Construction of flipping-invariant functions in higher dimensions

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Recently, there have been several studies of transformations called 'flippings', which map n-copulas to n-copulas. The resulting transforms can be thought of as the multivariate cumulative distribution functions of random vectors that are obtained by replacing (called flipping) each of the original random variables from a given subset of the random vector by a countermonotonic counterpart. It is important to note that if all the variables are flipped, the resulting transform is the well-known survival n-copula (for more details, see [9]). In the bivariate case, these transformations have been studied from the algebraic point of view in [7], and have been further generalized to binary aggregation functions in [2, 3]. In the multivariate case, these operations have been studied in [5] for n-copulas, while in [4] the authors have studied the case of multivariate aggregation functions.

Inspired by the above results and the notion of invariant copula (i.e., a copula that coincides with one of its transforms [8]), we present two methods to construct flipping-invariant copulas in higher dimensions, given a lower-dimensional marginal copula. Both methods are partially based on an associative extension of an aggregation function, although not in the way that it is usually done, as it can be easily seen that there is no associative solution to the Frank functional equation in the n-dimensional case for $n \geqslant 3$ (see [1, 5]).

In the first method, we construct a 3-dimensional function that is flipping invariant, starting from a bivariate flipping-invariant symmetric copula. We show that if the function that is obtained by this transformation is increasing, then it is a 3-quasi-copula. We also present some numerical examples of this method for well-known families of flipping-invariant 2-copulas, such as the Frank copula family and the Farlie-Gumbel-Morgenstern copula family. In the second method, we construct a 3-dimensional aggregation function that it is flipping invariant in the last variable starting from an arbitrary 2-copula. We study some properties of the aggregation function that is obtained by this transformation, as well as conditions that guarantee that it is a 3-(quasi)-copula. Finally, we discuss several possible generalizations of both methods in higher dimensions.

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