechanism is outlined in detail and compared with the rational expectations solution and the PEA solution of the model. Section 4.4 shows how learning behaviour in our model leads to time variation in the government spending multipliers.

Finally, Section 4.5 concludes.

4.2 Model

To study the transitional dynamics of fiscal policy changes, we use the standard RBC model with elastic labour supply. In this section, we briefly introduce the different components of the model.

4.2.1 Households

The maximisation problem of the representative household consists in maximising

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + b \frac{(1-n_t)^{1-\theta}}{1-\theta} \right] \tag{4.1}$$

subject to its budget constraint:

$$c_t + k_{t+1} = w_t n_t + (1+r_t)k_t - T_t. \tag{4.2}$$

In these equations, c_t represents the household's consumption, k_{t+1} denotes the capital stock, n_t is its labour supply, and w_t and r_t are the real wage and the real interest rate. The latter is equal to the rental charge on capital after depreciation ($r_t = r_t^k - \delta$). Furthermore, b is the taste for leisure, β the discount factor, σ the coefficient of relative risk aversion, and θ the inverse of the inter-temporal elasticity of substitution in leisure. T_t is the lump sum tax in period t.

The optimality conditions of the household with respect to labour and consumption are respectively given by

$$c_t^{-\sigma} = \frac{b\,(1-n_t)^{-\theta}}{w_t} \tag{4.3}$$

and

$$\beta E_t \left\{ c_{t+1}^{-\sigma} \left[r_{t+1}^k + 1 - \delta \right] \right\} = c_t^{-\sigma}. \tag{4.4}$$

4.2.2 Firms

The representative firm produces the final good according to

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{4.5}$$

where z_t evolves according to

$$z_t = z_{t-1}^{\rho_z} \exp(\varepsilon_t^Z). \tag{4.6}$$

with $\rho_z \in (0,1)$. In these equations, $\varepsilon_t^Z \sim \mathcal{N}(0, \sigma_Z^2)$ is an innovation in technology. Profits are given by

$$\mathcal{L} = z_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - r_t^k k_t \tag{4.7}$$

and the corresponding first-order conditions with respect to labour and capital respectively are:

$$w_t = (1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha}, \tag{4.8}$$

$$r_t^k = \alpha z_t k_t^{\alpha-1} n_t^{1-\alpha}. \tag{4.9}$$

4.2.3 Government

The fiscal government finances its expenditures on goods by levying lump sum taxes. Formally, we have

$$g_t = T_t. \tag{4.10}$$

Government spending g_t evolves according to

$$g_t = g_{t-1}^{\rho_g} \exp(\varepsilon_t^G), \tag{4.11}$$

with $\rho_g \in (0,1)$ and where $\varepsilon_t^G \sim \mathcal{N}(0, \sigma_G^2)$ is a government spending shock.

4.3 Real-time non-linear learning

4.3.1 Set-up

We assume that agents, instead of having rational expectations, form forecasts by means of a non-linear learning mechanism. In particular, agents in our model approximate the conditional expectational function in the consumption Euler equation

$$E_t \phi\left(s_{t+1}\right) = E_t\left[c_{t+1}^{-\sigma}\left(1 - \delta + \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha}\right)\right], \tag{4.12}$$

where $s_t = [c_t,\, k_t,\, z_t]$, by a parametric function $\psi\left(x_t,\, \boldsymbol{\gamma}_{t-1}\right)$ of the state variables $x_t = [1, k_t,\, z_t,\, g_t]$, and update the parameters $\boldsymbol{\gamma}_{t-1}$ using a constant-gain variant of recursive least squares. We follow the parameterised expectations literature and use an exponentiated polynomial to approximate the expectational function. More precisely, we consider the following first-order polynomial in the state variables of the model

$$E_t \phi\left(s_{t+1}\right) \simeq \psi\left(x_t, \boldsymbol{\gamma}_{t-1}\right) = \exp\left[\gamma_0 + \gamma_1 \log k_t + \gamma_2 \log z_t + \gamma_3 \log g_t\right]. \tag{4.13}$$

Agents update the vector of belief parameters $\boldsymbol{\gamma}_t$ in real time according to this learning rule:

$$\boldsymbol{\gamma}_t = \boldsymbol{\gamma}_{t-1} + \kappa S_t^{-1} x_{t-1} \left[\log\left(\phi\left(s_t\right)\right) - \log\left(\psi\left(x_{t-1}, \boldsymbol{\gamma}_{t-2}\right)\right)\right], \tag{4.14}$$

$$S_t = S_{t-1} + \kappa \left[x_{t-1} x_{t-1}^{'} - S_{t-1}\right], \tag{4.15}$$

where S_t is the moment matrix for x_t and $\kappa \in (0,1)$ is the gain parameter. If the gain parameter κ would be equal to t^{-1}, equations (4.14)–(4.15) are the recursive formulas of the ordinary least squares (OLS) estimator for the coefficients $\boldsymbol{\gamma}$ in the log-linear specification of equation (4.13), i.e. $\log\left(\phi\left(s_t\right)\right) \simeq \log\left(\psi\left(x_{t-1}, \boldsymbol{\gamma}\right)\right)$. The update for $\boldsymbol{\gamma}$ in equation (4.14) uses the most recent forecast error$\log\left(\phi\left(s_t\right)\right) - \log\left(\psi\left(x_{t-1}, \boldsymbol{\gamma}_{t-2}\right)\right)$.

Instead of adopting the (decreasing-gain) OLS algorithm, where $\kappa = t^{-1}$, we set the gain κ to a small constant. The constant-gain case is the most relevant one for our analysis and is widely used in the adaptive learning literature (see Eusepi and Preston, 2011; Milani, 2007; Slobodyan and Wouters, 2012, for example). As argued by, for example, Sargent (1999), Cho et al. (2002), Branch and Evans (2007), Milani (2007), and Orphanides and Williams (2007), it is a natural way of accomplishing "perpetual learning" as it places a greater weight on more recent observations. Hence, the constant gain makes sure that the variation in the agents' beliefs does not die out over time.

We are now able to formulate the dynamics of our model under non-linear learning. For some initial beliefs $\boldsymbol{\gamma}_0$ and an initial state vector $x_0 = [1,\ k_0,\ z_0,\ g_0]$, substituting the approximating function $\psi\left(x_t, \boldsymbol{\gamma}_{t-1}\right)$ for the expectation function in equation (4.4) gives consumption $c_t = \beta\left[\psi\left(x_t, \boldsymbol{\gamma}_{t-1}\right)\right]^{-1/\sigma}$, equation (4.3) determines labour supply n_t, and k_{t+1} follows from the resource constraint (4.2). The procedure is detailed in Appendix 4.B.

4.3.2 Discussion

Originally, the parameterised expectations approach was used as a method to approximate the rational expectations equilibrium of non-linear stochastic dynamic models. The original parameterised expectations algorithm (PEA) starts from a large sequence of shocks $\{z_t, g_t\}_{t=0}^{T}$, computes the corresponding endogenous variables consistent with the parameterised expectations, and iterates on the vector of parameters $\boldsymbol{\gamma}$ until the approximation becomes sufficiently accurate. A detailed step-by-step description of the algorithm is given in Appendix 4.A.

In this paper, we follow the suggestion of Marcet and Marshall (1994) and interpret the parameterised expectations approach as a real-time learning mechanism. Instead of holding the parameters $\boldsymbol{\gamma}$ constant over the stochastic simulation of the model, we let agents update them according to the learning rule above each time new information becomes available. In doing so we build on Berardi and Duffy (2015), who applied a similar learning mechanism to an optimal growth model.

4.3.3 Non-linear learning simulation

Before turning to the effects of government spending shocks, we compare the non-linear learning solution of the model with two conventional alternatives: the rational expectations solution and the solution provided by the original parameterised expectations algorithm (PEA).[3] Recall that in the latter case, the parameters of the approximating function are held fixed whereas in the non-linear learning model the parameters are updated over time. Given a sequence of 10,000 structural shocks, we generate a series of endogenous variables for these three different solutions.

Our assumptions on the parameter values are listed in Table 2 and are in line with the literature. The baseline value for the gain parameter, κ, is set to 0.02.[4] Given that this is a crucial parameter for the learning dynamics, we discuss the implications of different choices for κ in Section 4.4. The other parameters are set to values commonly used in the literature. The output elasticity of capital α is set to 1/3. According to Rogerson (2007), a reasonable range for θ in models with a macro focus is $[1, 3]$. We set θ equal to 1. The coefficient of risk aversion σ equals 1 and the discount factor β is set to 0.98. The depreciation rate δ equals 0.025. The AR(1) coefficients of technology, ρ_Z, and government spending, ρ_G, are set to 0.9. The share of government spending in GDP, $\bar{g}/\bar{y}$, is set to 0.2. For the standard deviations of the technology and fiscal disturbances, σ_Z and σ_G, the value 0.01 is chosen. The parameter capturing the taste for leisure, b, is determined such that individuals work on average 1/3 of their time endowment.

It is clear from Figure 4.1 that the simulated series of both the learning model and the parameterised expectations algorithm are close to the rational expectations series. The figure shows the evolution of consumption, output, labour, and investment for 500 periods of the full sample.[5] The relatively small differences between the rational expectations and parameterised expectations solution, indicate that the latter is a reasonably good approximation of the former. This comforts us in the choice of the functional form of the

[3] We use the deterministic extended path method to find the rational expectations solution of the model.

[4] This is a value well within the range of estimates reported in the literature. According to Orphanides and Williams (2005, 2007) a gain parameter in the range 0.01–0.04 provides the best fit between the agents' forecasts in the model and the expectations data from the Survey of Forecasters. Using a similar strategy, Branch and Evans (2006) obtain a value of 0.0345. The estimate of Milani (2007) equals 0.0183 and hence lies within the same range. The estimated gain in Eusepi and Preston (2011) is 0.0029. The Bayesian estimation results from Slobodyan and Wouters (2012) provide values for κ going from 0.001 to 0.06 depending on the particular learning scheme.

[5] Investment is defined as $i_t = y_t - c_t - g_t$.

Parameter	Description	Value
α	Output elasticity of capital	$1/3$
β	Discount factor	0.98
σ	Coefficient of risk aversion	1
δ	Depreciation rate	0.025
b	Taste for leisure	1.44
θ	Preference parameter	1
$\bar{g}/\bar{y}$	Steady state government expenditure to output ratio	0.2
ρ_Z	Technology shock AR(1) coefficient	0.9
ρ_G	Government spending AR(1) coefficient	0.9
σ_Z	Standard deviation of the technology disturbance ε^Z	0.01
σ_G	Standard deviation of the fiscal disturbance ε^G	0.01
κ	Gain parameter in the learning mechanism	0.02

Table 4.1: Calibration.

approximating function. The simulation results also illustrate the local stability of the non-linear learning mechanism.[6] Over our long simulation horizon, the recursive estimation of the belief parameters does not drive the economy towards diverging paths. This stability is also reflected in the evolution of the belief parameters over time, depicted in Figure 4.2. Although the parameters can deviate from their PEA values for a sustained period of time, they remain in the neighbourhood of those values.[7]

4.4 The effects of government spending under non-linear learning

In sharp contrast with the rational expectations and PEA solutions of the model, the non-linear learning solution generates time-variation in the transmission of structural shocks in the model economy. In this section we illustrate how this feature provides an attractive mechanism for generating variation in the government spending multipliers over time.

In standard rational expectations models, variation in the transmission mechanism of shocks is typically obtained by allowing structural parameters to vary over time. One common strategy is to estimate a fixed-parameter model on different samples and test for breaks. A downside of this strategy is that the time variation is by assumption infrequent,

[6]Since our analysis is numerical, establishing analytical convergence results is beyond the scope of this paper. Stability results for constant-gain algorithms can be found in (inter alia) Evans and Honkapohja (2001). Loosely speaking, constant-gain learning mechanisms are locally stable only if the gain parameter is sufficiently small. We discuss the sensitivity of our results with respect to this parameter in Section 4.4.

[7]Constant gain learning implies that the parameters do not converge to a point estimate but only to a distribution around the rational expectations beliefs.

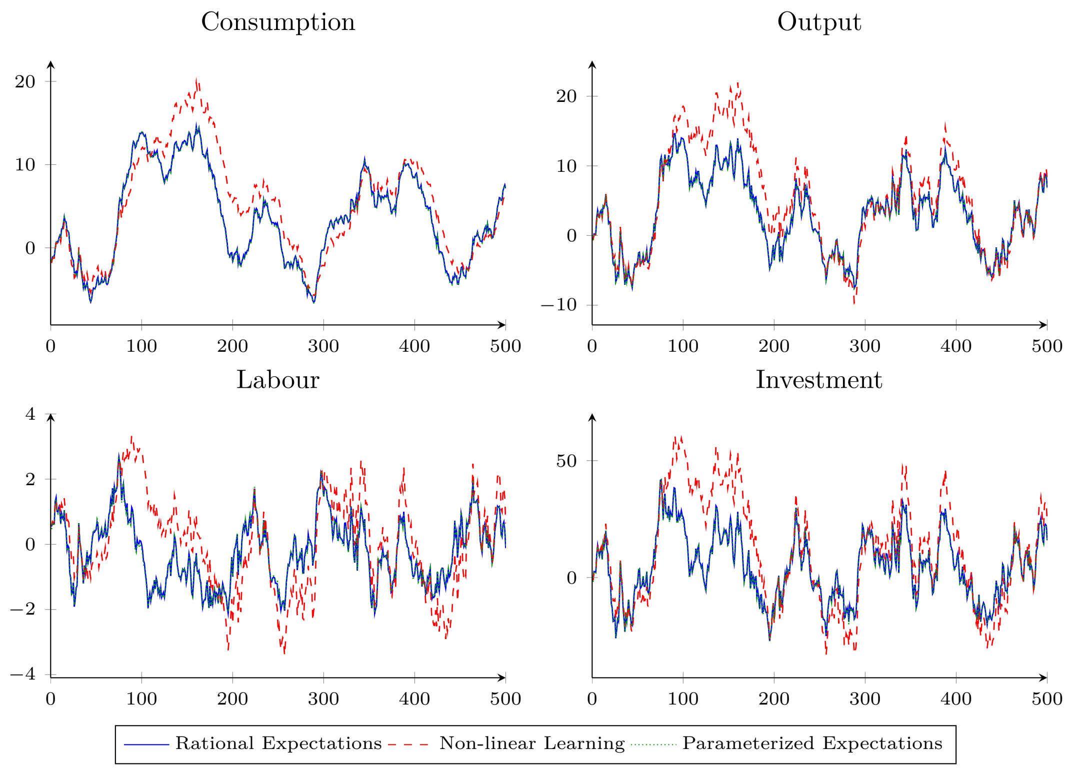

Figure 4.1: Simulated series of model variables under different assumptions regarding the formation of expectations. The series are measured in percentage deviations from the steady state. The horizontal axis measures time in quarters.

whereas our approach allows for time variation at a much higher frequency. Another popular strategy is to use stochastic time-varying coefficient models. In this approach, (some) model parameters are assumed to follow a stochastic process.[8] Both strategies have the disadvantage that the time variation is not endogenously determined by the model.

By contrast, our approach allows for endogenous time variation of the government spending multipliers at a relatively high frequency. Figure 4.3 shows the impact multipliers that correspond to the series in Figure 4.1. The transmission mechanism that underlies the multipliers changes as agents recursively update their beliefs over time. It is well understood that expectations play a crucial role in the impact of fiscal policies, and our modelling approach highlights this expectations channel for the transmission government spending shocks.

Figure 4.4 illustrates the time variation of the government spending multipliers generated by learning behaviour in our model. It reports the distribution of the impact multipli-

[8]Examples of the first strategy are Benati (2008) and Canova (2009); examples of the second strategy include Cogley and Sbordone (2008) and Justiniano and Primiceri (2008). A more extensive overview of the existing literature goes beyond the scope of this paper.

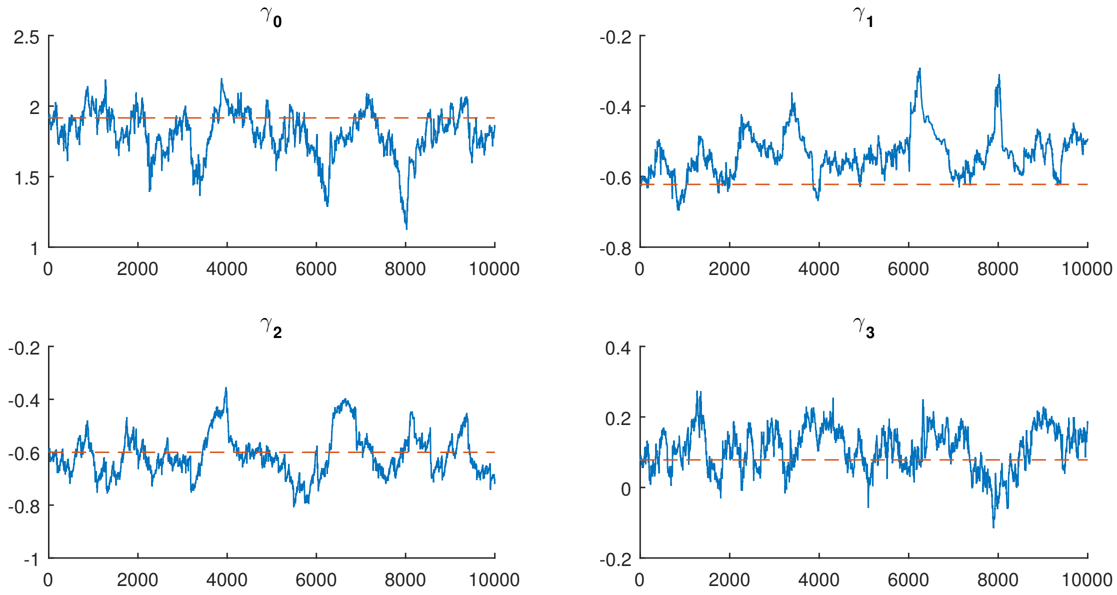

Figure 4.2: Evolution of the belief coefficients in the non-linear learning model. The series of solid lines correspond to the coefficients of the approximating function given by equation (4.13). The dashed lines represent the coefficients provided by the parameterised expectations algorithm.

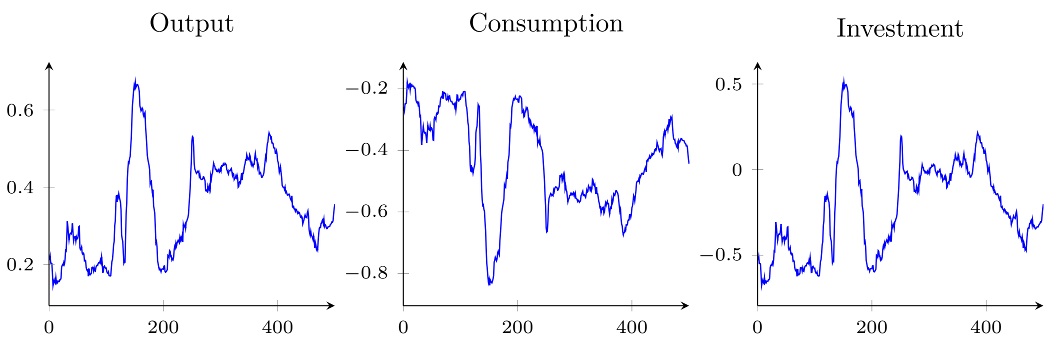

Figure 4.3: Evolution of the impact multipliers in the non-linear learning model. The series represent the impact multipliers in non-linear learning simulation for the same 500 periods as in Figure 4.1.

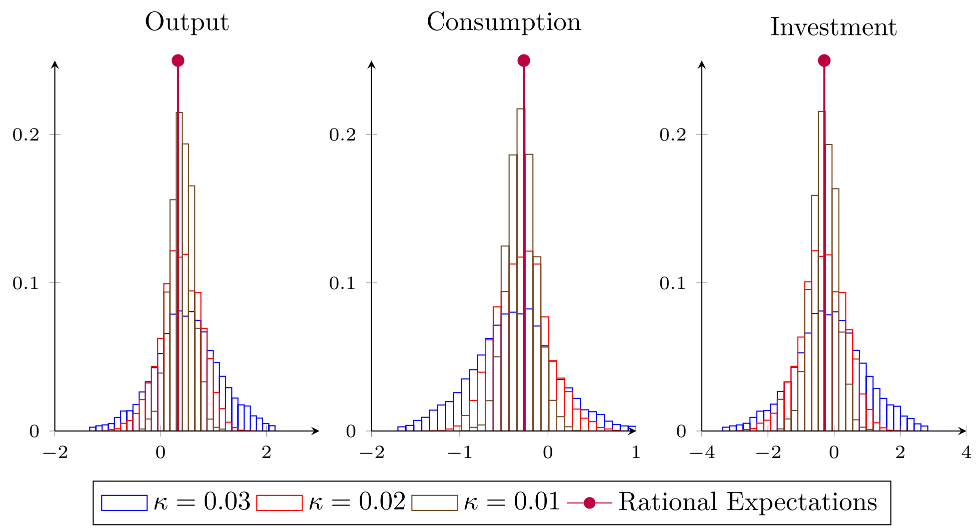

Figure 4.4: Distribution of impact multipliers based on 10 simulations of the non-linear learning model over 10,000 periods. The histograms represent the government spending multipliers for output, consumption, and investment in the non-linear learning model for different values of the gain parameter (κ). The purple lines represent the rational expectations multipliers. The vertical axis measures relative frequencies.

ers for output, consumption, and investment.[9] In the rational expectations equilibrium, the multipliers are time-invariant. The multipliers for output, consumption, and investment are, respectively, 0.33, -0.27 and -0.30. The relatively small output multiplier, mainly driven by a significant crowding out of private consumption, is a standard result of the real business cycle model. Under non-linear learning behaviour, however, the multipliers vary significantly around their rational expectations values. The degree of time variation is governed by the gain parameter κ in the recursive learning rule (4.14) since that parameter determines the volatility of the belief parameters. Overall, learning behaviour in our model generates substantial time variation in the multipliers and a higher gain increases this variation considerably. Figure 4.5, on the other hand, describes the evolution of the government spending multiplier in a sample of 160 periods. This figure illustrates once more that a higher value for κ leads to more time variation. If κ is equal to 0.01, the time variation is very small. This changes drastically when κ is equal to 0.03. Even within the simple RBC model, the government spending multiplier increases from 0.15 to nearly 0.35 over a period of 40 years.

[9] The results are based on 10 simulations of the non-linear learning model over 10,000 periods. For each simulation, the initial parameter vector is the vector provided by the parameterised expectations algorithm.

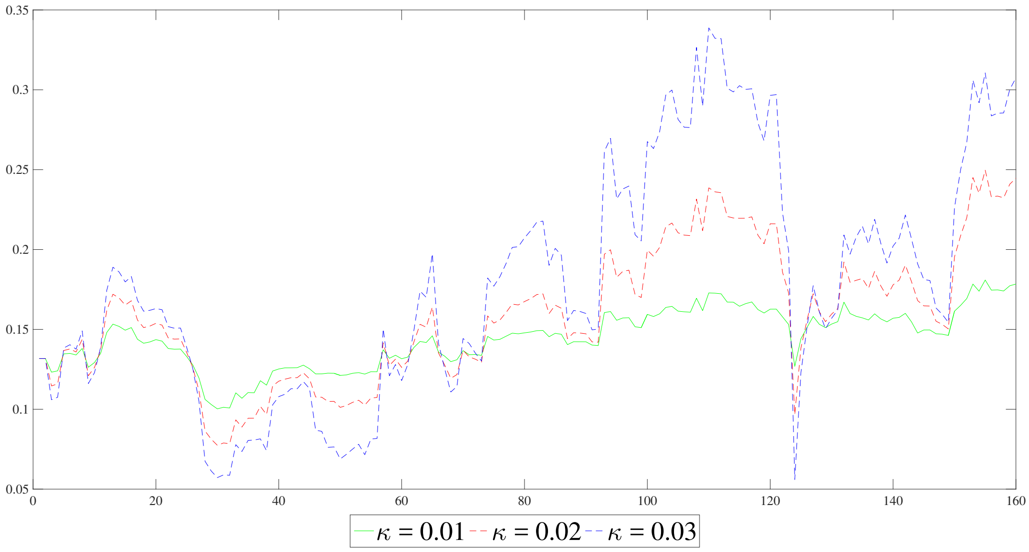

Figure 4.5: Evolution of the impact multipliers in a sample of 160 periods for different values of κ.

4.4.1 Discussion

The previous paragraph showed that our set-up leads to time-variation in the government spending multiplier. This result implies that the effectiveness of an increase in government spending varies over time. For this reason, this paragraph provides some insights into the drivers of the government spending multiplier. These determinants are of particular interest for fiscal policymakers. It allows them to assess the effectiveness of fiscal policy in the economy.

The evolution of the government spending multipliers is mainly driven by the evolution of γ_3, the coefficient for government spending in the approximating function 4.13. If γ_3 is high (low), the government spending multiplier for output is high (low) as well. Intuitively, this result makes sense. If individuals attach a higher weight to government spending in their approximating function, the impact of an increase in government spending on the decision variables will also be stronger. Figure 4.6 illustrates this relationship for the output, consumption, and investment multipliers. If γ_3 is high, the crowding out of private consumption after the government spending shock will be more severe. Hence, the consumption multiplier will be larger. Note, however, that in the context of an RBC model, a small multiplier for private consumption leads to a large multiplier for output. The output multiplier is driven by labour supply and a higher level of consumption leads, ceteris paribus, to a lower level of labour supply.

As the evolution of γ_3 is the most important driver of the multiplier, the next step is to explore the drivers of γ_3. Looking at Equation (4.14), two important channels can be identified. The first one is the forecasting error $\left[\log\left(\phi\left(s_t\right)\right) - \log\left(\psi\left(x_{t-1}, \boldsymbol{\gamma}_{t-2}\right)\right)\right]$. A

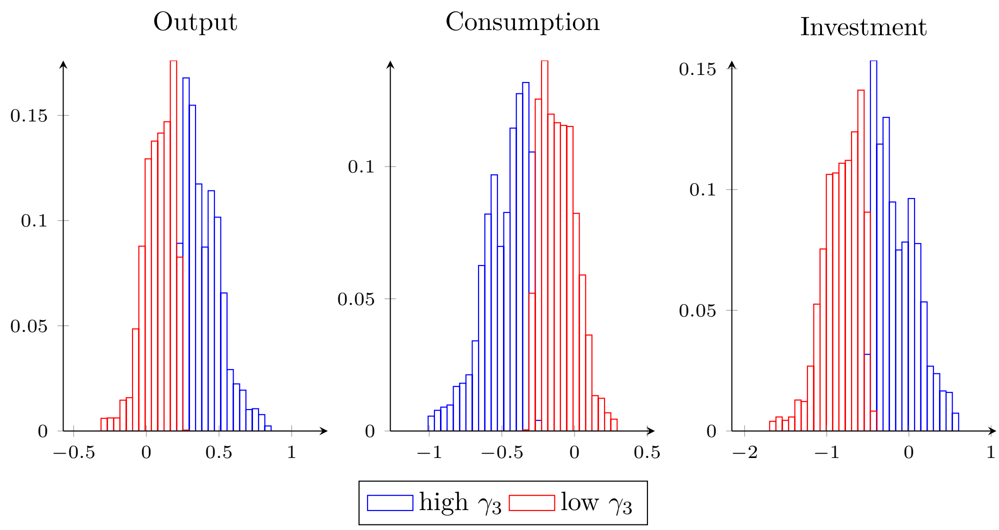

Figure 4.6: Distribution of impact multipliers for high and low values of belief parameter γ_3 . The histograms are based on a simulation of 10,000 periods of the non-linear learning model and represent the government spending multipliers for output, consumption, and investment. The high (low) γ_3 sub-sample contains the multipliers when γ_3 is above (below) its median value. The vertical axis measures relative frequencies.

positive (negative) forecasting error leads to an increase (decrease) in γ_3, everything else equal. The forecasting error itself is determined by the evolution of the technology shocks. More specifically, if $z_t > z_{t-1}$, the forecasting error is negative, whereas if $z_t < z_{t-1}$ it is positive. So, everything else equal, after a period of technological growth, the government spending multiplier is low and after a period of decreasing technology, the government spending multiplier is high. This result is, however, dependent on the level of government spending, which is the second channel. If government spending is above its steady state level, γ_3 will increase for a given level of technology, if it is below its steady state level, γ_3 will decrease for a given level of technology. Furthermore, the further government spending is from its steady state level, the stronger its influence on the evolution of γ_3 and thus the government spending multiplier.

Combined, these results lead to the following insights. The government spending multiplier is likely to be high if technology and government spending relative to its steady state level evolve in opposite direction. More specifically, the multiplier is high (increases) after (i) a considerable period of increasing technology and low government spending on the one hand, and (ii) after a considerable period of decreasing technology and high government spending on the other hand. The multiplier is low (i) after a considerable period of increasing technology and high government spending on the one hand and (ii) after a considerable period of decreasing technology and a low level of government spending on the other hand[10].

We illustrate the aforementioned results using Figure 4.7. In this Figure, we highlight three different episodes during which the government spending multiplier strongly changes. The corresponding levels of technology and government spending are displayed in respectively the middle and bottom panel. Furthermore, the red line in the bottom panel denotes the steady state level of government spending. Here, we discuss the evolution of the government spending multiplier between periods 176 and 204. The other ones follow the same reasoning. At first, technology strongly increases while the level of government spending is below its steady state level. We know that this combination leads to an increase in γ_3. This increase almost directly leads to an increase in the government spending multiplier. After some time, however, technology starts to decrease. At that point, government spending is higher than the steady state level. This combination pushes the government spending multiplier upwards.

4.4.2 Choice of the approximating function: linear versus non-linear learning

In the baseline simulation, we assume that agents approximate the expectation in the consumption Euler equation (cf. equation (4.12)) by the following non-linear function

$$\psi\left(x_t, \boldsymbol{\gamma}_{t-1}\right) = \exp\left[\gamma_0 + \gamma_1 \log k_t + \gamma_2 \log z_t + \gamma_3 \log g_t\right].$$

In this section, we compare this "non-linear" learning approach with "linear learning" where agents use the following linear approximating function

$$\psi\left(x_t, \boldsymbol{\gamma}_{t-1}\right) = \gamma_0 + \gamma_1 k_t + \gamma_2 z_t + \gamma_3 g_t.$$

As for the "non-linear" learning case, the parameters $\boldsymbol{\gamma}_{t-1}$ are updated using the constant-gain recursive least squares formulas (4.14)–(4.15) but the updating term between square brackets in equation (4.14) now becomes $\left[\phi\left(s_t\right) - \psi\left(x_{t-1}, \boldsymbol{\gamma}_{t-2}\right)\right]$.

When comparing the linear with the non-linear learning approach, two interesting observations can be made. First, non-linear learning outperforms linear learning in terms of forecasting performance, especially when (large) shocks drive the economy far away from its steady state. To illustrate this, Table 4.2 reports the root-mean square error (RMSE) of the forecasts under linear and non-linear learning. In a "high volatility" simulation,

[10]Recent contributions to the literature investigate the relationship between the size of the multiplier and the state of the business cycle – see e.g. Auerbach and Gorodnichenko (2012, 2013) and Owyang et al. (2013). We have investigated this relationship using different measures for the business cycle and found that this stylised RBC model is not adequate to uncover a structural relation. Investigating this issue in a more elaborate (more demand-driven) non-linear learning model is a promising direction for future research.

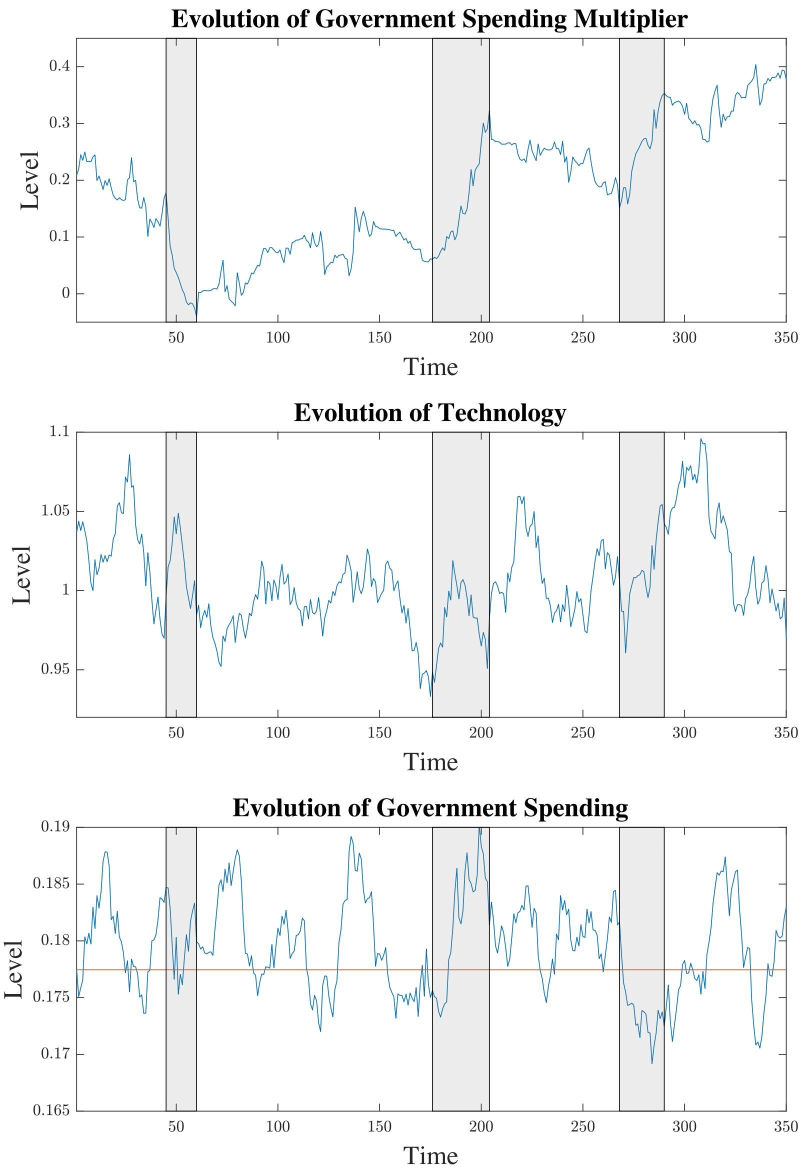

Figure 4.7: Endogenous drivers of the government spending multiplier

Learning scheme	Root-mean-square error	
	Baseline ($\sigma_Z = \sigma_G = 0.01$)	High volatility ($\sigma_Z = \sigma_G = 0.025$)
Non-linear learning	0.010	0.026
Linear learning	0.010	0.034

Table 4.2: Forecasting performance of linear and non-linear learning. Root-mean-squared errors are calculated over 10,000 simulation periods.

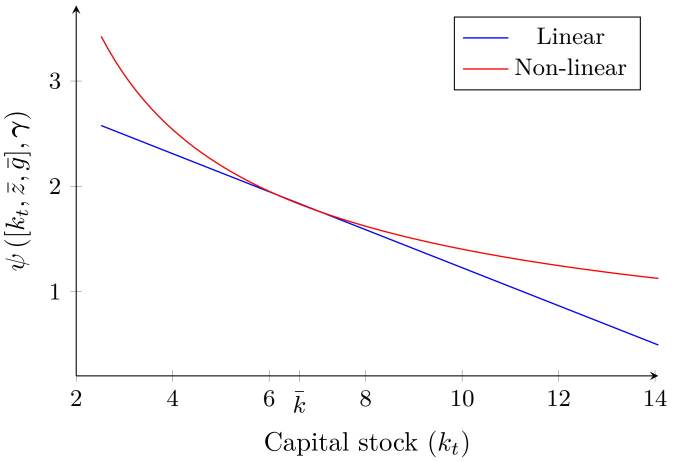

Figure 4.8: Linear and non-linear approximating function for different values of the capital stock. Government spending and technology are fixed at their steady state values and γ equals the coefficient vector of the parameterised expectations algorithm.

when structural shocks to the economy are larger than in the baseline simulation, non-linear learning improves the forecasting performance relative to linear learning. In the baseline simulation, however – when the structural shocks are relatively small – the forecasting performance of non-linear learning is (approximately) equal to linear learning. Hence, when shocks are relatively large, non-linear learning leads to better forecasting. To provide some intuition for this result, Figure 4.8 plots the linear and non-linear approximating function for different values of the capital stock. For values close to the steady state ($k = \bar{k}$), the approximation by the linear and non-linear function is very similar. For capital (and government spending) far away from the steady state, the linear and non-linear approximation differ quite substantially. In both cases, agents can update the coefficients of the approximating function to improve their forecasting, but the results presented in Table 4.2 show that non-linear learning outperforms linear learning in terms of RMSE.

Second, the linear learning approach also generates time variation in the government spending multipliers. Figure 4.9 shows how the impact multipliers for output, consump-

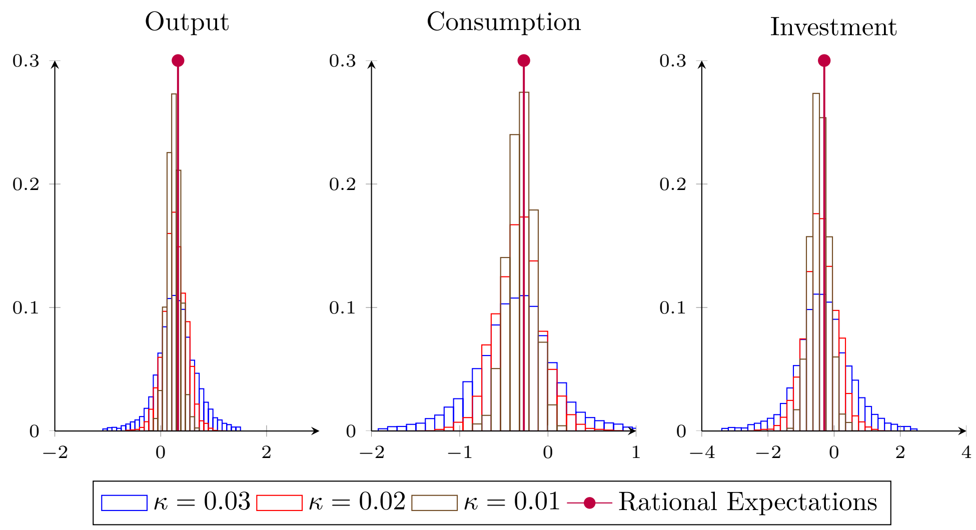

Figure 4.9: Distribution of impact multipliers based on 10 simulations over 10,000 periods of the model with a linear approximating function. The histograms represent the government spending multipliers for output, consumption, and investment for different values of the gain parameter (κ). The purple lines represent the rational expectations multipliers. The vertical axis measures relative frequencies.

tion, and investment again fluctuate around the rational expectations multipliers, but their values are less dispersed. Under non-linear learning, the updating of the belief coefficients generate more variation in the consumption response after the government spending shock. Consequently, the effects on output and investment will also vary more.

4.5 Conclusion

In this paper, we explore the transitional dynamics following fiscal policy changes in a stochastic macroeconomic framework where agents use adaptive learning to update non-linear forecast rules to form expectations. Several papers have already studied the effect of fiscal policy using a learning framework (see e.g. Evans and Honkapohja, 2009; Gasteiger and Zhang, 2014; Benhabib et al., 2014). All of these papers, however, use linear forecast rules. In this paper, we apply a different approach. Following, inter alia, Marcet and Marshall (1994) and Berardi and Duffy (2015), we interpret the parameterised expectations algorithm (PEA) as a real-time learning mechanism used by agents to update their expectations over time in the original non-linear model.

Our main contribution is to study the dynamics of government spending in the standard RBC model where agents use this non-linear learning mechanism to form expectations. To the best of our knowledge, ours is the first study to compare the transitional dynamics resulting from this framework with the dynamics under rational expectations.

In our non-linear learning set-up, the effects of government spending shocks vary substantially over time. We have shown that this variation is endogenously driven by agents' expectations about the future. The resulting fluctuations in the government spending multipliers are in marked contrast to the time-invariant multipliers of the rational expectations solution of the model. We also show that the forecasting performance of the learning mechanism is better if agents use a non-linear approximating function instead of a linear one. In particular in the context of relatively high structural shocks which drive the economy far away from its steady state, using a non-linear approximating function to form expectations is advantageous.

Our findings provide several avenues for future research. First, it may be very fruitful to go beyond the standard RBC model. Understanding how learning affects the variability of fiscal multipliers in more elaborate models is an important topic. It may, for example, shed further light on the dependency of multipliers on the state of the business cycle and the stance of monetary policy. Second, the non-linear learning model provides a natural framework for studying the consequences of a structural change in fiscal policy. Comparing the dynamics of this learning model with the rational expectations dynamics is, in our view, a promising direction for further research.

Appendices

Appendix 4.A Parameterised expectations algorithm

Under the parameterised expectations algorithm (PEA) the vector of coefficients $\boldsymbol{\gamma}$ of the approximating function $\psi(\cdot, \boldsymbol{\gamma})$ is calculated with the following iterative procedure.

Step 1 Draw a large sequence of shocks $\left\{\epsilon_t^G, \epsilon_t^Z\right\}_{t=0}^T$ and compute $\{g_t, z_t\}_{t=0}^T$ as defined in (4.6) and (4.11).

Step 2 Choose an initial guess $\boldsymbol{\gamma}_0$ and an initial value k_0 for the capital stock.

Step 3 At iteration $i \in \{0, \ldots, i_{max}\}$, use $\boldsymbol{\gamma}_i$ to generate the endogenous variables $\{c_t(\gamma_i), n_t(\gamma_i), k_{t+1}(\gamma_i)\}_{t=0}^T$. In particular, (i) substituting the approximating function $\psi(x_t, \boldsymbol{\gamma}_i)$ for the expectation function in equation (4.4) gives consumption $c_t = \beta\left[\psi(x_t, \boldsymbol{\gamma}_i)\right]^{-1/\sigma}$, (ii) the first-order condition (4.3) determines labour supply n_t, and (iii) k_{t+1} follows from the resource constraint (4.2).

Step 4 Use the data for $t = 0, \ldots, T-1$ to run the non-linear least squares regression of $c_{t+1}^{-\sigma}\left(1 - \delta + \alpha z_{t+1} k_{t+1}^{\alpha-1} n_{t+1}^{1-\alpha}\right)$ on the approximating function $\psi(x_t, \cdot)$ to obtain an estimate $\hat{\boldsymbol{\gamma}}$.

Step 5 Use this estimate to update the guess for $\boldsymbol{\gamma}$ according to

$$\boldsymbol{\gamma}_{i+1} = (1 - \mu)\boldsymbol{\gamma}_i + \mu\hat{\boldsymbol{\gamma}}, \tag{4.16}$$

where $(0, 1]$ is a damping parameter.

Step 6 Apply steps 3-5 iteratively until the convergence criterion $\sum_k \left|\gamma_{i+1}^k - \hat{\gamma}^k\right| < \tau$ is met, where k is the number of parameters in $\boldsymbol{\gamma}$.

In our experiments, we set $T = 10,000$, $i_{max} = 200$, $\mu = 0.8$, and $\tau = 1 \times 10^{-5}$. The initial value for the capital stock is the steady state: $k_0 = \bar{k}$.

Appendix 4.B Learning algorithm

To simulate the learning model, we follow the following steps.

Initialisation

1. Draw a sequence of shocks $\left\{\epsilon_t^G,\ \epsilon_t^Z\right\}_{t=0}^S$ and compute $\{g_t,\ z_t\}_{t=0}^S$ as defined in (4.6) and (4.11).

2. Choose an initial guess $\boldsymbol{\gamma}_0$ and an initial value k_0 for the capital stock.

Simulation Simulate the model S periods forward using the following scheme.

1. Approximate the conditional expectational function in the consumption Euler equation
$$E_t\phi\left(s_{t+1}\right) = E_t\left[c_{t+1}^{-\sigma}\left(1-\delta+\alpha z_{t+1}k_{t+1}^{\alpha-1}n_{t+1}^{1-\alpha}\right)\right], \tag{4.17}$$
by the parametric function $\psi\left(x_t,\ \boldsymbol{\gamma}_{t-1}\right)$ of the state variables $x_t = [1, k_t,\ z_t,\ g_t]$.

2. Calculate the corresponding endogenous variables determined by the following system
$$c_t = \beta\left[\psi\left(x_t, \boldsymbol{\gamma}_{t-1}\right)\right]^{-1/\sigma}, \tag{4.18}$$
$$c_t^{-\sigma} = \frac{b\left(1-n_t\right)^{-\theta}}{w_t}, \tag{4.19}$$
$$w_t = (1-\alpha)z_t k_t^{\alpha} n_t^{-\alpha} \tag{4.20}$$
$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{4.21}$$
$$i_t = y_t - c_t - g_t, \tag{4.22}$$
$$k_{t+1} = \left(1-\delta\right)k_t + i_t. \tag{4.23}$$

3. Update the vector of parameters $\boldsymbol{\gamma}_{t-1}$ according to
$$\boldsymbol{\gamma}_t = \boldsymbol{\gamma}_{t-1} + \kappa R_t^{-1} x_{t-1}\left[\log\left(\phi\left(s_t\right)\right) - \log\left(\psi\left(x_{t-1}, \boldsymbol{\gamma}_{t-2}\right)\right)\right], \tag{4.24}$$
$$R_t = R_{t-1} + \kappa\left[x_{t-1}x_{t-1}^{'} - R_{t-1}\right]. \tag{4.25}$$

Chapter 5

Beyond Rational Expectations: The Effects of Heuristic Switching in an Overlapping Generations Model[1]

Abstract

I explore the transitional dynamics in an Overlapping Generations framework with and without heuristic switching. Agents use these simple heuristics to forecast the interest rate and the real wage. The fraction of agents using a specific heuristic depends on its relative forecasting performance. In the absence of heuristic switching, the results indicate that there is a lot of variation in the transitional dynamics over different parameter values and heuristics. They might even oscillate or diverge. Including heuristic switching has two advantages. First, it decreases the variation in the transitional dynamics significantly. Second, it has a stabilising effect on oscillating or diverging transitional dynamics.

Keywords: Heuristic Switching, Heterogeneous Agents, Fiscal Policy, Transitional Dynamics, Overlapping Generations

JEL classification codes: D83, D84, E03

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5.1 Introduction

This paper explores the stability and transitional dynamics of an Overlapping Generations model with heuristic switching. Accordingly, this paper contributes to a growing literature that goes beyond the rational expectations paradigm. After all, the transitional and equilibrium impact of public policy, for instance, on the macroeconomy depends crucially on the behavioural response of households to these policies. An important determinant of this behavioural response is the procedure households apply to form expectations regarding the future course of different variables. Moreover, these expectations themselves are typically a key determinant of the current realisation of these variables. Several authors have argued that assuming that economic agents have rational expectations (RE) is unrealistic. An alternative is provided by the learning literature (see e.g. Evans and Honkapohja, 2001). In this literature, boundedly rational agents form expectations using a perceived law of motion. In the lion's share of this literature, agents act as econometricians who update the coefficients of their perceived law of motion as new realisations of the variables of interest become available over time.

A number of papers studied the effects of least-squares adaptive learning within an Overlapping Generations framework (henceforth OLG) - see e.g. Schönhofer (1998), Adam (2003), Tuinstra (2003) and Tuinstra and Wagener (2007). In Schönhofer (1998), for example, it is shown that if one explicitly considers learning in a monetary OLG model, the dynamic system exhibits chaotic behaviour. Tuinstra (2003), on the other hand, introduces the notion of beliefs equilibria. These are equilibria where the belief of the agents best fits the time series data, which itself is generated by the model where agents have this belief. Although the learning dynamics might converge, the author shows that the corresponding inflation dynamics might be erratic. Different from these studies, Chen et al. (2008) study the dynamic behaviour of an OLG model with capital accumulation under three different types of expectations: rational, myopic and adaptive expectations. They conclude that the dynamics can be complex when using the latter two types. Moreover, the dynamic properties of the model crucially depend on the value of the intertemporal elasticity of substitution in consumption and, in the case of adaptive expectations, on the weight agents attach to past observations when forming expectations.

All these papers have enriched our knowledge of the properties that characterise OLG models when moving beyond the scope of rational expectations. The majority of the papers using an OLG framework, however, focus on the (mostly local) stability properties of the equilibrium, not on the transitional dynamics following policy shocks. A second shortcoming is that often only one forecasting rule is studied. If the agents act as econometricians, they constantly update the coefficients of the same equation, but they cannot distinguish or switch between different rules. Furthermore, even if multiple rules are studied simultaneously, it is virtually always assumed that all agents use the same rule at a given point in time. In these papers, the focus often lies exclusively with one-period-ahead forecasts as well, ruling out the possibility of multi-period-ahead forecasts.

It is, however, plausible that a fraction of the economic agents does not have the cognitive capacities to act as an econometrician. Just as it can not be ruled out a priori that different forecasting rules are being used by the economic agents at one point in time. So then, what is the macroeconomic impact if one would assume that economic agents use simple rules to forecast wages and interest rates? Additionally, how do the transitional dynamics behave when individuals have multiple rules to choose from? And are the transitional dynamics sensitive to the rule being used? Finally, how do they compare to the transitional dynamics in the rational expectations case?

In this paper, I assume that agents use such simple rules, heuristics, to forecast the future course of the interest rate and the real wage. Agents use these heuristics because, in general, they do not possess the cognitive abilities as assumed by the RE literature nor to act as econometricians. Instead, the agents have a certain number of different heuristics at their disposal. On a regular basis, they assess the predictive power of the heuristic they are currently applying. If it performs well, the probability that agents will use the same heuristic in the next period will be higher. If it does not perform well, there is a higher probability that they switch to another rule.

The framework used to answer these questions is in line with the evidence provided by several laboratory experiments - see e.g. Adam (2007), Hommes (2011) , Heemeijer et al. (2012) and Hommes (2014). In Heemeijer et al. (2012), for example, the authors use a standard OLG framework to conduct an individual experiment in order to assess the ability of individuals to form expectations and the degree to which these individuals learn about the accuracy of their forecasts. In the experiment, participants are asked to submit fifty one-step-ahead forecasts for the inflation rate. Over time, the participants also observe the actual realisations of the inflation rate. These can be used by the participants to forecast the remaining future inflation rates. The authors argue that their experimental results cannot be explained using the rational expectations approach. Rather, they are consistent with the use of constant gain algorithms or average expectations. Their results also indicate that individuals switch between different heuristics according to the relative forecasting performance of these rules.

The aim of this paper is to contribute to the literature that goes beyond the rational expectations paradigm by exploiting the heuristic switching approach within an Overlapping Generations model. Triggered by a fiscal policy shock, the objective is to study the transitional dynamics of the model for a large number of settings including one or multiple heuristics and compare the behaviour of the dynamics with their rational expectations counterpart.

The simulations lead to three main findings. First, in a context without heuristic switching (i.e. a context where individuals only have one heuristic at their disposal to form expectations), the evolution of transitional dynamics can be substantially different from the rational expectations case, especially in the first periods of the transition. Rational expectations is thus not always a good approximation. Furthermore, there is a lot of variation in the dynamics over different parameter values and heuristics. This finding implies that if only heuristic is used, the macroeconomic impact of fiscal policy is highly sensitive to the heuristic being used. What is more, as the discount rate and the degree of risk aversion decrease, the model becomes unstable and the corresponding transitional dynamics oscillate or even diverge.

Second, after activating the heuristic switching regime, the variation in the transitional dynamics decreases significantly. Consequently, the sensitivity of the transitional effects of fiscal policy is much lower now and its exact impact is thus less uncertain for policy makers.

Third and last, the heuristic switching has a stabilising effect on the transitional dynamics. For certain configurations of the parameter values for which the dynamics were very unstable in the absence of heuristic switching, the dynamics now converge to the steady state in most cases.

These findings are important. They imply that allowing individuals to choose from a wide range of forecasting rules is actually a better option than constraining them to use only forecasting rule. It allows them to select the rules that perform relatively better, a feature that not only enhances the stability of the model, but reduces the uncertainty in the transitional dynamics as well. It implies that going beyond rational expectations does not lead to a wide range of possible trajectories. Furthermore, it turns out that rational expectations is a better approximation when the switching mechanism is activated.

The remainder of this paper is structured as follows. Section 5.2 outlines the different model blocks. Section 5.3 focuses on the heuristic switching. In Section 5.4, I provide some details on the timing in the model. The calibration and parameterisation of the model is described in Section 5.5. Section 5.6 consists of the description and a detailed look into the results of the different simulations. Section 5.7 concludes.

5.2 Model

I consider a closed economy in which time is discrete and runs from 0 to ∞. Each period lasts for 4 years. At each moment in time, the economy is populated by J overlapping generations. The model consists of three actors: heterogeneous agents, firms, and a fiscal government. Markets are incomplete meaning that individuals cannot explicitly insure themselves against productivity shocks.

5.2.1 Demographics

At the beginning of each period, a continuum of new agents with measure one enters the model. Individuals have an uncertain lifespan. They face an age-specific survival probability φ_j between the age of j and $j+1$. The demographics of the model are exogenous and given by:

$$N_{j+1,t+1} = \varphi_j N_{j,t} \tag{5.1}$$

where $N_{j,t}$ represents the number of individuals of age j at time t. Every individual who survives J periods will die with certainty after the J-th period.

5.2.2 Individuals

Individuals enter the model at the age of 18. Ex-ante, before any decisions are made, individuals only differ with respect to the heuristic they apply to form expectations. Furthermore, the economic agents face idiosyncratic income risk during their active period of life. At any given point in time, individuals are characterised by a state vector (j, a, η, h), where j is the age of the agent, a the accumulated non-human wealth at the beginning of period t, η the productivity shock and h the heuristic that the agents is currently applying. Let $\Phi_{j,t}(a, \eta, h)$ denote the share of agents aged j of type (a, η, h) at date t. For each t and j we have $\int \Phi_{t,j}(da \times d\eta \times dh) = 1$.

Individuals choose sequences of (n, c, a'), i.e. labor supply, consumption and accumulated non-human wealth, to maximise their expected lifetime utility. The latter is given by

$$U = E\left\{\sum_{j=1}^{J} \beta^{j-1} \frac{\left(c_j{}^{(1-\mu)}(1-n_j)^{\mu}\right)^{1-\theta}}{1-\theta}\right\}. \tag{5.2}$$

The share of consumption is given by $1-\mu$. The degree of relative risk aversion is governed by θ. The time discount factor is denoted by β. Individuals reaching the age of J_R retire.

The dynamic budget constraint of an individual aged $j < J_R$ with state (a, η, h) at time t is given by

$$(1+\tau_c)c_j + a'_j = w_t \eta \varepsilon_j n_j (1-\tau) + (1 + r_t(1-\tau_k))(a_j + Tr_t) \tag{5.3}$$

He or she earns an after-tax wage of $w_t \varepsilon_j \eta n_j (1-\tau)$, where w_t is the real wage per unit of effective labour at time t, ε_j is an age-specific productivity parameter, η is the labour productivity shock and τ is the average tax rate on labour income. The consumption tax rate is τ_c. The real interest rate is given by r_t. Individuals pay taxes on capital income where the capital tax rate is denoted by τ_k. Individuals enter the model without wealth and leave no intentional bequests. Due to accidental bequests, individuals receive a transfer Tr_t from the government. The accumulated non-human wealth at the end of the period is denoted by a'. I impose that individuals are not able to borrow: $a' \geq 0$. This individual maximises the following recursive problem:

$$V(j, a, \eta, h) = \max_{c_j, n_j, a'_j} U(c_j, n_j) + \beta \varphi_j \sum_{\eta'} \pi(\eta'|\eta) V(j+1, a', \eta', h) \tag{5.4}$$

The stochastic process regarding the labor productivity shock is denoted by $\pi(\eta'|\eta)$. The heuristic used is given by h. From the age of J_R onwards, individuals receive a public pay-as-you-go pension. Their budget constraint for the ages $j \geq J_R$ is given by

$$(1+\tau_c)c_j + a'_j = (1 + r_t(1-\tau_k))(a_j + Tr_t) + pp_t \tag{5.5}$$

The maximisation problem is now given by

$$V(j, a, h) = \max_{c, a'} U(c_j) + \beta \varphi_j V(j+1, a', h) \tag{5.6}$$

A final note on the basic PAYG-pension pp_t received by the retired households in the model. For simplicity, it is assumed that all individuals receive the same pension, i.e. a fraction b_p of the average after-tax wage in the economy.

5.2.3 Firms

The production function of the representative firm is given by

$$Y_t = A K_t^{\alpha} L_t^{1-\alpha} \tag{5.7}$$

where A is the level of technology that assumed to be constant over time, K_t is the capital used by the firm, and L_t is given by

$$L_t = \sum_{j=1}^{J_R-1} N_{j,t} \int n_j(a,\eta,h)\eta\varepsilon_j \Phi_{t,j}(da \times d\eta \times dh) \tag{5.8}$$

5.2.4 Government

Government expenditures on goods and public pensions are financed by taxes on labour, capital and consumption. The fraction of output that is devoted to government consumption g_c is adjusted such that the government budget is balanced every period. Formally, the government budget constraint is given by

$$G_{c,t} + P_t = T_{n,t} + T_{c,t} + T_{k,t} \tag{5.9}$$

$$\text{with:} \begin{cases} G_{ct} = g_c Y_t \\ P_t = \sum_{j=j_R}^{J} N_{j,t} \int pp_t \Phi_{j,t}(da \times dh) \\ T_{kt} = \tau_k r_t K_t \\ T_{nt} = \tau \sum_{j=1}^{J_R-1} N_{j,t} \int w_t n_j(a,\eta,h)\eta\varepsilon_j \Phi_{j,t}(da \times d\eta \times dh) \\ T_{ct} = \tau_c \sum_{j=1}^{J} N_{j,t} \int c_j(a,\eta,h) \Phi_{j,t}(da \times d\eta \times dh) \end{cases}$$

5.3 Heuristic switching

In this paper, I take the view that economic agents have limited cognitive capabilities. In such a world, individuals use simple rules, heuristics, to forecast the evolution of aggregate macroeconomic variables and to form expectations. Notwithstanding their limited cognitive capabilities, economic agents are willing to learn from their mistakes. To this end, I combine the use of heuristics with a trial-and-error learning approach[2]. More specifically, economic agents have different heuristics at their disposal and they endogenously select the heuristic or forecasting rule that performed the best in previous periods. On a regular basis, individuals assess the predictive power of the heuristic they are currently using vis-à-vis the predictive power of the other rules. If the current rule performs well, the probability that an individual will keep on using the same rule is higher. If not, there is a higher probability that he or she will switch.

[2]For an in-depth analysis of the use of heuristics and heuristic switching, I refer the reader to De Grauwe (2012), Heemeijer et al. (2012) and Hommes (2014).

The different heuristics at the disposal of an individual to form expectations are the following[3]:

$$r^{1,e}_{t+s,t} = r^{1,e}_{t,t-1} + \psi_1(r_t - r^{1,e}_{t,t-1}) \text{ (Adaptive (1))} \tag{5.10}$$

$$r^{2,e}_{t+s,t} = r_t + \psi_2(r_t - r_{t-1}) \text{ (Trend (1))} \tag{5.11}$$

$$r^{3,e}_{t+s,t} = \frac{1}{\psi_3}\sum_{j=0}^{\psi_3 - 1} r_{t-j} \text{ (Average (1))} \tag{5.12}$$

$$r^{4,e}_{t+s,t} = \sum_{j=0}^{\psi_4 - 1} \phi_j r_{t-j} \text{ (Average (2))} \tag{5.13}$$

$$r^{5,e}_{t+s,t} = r_t \text{ (Myopic)} \tag{5.14}$$

with $s \in \{1, ..., J\}$. In these equations, both actual realisations (r_t) and expected values ($r^{h,e}_{t+s,t}$) of the interest rate are given. The subscript t in r_t denotes the historical period t in which the realisation occurred. On the other hand, $r^{h,e}_{t+s,t}$ is the expectation at time t of the interest rate in period $t+s$ using heuristic h. For example, $r^{1,e}_{t,t-1}$ is the expectation at time $t-1$ using the first heuristic of the interest rate at time t.

The first heuristic boils down to an adaptive expectations approach to form forecasts. It states that the expectation of an individual regarding the evolution of the interest rate equals $r^{1,e}_{t,t-1}$, the expectation of the interest rate in the current period t made at time $t-1$, and a fraction ψ_1 of the forecast error, i.e. the difference between the actual realisation of the interest rate r_t and $r^{1,e}_{t,t-1}$. The second one is a trend rule. Here, $r^{2,e}_{t+1,t}$ equals the actual realisation of the interest rate r_t plus a fraction ψ_2 of the difference between the current and previous realisation of the interest rate. Agents expect higher interest rates in the future when the current interest rate r_t is higher than the previous interest rate r_{t-1} and vice versa. When ψ_1 and ψ_2 are low, individuals are less inclined to adjust their expectations. When ψ_1 and ψ_2 are high, individuals will be more inclined when adjusting their expectations. The third heuristic implies that the expected interest rate for the next period equals an unweighted average of the last ψ_3 realisations of the interest rate, while in the fourth heuristic $r^{4,e}_{t+1,t}$ is determined using a weighted average of the last ψ_4 realisations of the interest rate. Finally, the fifth heuristic is equivalent to the use of

[3]In this section, I only provide the different heuristics for the evolution of the interest rate. Note that agents form expectations about wages as well using the same heuristic, and the heuristics that they use to do so are equivalent to the ones stated in this section. Furthermore, these heuristics are based on the heuristics provided in Hommes (2014).

myopic expectations. Individuals simply assume that the interest in the next period will equal the current realisation of the interest rate.

Agents using heuristic h at t use this heuristic to form $r^{h,e}_{t+s,t}$ ($s \in \{1, .., J-j\}$, J denoting the maximum age an individual can reach and j denoting the actual age), i.e. the expected values at time t of the interest rate and the real wage in the remaining periods of their life. In period $t+1$, they will update their expectations of these values as new information becomes available. For the last three heuristics (Equations (5.12-5.14)), I assume that the agents using these heuristics expect that the new value applies for the remainder of their life. Thus, for example, if the individual has at most three more periods to live after the current period, expectations for these periods held at time t are the following: $r^{h,e}_{t+1,t} = r^{h,e}_{t+2,t} = r^{h,e}_{t+3,t}$. In the next period, however, the updated expected values for the last two periods of life ($r^{h,e}_{t+2,t+1} = r^{h,e}_{t+3,t+1}$) might differ from the expected values for these periods at time t ($r^{h,e}_{t+2,t} = r^{h,e}_{t+3,t}$). In other words, agents assume that the expected value at time t applies for the remainder of their life. In time $t+1$, the expected value itself might change, but they still assume that this new expected value applies for the remainder of their life.

For the first two heuristics, I use two different versions. In one version, I use the same assumption as for the last three heuristics, namely that the forecast for the next period $r^{h,e}_{t+1,t}$ equals the forecast for the subsequent periods as well. In the second version, though, to form expectations for the periods $t+2, t+3, t+4, ...$ the heuristics are adjusted as follows:

$$r^{6,e}_{t+s,t} = r^{6,e}_{t+s-1,t} + \frac{\psi_1}{\psi_5 s}(r_t - r^{6,e}_{t,t-1}),\ s \in \{1, ..., J-j\}\ ,\ \text{(Adaptive (2))} \tag{5.15}$$

$$r^{7,e}_{t+s,t} = r^{7,e}_{t+s-1,t} + \frac{\psi_2}{\psi_6 s}(r_t - r_{t-1}),\ s \in \{1, ..., J-j\}\ ,\ \text{(Trend (2))} \tag{5.16}$$

where s is the number of remaining periods of life. Heuristic 6 implies that the individual observes the forecast error $r_t - r^{6,e}_{t,t-1}$ and acknowledges that he or she might make forecasting errors in the future. Using the forecast error, he or she iterates forward to form expectations for the interest rate in period $t+j$. The term $\psi_5 s$ then captures to what extent he or she thinks that they will make the same forecasting error in the future. For the last heuristic, the basic idea is the same, but now it captures to what extent an individual thinks that the trend will continue in the future.

In total agents have seven different heuristics at their disposal. Consistent with the literature on Heuristic Switching Models, the performance of heuristic $h \in (1, 2, 3, 4, 5, 6, 7)$ is measured by the squared prediction error of that specific rule $\Theta_{t,h}$ in a specific period t:

$$\Theta_{t,h} = -(r_t - r^{h,e}_{t,t-1})^2 + \upsilon\Theta_{t-1,h} \tag{5.17}$$

All that is left to be specified is the fraction of agents using a specific heuristic h. Using a discrete choice model, this is given by

$$\Gamma_{h,t} = \xi\Gamma_{h,t-1} + (1-\xi)\frac{exp(k\Theta_{t-1,h})}{\sum_{h=1}^{7} exp(k\Theta_{t-1,h})} \tag{5.18}$$

Here, $\Gamma_{h,t}$ measures the fraction of individuals using heuristic h at period t. This means that $\sum_{h=1}^{7}\Gamma_{h,t} = 1$. Furthermore, υ ∈[0,1] is a parameter measuring the memory of the economic agents. The lower υ, the less economic agents take past periods into account when comparing heuristics. Furthermore, $k \geq 0$ is the intensity of choice. The larger k, the faster agents switch between heuristics. The last parameter is $\xi \in [0,1]$, measuring inertia. If this parameter is low, economic agents switch less to other heuristics even if they clearly perform better. In other words, the habit of using a certain heuristic is stronger.

5.4 Timing

Each period, a number of decisions have to be made by the individuals populating the economy. These sequential steps are:

1. Given their expectations, individuals decide on the amount of labour they want to supply to the labour market.

2. Based on K_t and L_t, r_t and w_t are determined. These values of w_t and r_t do not change the value of n. Labour has already been supplied to the labour market.

3. Individuals receive their labour (based on n as determined in step 2) and capital income. Afterwards, they decide on c and a'.

4. They evaluate the heuristic they are applying using Equation (5.17). Afterwards, the heuristic switching takes place.

5. Having observed the actual realisation of the interest rate and the wage, individuals update their expectations about the future values of the interest rate and the real wage rate using the heuristic they are using.

5.5 Definition of Equilibrium

Let $\Phi_{j,t}(a,\eta,h)$ denote the share of agents aged j at time t with state (a,η,h). For each j and t we have $\int \Phi_{j,t}(da \times d\eta \times dh) = 1$.

Definition 1 Given an initial capital stock K_0, a given vector of exogenous fiscal policy variables $\{\tau_{c,t}, \tau_{k,t}, \tau_t, b_{p,t}\}_{t=1}^{\infty}$ and initial measures $\{\Phi_{j,0}\}_{j=1}^{J}$, an intertemporal equilibrium consists of sequences of value and policy functions $\{V_t, a'_{j,t}, c_{j,t}, n_{j,t}\}_{t=0}^{\infty}$, sequences of transfers $\{Tr_t\}_{t=0}^{\infty}$, sequences of prices $\{w_t, r_t\}_{t=0}^{\infty}$, sequences of expectations $\left\{r_{t+k,t}^{h,e}, w_{r+k,t}^{h,e}\right\}_{t=1}^{\infty}$, sequences of taxes, social security policies, tax aggregates and government spending $\{G_c, T_{n,t}, T_{k,t}, T_{c,t}, pp_t\}_{t=1}^{\infty}$, aggregate variables $\{Y_t, K_t, L_t\}_{t=0}^{\infty}$ and sequences of measures $\{\Phi_{j,t}\}_{t=0}^{\infty}$ such that:

1. Given expectations $r_{t+k,t}^{h,e}$ and $w_{t+k,t}^{h,e}$ and government policies, $n(j,a,\eta,h)$ is the optimal labour supply of an individual with state $n(j,a,\eta,h)$.

2. Given prices r_t and w_t, government policies, expectations $r_{t+k,t}^{h,e}$ and $w_{t+k,t}^{h,e}$, and the labour supply $n(j,a,\eta,h)$ as decided in the beginning of period t, V_t satisfies the Bellman equations as given in subsection 5.2.2 for all states (j,a,η,h), where $\{a'_{j,t}, c_{j,t}\}$ are the related policy functions.

3. Interest rates and wages are given by:
$$r_t = \alpha A \left(\frac{L_t}{K_t}\right)^{(1-\alpha)} - \delta \tag{5.19}$$
and
$$w_t = (1-\alpha) A \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{5.20}$$

4. The pension pp_t is determined by:
$$pp_t = b_{p,t} \left(\frac{\sum_{j=1}^{J_{R-1}} L_{j,t} \int w_t n(j,a,\eta,h) \eta \varepsilon_j (1-\tau) \Phi_{j,t}(da \times d\eta \times dh)}{\sum_{j=1}^{J_{R-1}} L_{j,t}} \right) \tag{5.21}$$
where $b_{p,t}$ is the net replacement rate for the individuals.

5. The transfer Tr_{t+1} is determined by:
$$Tr_{t+1} = \left(\frac{\sum_{j=1}^{J} (1-\varphi_j) N_{j,t} \int a'(j,a,\eta,h) \Phi_{j,t}(da \times d\eta \times dh)}{\sum_{j=1}^{J} N_{j,t+1}} \right) \tag{5.22}$$

6. Expectations $r^{h,e}_{t+k,t}$ and $w^{h,e}_{t+k,t}$ are updated using Equations (5.10-5.16).

7. The performance of the heuristic and the fraction of individuals using a certain heuristic is determined via Equations (5.17-5.18).

8. The capital market, the labour market and the goods market clear every period t:

$$K_{t+1} = \sum_{j=1}^{J} N_{j,t} \int a'(j,a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh) \tag{5.23}$$

$$L_t = \sum_{j=1}^{J_{R-1}} N_{j,t} \int n(j,a,e,\pi,\eta,H)\varepsilon_j \eta \Phi(da \times d\eta \times dh) \tag{5.24}$$

$$Y_t = \sum_{j=1}^{J} N_{j,t} \int c(j,a,\eta,h)\Phi_{j,t}(da \times d\eta \times dh) + G_{c,t} + (K_{t+1} - (1-\delta)K_t) \tag{5.25}$$

9. Government policies $\{G_c, T_{n,t}, T_{k,t}, T_{c,t}\}$ are determined using Equation (5.9) and the fraction of Y_t used for government spending (g_c) is endogenously determined such that the government budget is balanced each period:

$$g_c = \frac{T_{n,t} + T_{c,t} + T_{k,t} - PP_t}{Y_t} \tag{5.26}$$

10. Y_t is determined by equation (5.7).

11. $\Phi_{j+1,t+1} = Z_{j,t}(\Phi_{j,t})$ where $Z_{j,t}$ is the law of motion induced by the exogenous mortality rates, the exogenous Markov process for labour productivity, the endogenous asset accumulation and the heuristic switching regime.

5.6 Data and calibration

In this section, the parameterisation and calibration of the model is outlined in detail. The lion's share of the calibration is in line with the literature on quantitative OLG models with idiosyncratic risk. The model is calibrated to Belgium for the period 2000-2007.

5.6.1 Demographics

Agents enter the economy at the age of 18 (model age = 1), retire at the age of 66 (model age = 13) and live at most until the age of 94 years. Each period in the model lasts for four years. The conditional survival probabilities $\{\varphi_j\}$ are taken from the Human Mortality Database and are for 2000.

5.6.2 Technology and employment

The parameters regarding technology are $\{\alpha, A, \delta\}$. The capital share in production α equals 0.36. The level of technology A is constant and normalised such that the equilibrium real wage rate in the benchmark model w is equal to 1. The depreciation rate δ is calibrated using a target for the equilibrium annual real interest rate of $r = 4.5\%$.

5.6.3 Labour productivity shocks and parameters

In the model, an individual of age j and idiosyncratic shock η who works n hours will earn a pre-tax wage of

$$w n_j \eta \varepsilon_j \tag{5.27}$$

I use the specification reported by Cournède and Gonand (2006) to calibrate the age-specific productivity profile ε_j. The resulting profile is hump-shaped.

The productivity shock η can take three values: $\eta \in \{\eta_1, \eta_2, \eta_3\}$. The Markov transition matrix is a (3x3)-matrix:

$$\Omega = \begin{bmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{bmatrix} \tag{5.28}$$

where ρ_{ij} is the probability $Pr(j|i)$ to end up in state j in the next period given state i in the current period. Taking all this information together, the states of the Markov chain $\{\eta_1, \eta_2, \eta_3\}$ and the Markov transition matrix Ω still have to be determined. For the labor productivity states $\{\eta_1, \eta_2, \eta_3\}$ in the labor earnings process, I use a discretised Markov chain for a continuous AR(1)-process with persistence ζ_s and variance σ^2_η. The persistence is chosen to be 0.969 and the variance 0.01 (Krueger and Ludwig, 2013).

The markov transition matrix for the idiosyncratic productivity risk is then given by

$$\Omega = \begin{bmatrix} 0.8851 & 0.1113 & 0.0035 \\ 0.0557 & 0.8887 & 0.0557 \\ 0.0035 & 0.1113 & 0.8851 \end{bmatrix}$$

while the values for η_1, η_2 and η_3 are respectively 0.6029, 1 and 1.6587.

5.6.4 Preferences

The instantaneous utility function of the individuals is given by Equation (5.2)[4]. The parameters to be calibrated are $\{\beta, \mu, \theta\}$. As in Conesa and Krueger (2006) and Krueger

[4]This functional form is often used in the quantitative OLG literature with idiosyncratic risk: see e.g. Conesa and Krueger (2006) and Krueger and Ludwig (2013)

Table 5.1: Calibration summary

Parameter values			
Weight on leisure in utility function	μ	0.6164	Average fraction of time spent working = 1/3
Discount factor in utility function	β	0.96	
Coefficient of risk aversion	θ	4	Conesa and Krueger (2006); Krueger and Ludwig (2013)
Level of technology	A	4.478	$w = 1$
Capital share in production	α	0.36	
Depreciation rate	δ	0.36	$r = 4.5\%$
Age-specific component of wages	ε_j		Cournède and Gonand (2006)
Tax rate on labour	τ	52.2%	Heylen and Van de Kerckhove (2013)
Tax rate on consumption	τ_c	13.4%	Heylen and Van de Kerckhove (2013)
Tax rate on capital	τ_k	26.8%	McDaniel (2007)
Net replacement rate pension	b_p	0.631	OECD, Pensions at a Glance (2005)

and Ludwig (2013), θ is chosen to be 4. The relative weight on leisure μ, on the other hand, is determined such that Belgian employed individuals work on average $1/3$ of their time. Using the values of μ and θ, a coefficient of relative risk aversion of approximately 2 is obtained. Finally, β is set to 0.96.

5.6.5 Fiscal policy variables

The government in the model finances spending on goods and PAYG pensions with taxes on consumption, capital and labour. For the tax rates τ_c and τ, I use the same data as Heylen and Van de Kerckhove (2013). For details on the construction of these fiscal policy variables, I refer to Heylen & Van de Kerckhove (2013, their Appendix 1). The value for the capital tax rate τ_k is determined using the tax series constructed by Cara McDaniel. I use the average between 2000-2007[5]. Regarding the basic PAYG-pension received by the retired households in the model, I use data on the average net replacement rate after retirement obtained from the OECD (Pensions at a Glance, 2005).

5.7 Simulations

5.7.1 Set-up

The goal of this paper is go beyond the rational expectations paradigm and use heuristics to study the transitional dynamics following fiscal policy shocks. In this section, I explore the effects of an unanticipated permanent labour tax decrease financed by gov-

[5] The updated tax series can be downloaded from www.caramcdaniel.com/researchpapers. The methodology is discussed in McDaniel (2007).

ernment spending. I proceed in two steps. In the first step, I assume that economic agents have only one of the available heuristics to form expectations at their disposal. Thus, at this point there is no heuristic switching. In a second step, I perform the same analysis, however, now I do use the heuristic switching framework: i.e. I assume that agents evaluate the performance of the heuristic they are currently using. According to its relative performance to other heuristics, individuals might switch to a different heuristic. All the results discussed in the following subsections are for the scenario in which τ is reduced from 52.2% to 42.2%.

5.7.1.1 No heuristic switching

I perform the analysis in step 1 for different parameter values for the discount factor β, the coefficient of relative risk aversion θ and the parameters ψ_1, ψ_2, ψ_5 and ψ_6 governing the adaptive and trend heuristics. For β, I use two different and rather extreme values: 0.98 and 0.86, whereas θ takes the values 2 and 4[6]. The lower (higher) the value of θ, the higher (lower) the Frish elasticity of labour supply and the more (less) individuals respond to different real wage rates. For ψ_1 and ψ_2, the two values are 0.1 and 0.9. In the latter case, individuals are much faster in adjusting their expectations to current evolutions. To conclude, the values used for ψ_5 and ψ_6 are 1 and 2. Higher values for the latter two variables indicate that current evolutions matter less in the future. The values for ψ_3, ψ_4 and ϕ_j remain constant over all scenarios. For the first two, I attach the value of 5. That means that individuals take a total period of 20 years to average. The weights ϕ_j are determined using the formula $\frac{9-2j}{25}$, meaning that the weights are 9/25 for the current period, 7/25 for the previous period, and 5/25, 3/25 and 1/25 for the other periods.

Thus, I end up with 4 parameter combinations of β and θ. For each of these combinations, I have 4 different combinations for each of the adaptive expectations heuristics (Equations (5.10) and (5.15)), 4 different combinations for each of the trend rule heuristics (Equations (5.11) and (5.16)) and 1 single specification for the other three heuristics (Equations (5.12-5.14)), leading to a total number of 76 scenarios.

5.7.1.2 Heuristic switching

Of course, not all individuals use the same heuristic to form expectations about the future realisations of the interest rate and the real wage rate. It is more realistic to assume that individuals have more than one heuristic at their disposal to form expectations. Therefore, a next step is activating the heuristic switching framework. For these simulations, I use the same range of parameter values for β, θ, ψ_1, ψ_2, ψ_5 and ψ_6. The values for ψ_3, ψ_4

[6] I performed the analysis as well for $\theta = 10$ but the results are highly similar to $\theta = 4$.

Table 5.2: Different initial distributions for the weights (fractions of individuals using the specific heuristics).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\Gamma_{1,0}$ (Adaptive (1))	1	0	0	0	0	0	0	1/7	1/4	0
$\Gamma_{2,0}$ (Adaptive (2))	0	1	0	0	0	0	0	1/7	1/4	0
$\Gamma_{3,0}$ (Trend (1))	0	0	1	0	0	0	0	1/7	1/4	0
$\Gamma_{4,0}$ (Trend (2))	0	0	0	1	0	0	0	1/7	1/4	0
$\Gamma_{5,0}$ (Average (1))	0	0	0	0	1	0	0	1/7	0	1/3
$\Gamma_{6,0}$ (Average (2))	0	0	0	0	0	1	0	1/7	0	1/3
$\Gamma_{7,0}$ (Myopic)	0	0	0	0	0	0	1	1/7	0	1/3

and ϕ_j are the same as well. Furthermore, I have to choose values for the parameters υ, ξ and k. For the first parameter υ, measuring the degree to which individuals take the past performance of heuristics into account, the values 0.2 and 0.9 are used. In the first (last) case, individuals attach low (high) weight to errors in the past. For the second parameter ξ, measuring the degree of inertia in the switching of individuals between different heuristics, the values 0.1 and 0.9 are used. The latter value indicates that the fraction of individuals using a specific weight will change slowly, even if its performance is especially good or bad. The last parameter k indicates the intensity of choice. I use two values: 0.1 and 100. The higher, the more people are inclined to switch to the best performing heuristic.

I use all the combinations of these parameters for 10 different initial distributions of weights, i.e. initial fractions of individuals using the different heuristics. These are given in Table 5.2. For example, in (1), all agents start by using Adaptive (1) to form expectations. When one heuristic is used extensively in the first periods of the transition period, this could lead to different dynamics compared to when a second heuristic is used heavily in the beginning.

5.7.2 Results

5.7.2.1 With vs without heuristic switching for $\beta = 0.98$ and $\theta = 4$

The results for $\beta = 0.98$ and $\theta = 4$ with and without heuristic switching are given in Figure 5.1[7]. The three panels on the left concern the scenarios without heuristic switching. They display the median transitional dynamics for output, the real wage rate and the

[7]The permanent decrease in the labour tax occurs in period 1. The value for output in the old steady state is 16.6. The initial steady state level for the pre-tax wage is 1. The results for $\beta = 0.86/\theta = 4$, $\beta = 0.98/\theta = 10$, $\beta = 0.86/\theta = 10$ and $\beta = 0.98/\theta = 2$ are similar and are therefore not displayed in this text. The conclusions mentioned in the text apply to these configurations as well. The results for $\beta = 0.86/\sigma = 2$ are discussed in the next subsection.

forecast errors for all the different heuristics over all the different parameter values for ψ_1, ψ_2, ψ_5 and ψ_6 as discussed in subsection 5.7.1.1. The three panels on the right concern the results with heuristic switching for $\beta = 0.98$ and $\theta = 4$. For each combination of respectively ξ, υ and k, they show the median transitional path over all the combinations for ψ_1, ψ_2, ψ_5 and ψ_6 and over all initial distributions as given in Table 5.2. Furthermore, for the panels with heuristic switching, I also give an indication of the variation in the transitional dynamics over all different scenarios. Note that the RE case is displayed using the dashed line.

I start with the discussion for the results without heuristic switching. In the first period, the difference between output in the rational expectations case (dashed line) and the heuristic scenarios is most pronounced. In the RE case, the value for w_t is lower than the value in the old steady state due to the higher labor supply. For the different heuristics, however, the expectation for the wage equals the value for the real pre-tax wage rate in the old steady state. As the individuals base their labour supply on their expectation for w, they supply more labour relative to the RE case leading to a higher level of output. Afterwards, output decreases as employment decreases, but the increase in K_t counteracts this decrease to some extent. These panels indicate that substantial output gains can be achieved compared to the rational expectations case. In all heuristic cases, output overshoots its RE counterpart. This effect is most pronounced in the adaptive expectations case, since expectations are lagging behind the most. When individuals use the trend heuristic, there is some oscillation in the beginning. Overshooting is the largest when average expectations are used.

Unsurprisingly, the dynamics of the actual real wage rate are closely related to the heuristic used to form expectations. At first, the real wage rate is lower than its RE counterpart, but it quickly catches up. The panels in the last row show the forecast errors. The forecast error is negative when the expected value is bigger than the actual value and positive otherwise. The oscillation following the trend heuristic is also clear from the panels in the last row of Figure 5.1. The forecast error changes in sign compared to the period before and gradually it converges to zero. In the adaptive expectations case, forecast errors are always negative. Average expectations lead to a negative forecast error in the beginning. Around the fifteenth period, they become positive for some periods.

Most importantly, these panels indicate that even for very common parameter values for β and θ, the resulting dynamics, both for output and wages but also for all the other variables, are substantially different from the dynamics in the rational expectations case. Second, they show that the resulting dynamics depend on the heuristic that is used and the parameterisation of that heuristic. Thus, there is a certain degree of variation in the dynamics.

As for the results with heuristic switching, one can see that the distance between the values of respectively + 1 standard deviation and -1 standard deviation from the mean value in each period is small and quickly becomes smaller as time goes by. Thus, in a framework without rational expectations and with heuristic switching, the effects of a labour tax decrease become much more predictable and monotonic even though a lot of individuals have different ways of forming expectations and even considering different initial distributions. The heuristic switching framework enables policy makers to better anticipate the effects of tax changes.

Figure 5.2 displays the boxplot of the value of Equation (5.17) for each period for each heuristic over the transition. This figure gives a lot of information on how the different heuristics are performing over the transition period. The two adaptive heuristics and the two average heuristics perform the best in the beginning, as they are overall the slowest ones to adjust their expectations. The trend rules are the worst performers in the beginning. Over time, the myopic expectations heuristic gradually becomes the best performing heuristic based on the median value of Equation (5.17). Another aspect worth mentioning is the fact that for the adaptive and trend heuristics, the range of the values for Equation (5.17) is much bigger compared to the others. This means that in some cases, especially when the adjustment factors ψ_1 and ψ_2 are high, these heuristics perform extremely bad. For the adaptive heuristics, this is typically after the tenth period.

The main message here is that after the heuristic switching regime has been activated, the variation in the transitional dynamics decreases significantly, meaning that the transitional effects of a decrease in labour taxes become more predictable and monotonic over all different parameter values in the specifications of the heuristics (both extreme and moderate) and of the parameters in Equations (5.17) and (5.18) and over all possible initial distributions. Thus, allowing for alternative expectations and a lot of heterogeneity in terms of initial fractions does not lead to a wide range of possible transitional paths, but decreases in fact the range in which the transitional dynamics are located.

5.7.2.2 With vs without heuristic switching for $\beta = 0.86$ and $\theta = 2$

In the previous subsection, I discussed the results for β =0.98 and $\theta = 4$. As mentioned previously, I also performed the same analysis for $\beta = 0.86$ and $\theta = 2$. I will, however, discuss the results in a different way as I want to highlight a different feature of the heuristic switching approach compared to the previous subsection.

Figure 5.1 showed that in all of the scenarios, the transitional dynamics converge to the steady state. This is not always the case, however. In Figure 5.3, I include the transitional dynamics for output for all the different heuristics but without heuristic switching when

$\beta = 0.86/\theta$=2 and over all the different parameter values as discussed in subsection 5.7.1.1. For a lot of the scenarios, the transitional dynamics oscillate around the steady state or even diverge. The oscillations are very large in some cases. Not surprisingly, in all the adaptive and trend scenarios in which individuals adjust their expectations faster (ψ_1 and ψ_2 equal to 0.9), the dynamics diverge or experience large oscillations. For smaller weights, the dynamics oscillate to a smaller extent. The variation in the dynamics is thus very big.

With heuristic switching, the conclusion is different. Figure 5.4 reveals that the heuristic switching approach has a stabilising effect on the dynamics. Only when the agents using the adaptive heuristic and the ones using the trend heuristic are simultaneously aggressive in adjusting their expectations ($\psi_1 = \psi_2 = 0.9$), the dynamics oscillate in some cases. In all other cases, the heuristic switching approach stabilises the dynamics.

Figures 5.5 and 5.6 further examine the sensitivity of the results to the memory parameter (υ), the noise parameter (k) and the inertia parameter (ξ) for $\beta = 0.86$ and $\theta = 2$. First, the results for $\upsilon = 0.2$. Only when $k = 100$ and $\xi = 0.9$ do all the different simulations over the parameter values for the specification of the heuristics and the initial distributions converge. So even when $\psi_1 = \psi_2 = 0.9$, the scenario which appeared to be problematic in the previous figure. When the noise is low, the economic agents will be more able to detect the better performing heuristics. Combined with high inertia, meaning that economic agents don't quickly switch from heuristic, this leads to a stabilising effect for all cases. The value of k is more important than ξ. Even for low inertia, if there is not a lot of noise, the lion's share of the simulations converge.

When the memory parameter is higher, the conclusions from the previous figure are even stronger. In this case, whenever $k = 100$, the dynamics will converge to the steady state, even for low inertia. The oscillations in the cases where $k = 0.1$ are also smaller than their counterparts in Figure 5.5.

Figure 5.1: Transitional dynamics after a labour tax decrease for different heuristics and under rational expectations scenario

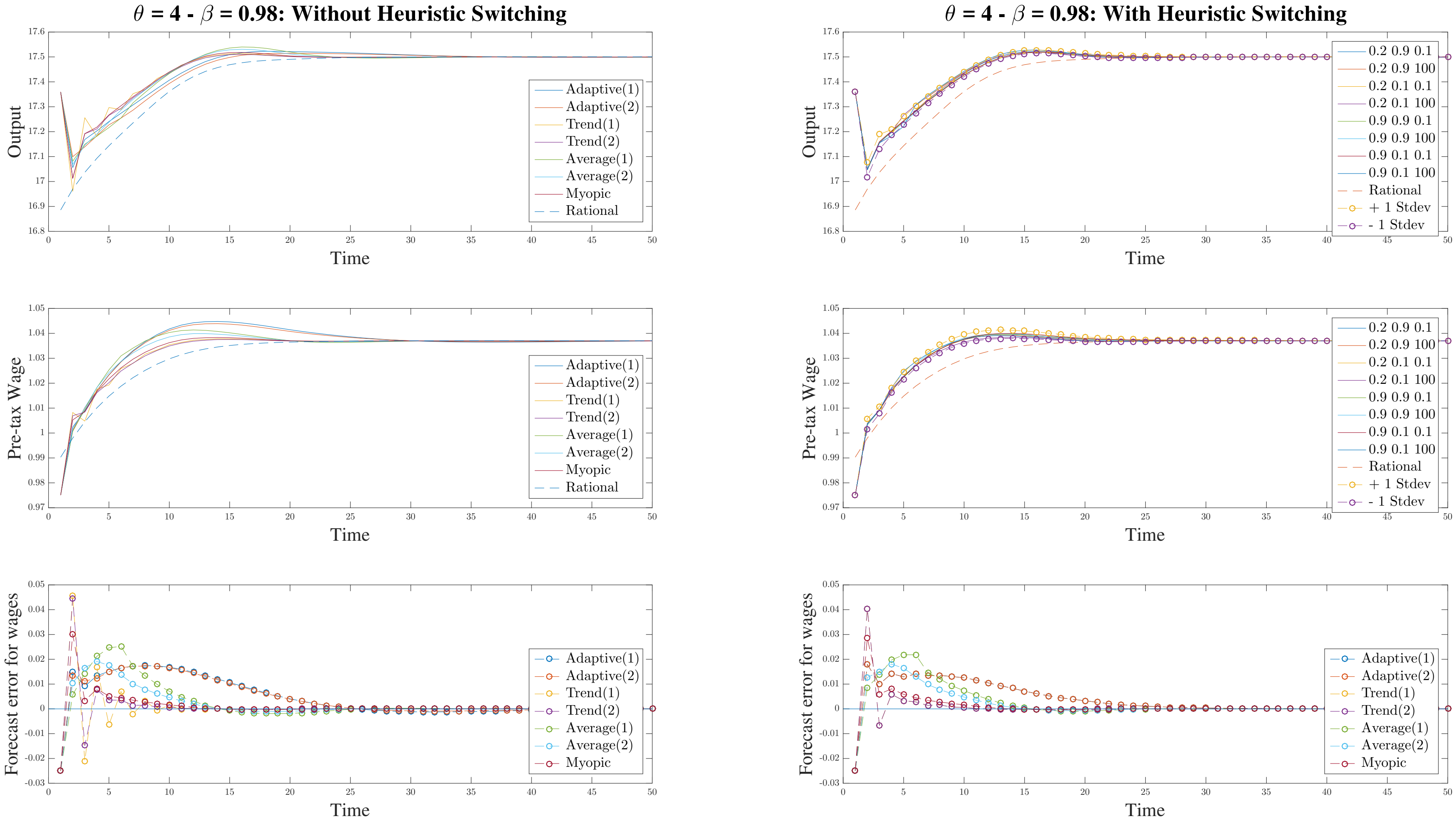
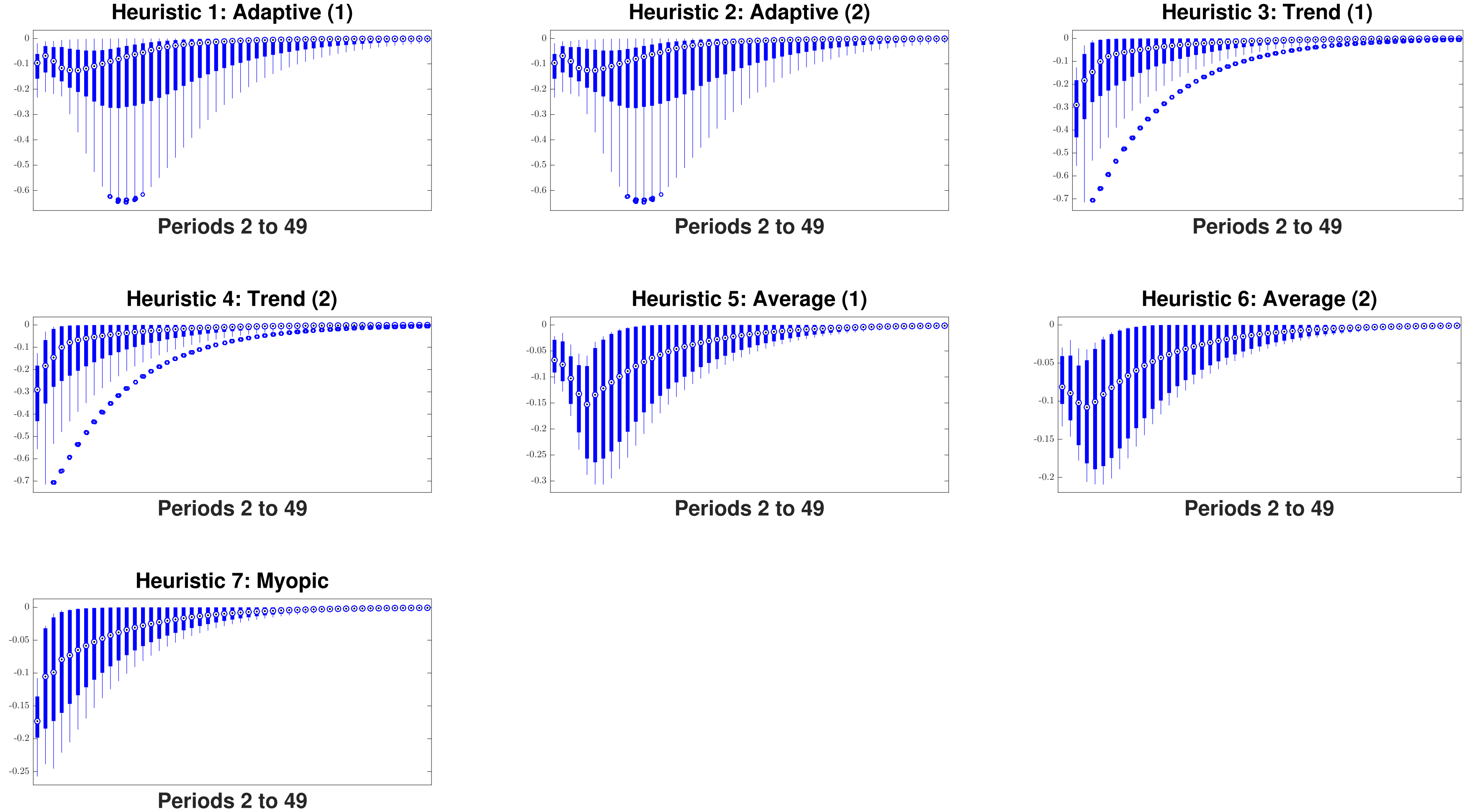


Figure 5.2: Boxplot of the Squared Prediction Error for w for the different heuristics over the transition period



Note: these boxplots are constructed using the Squared Prediction Error for a given heuristic in a given period over all parameter values discussed in Section 5.7.1.2.

Figure 5.3: Transitional dynamics after labour tax decrease for $\beta = 0.86$ and $\theta = 2$ without heuristic switching

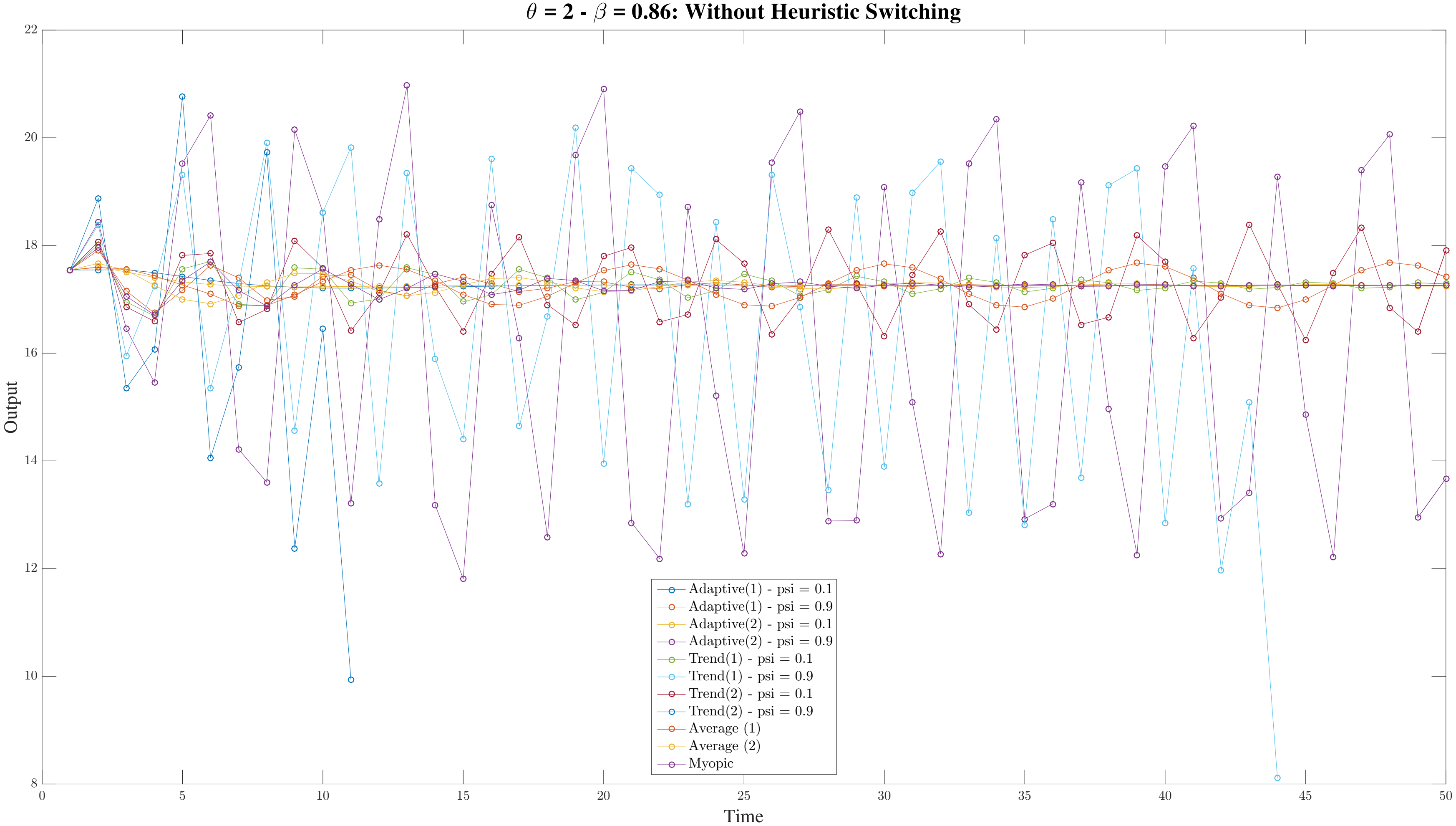
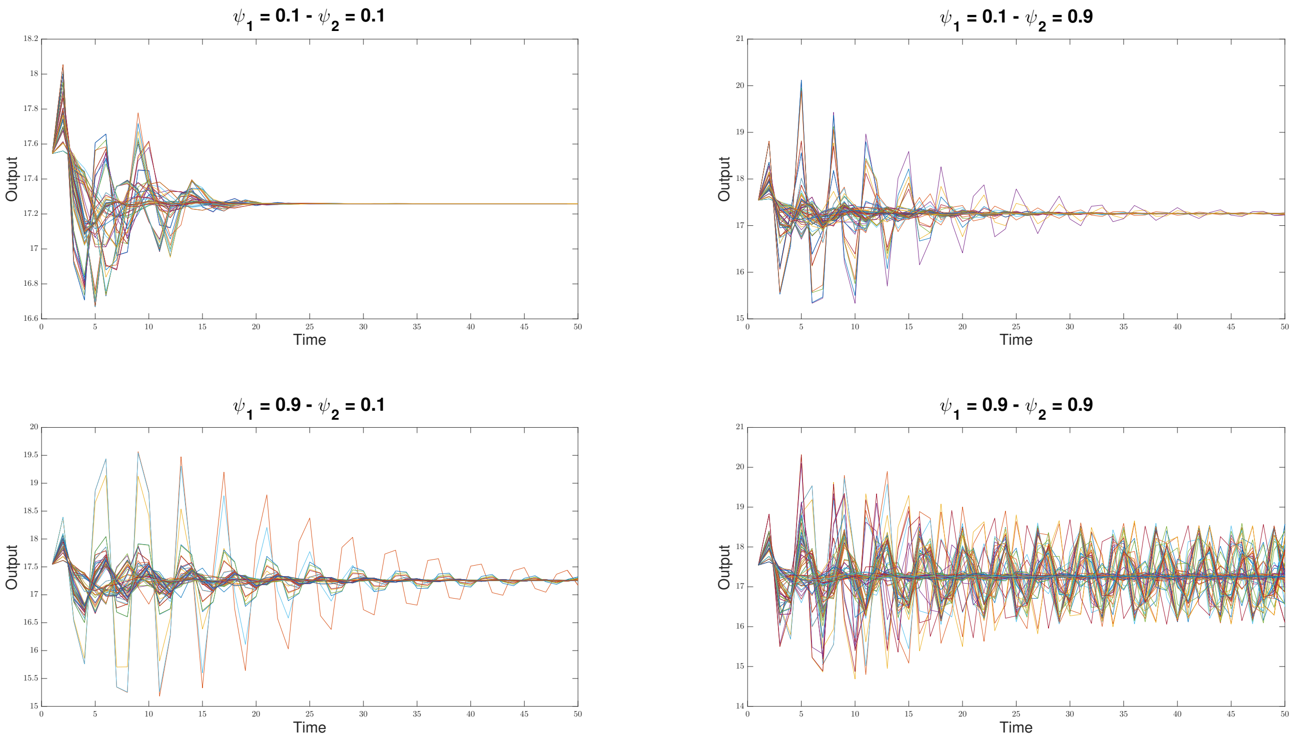


Figure 5.4: Transitional dynamics after labour tax decrease for $\beta = 0.86$ and $\theta = 2$ with heuristic switching split up for different combinations of ψ_1 and ψ_2



5.8 Conclusion

This paper explores the stability and transitional dynamics of an overlapping generations model with heuristic switching. Accordingly, this paper contributes to a growing literature that goes beyond the rational expectations paradigm. I assume that agents use simple rules, heuristics, to forecast the future course of the interest rate and the real wage. Agents use these heuristics because, in general, they do not possess the cognitive abilities as assumed by the RE literature nor to act as econometricians. Instead, the agents have a certain number of different heuristics at their disposal. On a regular basis, they assess the predictive power of the heuristic they are currently applying. If it performs well, the probability that agents will use the same heuristic in the next period will be higher. If it does not perform well, there is a higher probability that they switch to another rule.

The aim of this paper is to contribute to the literature that goes beyond the rational expectations paradigm by exploiting the heuristic switching approach within an Overlapping Generations model. Triggered by a fiscal policy shock, the objective is to study the transitional dynamics of the model for a large number of settings including one or multiple heuristics and compare the behaviour of the dynamics with their rational expectations counterpart.

Figure 5.5: Transitional dynamics after labour tax decrease for $\beta = 0.86$, $\theta = 2$ and $\upsilon = 0.2$: sensitivity of results to different values of the inertia and the noise

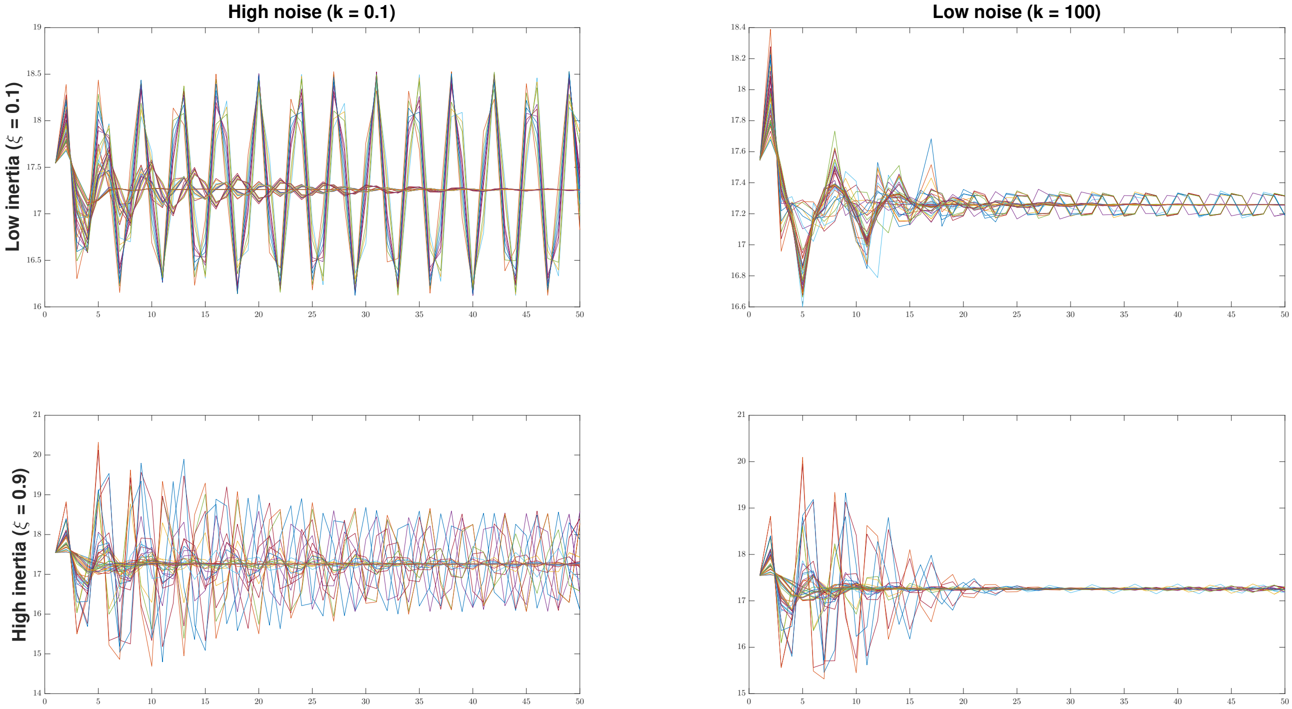
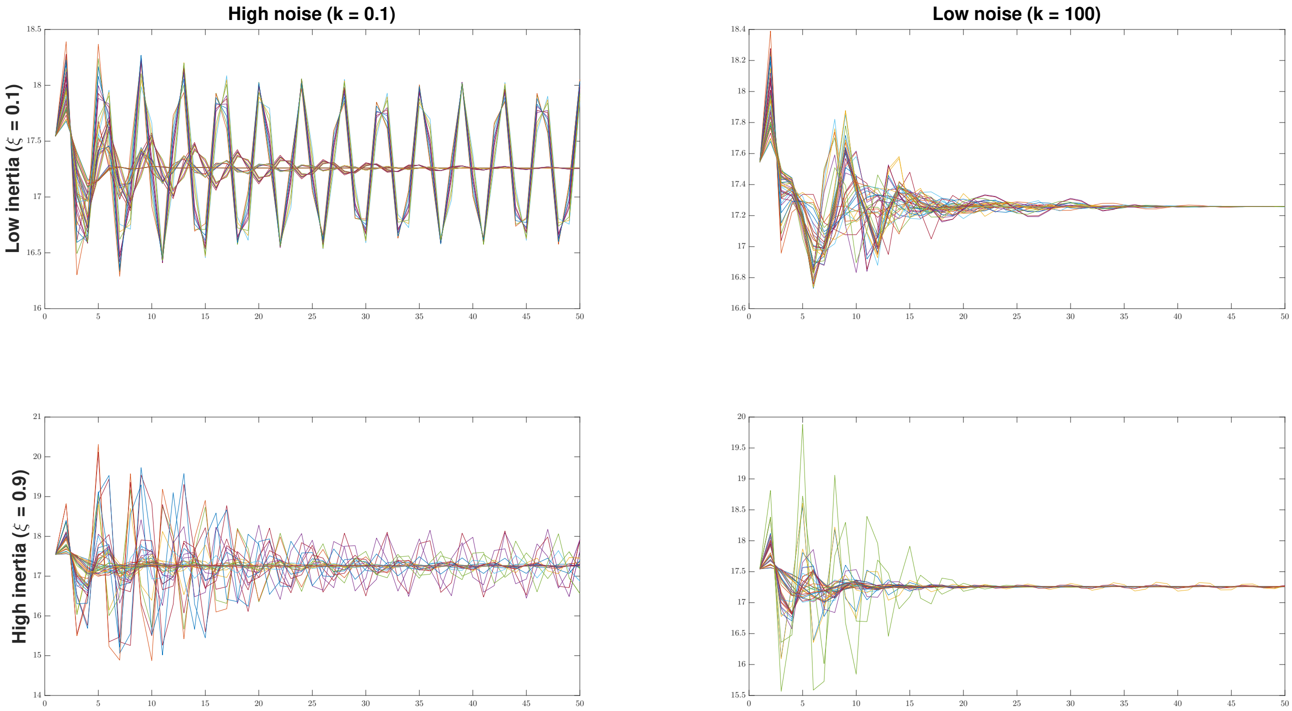


Figure 5.6: Transitional dynamics after labour tax decrease for $\beta = 0.86$, $\theta = 2$ and $\upsilon = 0.9$: sensitivity of results to different values of the inertia and the noise



The simulations lead to three main findings. First, in a context without heuristic switching (i.e. a context where individuals only have one heuristic at their disposal to form expectations), the evolution of transitional dynamics can be substantially different from the rational expectations case, especially in the first periods of the transition. Furthermore, there is a lot of variation in the dynamics over different parameter values and heuristics. This finding implies that if only heuristic is used, the macroeconomic impact of fiscal policy is highly sensitive to the heuristic being used. What is more, as the discount rate and the degree of risk aversion decrease, the model becomes unstable and the corresponding transitional dynamics oscillate or even diverge. Second, after activating the heuristic switching regime, the variation in the transitional dynamics decreases significantly. Consequently, the sensitivity of the transitional effects of fiscal policy is much lower now and its exact impact is thus less uncertain for policy makers. Third and last, the heuristic switching has a stabilising effect on the transitional dynamics. For certain configurations of the parameter values for which the dynamics were very unstable in the absence of heuristic switching, the dynamics now converge to the steady state in most cases.

These findings are important. They imply that allowing individuals to choose from a wide range of forecasting rules is actually a better option than constraining them to use only forecasting rule. It allows them to select the better performing rules, a feature that not only enhances the stability of the model, but reduces the uncertainty in the transitional dynamics as well.

Chapter 6

Conclusions, Policy Recommendations, and Avenues for Future Research

The final chapter of this dissertation aims to provide the reader with the overall conclusions that can be drawn from this dissertation and the derived policy recommendations. Its second objective is to highlight the avenues for future research that are potentially interesting in my opinion.

Fiscal policy, labour market institutions, and the labour market performance of low-skilled individuals

When it comes to the main conclusion of the first two papers, the title of this section says it all. In other words, the results of these papers clearly indicate that both fiscal policy and labour market institutions are crucial components (i) to explain the cross-country variation in low-skilled unemployment and (ii) to design public policies that improve the labour market position of the low-skilled significantly without compromising the welfare of the unemployed.

Going into more detail with respect to (i), the Shapley decomposition performed in the first paper reports an almost equal role for differences in fiscal policy variables and in union preferences. Nor differences in fiscal policy nor differences in union preferences are thus sufficient to explain why OECD countries differ in terms of labour market performance of the low-skilled. The interaction between the two does the trick. More specifically, the source of unemployment is the above market-clearing wage chosen by the unions, while fiscal policy variables explain a significant part of the magnitude of unemployment. Despite the overall importance of fiscal policy, the different fiscal policy variables differ significantly in terms of explanatory power, though. The key variable within the fiscal policy component is the net unemployment benefit replacement rate. Somewhat more surprising, on the other hand, is the lack of explanatory power of labour taxes.

Not only do we need both components to explain low-skilled unemployment, the design of public policy that aims at improving the labour market performance of the low-skilled benefits from the interaction of the two as well, the implication being that fiscal governments and unions have a mutual interest in joining forces for the design and execution of such policies. Preferable policies feature a reduction in labour taxes for the low-skilled combined with an obligation for the unions to accept lower wages. Policies imposing a decrease in the unemployment benefit yield a similar improvement in the employment prospects of the low-skilled, but they are less preferable as they are very likely to go hand in hand with welfare losses for those low-skilled individuals who remain unemployed after all. Chapter 3 also demonstrates that the different fiscal policy instruments may have a very different impact on unemployment, hours worked, and participation. In other words, policymakers thus have to choose the instrument depending on the variable they want to target. Only decreasing labour taxes, for example, has no use when the fiscal government aims to decrease low-skilled unemployment. Decreasing the net unemployment benefit replacement rate and less aggressive wage policies by the union, on the other hand, have a strong impact on low-skilled unemployment. But they have no use if the target variable is the amount of hours worked per employed individual. Even more, if policymakers choose labour taxes as the desired instrument, a decrease in the tax rate faced by the employees is preferred over a decrease in the tax rate faced by employers as the behavioural effects are much stronger in the former case.

Nevertheless, there are many roads ahead for future research. First, the richness of the theoretical model used in the Chapter 3 is not yet used to the fullest. For example, it would be interesting to study the importance of labour market frictions in the context of pension reform. Using a search and matching set-up, de la Croix et al. (2013) find that the macroeconomic effects of a shift from a pay-as-you-go to a fully funded pension scheme do indeed differ if labour market imperfections are considered. Furthermore, the models can also be applied to determine the extent to which the decision to retire early is influenced by the generosity of the early retirement benefits on the one hand and the probability that a low-skilled individual is employed when he or she participates on the other hand. A next avenue for future research is equally interesting and challenging. So far, the theoretical model constructed in Chapter 3 is only used for an in-depth comparison of steady states. The reason being that the computational complexity to solve the model is very high. Moving beyond a steady state comparison and applying the model to construct a transition path for each of the different policies would benefit hugely from a reduction in this computational burden. A full transition path would be especially rewarding as it would allow for a detailed welfare analysis of consecutive generations, both from an ex ante and an ex post perspective. As a result, the political feasibility of the policy changes could be studied. These directions for future research would, if executed, definitely sharpen the recommendations for policy makers.

Fiscal policy and alternative expectations

Compared to the previous subsection, the structure is now somewhat different. For the sake of clarity, I will first elaborate on the conclusions that can be drawn from Chapters 4 and 5 with respect to the effectiveness and impact of fiscal policy under alternative expectations and the related policy recommendations. Thereafter, I provide some insights on the use of rational versus alternative expectations. These recommendations and conclusions are mainly targeted at researchers. Finally, and similar to the previous section, I will identify some avenues for future research.

The central result emerging from Chapter 4 is that a switch from rational expectations to a learning set-up within the standard Real Business Cycle model is able to generate a time-varying government spending multiplier, a feature which has been reported by numerous empirical studies. What this ultimately implies, is that the effectiveness of fiscal policy evolves over time.

Sensible policy recommendations, however, require insights into the drivers of the government spending multiplier. One variable that is of particular interest to explain fluctuations in the government spending multiplier is the importance households attach to government spending when making decisions. A result that makes sense intuitively. If households value information on government spending more when making decisions, the impact of fiscal policy on the macroeconomy is bigger. So when do households attach a large weight to government spending? Our analysis shows that the government spending multiplier is high after (i) a considerable period of increasing technology and low government spending on the one hand, and (ii) after a considerable period of decreasing technology and high government spending. In the basic RBC model, these conditions are not often fulfilled. They are rather rare events in the current set-up. For shorter samples, we see that the fluctuations in the government spending multiplier are smaller and that these fluctuations occur around a rather steady level for the multiplier, where the level is determined following the aforementioned conditions.

Do the results of Chapters 4 and 5 tell us something about whether rational expectations or alternative expectations have to be used? If the aim is to compare different steady states, rational expectations is the better choice as learning is especially relevant for the transitional dynamics. So what about the transitional dynamics then? At first sight, our results indicate that, in general, the transitional dynamics for both the learning framework in Chapter 4 and the heuristic framework in Chapter 5 are fairly close to their rational expectations counterpart. As such, one might conclude that rational expectations seems a reliable set-up. However, Chapter 4 reveals that using rational expectations ignores the time-varying government spending multiplier. The level of the multiplier could, however, be of particular interest for the fiscal government. Chapter 5, on the other hand, indicates that the transitional dynamics can be substantially different from their rational expectations counterpart, especially in the first periods of the transition. In this case, rational expectations is thus not always a good approximation. If the researcher decides to use heuristics, he or she has to use the heuristic switching approach. This approach yields better results compared to the set-ups in which only one heuristic is used. Indeed, allowing individuals to switch to better performing heuristics actually reduces the number of possible trajectories. The transition of the macro-economy following a fiscal policy shock is thus far less sensitive to how individuals form expectations if there are multiple heuristics at their disposal. As a result, rational expectations is a better approximation when the switching mechanism is activated.

There are numerous avenues for further research. For example, the model that is being used in Chapter 4 is a standard RBC model. Adding additional building blocks, in particular price rigidity and occasionally binding constraints such as the zero lower bound, could sharpen our recommendations. Regarding Chapter 5, it would be highly interesting to study other types of shocks, both transitional and permanent. In addition, adding other heuristics and study their performance would also benefit the analysis in Chapter 5. A next step could be to introduce aggregate risk in the model. So far, only a few lifecycle models featuring idiosyncratic risk have introduced aggregate risk. The reason being that the computational complexity of the model increases strongly. However, the heuristics provide a elegant and useful approach to simplify the analysis. Furthermore, while laboratory research found that individuals do use the heuristic switching mechanism, it would still be highly interesting to empirically test whether rational expectations or the heuristic switching approach fits the data best.

Bibliography

Adam, K. (2003). Learning and equilibrium selection in a monetary overlapping generations model with sticky prices. *Review of Economic Studies*, 70:887–907.

Adam, K. (2007). Experimental Evidence on the Persistence of Output and Inflation. *Economic Journal*, 117(520):603–636.

Age Wave and Merrill Lynch (2016). *Retirement in retirement: beyond the bucket list*. Bank of America Corporation.

Alesina, A., Glaeser, E., and Sacerdote, B. (2005). Work and leisure in the U.S. and Europe: why so different? *NBER working paper*, No. 11278.

Alonso-Ortiz, J. (2014). Social security and retirement across the OECD. *Journal of Economic Dynamics and Control*, 47:300–316.

Andolfatto, D. (1996). Business cycles and labor-market search. *The American Economic Review*, 86(1):112–132.

Auerbach, A. J. and Gorodnichenko, Y. (2012). Measuring the output responses to fiscal policy. *American Economic Journal: Economic Policy*, 4(2):1–27.

Auerbach, A. J. and Gorodnichenko, Y. (2013). Fiscal Multipliers in Recession and Expansion. In *Fiscal Policy after the Financial Crisis*, NBER Chapters, pages 63–98. National Bureau of Economic Research, Inc.

Balleer, A., Gómez-Salvador, R., and Turunen, J. (2009). Labour force participation in the euro area: a cohort based analysis. *ECB Working Paper Series*, (1049).

Bassanini, A. and Duval, R. (2009). Unemployment, institutions, and reform complementarities: re-assessing the aggregate evidence for OECD countries. *Oxford Review of Economic Policy*, 25:40–59.

Benati, L. (2008). Investigating inflation persistence across monetary regimes. *The Quarterly Journal of Economics*, 123(3):1005–1060.

Benhabib, J., Evans, G. W., and Honkapohja, S. (2014). Liquidity traps and expectation dynamics: Fiscal stimulus or fiscal austerity? *Journal of Economic Dynamics and Control*, 45:220 – 238.

Berardi, M. and Duffy, J. (2015). Real-Time, Adaptive Learning Via Parameterized Expectations. *Macroeconomic Dynamics*, 19(02):245–269.

Blanchard, O. (2004). The economic future of Europe. *NBER working paper*, No. 10310.

Boone, B. and Heylen, F. (2015). Cross-country differences in unemployment: fiscal policy, unions and household preferences in general equilibrium. *FEB UGent Working Paper*, (2015/899).

Boone, B. and Heylen, F. (2017). Cross-country differences in unemployment: fiscal policy, unions and household preferences in general equilibrium. Revised and resubmitted to *Scandinavian Journal of Economics*.

Borjas, G. (2003). The labor demand curve is downward sloping: re-examining the impact of immigration on the labor market. *Quarterly Journal of Economics*, 118:1335–1374.

Bouzahzah, M., de la Croix, D., and Docquier, F. (2002). Policy reforms and growth in computational OLG economies. *Journal of Economic Dynamics and Control*, 19:2093–2113.

Branch, W. and Evans, G. W. (2007). Model uncertainty and endogenous volatility. *Review of Economic Dynamics*, 10(2):207–237.

Branch, W. A. and Evans, G. W. (2006). A simple recursive forecasting model. *Economics Letters*, 91(2):158–166.

Buyse, T., Heylen, F., and Van de Kerckhove, R. (2017). Pension reform in an OLG model with heterogeneous abilities. *Journal of Pension Economics and Finance*, 16:144–172.

Cahuc, P. and Michel, P. (1996). Minimum wage unemployment and growth. *European Economic Review*, 40:1463–1482.

Calmfors, L. and Driffill, J. (1988). Bargaining structure, corporatism and macroeconomic performance. *Economic Policy*, 6:13–61.

Canova, F. (2009). What Explains The Great Moderation in the U.S.? A Structural Analysis. *Journal of the European Economic Association*, 7(4):697–721.

Card, D., Lemieux, T., and Riddell, W. (2004). Unions and wage inequality. *Journal of Labor research*, 25:519–559.

Caselli, F. and Coleman, W. (2006). The world technology frontier. *American Economic Review*, 96:499–522.

Chang, P., Wray, L., and Lin, Y. (2014). Social relationships, leisure activity, and health in older adults. *Health Psychology*, 33(6):516–523.

Checci, D. and García-Peñalosa, C. (2008). Labour Market institutions and income inequality. *Economic Policy*, 56:600–651.

Chen, H., Li, M., and Lin, Y. (2008). Chaotic dynamics in an overlapping generations model with myopic and adaptive expectations. *Journal of Economic Behavior and Organization*, 67:48–56.

Cho, I.-K., Williams, N., and Sargent, T. J. (2002). Escaping Nash Inflation. *Review of Economic Studies*, 69(1):1–40.

Cogley, T. and Sbordone, A. M. (2008). Trend inflation, indexation, and inflation persistence in the new keynesian phillips curve. *American Economic Review*, 98(5):2101–26.

Conesa, J., Kitao, S., and Krueger, D. (2009). Taxing capital? Not a bad idea after all! *American Economic Review*, 99:25–48.

Conesa, J. and Krueger, D. (2006). On the optimal progressivity of the income tax code. *Journal of Monetary Economics*, 53:1425–1450.

Corneo, G. and Marquardt, A. (2000). Public pensions, unemployment and growth. *Journal of Public Economics*, 75:293–311.

Cournède, B. and Gonand, F. (2006). Restoring fiscal sustainability in the euro area: raise taxes or curb spending. *OECD Economics Department Working Papers*, No. 270.

Daveri, F. and Tabellini, G. (2000). Unemployment, growth and taxation in industrial countries. *Economic Policy*, 15(30):49–104.

De Grauwe, P., editor (2012). *Lectures on Behavioral Economics*. Princeton: Princeton University Press.

de la Croix, D., Palm, F. C., and Pfann, G. (1996). A dynamic contracting model for wages and employment in three european countries. *European Economic Review*, 40:429–448.

de la Croix, D., Pierrard, O., and Sneessens, H. R. (2013). Aging and pensions in general equilibrium: Labor market imperfections matter. *Journal of Economic Dynamics and Control*, 37(1):104–124.

den Haan, W. J. and Marcet, A. (1990). Solving the Stochastic Growth Model by Parameterizing Expectations. *Journal of Business & Economic Statistics*, 8(1):31–34.

Dhont, T. and Heylen, F. (2008). Why do Europeans work (much) less? it is taxes and government expenditures. *Economic Inquiry*, 46:197–207.

Erosa, A., Fuster, L., and Kambourov, G. (2012). Labor supply and government programmes: a cross-country analysis. *Journal of Monetary Economics*, 59:84–107.

Eusepi, S. and Preston, B. (2011). Expectations, learning, and business cycle fluctuations. *American Economic Review*, 101(6):2844–72.

Evans, G. W. and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*. Princeton University Press, Princeton, New Jersey.

Evans, G. W. and Honkapohja, S. (2009). Learning and macroeconomics. *Annual Review of Economics*, 1(1):421–451.

Evans, G. W., Honkapohja, S., and Mitra, K. (2009). Anticipated fiscal policy and adaptive learning. *Journal of Monetary Economics*, 56(7):930–953.

Faia, E. and Rossi, L. (2013). Union power, collective bargaining and optimal monetary policy. *Economic Inquiry*, 51:408–427.

Fehr, H. (2009). Computable stochastic equilibrium models and their use in pension- and ageing research. *De Economist*, 157:359–416.

Gali, J., Smets, F., and Wouters, R. (2011). Unemployment in an estimated New Keynesian model. *NBER Working Paper*, No. 17084.

Gasteiger, E. and Zhang, S. (2014). Anticipation, learning and welfare: the case of distortionary taxation. *Journal of Economic Dynamics and Control*, 39:113 – 126.

Guo, J. T. and Lansing, K. J. (1998). Indeterminacy and stabilization policy. *Journal of Economic Theory*, 82(2):482–490.

Heathcote, J. (2005). Fiscal policy with heterogeneous agents and incomplete markets. *Review of Economic Studies*, 72:161–188.

Heathcote, J., Storesletten, K., and Violante, G. (2009). Quantitative macroeconomics with heterogeneous households. *Annual Review of Economics*, 1:319–354.

Heathcote, J., Storesletten, K., and Violante, G. (2010). The macroeconomic implications of rising wage inequality in the United States. *Journal of Political Economy*, 118:681–722.

Heckman, J. (2001). Micro data, heterogeneity, and the evaluation of public policy: Nobel lecture. *Journal of Political Economy*, 109:673–748.

Heemeijer, P., Hommes, C., Sonnemans, J., and Tuinstra, J. (2012). An experimental study on expectations and learning in overlapping generations models. *Studies in Nonlinear Dynamics and Econometrics*, 16(4).

Heijdra, B. and Ligthart, J. (2009). Labor tax reform, unemployment, and search. *International Tax and Public Finance*, 16:82–104.

Heylen, F. and Van de Kerckhove, R. (2013). Employment by age, education, and economic growth: effects of fiscal policy composition in general equilibrium. *B. E. Journal of Macroeconomics*, 13(1):49–103.

Hobijn, B. and Sahin, A. (2009). Job-finding and separation rates in the oecd. *Economics Letters*, 104:107–111.

Hommes, C. (2011). The heterogeneous expectations hypothesis: Some evidence from the lab. *Journal of Economic Dynamics and Control*, 35(1):1 – 24.

Hommes, C. H. (2014). Behaviorally rational expectations and almost self-fulfilling equilibria. *Review of Behavioral Economics*, 1:75–97.

Honkapohja, S., Mitra, K., and Evans, G. W. (2013). Notes on agents' behavioral rules under adaptive learning and studies of monetary policy. In Sargent, T. and Vilmunen, J., editors, *Macroeconomics at the Service of Public Policy*, chapter 4, pages 63–79. Oxford University Press, Oxford, United Kingdom.

Huggett, M., Ventura, G., and Yaron, A. (2006). Human capital and earnings distribution dynamics. *Journal of Monetary Economics*, 53:265–290.

Huggett, M., Ventura, G., and Yaron, A. (2011). Sources of lifetime inequality. *American Economic Review*, 101:2923–2954.

Israeli, O. (2006). A shapley-based decomposition of the R-square of a linear regression. *The Journal of Economic Inequality*, 5:199–212.

Justiniano, A. and Primiceri, G. E. (2008). The time-varying volatility of macroeconomic fluctuations. *American Economic Review*, 98(3):604–41.

Katz, L. and Murphy, K. (1992). Changes in relative wages, 1963-1987: supply and demand factors. *Quarterly Journal of Economics*, 107:35–78.

Keane, M. and Wolpin, K. (1997). The career decisions of young men. *Journal of Political Economy*, 105:473–521.

Kindermann, F. and Krueger, D. (2014). High marginal tax rates on the top 1%? Lessons from a life cycle model with idiosyncratic income risk. *NBER working paper*, (20601).

Kitao, S. (2014). Sustainable social security: Four options. *Review of Economic Dynamics*, 17:756–779.

Kitao, S. and Imrohoroglu, S. (2012). Social security reforms: benefit claiming, labor force participation and long-run sustainability. *American Economic Journal: Macroeconomics*, 4:96–127.

Koyuncu, M. (2011). Can progressive taxation account for cross-country variation in labor supply? *Journal of Economic Dynamics and Control*, 35:1474–1488.

Krueger, D. and Ludwig, A. (2013). Optimal progressive taxation and education subsidies in a model of endogenous human capital formation. *American Economic Review: Papers and Proceedings*, 103:496–501.

Lucas, R. E. (1990). Supply-side economics: an analytical review. *Oxford Economic Papers*, 42:292–316.

Ludwig, A. (2007). The Gauss-Seidel-Quasi-Newton method: a hybrid algorithm for solving dynamic economic models. *Journal of Economic Dynamics and Control*, 31:1610–1632.

Ludwig, A., Schelke, T., and Vogel, E. (2012). Demographic change, human capital and welfare. *Review of Economic Dynamics*, 15:94–107.

Marcet, A. and Lorenzoni, G. (1999). *Computational Methods for the Study of Dynamic Economies*, chapter The Parameterized Expectations Approach Some Practical Issues, pages 143–171. Number 9780198294979 in OUP Catalogue. Oxford University Press.

Marcet, A. and Marshall, D. A. (1994). Solving nonlinear rational expectations models by parameterized expectations: Convergence to stationary solutions. *Federal Reserve Bank of Minneapolis Discussion Paper*, (91).

McDaniel, C. (2007). Average tax rates on consumption, investment, labor and capital in the OECD 1950-2003. *Working Paper*.

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36:300.

Milani, F. (2007). Expectations, learning and macroeconomic persistence. *Journal of Monetary Economics*, 54(7):2065–2082.

Miles, D. (1999). Modelling the impact of demographic change upon the economy. *Economic Journal*, 109:1–36.

Mitra, K., Evans, G. W., and Honkapohja, S. (2013). Policy change and learning in the RBC model. *Journal of Economic Dynamics and Control*, 37(10):1947–1971.

Nickell, S., Nunziata, L., and Ochel, W. (2005). Unemployment in the OECD since the 1960s. what do we know? *Economic Journal*, 115:1–27.

Nishiyama, S. and Smetters, K. (2005). Consumption taxes and economic efficiency with idiosyncratic wage shocks. *Journal of Political Economy*, 113:1088–1115.

Nishiyama, S. and Smetters, K. (2007). Does social security privatization produce efficiency gains? *Quarterly Journal of Economics*, 122:1677–1719.

Nymoen, R. and Sparrman, V. (2015). Equilibrium unemployment dynamics in a panel of OECD countries. *Oxford Bulletin of Economics and Statistics*, 77:164–190.

OECD (2014). *Education at a Glance*. OECD, Paris.

Ohanian, L., Raffo, A., and Rogerson, R. (2008). Long-term changes in labor supply and taxes: evidence from OECD countries, 1956-2004. *Journal of Monetary Economics*, 55:1353–1362.

Ono, T. (2007). Unemployment dynamics in an OLG economy with public pensions. *Economic Theory*, 33:549–577.

Ono, T. (2010). Growth and unemployment in an OLG economy with public pensions. *Journal of Population Economics*, 23:737–767.

Orphanides, A. and Williams, J. C. (2005). The decline of activist stabilization policy: Natural rate misperceptions, learning, and expectations. *Journal of Economic Dynamics and Control*, 29(11):1927–1950.

Orphanides, A. and Williams, J. C. (2007). Robust monetary policy with imperfect knowledge. *Journal of Monetary Economics*, 54(5):1406–1435.

Owyang, M. T., Ramey, V. A., and Zubairy, S. (2013). Are Government Spending Multipliers Greater during Periods of Slack? Evidence from Twentieth-Century Historical Data. *American Economic Review*, 103(3):129–34.

Pencavel, J. H. (1984). The trade-off between wages and employment in trade union objectives. *Quarterly Journal of Economics*, 99(2):215–231.

Prescott, E. C. (2004). Why do Americans work do much more than Europeans. *Federal Reserve Bank of Minneapolis Quarterly Review*, 28:2–13.

Prescott, E. C., Rogerson, R., and Wallenius, J. (2009). Lifetime aggregate labor supply with endogenous workweek length. *Review of Economic Dynamics*, 12:23–36.

Preston, B. (2005). Learning about Monetary Policy Rules when Long-Horizon Expectations Matter. *International Journal of Central Banking*, 1(2).

Ravn, M. and Sørensen, J. (1999). Schooling, training, growth and minimum wages. *Scandinavian Journal of Economics*, 101:441–457.

Rogerson, R. (2007). Taxation and market work: is Scandinavia an outlier? *Economic Theory*, 32(1):59–85.

Rogerson, R. and Wallenius, J. (2009). Micro and macro elasticities in a life cycle model with taxes. *Journal of Economic Theory*, 144:2277–2292.

Sargent, T. J. (1999). *The Conquest of American Inflation*. Princeton University Press.

Schönhofer, M. (1998). Chaotic learning equilibria. *Journal of Economic Theory*, 89:1–20.

Slobodyan, S. and Wouters, R. (2012). Learning in an estimated medium-scale DSGE model. *Journal of Economic Dynamics and Control*, 36(1):26–46.

Sommacal, A. (2006). Pension systems and intragenerational redistribution when labour supply is endogenous. *Oxford Economic Papers*, 58:379–406.

Tomas, C. (2008). Search and matching frictions and optimal monetary policy. *Journal of Monetary Economics*, 55(5):936–956.

Tuinstra, J. (2003). Beliefs equilibria in an overlapping generations model. *Journal of Economic Behavior and Organization*, 50:145–164.

Tuinstra, J. and Wagener, F. (2007). On learning equilibria. *Economic Theory*, 30:493–513.

Wallenius, J. (2013). Social security and cross-country differences in hours: A general equilibrium analysis. *Journal of Economic Dynamics and Control*, 37:2466–2482.