Expectations and the Macroeconomic Dynamics of Fiscal Policy

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Nederlandstalige Samenvatting

Kan de overheid de economie een duwtje in de rug geven door tijdelijk meer uit te geven? De voorbeelden uit het verleden leren ons dat het succes van zo een stimuleringsbeleid sterk kan variëren. Dit proefschrift toont aan dat wisselende verwachtingen van gezinnen en bedrijven deze variatie kunnen verklaren. Door in rekening te brengen dat deze verwachtingen vaak op beperkte informatie gebaseerd zijn, kunnen we bovendien begrijpen waarom het effect van tijdelijke overheidsuitgaven doorgaans groter is dan wat traditionele macro-economische modellen voorspellen.

In de loop van de geschiedenis hebben beleidsmakers getracht de vraag naar goederen en diensten in de economie te stimuleren door tijdelijk meer uit te geven. Of ze daarin slagen, hangt in belangrijke mate af van de vooruitzichten van gezinnen en bedrijven. Aanhangers van de theorie van de 'rationele verwachtingen' gaan ervan uit dat vooruitziende belastingbetalers het succes van de stimulus zullen ondergraven. De overheid zal haar hogere uitgaven vroeg of laat moeten financieren met hogere belastingen. Zelfs als de overheid de stimulus met leningen financiert, zijn hogere belastingen in de toekomst noodzakelijk zodat de overheid haar leningen kan terugbetalen. Rationele consumenten zullen hierop vooruitlopen en meer sparen om de toekomstige belastingfactuur te kunnen betalen. Terwijl de overheid dus meer uitgeeft, geven de gezinnen minder uit.

Als elke euro die de overheid in de economie pompt grotendeels door de private sector wordt opgespaard, helpen extra overheidsuitgaven de economie dus weinig vooruit. Deze voorspelling klopt echter niet met de feiten. Volgens een gezaghebbende literatuurstudie van Ramey (2011) zorgt elke tijdelijke uitgavenverhoging van één dollar door de overheid in de Verenigde Staten waarschijnlijk voor een toename in de Amerikaanse economische activiteit van 0.8 à 1.5 dollar. De toename in de overheidsuitgaven heeft dus een veel groter effect op de economische activiteit, dan wat de theorie van 'rationele verwachtingen' voorspelt.

Een voor de hand liggende verklaring is dat – in tegenstelling tot wat de aanhangers van de theorie van de 'rationele verwachtingen' veronderstellen – gezinnen en bedrijven in werkelijkheid niet perfect vooruitziend zijn. Bijvoorbeeld omdat ze onvoldoende geïnformeerd zijn of omdat ze onmogelijk de werking van de economie in al haar complexiteit volledig kunnen doorgronden. Hierdoor kunnen ze de toekomstige gevolgen van het beleid niet perfect inschatten.

Dit proefschrift vertrekt van dit inzicht en plaatst een meer plausibele ‘leerbenadering’ van verwachtingsvorming tegenover ‘rationele verwachtingen’. De benadering is op twee principes gestoeld. Ten eerste baseren gezinnen en bedrijven zich op beperkte informatie om vooruitzichten te maken. Ze gebruiken deze informatie naar best vermogen, maar doorgronden de algehele werking van de economie niet, of althans niet volledig. Ten tweede herzien ze hun voorspellingen telkens nieuwe informatie beschikbaar wordt. Ze leren met andere woorden uit hun fouten.

Hoe wordt het succes van het beleid door dit leergedrag beïnvloed? Het proefschrift vertrekt hiervoor van de heersende macro-economische modellen, zeg maar het instrumentarium dat beleidsmakers gebruiken om de gevolgen van hun beleid door te rekenen. De resultaten in dit proefschrift tonen aan dat het loont om de ‘rationele verwachtingen’ in deze modellen te vervangen door verwachtingen gevormd door leergedrag.

Allereerst sluiten de voorspelde effecten van een toename van de overheidsuitgaven in een model met leergedrag beter aan bij de werkelijkheid. Het eerste hoofdstuk illustreert dit voor de Verenigde Staten. Volgens het leermodel doet een tijdelijke toename van de Amerikaanse overheidsuitgaven de economische activiteit ongeveer evenredig toenemen. Om precies te zijn: elke extra dollar overheidsconsumptie laat het bruto binnenlands product groeien met één dollar en één dollarcent. Bij ‘rationele verwachtingen’ is dat ongeveer de helft, oftewel 51 dollarcent. De voorspelling van het leermodel valt dus mooi binnen de grenzen van de literatuurstudie van Ramey (2011), terwijl het model met ‘rationele verwachtingen’ het effect onderschat.

Cruciaal is de veronderstelling dat gezinnen en bedrijven de toekomstige gevolgen van de beleidswijziging niet helemaal in rekening brengen. Wat zijn deze gevolgen? Er is natuurlijk het directe gevolg van hogere belastingen op het inkomen. Als de overheid de extra uitgaven met hogere (toekomstige) belastingen financiert, zullen de gezinnen netto minder overhouden. Het doorsnee gezin is zich hier waarschijnlijk van bewust.[1] Maar de effecten voor de algehele economie reiken veel verder dan dat. Zo voorspelt het model in het eerste hoofdstuk dat bedrijven meer werknemers zullen aanwerven of de bestaande werknemers langer zullen laten werken om aan de verhoogde vraag te voldoen. Dit zet opwaartse druk op de lonen. Daarnaast zal ook de rente stijgen. Dit is goed nieuws voor wie spaart of belegt, maar slecht nieuws voor wie geld leent. De resultaten van het leermodel laten zien dat wanneer deze indirecte toekomstige gevolgen niet geanticipeerd worden, gezinnen niet minder maar juist méér zullen besteden. Precies omgekeerd dus aan wat de theorie van de ‘rationele verwachtingen’ voorspelt. Vooral het onderschatten van het rente-effect speelt een belangrijke rol. Hogere rentes maken het voor

[1] Bijvoorbeeld omdat de overheid de toekomstige belastingverhoging duidelijk communiceert. Het referentiescenario in het eerste hoofdstuk maakt deze veronderstelling, maar ook als het gezin dit effect niet in rekening brengt, blijven de (kwalitatieve) conclusies die hier gepresenteerd worden, overeind.

gezinnen interessant om te sparen of te beleggen in plaats van te consumeren. Maar als de gezinnen de toekomstige rentestijging niet anticiperen, is dit rente-effect op de consumptie veel zwakker. In het leermodel domineren de positieve effecten van de hogere werkgelegenheid en het hogere loon op het negatieve rente-effect waardoor de gezinnen meer besteden. De toename van de private consumptie verklaart meteen ook waarom de totale economische activiteit sterk toeneemt. De private bestedingen maken immers meer dan de helft van de totale economische vraag uit.

Waar het eerste hoofdstuk de effecten van leergedrag illustreert voor de Verenigde Staten, kijkt het tweede hoofdstuk naar de eurozone. Dit hoofdstuk toetst het leermechanisme aan de feitelijke macro-economische ontwikkelingen in de eurozone in de laatste decennia. De resultaten tonen aan dat een macro-economisch model met leergedrag deze ontwikkelingen beter verklaart dan een model met ‘rationele verwachtingen’. Net zoals in het eerste hoofdstuk is het effect van extra overheidsuitgaven op de economie merkelijk groter dan onder ‘rationele verwachtingen’. Over de periode 1980-2013 deed een tijdelijke verhoging van de overheidsuitgaven met één euro de economische activiteit in de eurozone, volgens het leermodel, stijgen met gemiddeld 1.12 euro. Het uitgangspunt dat verwachtingen op beperkte informatie gebaseerd zijn, ligt opnieuw aan de basis van dit resultaat. Volgens het model met perfect geïnformeerde en ‘rationele’ individuen is het effect van een tijdelijke verhoging van de overheidsuitgaven met één euro amper 0.43 euro.

Omdat gezinnen en bedrijven de toekomstige gevolgen van stimuleringsbeleid niet volledig kunnen ontrafelen, kan de impact van dat beleid dus groter zijn dan wat modellen met perfect geïnformeerde individuen voorspellen. Een tweede belangrijke conclusie van dit proefschrift is dat wisselende verwachtingen van gezinnen en bedrijven kunnen verklaren waarom de effecten van overheidsuitgaven sterk variëren over de tijd. Volgens de leerbenadering worden deze verwachtingen immers voortdurend bijgewerkt. Telkens nieuwe informatie beschikbaar wordt, passen individuen hun prognoses aan. Het tweede en derde hoofdstuk gaan hier dieper op in.

Het leermodel in het tweede hoofdstuk laat zien dat de impact van extra overheidsuitgaven op de economische activiteit in de eurozone in de jaren ‘80 en het begin van de jaren ‘90 gedaald is. In het begin van de jaren ‘80 zorgde een extra euro overheidsuitgaven nog voor een toename in de economische activiteit van ongeveer 1.30 euro. In het begin van de jaren ‘90 zakte dit effect echter net onder de kritische grens van één euro. Deze daling is volgens het leermodel in belangrijke mate te wijten aan een meer pessimistische inschatting van de gevolgen van de extra overheidsuitgaven voor de toekomstige consumptiemogelijkheden. Vanaf het midden van de jaren ‘90 stijgt het effect echter weer. Eind 2013 zorgde elke extra euro uitgaven voor een toename van 1.13 euro in de economische activiteit van de eurozone.

Het derde hoofdstuk, ten slotte, levert een belangrijke methodologische bijdrage voor het onderzoek rond leergedrag en de effecten van overheidsuitgaven. Globaal gesproken richt dit proefschrift zich op de conjuncturele effecten van overheidsuitgaven, dat wil zeggen de effecten op de kortetermijnschommelingen van de economische activiteit rond haar langetermijngroeipad. Deze schommelingen worden in de wetenschappelijke literatuur doorgaans geanalyseerd in een lineaire benadering van een macro-economisch model rond dit groeipad. Voor kleine schommelingen kan deze benadering nuttig zijn, maar voor grote afwijkingen ten opzichte van het groeipad – denk aan een diepe recessie of een vastgoedzeepbel – kan een lineaire benadering erg onnauwkeurig zijn. Het derde hoofdstuk stapt af van deze lineaire benadering en toont aan dat het ook mogelijk is om de effecten van overheidstuitgaven te onderzoeken in het originele, niet-lineaire model. Net zoals in de eerste twee hoofdstukken, ontstaat er variatie in de macro-economische effecten van extra overheidsuitgaven omdat gezinnen en bedrijven hun prognoses updaten wanneer ze nieuwe informatie krijgen. Beleidsmakers en onderzoekers kunnen het kader dat we in het derde hoofdstuk aanreiken, gebruiken om de gevolgen van beleid te bepalen, ook wanneer de economie zich ver weg van haar langetermijnevenwicht bevindt.

Contents

Dankwoord / Acknowledgements 5

Nederlandstalige Samenvatting 7

Contents 11

List of Figures 15

List of Tables 17

Introduction 19

1 Learning and the Size of the Government Spending Multiplier 21

1 Introduction 21
2 The Model economy 24
3 Adaptive learning 27
4 The role of expectations for the effects of government spending shocks 29
4.1 Calibration 30
4.2 Impulse responses after a government spending shock 31
5 The government spending multiplier 36
6 Alternative specification of fiscal policy 39
7 Conclusion 40
Appendices 43
1.A Derivations of model equations 43
1.A.1 Household's optimization problem 43
1.A.2 Firms' optimization problem 44
1.A.2.1 Final goods sector 44
1.A.2.2 Intermediate goods sector 45
1.B Log-linearisation 46
1.B.1 New Keynesian specification 46
1.B.2 Neoclassical specification 47

1.C Learning dynamics . . . 47
1.C.1 Baseline model . . . 47
1.C.1.1 Household . . . 47
1.C.1.2 Firms . . . 49
1.C.2 Alternative specification of fiscal policy . . . 50
1.D Alternative specification of fiscal policy . . . 51

2 Adaptive Learning and the Transmission of Government Spending Shocks in the Euro Area 55
1 Introduction . . . 55
2 The Model Economy . . . 57
3 Rational Expectations Equilibrium . . . 60
4 Adaptive Learning . . . 61
5 Bayesian Estimation . . . 63
5.1 Data and Observation Equations . . . 63
5.2 Prior Distributions . . . 64
5.3 Estimation Results . . . 66
6 Dynamics of a Government Spending Shock . . . 68
7 The Government Spending Multiplier . . . 71
7.1 Results . . . 71
7.2 Discussion . . . 75
8 Robustness analysis . . . 77
8.1 Alternative learning schemes . . . 77
8.2 Alternative sample periods . . . 79
9 Conclusion . . . 80
Appendices . . . 83
2.A Model appendix . . . 83
2.A.1 Non-linear model . . . 83
2.A.1.1 Household . . . 83
2.A.1.2 Employment agencies and labour unions . . . 85
2.A.1.3 Final goods sector . . . 86
2.A.1.4 Intermediate goods sector . . . 87
2.A.1.5 Government . . . 89
2.A.1.6 Market clearing . . . 89
2.A.2 Stationary equilibrium . . . 89
2.A.2.1 Household . . . 89
2.A.2.2 Employment agencies and labour unions . . . 90
2.A.2.3 Intermediate goods sector . . . 91

2.A.2.4 Government 92
2.A.2.5 Market clearing 92
2.A.3 Steady state 92
2.A.4 Log-linearised model 93
2.A.4.1 Households 93
2.A.4.2 Employment agencies and labour unions 93
2.A.4.3 Intermediate goods sector 94
2.A.4.4 Government 95
2.A.4.5 Market clearing 95
2.B Additional results 96

3 Real-time Parameterized Expectations and the Effects of Government Spending 101
1 Introduction 101
2 Model 103
2.1 Households 103
2.2 Firms 104
2.3 Government 104
3 Real-time non-linear learning 105
3.1 Set-up 105
3.2 Discussion 106
3.3 Non-linear learning simulation 106
4 The effects of government spending under non-linear learning 108
4.1 Discussion 111
4.2 Choice of the approximating function: linear versus non-linear learning 114
5 Conclusion 116
Appendices 119
3.A Parameterized expectations algorithm 119
3.B Learning algorithm 119

4 General Conclusion 121

Bibliography 123

List of Figures

1.1 Impulse responses to an increase in government spending of 1% of GDP in the neoclassical specification of the model. 32
1.2 Impulse responses to an increase in government spending of 1% of GDP in the new Keynesian model. 34
1.3 Forecast errors after a government spending shock of 1% of GDP in the neoclassical and new Keynesian specification of the learning model. 35
1.4 Impulse responses to an increase in government spending of 1% of GDP for different degrees risk aversion (σ) in the new Keynesian model. 36
1.5 Impact multipliers for different degrees of price rigidity in the rational expectations model and the adaptive learning model. 38
1.6 Impulse responses to an increase in government spending of 1% of GDP of the new Keynesian model for different fiscal policy specifications. 52
1.7 Expectations on forward-looking variables after a government spending shock of 1% of GDP of the new Keynesian model for different fiscal policy specifications. . 53

2.1 Data used in the estimation. 65
2.2 Estimated beliefs in the forecasting models of the agents. 69
2.3 Impulse responses to an increase in government spending of 1% of GDP. 72
2.4 Present value government spending multipliers at selected horizons. 74
2.5 Impact multipliers for counterfactual evolutions of the belief parameters. 76
2.6 Box-plots for the impact multipliers for different model specifications. 78
2.7 Box-plots for the impact multipliers for different sample periods. 79
2.8 Impact multipliers for counterfactual evolutions of the belief parameters. 96
2.9 Impact multipliers for different model specifications and different sample periods. . 97

3.1 Simulated series of model variables under different assumptions regarding the formation of expectations. 108
3.2 Evolution of the belief coefficients in the non-linear learning model. 109
3.3 Evolution of the impact multipliers in the non-linear learning model. 110
3.4 Distribution of impact multipliers in the non-linear learning model. 110

3.5 Distribution of impact multipliers for high and low values of belief parameter γ_3 . . 111
3.6 Endogenous drivers of the government spending multiplier. 113
3.7 Linear and non-linear approximating function for different values of the capital stock. 115
3.8 Distribution of impact multipliers in the learning model with a linear approximating function. 115

List of Tables

1.1 Model parameters. 31

1.2 Steady-state values of main variables in the baseline model. 31

1.3 Present-value multipliers in the new Keynesian model under rational expectations and under adaptive learning. 37

1.4 Present-value multipliers for different specifications of fiscal policy in the rational expectations model and the adaptive learning model. 41

2.1 Prior and posterior distributions of the model parameters under Kalman filter learning and under rational expectations. 67

2.2 Prior and posterior distributions of the standard deviations of the shocks under Kalman filter learning and under rational expectations. 68

2.3 Log marginal likelihood of different model specifications. 78

2.4 Log marginal likelihood under Rational Expectations and under Kalman Filter Learning for different sample periods. 79

2.5 Comparison of the marginal likelihoods and the posterior modes of the model parameters for different sample periods. 98

2.6 Comparison of the posterior modes of the standard deviations of the shocks under Kalman filter learning and under rational expectations for different sample periods. 99

3.1 Calibration. 107

3.2 Forecasting performance of linear and non-linear learning. 116

Introduction

Can government spending stimulate the economy? This question has been the subject of one of the strongest and most unresolved debates in economics. Throughout modern history, governments raised spending during recessions in attempts to revive the economy. The large fiscal stimulus adopted by many governments during the recent economic crisis is a clear example of this.

Structural macroeconomic models, which explicitly feature optimizing businesses and consumers, are indispensable for understanding the effects of government spending shocks. They uncover the mechanisms underlying the effects of fiscal policies and provide counterfactual policy predictions. A major challenge is to match the fiscal policy predictions of those models with empirical evidence. The standard real business cycle model, for example, typically generates government spending multipliers for output that are outside the range of most empirical estimates. Moreover, the model is unable to account for the positive reaction of private consumption after a government spending shock found by several empirical studies. Another shortcoming is that the effects of government spending are usually assumed to be constant over time. This stands in sharp contrast with the observed time variation documented in a substantial, and still growing, body of literature.

The predictions of structural models crucially depend on how they treat private-sector expectations. In virtually all of these models, households and firms hold rational expectations. Under this strong assumption, agents have complete knowledge of the functioning of the economy. They fully understand the macroeconomic effects of fiscal stimulus and rationally incorporate this knowledge in their decision making. In fact, consumers and businesses do not only foresee the direct effects of fiscal stimulus, for instance the effect of changing taxes on their disposable incomes, but also foresee the indirect, general equilibrium effects on future wages, interest rates, and other macroeconomic variables.

It is well understood that the rational expectations assumption downplays the role of fiscal policy as a tool of stabilization policy. Take for example the case of a tax-financed expansion in government expenditure. If consumers anticipate the period of higher taxes during the stimulus, a drop in private consumption is almost inevitable which seriously undermines the effectiveness of the fiscal expansion.

Quite obviously, real-world decision makers have cognitive limitations and limited information, restricting their ability to understand the world. This dissertation goes beyond the rational expectations hypothesis and considers a more plausible view of rationality. In the models presented in the subsequent chapters agents are bounded by cognitive and informational constraints and rely on small forecast rules to form expectations. In addition, they learn from their mistakes as they update their forecast rules each time new information becomes available. This adaptive learning approach is a flexible way of combining insights from cognitive scientists with macroeconomic theory into a single analytical framework.

This dissertation comprises three papers that show how structural macroeconomic models with learning behaviour enhance our understanding of the effects of government spending shocks. Chapter 1 employs a new Keynesian model to examine the government spending multiplier. When agents form expectations using adaptive learning the effects of a government spending shock are substantially different from those under rational expectations. In contrast to the rational expectations model, private consumption can react positively to a government spending increase. Sticky prices and complementarity between consumption and labour are crucial for obtaining this result. Interaction between these two model features provides a channel by which private consumption can react positively to a government spending shock. Under learning, this channel is strong enough to generate a positive reaction of private consumption, whereas under rational expectations it is not.

Chapter 2 estimates a dynamic stochastic general equilibrium model for the euro area. For this purpose, the new Keynesian model from Chapter 1 is extended with a number of model features such as sticky wages and a time-varying inflation target. The model is such that agents use the Kalman filter to update their forecasting models. Based on the marginal likelihood criterion, this chapter finds that there is strong evidence in favour of this learning mechanism relative to rational expectations. Moreover, this chapter illustrates how learning behaviour generates significant time variation in the effects of government spending shocks. As agents update their forecasting models, the transmission of government spending shocks changes as well. As a consequence, the model generates endogenous time variation in the government spending multipliers.

Chapter 3, co-authored with Brecht Boone, shows that it is also possible to introduce learning behaviour in a non-linear macroeconomic model. The principal contribution of this chapter is methodological in nature. Following the vast majority of the literature, the models in Chapter 1 and Chapter 2 are linearised around the steady state. Chapter 3, on the other hand, considers a non-linear real business cycle model. We find that learning in this non-linear model can generate substantial time variation in the transmission of government spending shocks in the economy.

Finally, Chapter 4 offers a general conclusion of this dissertation. It highlights the recommendations for policy-makers and gives directions for future research.

Chapter 1

Learning and the Size of the Government Spending Multiplier

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Abstract

This paper examines the government spending multiplier when economic agents combine adaptive learning and knowledge about future fiscal policy to form their expectations. The analysis shows that the effects of a government spending shock substantially change when the rational expectations hypothesis is replaced by this learning mechanism. In contrast to the dynamics under rational expectations, a government spending shock in a small-scale new Keynesian DSGE model with learning crowds *in* private consumption and is associated with a positive co-movement between real wages and hours worked. In the baseline calibration, the output multiplier under learning is above one and about twice as large as under rational expectations.

1 Introduction

Since the outbreak of the global financial crisis in 2008, countries around the world have tried to fight the recession with a series of fiscal policy measures. Many governments have adopted a broad range of expansionary measures such as large tax cuts, boosts in direct spending and various investment programmes. Conversely, other countries have embarked on fiscal austerity measures, because of concerns about the sustainability of public finances. This revival of fiscal policy has renewed the debate on the effects of discretionary fiscal policy.

A central issue in this debate is the size of the government spending multiplier. Although the empirical estimates are dispersed over a broad range, the estimates are in many cases higher than those found in theoretical business cycle models. Based on a comprehensive review of the empirical literature Ramey (2011) concludes that the multiplier is probably between 0.8 and 1.5. Moreover, several studies find a large positive response of private consumption – see for example Blanchard and Perotti (2002), Fatás and Mihov (2001), Galí et al. (2007), and Perotti (2008). This stands in sharp contrast with the crowding *out* of consumption and the much smaller multipliers in most theoretical models.

An important question is to what extent these small multipliers depend on the hypothesis of rational expectations. This paper addresses this question by comparing the rational expectations benchmark with a model where agents form expectations using an adaptive learning mechanism and knowledge about future fiscal policy.

The hypothesis of rational expectations presumes that agents fully oversee the structure of the model and do not face any computational limitations in deriving expectations. The adaptive learning approach, discussed in Evans and Honkapohja (2001), introduces a more plausible view of rationality. According to this approach agents form expectations based on estimated forecasting models and update the coefficients in these models over time as new data becomes available.

It seems very natural to assume that agents have no perfect knowledge concerning the general equilibrium effects of fiscal policy. On the other hand, they presumably understand the direct implications for their current and future after-tax incomes. Therefore we follow the approach of Evans et al. (2009) and assume that agents understand the future path of taxes and other fiscal instruments implied by the government financing structure, while using infinite-horizon learning to forecast other variables.

This paper extends the existing literature by assessing the role of this learning set-up for the effects of government spending shocks in a new Keynesian DSGE model. The key result is that the multiplier under learning is about twice as large as under rational expectations. Hence, this paper provides a theoretical argument for the large multipliers in the recent empirical literature. Moreover, in contrast to the dynamics under rational expectations, government spending crowds *in* private consumption when agents engage in learning behaviour. The intuition for this result is that, in the learning model, households initially do not anticipate the general equilibrium effects of future government spending. In particular, expectations about future interest rates are considerably lower than under rational expectations and this has a positive effect on current consumption.

The analysis confirms the importance of price rigidity and the complementarity between labour and consumption for explaining the positive consumption response, as emphasized by Bilbiie (2011) and Christiano et al. (2011), for example. However, under rational expectations these features alone are not sufficient for government spending to crowd *in* private consumption. Only in the learning model consumption rises after the fiscal shock, resulting in a multiplier greater than one, even if price rigidity is limited and the degree of consumption-labour complementarity is small. Another result is that learning is crucial for generating a positive co-movement between hours worked and real wages after a government spending shock.

This paper also considers different fiscal financing strategies in an extended version of the model with distortionary taxes. Not surprisingly, the government spending multipliers are substantially smaller, or even negative, when government spending is financed through capital or labour income taxes. However, the output and consumption multipliers under learning are always larger than under rational expectations, irrespective of the financing strategy.

The work presented here is related to several other papers that build on the learning set-up of Evans et al. (2009). All these contributions emphasize the substantial differences between the responses to fiscal policy changes under learning and under rational expectations. Mitra et al. (2013) consider permanent policy changes in a real business cycle (RBC) model where agents also have to estimate the new steady state values. The authors show that under learning oscillatory dynamics can emerge and that the effects under learning depend on the specific form of the policy change. Recently, Mitra et al. (2016) have analysed the effects of a surprise two-year increase in government spending. An interesting result is that their learning model can generate a hump-shaped response in consumption. Gasteiger and Zhang (2014) study the impact of fiscal policy in a deterministic version of the RBC model with distortionary taxation. This paper generalizes the analysis of the cited works by examining the dynamics in a new Keynesian DSGE model with commonly used model features such as imperfect competition, price rigidity, and capital adjustment costs. This paper shows that these model features crucially affect the impact of adaptive learning on the dynamics of a government spending shock, in particular when it comes to the degree of price rigidity. The importance of these features is also emphasized in a recent contribution by Evans et al. (2015). The authors examine the possibility of stagnation in a new Keynesian model when the zero lower bound on the nominal interest rate is binding. They show that, under learning, pessimistic expectations can push the economy into recession. The results presented here, apply to "normal times" where the central bank maintains a standard Taylor rule.

The remainder of the paper is organized as follows. Section 2 sets forth the DSGE model that will be used throughout the paper. The adaptive learning mechanism is set out in Section 3. In Section 4, the effects of a temporary increase in government spending in the learning model are compared with the effects under rational expectations. A distinction is made between a neoclassical specification with fully flexible prices and a new Keynesian specification of the model. The role of learning for the government spending multipliers is discussed in Section 5. As an extension, Section 6 adds a richer specification of fiscal policy to the baseline model and discusses the role of different financing strategies. The last section concludes.

2 The Model economy

This section briefly describes the new Keynesian DSGE model that we will use in this paper. The model is based on the standard sticky-price framework analysed, for instance, in Woodford (2003) and Christiano et al. (2011). More elaborate specifications can be found in Smets and Wouters (2003, 2007) and Christiano et al. (2005).

The economy is populated by a representative household, a perfectly competitive final goods producer, a continuum of monopolistically competitive intermediate goods producers, a central bank, and a fiscal authority.[1]

Household The representative household maximizes expected lifetime utility. Preferences are defined over consumption, C_t, and hours worked, N_t, and described by the following utility function:

$$E_0^* \sum_{t=0}^{\infty} \beta^t \frac{\left[C_t^{\phi}(1-N_t)^{1-\phi}\right]^{1-\sigma} - 1}{1-\sigma}, \tag{1.1}$$

with $\beta \in (0,1)$, $\sigma > 0$, $\sigma \neq 1$, and $\phi \in (0,1)$.[2] Here $E_t^*(\cdot)$ denotes the subjective expectations of the household at time t. We consider King, Plosser and Rebelo (1988) preferences, which is standard in business cycle analysis. If $\sigma > 1$ consumption and labour are complements, which is an important model feature for the analysis in the subsequent sections.

The household's flow budget constraint is given by

$$C_t + I_t + B_{t+1} \leq W_t N_t + r_t^k K_t + R_{t-1}\Pi_t^{-1} B_t + D_t - T_t, \tag{1.2}$$

where I_t, W_t, r_t^k, D_t, and T_t denote period t gross investment, real wage rate, real rental rate of capital, dividends from intermediate firms, and lump-sum taxes, respectively. In addition, the variable B_t represents the quantity of one-period bonds carried over from period $t-1$. The variable R_{t-1} denotes the gross nominal interest rate on bonds purchased in period $t-1$, and Π_t denotes the gross inflation rate. The stock of physical capital, K_t, is owned by the household and accumulates according to

$$K_{t+1} = (1-\delta)K_t + I_t - \frac{\varsigma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \tag{1.3}$$

were δ denotes the physical rate of depreciation, and $\varsigma_I > 0$ is the Lucas and Prescott (1971) capital adjustment cost parameter.

As shown in Appendix 1.C.1, log-linearising the equilibrium conditions and substituting the consumption Euler equation into the household's inter-temporal budget constraint yields

[1] Appendix 1.A contains the derivations of the model equations.

[2] Money is not included explicitly in the analysis. A cashless limit economy is assumed. See Woodford (2003) for a detailed discussion of this approach.

the following consumption function:[3]

$$\Gamma_1 \hat{C}_t = \beta^{-1}\bar{K}\hat{K}_t + \Gamma_2 \hat{W}_t + \bar{K}\bar{r}^k \hat{r}^k_t + \bar{D}\hat{D}_t - \bar{G}\hat{G}_t - \Gamma_3 \hat{R}_t + SW^e_t - SR^e_t + S\Pi^e_t + Sr^{k,e}_t + SD^e_t - SG^e_t, \tag{1.4}$$

where

$$SW^e_t \equiv \Gamma_4 \sum_{j=1}^{+\infty} \beta^j E^*_t \hat{W}_{t+j}, \tag{1.5}$$

$$SR^e_t \equiv \Gamma_3 \sum_{j=1}^{+\infty} \beta^j E^*_t \hat{R}_{t+j}, \tag{1.6}$$

$$S\Pi^e_t \equiv \Gamma_3 \beta^{-1} \sum_{j=1}^{+\infty} \beta^j E^*_t \hat{\Pi}_{t+j}, \tag{1.7}$$

$$Sr^{k,e}_t \equiv \bar{K}\bar{r}^k \sum_{j=1}^{+\infty} \beta^j E^*_t \hat{r}^k_{t+j}, \tag{1.8}$$

$$SD^e_t \equiv \bar{D} \sum_{j=1}^{+\infty} \beta^j E^*_t \hat{D}_{t+j}, \tag{1.9}$$

$$SG^e_t \equiv \bar{G} \sum_{j=1}^{+\infty} \beta^j E^*_t \hat{G}_{t+j}, \tag{1.10}$$

under the assumption that the transversality condition

$$\lim_{j \to +\infty} \mathscr{R}_{t,t+j-1} E^*_t B_{t+j} = 0, \tag{1.11}$$

with $\mathscr{R}_{t,t+j} = \left(\prod_{s=0}^{j} E^*_t R_{t+s-1} E^*_t \Pi^{-1}_{t+s} \right)^{-1}$, holds.[4] The coefficients Γ_1, Γ_2, Γ_3, and Γ_4 are given in Appendix 1.C.1.

Equation (1.4) implies that the household's choice of current consumption depends on initial assets, factor prices, dividends, government expenditures, interest rates and expectations of future factor prices, interest rates, inflation rates, dividends, and government expenditures. It is assumed that agents understand the direct effect of government spending on their future disposable incomes, so that $E^*_t \hat{G}_{t+j} = \hat{G}_{t+j}$. The forecasts of the other variables, i.e. $E^*_t \hat{W}_{t+j}$, $E^*_t \hat{R}_{t+j}$, $E^*_t \hat{\Pi}_{t+j}$, $E^*_t \hat{r}^k_{t+j}$, and $E^*_t \hat{D}_{t+j}$, are based on an adaptive learning procedure, the details of which are described in Section 3.

Optimal investment requires that

$$\hat{Q}_t = \beta E^*_t \hat{Q}_{t+1} - \hat{R}_t + E^*_t \hat{\Pi}_{t+1} + \beta \bar{r}^k E^*_t \hat{r}^k_{t+1}, \tag{1.12}$$

[3]Throughout the paper, hatted variables denote log-deviations from the steady state. Barred variables refer to steady state values.

[4]Evans et al. (2012) provide a detailed analysis of the role of this condition for the validity of the Ricardian equivalence proposition.

where Q_t denotes Tobin's Q, the shadow value of existing capital.[5] Forward iteration gives the following infinite-horizon optimal investment rule

$$\hat{Q}_t = -\hat{R}_t + \sum_{j=1}^{\infty} \beta^j \left[\bar{r}^k E_t^* \hat{r}_{t+j}^k - E_t^* \hat{R}_{t+j} + \beta^{-1} E_t^* \hat{\Pi}_{t+j} \right]. \tag{1.13}$$

Firms A representative, perfectly competitive firm bundles a continuum of intermediate goods into a final good using the following CES technology:

$$Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1.14}$$

where $\varepsilon > 1$, and $Y_t(i)$ is the input of intermediate good $i \in [0,1]$. The firm chooses the quantities of inputs so as to maximize its profit, taking as given the final goods price P_t and the intermediate goods prices $P_t(i)$, for all $i \in [0,1]$. Profit maximization implies the demand equation for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t. \tag{1.15}$$

There is a continuum of monopolistically competitive intermediate goods producers populating the unit interval. Facing the real factor prices W_t and r_t^k, and the demand function (1.15), a typical intermediate goods firm $i \in [0,1]$ rents labour, $N_t(i)$, and capital, $K_t(i)$, in order to minimize costs. Its production function is given by

$$Y_t(i) = Z_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}, \tag{1.16}$$

where Z_t represents technology that follows an exogenous process given by

$$Z_t = Z_{t-1}^{\rho_Z} \exp\left(\varepsilon_t^Z\right), \quad \varepsilon_t^Z \sim \mathscr{N}\left(0, \sigma_Z^2\right), \tag{1.17}$$

with $\rho_Z \in (0,1)$.

Following Calvo (1983), intermediate goods producers set nominal prices in a staggered fashion. Each period an intermediate goods producer can adjust its price with a constant probability $1-\theta$. A firm i that is permitted to adjust prices at period t, will choose a new optimal price, $P_t^*(i)$, to maximize the expected present discounted value of future profits

$$E_t^* \sum_{j=0}^{\infty} (\beta\theta)^j \frac{U_{C,t+j}}{U_{C,t}} \left\{ \frac{P_t^*(i)}{P_{t+j}} Y_{t+j}(i) - MC_{t+j} Y_{t+j}(i) \right\}, \tag{1.18}$$

where $U_{C,t+j}$ is the j-period ahead marginal utility of consumption. At the end of each period, the intermediate firm distributes its profits as a real dividend, $D_t(i)$, to the representative household.

[5] Tobin's Q is defined as $Q_t \equiv q_t/\lambda_t$, where q_t is the Lagrangian multiplier with respect to the capital accumulation rule and λ_t the Lagrangian multiplier with respect to the household's budget constraint in the household's optimization problem. See Appendix 1.A for the derivations.

It is shown in Appendix 1.C.1 that optimal price setting yields the following representation of the infinite-horizon Phillips curve:

$$\hat{\Pi}_t = \varphi\theta^{-1}\widehat{MC}_t + \beta\varphi\sum_{j=0}^{+\infty}(\beta\theta)^j\left[(1-\alpha)E_t^*\hat{W}_{t+j+1} + \alpha E_t^*\hat{r}_{t+j+1}^k - E_t^*\hat{Z}_{t+j+1}\right] + \beta(1-\theta)\sum_{j=0}^{+\infty}(\beta\theta)^j E_t^*\hat{\Pi}_{t+j+1}, \quad (1.19)$$

with $\varphi \equiv (1-\theta)(1-\beta\theta)$.[6]

Government Policies The fiscal authority finances expenditure through lump-sum taxes and bond sales. The government budget constraint is given by

$$T_t + B_{t+1} = G_t + R_{t-1}\Pi_t^{-1}B_t. \quad (1.20)$$

The central bank sets the nominal interest rate according to the following Taylor rule:

$$R_t = \Pi_t^{\rho_\Pi}u_t^R \quad (1.21)$$

with $u_t^R = \left(u_{t-1}^R\right)^{\rho_R}\exp\left(\varepsilon_t^R\right)$, and $\varepsilon_t^R \sim \mathcal{N}(0,\sigma_R^2)$. It is assumed the Taylor principle to hold, i.e. $\rho_\Pi > 1$.

Market Clearing Market clearing in the goods market and the markets of production factors requires that the following conditions are met:

$$Y_t = C_t + I_t + G_t, \quad (1.22)$$

$$N_t = \int_0^1 N_t(i)di, \quad (1.23)$$

$$K_t = \int_0^1 K_t(i)di. \quad (1.24)$$

Linear approximation The remainder of the paper considers the log-linear approximation of the model about its steady state. The equations of the linearised model are given in Appendix 1.B.

3 Adaptive learning

We now turn to the specification of the adaptive learning model. We use the standard case of rational expectations as a benchmark to compare against the learning model. In the absence of a

[6]The results in this paper are independent of whether the forecasts of the technology shock $E_t^*\hat{Z}_{t+j}$ are determined by adaptive learning or by the shock process (1.17).

policy change, the dynamics of the linearised model under rational expectations can be written as a function of the capital stock $\hat{K}_t$ and the technology shock $\hat{Z}_t$

$$y_t = \bar{\Psi} \begin{bmatrix} \hat{K}_t \\ \hat{Z}_t \end{bmatrix}, \tag{1.25}$$

where y_t is the vector of log-linearised endogenous variables of the model.[7]

We go beyond the rational expectations hypothesis and assume agents combine limited structural knowledge with adaptive learning to forecast future variables. In particular, agents understand the structure of government financing and use the government budget constraint (1.20) and the announced future change in government spending to forecast future taxes. In forecasting other variables they rely on forecasting models estimated using least-squares learning.

As argued by Evans et al. (2009) this set-up is a natural way to proceed. When it comes to the general equilibrium effects of fiscal policy, it is hard to believe that households and firms have perfect knowledge on how fiscal policy shocks affect future aggregate variables. On the other hand, agents presumably understand the direct implications of higher future taxes for their future disposable incomes.

This approach implies that $E_t^* \hat{G}_{t+j} = G_{t+j}$ in the consumption function (1.4), since households know the future path of government spending and understand the direct effect of this path on their future disposable incomes.

Forecasts on wages $E_t^* \hat{W}_{t+j}$, interest rates $E_t^* \hat{R}_{t+j}$, inflation rates $E_t^* \hat{\Pi}_{t+j}$, rental rates of capital $E_t^* \hat{r}^k_{t+j}$, and dividends $E_t^* \hat{D}_{t+j}$ appearing in the conditions (1.4), (1.13), and (1.19) are determined by adaptive learning.[8] These forecasts depend on the perceived laws of motion (PLMs) held by the agents. Following Mitra et al. (2013, 2016), it is assumed that the form of these laws correspond to (1.25) so that they are linear functions of the capital stock $\hat{K}_{t+1}$ and the technology shock $\hat{Z}_t$:

$$E_t^* y^f_{t+1} = \psi_t \begin{bmatrix} \hat{K}_{t+1} \\ E_t^* \hat{Z}_{t+1} \end{bmatrix} = \psi_t \begin{bmatrix} \hat{K}_{t+1} \\ \rho_Z \hat{Z}_t \end{bmatrix}, \tag{1.26}$$

where $E_t^* y^f_{t+1} = \left(E_t^* \hat{W}_{t+1}; E_t^* \hat{R}_{t+1}; E_t^* \hat{\Pi}_{t+1}; E_t^* \hat{r}^k_{t+1}; E_t^* \hat{D}_{t+1}; E_t^* \hat{K}_{t+2} \right)$.[9]

[7]The 13 endogenous log-linearised variables of the model are private consumption ($\hat{C}_t$), dividends ($\hat{D}_t$), investment ($\hat{I}_t$), capital ($\hat{K}_t$), marginal cost ($\hat{MC}_t$), labour ($\hat{N}_t$), inflation ($\hat{\Pi}_t$), Tobin's Q ($\hat{Q}_t$), nominal interest rate ($\hat{R}_t$), rental rate of capital ($\hat{r}^k_t$), lump-sum taxes ($\hat{T}_t$), wage rate ($\hat{W}_t$), and output ($\hat{Y}_t$).

[8]Following the *infinite-horizon* learning approach, it is assumed that agents make forecasts infinitely many periods ahead. By contrast, the *Euler equation* learning approach assumes that agents make one-step ahead forecasts which are typically present in Euler equations. For a discussion of the two approaches see Honkapohja et al. (2013). An earlier version of the paper considered *Euler equation* learning in a model where agents did not incorporate the future path of government spending into their behavioural rules. The results of this approach, which are available upon request, are very similar to those presented here.

[9]As is standard in the learning literature, it is assumed that the agents know the parameters of the observed exogenous processes.

Agents estimate the coefficients ψ_t using a constant-gain variant of Recursive Least Squares:

$$\begin{aligned}\psi_t &= \psi_{t-1} + \gamma S_t^{-1} X_{t-1} \left(y_{t-1}^f - \psi_{t-1}^T X_{t-1} \right)^T, \\ S_t &= S_{t-1} + \gamma \left(X_{t-1} X_{t-1}^T - S_{t-1} \right),\end{aligned} \tag{1.27}$$

where $X_t = \left(\hat{K}_{t+1}; \hat{Z}_t\right)$ is the data vector used to estimate the beliefs, S_t is the moment matrix for X_t, and $\gamma > 0$ is the gain parameter.

Because the gain parameter is assumed to be a positive constant, the learning algorithm weighs recent data more heavily. Orphanides and Williams (2005, 2007) refer to this approach as "perpetual learning" because agents forget past data over time and hence learn permanently. The constant-gain recursive least squares algorithm is therefore widely used in the adaptive learning literature (see Eusepi and Preston, 2011; Milani, 2007; Slobodyan and Wouters, 2012b, for example).

Given the expectations on the variables the household needs to forecast, we can explicitly calculate the sums of future expected terms in (1.4), (1.13), and (1.19), and determine the dynamics under learning. See Appendix 1.C.1 for further details.

The forecast rule (1.26) cannot converge to the rational expectations equilibrium, because it is not in the same space.[10] However, it can converge to a so-called restricted perceptions equilibrium (RPE). In this equilibrium the agents' forecasts are optimal relative to the restricted information set. That is, although agents use an underparameterised forecasting model, their forecast errors are uncorrelated with the (restricted) information set X_t used in the expectation formation. Guse (2008) provides a general technique to project the actual law of motion into the same class as the underparameterised forecasting model. The technique defines a projected T-map which maps the restricted forecast rule to the projected actual law of motion. The RPE can be found as a fixed point of this map. In the next section, the initial coefficients of the forecast rule, ψ_0, are pinned down to the RPE-implied coefficients.

4 The role of expectations for the effects of government spending shocks

This section examines the effects of a temporary increase in government spending under different assumptions with respect to agents' expectations. In particular, the macroeconomic effects of the shock under rational expectations are compared with those under adaptive learning. Be-

[10] Using the terminology of Evans and Honkapohja (2001, Chapter 13) the PLMs are "restricted" or "underparameterised" because they do not include $\hat{G}_t$ and $\hat{u}_t^R$. Adding the monetary policy shock $\hat{u}_t^R$ does not alter the results of the paper. The exclusion of government spending $\hat{G}_t$, reflects the assumption that agents have imperfect knowledge on the general equilibrium effects of fiscal policy. If this variable were added to the PLMs, the impulse response functions under learning presented in this paper would coincide with those under rational expectations.

cause the role of price rigidity is of crucial importance, the new Keynesian model is examined in comparison with a neoclassical specification of the model where prices are fully flexible.

4.1 Calibration

The model is calibrated to quarterly periods. The parameters receive the values presented in Table 1.1. Most parameters are set to values that are typical in the business cycle literature. The elasticity of output with respect to capital, α, is fixed to 1/3. The subjective discount factor, β, is calibrated to match an annualized steady state real interest rate of 4.0%. The value of δ is 0.025 so that the depreciation rate of capital is 2.5% per quarter. The elasticity of substitution between intermediate goods, ε, is such that the mark-up of price over marginal cost is equal to 20% in steady state. The Calvo (1983) parameter, θ, is 0.75, implying an average frequency of price re-optimization of 4 quarters. The Taylor rule coefficient on inflation, ρ_{Π}, is 1.5, a standard value in the literature. The AR(1) coefficient of technology, ρ_Z, receives a value of 0.90. The coefficient of risk aversion, σ, is set to 2.0. This value is roughly in the middle of the range of the empirical estimates and consistent with the estimates obtained by Basu and Kimball (2002). Following Christiano et al. (2011), the capital adjustment cost parameter, ς_I, is equal to 17. The share of government expenditure in GDP, $\bar{G}/\bar{Y}$, is set at 0.20 to match the postwar U.S. government spending share. For the ratio $\bar{B}/\bar{Y}$ the average general government gross financial liabilities for the U.S. provided in the OECD (2014b) database over the period 2000–2013 are used. The preference parameter ϕ is calibrated such that the share of time devoted to work in the steady state is fixed to 1/3. As a benchmark, the gain parameter, γ, is set to 0.02, which is a value well within the range of estimates reported in the literature. However, the particular value of the gain parameter is not crucial for our impulse response analysis.[11]Following King and Rebelo (2000), the standard deviation of the technology shock is set to 0.72. The interest rate disturbance is assumed to have a standard deviation of 0.05. For simplicity, the AR(1) coefficient of the nominal interest rate, ρ_R, is assumed to be zero.

Table 1.2 shows the model values of some important macroeconomic aggregates. The calibration produces shares of private consumption and investment in GDP close to those observed in most industrialized countries. The steady-state labour's share of total income is 0.56, a value roughly consistent with the observed U.S. labour income share.[12]

[11] Orphanides and Williams (2005, 2007) found that a gain parameter in the range 0.01–0.04 provides the best fit between the agents' forecasts in the model and the expectations data from the Survey of Forecasters. Using a similar strategy, Branch and Evans (2006) obtain a value of 0.0345. The estimate of Milani (2007) equals 0.0183 and hence lies within the same range. However, the estimated gain of 0.0029 in Eusepi and Preston (2011) is much smaller. The estimation results from Slobodyan and Wouters (2012b) provide values for γ going from 0.001 to 0.06 depending on the particular learning scheme. Within the range of values mentioned here, the effect of a different value for the gain parameter is negligible.

[12] The U.S. labour income share in the industrial sector over the period 2000-2010 was on average 57% (OECD, 2014b).

Parameter	Description	Value
α	Output elasticity with respect to capital	1/3
β	Households subjective discount factor	$1.04^{-0.25}$
γ	Gain parameter	0.02
δ	Rate of physical capital depreciation	0.025
ε	Elasticity of substitution between intermediate goods	6.0
θ	Degree of nominal price rigidity	0.75
ρ_Π	Taylor rule inflation rate coefficient	1.5
ρ_R	Interest rate AR(1) coefficient	0.00
ρ_Z	Technology shock AR(1) coefficient	0.90
σ	Coefficient of risk aversion	2.0
σ_R	Standard deviation of the interest rate disturbance ε^R	0.05
σ_Z	Standard deviation of the technology disturbance ε^Z	0.72
ς_I	Capital adjustment cost parameter	17
ϕ	Preference parameter	0.35
$\bar{B}/\bar{Y}$	Steady state government debt to output ratio	0.74
$\bar{G}/\bar{Y}$	Steady state government expenditure to output ratio	0.20

Table 1.1: Model parameters.

Variable	Value	Variable	Value
$\frac{C}{Y}$	0.601	$\frac{r^k K}{Y}$	0.278
$\frac{I}{Y}$	0.199	Annualised r	0.040
$\frac{WN}{Y}$	0.556	Annualised r^k	0.147

Table 1.2: Steady-state values of main variables in the baseline model. Annualised r and r^k are defined as $\bar{R}^{1/4} - 1$ and $\left(1+\bar{r}^k\right)^{1/4} - 1$, respectively.

4.2 Impulse responses after a government spending shock

We now turn to the impulse responses of economic variables following a temporary increase in government spending of 1% of GDP at the beginning of period $t = 1$ that is financed through an increase in lump-sum taxes. After the shock, government spending gradually converges back to its steady-state value according to the following autoregressive process:

$$\hat{G}_t = \rho_G \hat{G}_{t-1}, \quad t > 1, \tag{1.28}$$

where ρ_G is assumed to be equal to 0.9. Then the expression (1.10) for the present value of future government expenditures becomes

$$SG_t^e = \bar{G}\frac{\beta\rho_G}{1-\beta\rho_G}\hat{G}_t. \tag{1.29}$$

Throughout the paper, it is assumed that real public debt remains constant and lump-sum taxes adjust to maintain budget balance in each period.

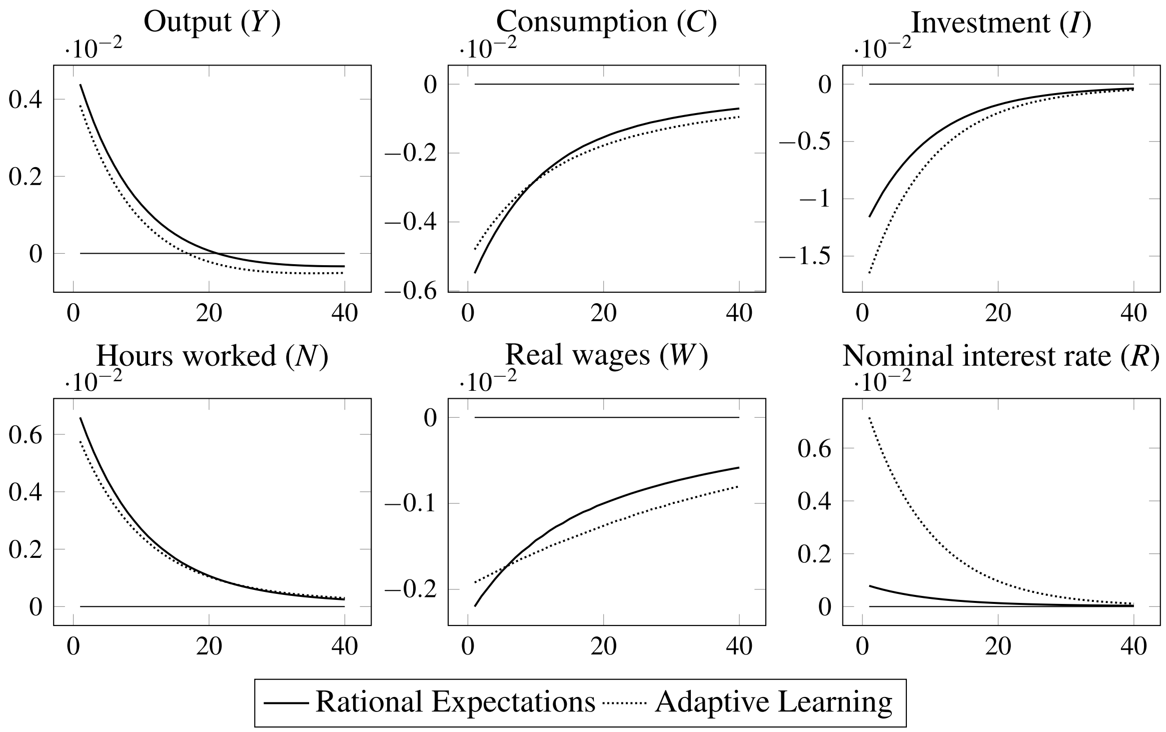

Figure 1.1: Impulse responses to an increase in government spending of 1% of GDP in the neoclassical specification of the model. The impulse response functions are measured in percentage deviations from steady-state. The horizontal axis measures quarters.

Neoclassical specification Figure 1.1 shows the responses to the government spending shock when prices are fully flexible ($\theta \to 0.00$). The solid and dotted lines depict the impulse responses under rational expectations and adaptive learning, respectively. The forecast errors in the learning model are displayed as dotted lines in Figure 1.3.[13]

Under rational expectations the effects of fiscal policy in a neoclassical model are well-understood – see for instance Baxter and King (1993). A temporary increase in government spending has a negative wealth effect, through additional taxes, resulting in a fall in private consumption and leisure. As a consequence, labour supply increases which causes a fall in the real wage. The government absorption of resources reduces private investment. The fall in the capital-labour ratio raises the rental rate of capital r_t^k. The ratio converges to its steady-state value as investment recovers and both W_t and r_t^k return to their steady-state values.

Under learning, consumption falls less than under rational expectations. The reason is that agents anticipate the increase in future taxes, but fail to correctly foresee the paths of lower future wages and higher future interest rates. The forecast errors in Figure 1.3 clearly illustrate that agents are too optimistic about future wages and underestimate the rise in interest rates that will result from the policy change. More generally, the responses of expected future interest

[13]The impulse responses of all variables in the model and the expectations on all forward-looking variables are included in Appendix 1.D.

rates, factor prices, and dividends to the fiscal shock are only limited because these expectations are determined by adaptive learning and only gradually adjust to the observed fall in the capital stock. Those expectations determine the household's consumption choice, as they appear in the expectational terms SW_t^e, SR_t^e , $Sr_t^{k,e}$, and SD_t^e in the consumption function (1.4). Under learning expected wages (SW_t^e) are higher and expected interest rates (SR_t^e) are lower than under rational expectations. That is why the fall in consumption is less severe, even though expected future rental rates ($Sr_t^{k,e}$) and dividends (SD_t^e) are lower than under rational expectations. Another consequence is the smaller increase in labour supply. The drop in disposable income is captured by a stronger contraction of investment. This leads to a larger increase in the interest rate. In the aggregate, the net impact of a government spending shock on output is slightly smaller under adaptive learning than under rational expectations.

New Keynesian specification Figure 1.2 depicts the impulse responses after a government spending shock in the economy where prices are rigid. When agents have rational expectations, the effects of a fiscal expansion are similar to those under fully flexible prices. Quantitatively, however, the effect on hours worked is stronger because the rise in labour supply is accompanied by an outward shift in labour demand. As set forth by Linnemann and Schabert (2003), Perotti (2008) and others, nominal rigidities generate a fall in the mark-up when the government boosts aggregate demand. This induces a rise in labour demand, which amplifies the increase in employment and reduces the fall in the real wage rate.

When agents form expectations using an adaptive learning mechanism, the effects of a government spending shock change substantially, especially with respect to the response of private consumption and real wages. In contrast to the neoclassical specification, government spending crowds *in* private consumption. This finding is particularly interesting since it is in accordance with the empirical evidence found in Auerbach and Gorodnichenko (2013), Blanchard and Perotti (2002), Fatás and Mihov (2001), Galí et al. (2007), Perotti (2008), and Tagkalakis (2008), for example.

Expectations, in particular those of future real interest rates, are key for understanding the response of consumption under learning. Just as in the neoclassical specification of the model, the inter-temporal substitution effect of higher future interest rates is much weaker under learning. The forecast errors, displayed as dashed lines in Figure 1.3, show that agents' inflation and interest rate forecasts are considerably lower than under rational expectations and this acts in favour of a positive response of consumption. At the same time, agents underestimate the effect of government spending on future factor prices and dividends, although this has only a minor effect on private consumption.

Comparing the neoclassical and new Keynesian specification of the learning model, it is apparent that price rigidity is crucial for generating crowding *in* of private consumption. In particular, as argued by Bilbiie (2011) and Christiano et al. (2011), staggered price setting

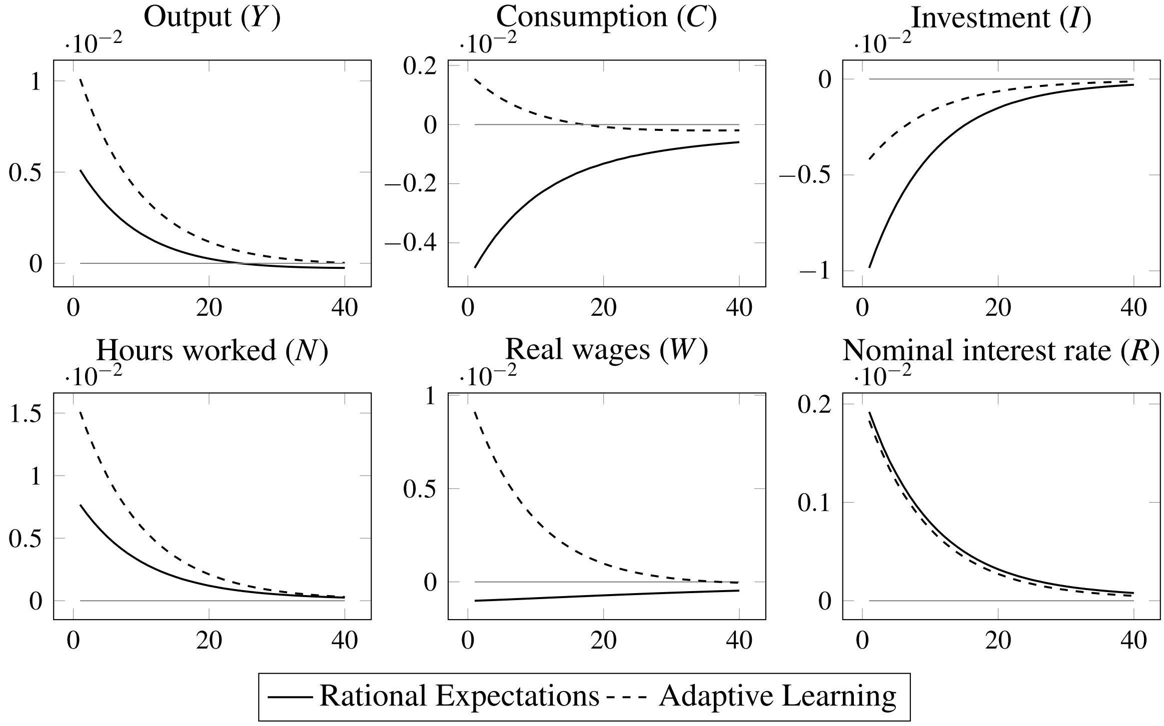

Figure 1.2: Impulse responses to an increase in government spending of 1% of GDP in the new Keynesian model. The impulse response functions are measured in percentage deviations from steady-state. The horizontal axis measures quarters.

by monopolistically competitive firms and complementarity between consumption and labour provides a channel by which private consumption can react positively to a government spending shock. If prices are sticky, a government spending shock induces an outward shift in the demand for labour, strengthening the rise in employment. Because consumption and labour are complements, i.e. $\sigma > 1$, this increase in employment raises the marginal utility of consumption. Under learning, this channel is strong enough for private consumption to crowd *in* after the government spending shock, whereas under rational expectations it is not.

Given our preference specification, higher values of σ imply stronger complementarity between consumption and labour. At the same time, a higher value of σ, i.e. a lower intertemporal elasticity of substitution, makes households less willing to postpone consumption in response to the expected real interest rate. Thus, the response of consumption is stronger, the larger the value of σ.[14] Figure 1.4 illustrates this result. The figure shows the impulse responses

[14] This result is particularly interesting given the discussion in the literature on preference-based explanations for government spending crowding *in* private consumption and the positive co-movement of consumption and hours worked. Linnemann (2006) argues that a certain type of non separable utility, where labour and consumption are complements, can generate these results in a standard real business cycle model. However, Bilbiie (2009, 2011) points out that the preferences considered by Linnemann rely on a downward-sloping labour supply schedule. By contrast, a standard King et al. (1988) utility specification is considered here and we find that in the adaptive learning model government spending crowds *in* private consumption even if the degree of complementarity, σ, is small.

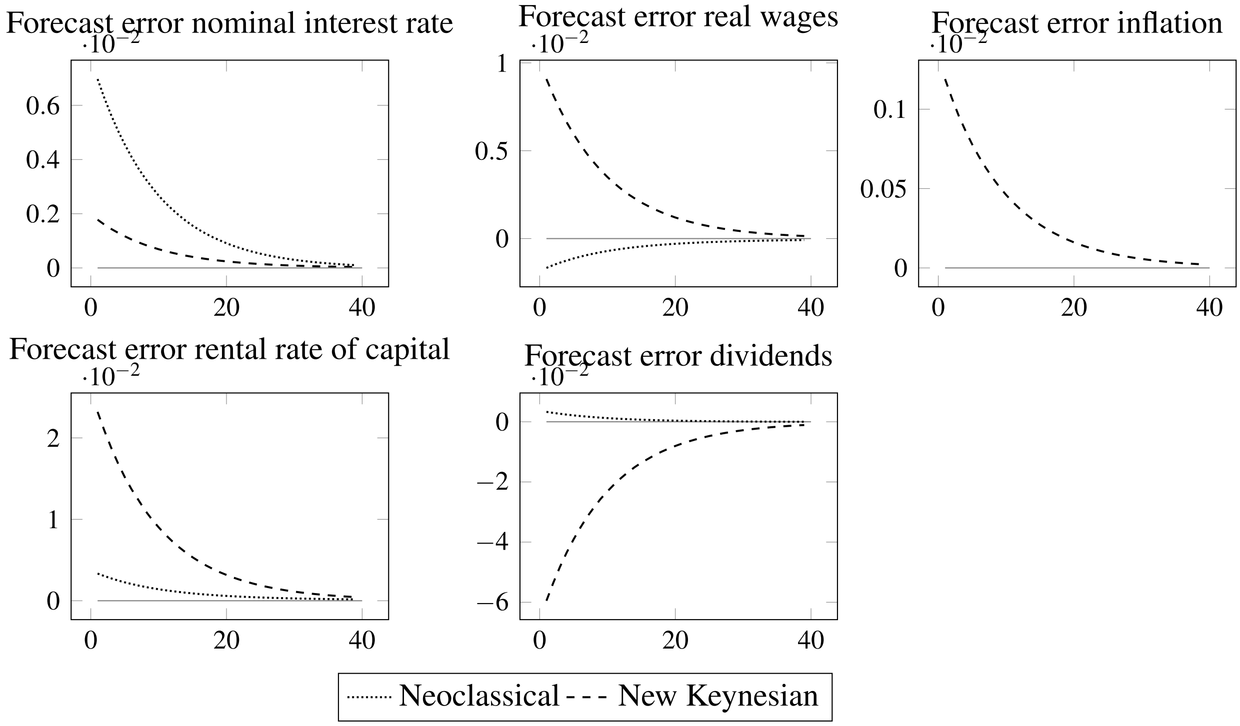

Figure 1.3: Forecast errors after a government spending shock of 1% of GDP in the neoclassical and new Keynesian specification of the learning model. Forecast errors for any variable y are defined as $y_{t+1} - E_t y_{t+1}$. The horizontal axis measures quarters.

of private consumption and output for different values of σ. The grey (light) shaded area in the right-hand plot shows that when economic agents have rational expectations, the impact of government spending on private consumption is negative for all considered values of σ. This is in sharp contrast with the impulse responses of the adaptive learning model depicted by the blue (dark) shaded area. It is clear that under the learning mechanism, the crowding *in* effect on consumption occurs for every $\sigma > 1$. However, in the limit case of $\sigma = 1$, when preferences are separable over leisure and consumption, this effect does not occur.

Another notable observation is the positive response of real wages under learning. Only when agents use the adaptive learning mechanism, the increase in aggregate hours after a positive government spending shock coexists with an increase in real wages. That is because the learning behaviour reduces the labour supply effect of the government spending shock, while price rigidity leads to a rise in labour demand. Considering this adaptive learning mechanism brings the theoretical impulse responses again into line with those observed empirically. Evidence on the positive co-movement between real wages and hours worked after a government spending shock can be found in Rotemberg and Woodford (1992), Galí et al. (2007), and Fatás and Mihov (2001), for example. However, the empirical evidence is not entirely unambiguous (see, for instance, Ramey and Shapiro (1998), and Perotti (2008)). Another difference with the neoclassical specification, is the dampening effect of learning on the fall in investment.

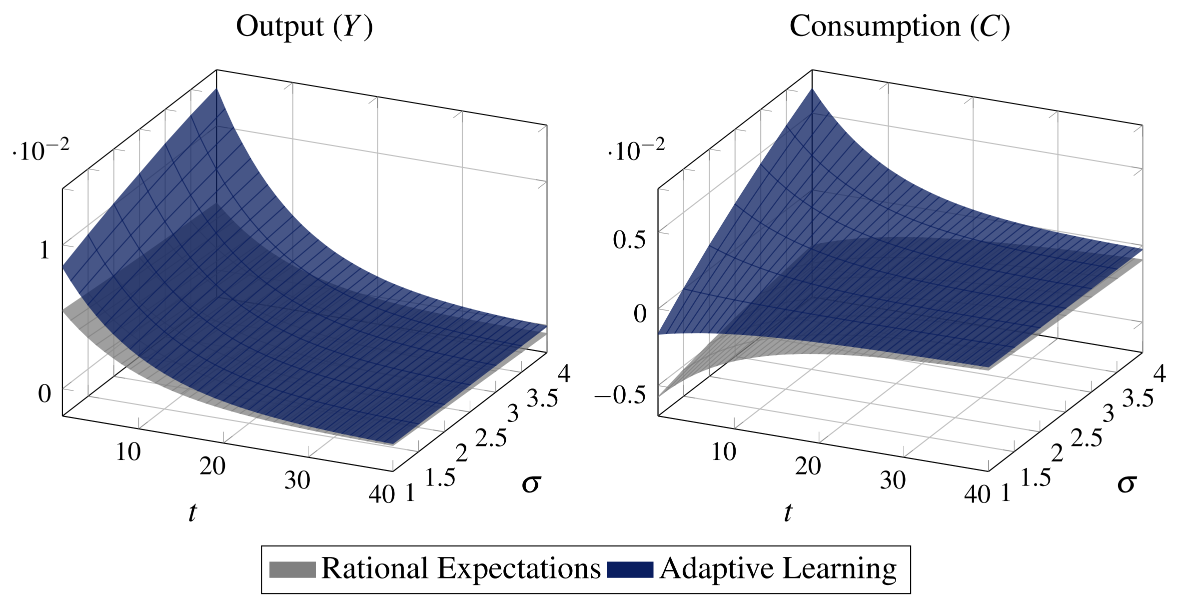

Figure 1.4: Impulse responses to an increase in government spending of 1% of GDP for different degrees of non-separability (σ) in the new Keynesian model. The impulse response functions are measured in percentage deviations from steady-state. The x-axis measures quarters.

5 The government spending multiplier

We now turn to the analysis of the government spending multiplier in the new Keynesian model. The question of the size of the government spending multiplier has been addressed by many authors in the literature. The growing empirical evidence that the size of the output multiplier can be much larger than one, especially when the economy is in recession and the zero bound on the nominal interest rate binds, confronts the theoretical literature with an important challenge. In response to this, several authors have proposed different mechanisms such as alternative preference specifications (Linnemann, 2006), the existence of rule-of-thumb consumers (Galí et al., 2007), different kinds of rigidities, and the stance of monetary policy (Christiano et al., 2011; Coenen et al., 2012; Leeper et al., 2015).

Against that background, the discussion in the previous section shows that expectation formation too is a key factor for the impact of fiscal policy. Adaptive learning can amplify this impact substantially, even in the absence of accommodative monetary policy. As explained in the previous section, because under learning agents underestimate the general equilibrium effects of tax-financed government spending expansion in their expectation formation, private consumption can respond positively and in this way amplify the response of aggregate economic activity.

Table 1.3 illustrates this result. It reports the present-value government spending multipliers for output, consumption, and investment in the rational expectations model and the adaptive learning model. Following Mountford and Uhlig (2009), the present-value multiplier for vari-

	Rational expectations				Adaptive learning			
	Impact	1 year	4 years	6 years	Impact	1 year	4 years	6 years
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	0.51	0.50	0.46	0.43	1.01	1.00	0.99	0.97
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−0.29	−0.30	−0.34	−0.37	0.09	0.09	0.07	0.06
$\frac{PV(\Delta I)}{PV(\Delta G)}$	−0.20	−0.20	−0.20	−0.20	−0.08	−0.08	−0.09	−0.09

Table 1.3: Present-value multipliers in the new Keynesian model under rational expectations and under adaptive learning.

able X over a k-period horizon is calculated as

$$\frac{PV(\Delta X)}{PV(\Delta G)} = \frac{\sum_{t=0}^{k} \bar{R}^{-t} X_t}{\sum_{t=0}^{k} \bar{R}^{-t} G_t} \frac{1}{\bar{G}/\bar{X}}, \tag{1.30}$$

where X_t is the response of variable X at period t, G_t is government spending at period t, $\bar{R}$ is the steady state gross nominal interest rate, and $\bar{G}/\bar{X}$ is the steady state government expenditure to X ratio.

Table 1.3 shows that the present-value multipliers for output are significantly bigger under learning than under rational expectations, also at longer horizons. Thus, the learning model is capable of generating multipliers that are well within the range of empirical estimates reviewed by Ramey (2011). Moreover, the short- and longer-term consumption multiplier is always positive under learning, whereas it is always negative under rational expectations. An important result is that it is possible to achieve this outcome even if the degree of complementarity between labour and consumption in the utility function is weak, whereas in a model with rational expectations it is often necessary to assume high values for this parameter (see Linnemann, 2006; Bilbiie, 2009, 2011, for example).

In addition, adaptive learning provides a theoretical mechanism for generating government spending multipliers bigger than one, even if the price stickiness is relatively small. This is particularly relevant since the discussion in Nakamura and Steinsson (2008) points out that the extent of price rigidity is often overestimated. Figure 1.5 reports the multipliers for output, consumption, and investment for different degrees of price rigidity θ. Moreover, the figure allows to compare the multipliers under rational expectations with those under adaptive learning. As noted earlier, throughout this analysis the central bank maintains a standard Taylor rule. The figure shows that for the benchmark case with $\theta = 0.75$, the output multiplier under learning is bigger than one and about twice as large as the multiplier under rational expectations.

The consumption multiplier is increasing with the degree of price rigidity. As noted earlier, it is optimal for an intermediate firm that cannot change its price, to hire more labour when the demand for its intermediate good increases. This amplifies the rise in employment after a government spending increase, and encourages the household to consume more when preferences

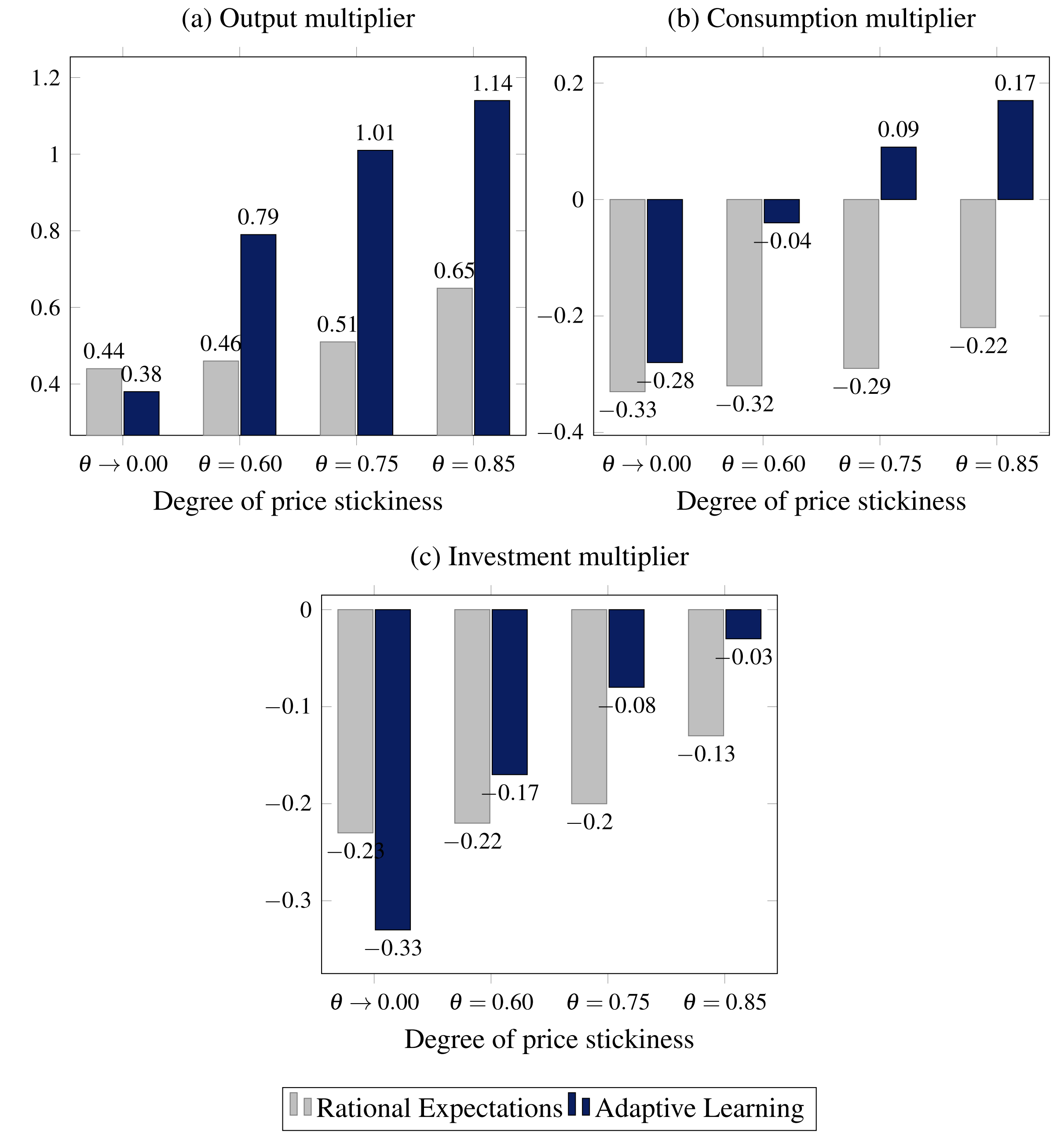

Figure 1.5: Impact multipliers for different degrees of price rigidity in the rational expectations model and the adaptive learning model.

are non-separable. Figure 1.5 shows that government spending crowds *in* private consumption when prices are sufficiently rigid. For example, if $\theta = 0.75$ the consumption multiplier equals 0.09. Moreover, notice that the crowding out of investment becomes smaller as prices become more sticky. Nevertheless, the investment multiplier always remains negative.

6 Alternative specification of fiscal policy

In the baseline impulse response analysis, the increase in government spending was financed through an increase in lump-sum taxes. This makes the results comparable with the policy experiments typically considered in the literature. As an extension, this section considers a richer specification of fiscal policy in which the fiscal authority finances expenditure, interest payments, and lump-sum transfers through the emission of one-period debt and through taxation on private consumption and capital and labour income.

The government budget constraint is now given by

$$B_{t+1} + \tau_t^c C_t + \tau_t^w W_t N_t + \tau_t^k r_t^k K_t = G_t + R_{t-1} \Pi_t^{-1} B_t + TR_t, \tag{1.31}$$

where τ_t^c, τ_t^w, and τ_t^k are the tax rates on private consumption, labour income, and capital income, respectively, and TR_t are lump-sum transfers.[15]

The steady state tax rates are set equal to U.S. averages over the period 2000–2013: $\bar{\tau}^c = 0.01$, $\bar{\tau}^w = 0.39$, and $\bar{\tau}^k = 0.39$. The consumption tax rate is calculated as in Leeper et al. (2010). The labour income tax rate and the corporate income tax rate are retrieved from the OECD (2014c) and OECD (2014a) databases.[16]

The rich specification of fiscal policy allows us to compare government spending multipliers for different fiscal financing strategies. Table 1.4 includes the results for three strategies. "Strategy 1" corresponds to the baseline analysis of a government spending increase financed through lump-sum taxation. In "Strategy 2" the spending increase is associated with a rise in the capital income tax. In particular, the government raises the tax rate on capital such that in each period the spending increase is matched by an equal increase in the steady-state tax revenues from capital. In the same manner, "Strategy 3" corresponds to an increase in the labour income tax. Throughout all simulations real public debt remains constant and lump-sum transfers adjust to maintain budget balance in each period. The derivation of the equations governing the dynamics under learning is detailed in Appendix 1.C.2.

[15] In contrast to the baseline model, a lump-sum *transfer* TR_t is considered instead of a lump-sum *tax* T_t, but this is just a matter of definition since $T_t = -TR_t$. It is more natural to proceed in this way since, with this alternative fiscal policy specification, the parametrization of the model implies a *negative* lump-sum tax.

[16] The labour income tax is the combined central and sub-central government income tax rate plus employee social security contribution, as a percentage of average gross wage earnings. The capital income tax rate is the basic combined central and sub-central (statutory) corporate income tax rate given by the adjusted central government rate plus the sub-central rate. Both tax rates are marginal rates. See OECD (2014c) and OECD (2014a) for explanatory notes.

Table 1.4 shows that the effects of a government spending shock depend quite dramatically on the fiscal financing strategy. The output and consumption multipliers are smaller when the shock is financed by the tax rate on capital income (Strategy 2). In the learning model, the output multiplier at impact is less than half of the multiplier under lump-sum financing. Naturally, investment declines more strongly as the high capital tax rates make saving and investing less attractive. The adverse effects of this on the capital stock suppress the multipliers in the medium and long run. The consumption multiplier under learning is now slightly negative at impact and reaches a value of -0.37 after six years.

The most striking difference between rational expectations and learning occurs in the presence of labour income tax financing (Strategy 3). Under rational expectations, labour supply falls considerably as the intra- and inter-temporal decision is distorted by the (temporary) increase in the labour income tax rate. The output multiplier is negative at every horizon. Now the consumption-labour complementarity works in the opposite direction as before: the sharp drop in employment lowers the marginal utility of consumption significantly, resulting in deeply negative consumption multipliers. The same mechanism is at play in the learning model, but the drop in consumption is much weaker. More importantly, learning completely reverses the sign of the output multipliers. Under rational expectations, the impact multiplier is negative and equals -0.64, whereas under learning it is positive and equal to 0.48. The impulse responses for the different financing strategies are depicted in Figure 1.6 of the Appendix.

7 Conclusion

This paper assesses the role of expectations for the macroeconomic dynamics of a government spending shock and, in particular, for the size of the government spending multiplier. There is no doubt that it is implausible to assume that agents have complete knowledge of the structure of the economy. Therefore, this paper considers a model where agents understand the direct wealth effects from the change in government spending and taxes, but fail to fully foresee the general equilibrium effects on factor prices and other aggregate variables. To forecast these variables they rely on small forecasting models estimated using least-squares learning. The impulse responses under this type of learning show that the effects of expansionary fiscal policy crucially depend on the agents' beliefs about the future.

Expectations significantly influence the size of the short- and longer-term multipliers of output, private consumption, and investment. The new Keynesian adaptive learning model generates an output multiplier at impact of 1.01, a value that is about twice as large as the multiplier under rational expectations. Expectations of future real interest rates are crucial for understanding this result. Under rational expectations, the inter-temporal substitution effect of higher future interest rates causes consumption to fall. Under learning, however, agents

	Rational expectations				Adaptive learning			
	Impact	1 year	4 years	6 years	Impact	1 year	4 years	6 years
Strategy 1: lump-sum financing (baseline model)								
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	0.51	0.50	0.46	0.43	1.01	1.00	0.99	0.97
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−0.29	−0.30	−0.34	−0.37	0.09	0.09	0.07	0.06
$\frac{PV(\Delta I)}{PV(\Delta G)}$	−0.20	−0.20	−0.20	−0.20	−0.08	−0.08	−0.09	−0.09
Strategy 2: capital tax financing								
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	0.32	0.29	0.12	0.02	0.43	0.38	0.20	0.08
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−0.23	−0.26	−0.40	−0.48	−0.08	−0.12	−0.27	−0.37
$\frac{PV(\Delta I)}{PV(\Delta G)}$	−0.68	−0.69	−0.70	−0.71	−0.80	−0.80	−0.81	−0.82
Strategy 3: labour tax financing								
$\frac{PV(\Delta Y)}{PV(\Delta G)}$	−0.64	−0.68	−0.85	−0.95	0.48	0.47	0.43	0.40
$\frac{PV(\Delta C)}{PV(\Delta G)}$	−1.07	−1.10	−1.24	−1.33	−0.37	−0.38	−0.41	−0.43
$\frac{PV(\Delta I)}{PV(\Delta G)}$	−0.71	−0.71	−0.73	−0.74	−0.17	−0.18	−0.18	−0.18

Table 1.4: Present-value multipliers for different specifications of fiscal policy in the rational expectations model and the adaptive learning model. See main text for a description of the different financing strategies.

underestimate the increase in future interest rates and consumption rises at impact. Additionally, the learning mechanism induces a positive co-movement between real wages and hours worked after a government spending shock in the new Keynesian model. Also the investment multiplier for this model is larger than for the rational expectations model, but remains negative.

This paper confirms the findings of Bilbiie (2011), Christiano et al. (2011), and others, that emphasize the importance of sticky prices and consumption-labour complementarity for government spending to crowd *in* private consumption. However, in the parametrisation considered, these model features alone are not sufficient to generate a rise in consumption. Only in the learning model, government spending crowds *in* private consumption. Hence, this paper provides a new explanation for a positive consumption response to a temporary government spending increase.

Acknowledgements

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Appendices

Appendix 1.A Derivations of model equations

1.A.1 Household's optimization problem

The Lagrangian associated with the household's optimization problem is given by

$$\mathscr{L} = E_0^* \sum_{t=0}^{\infty} \beta^t \Bigg\{ U\left(C_t, 1-N_t\right) + \lambda_t \left[W_t N_t + r_t^k K_t + R_{t-1}\Pi_t^{-1} B_t + D_t - T_t - C_t - I_t - B_{t+1}\right] + q_t \left[(1-\delta)K_t + I_t - \mathscr{S}\left(K_t, I_t, I_{t-1}\right) - K_{t+1}\right] \Bigg\}.$$

The associated optimality conditions are

$$\frac{\partial \mathscr{L}_t}{\partial C_t} = 0 \quad \Leftrightarrow \quad U_{C,t} = \lambda_t, \tag{1.32}$$

$$\frac{\partial \mathscr{L}_t}{\partial (1-N_t)} = 0 \quad \Leftrightarrow \quad U_{1-N,t} = \lambda_t W_t, \tag{1.33}$$

$$\frac{\partial \mathscr{L}_t}{\partial B_{t+1}} = 0 \quad \Leftrightarrow \quad \beta E_t^* \left(\lambda_{t+1} R_t \Pi_{t+1}^{-1}\right) = \lambda_t \Leftrightarrow R_t = E_t^* \left(\frac{\lambda_t \Pi_{t+1}}{\beta \lambda_{t+1}}\right), \tag{1.34}$$

$$\frac{\partial \mathscr{L}_t}{\partial I_t} = 0 \quad \Leftrightarrow \quad \lambda_t = q_t(1-\mathscr{S}_{I_t,t}) - \beta E_t^* \left(q_{t+1} \mathscr{S}_{I_{t-1},t+1}\right), \tag{1.35}$$

$$\frac{\partial \mathscr{L}_t}{\partial K_{t+1}} = 0 \quad \Leftrightarrow \quad \beta E_t^* \left(\lambda_{t+1} r_{t+1}^k\right) + \beta E_t^* \left[q_{t+1}\left(1-\delta-\mathscr{S}_{K_t,t+1}\right)\right] = q_t, \tag{1.36}$$

$$\frac{\partial \mathscr{L}_t}{\partial \lambda_t} = 0 \quad \Leftrightarrow \quad W_t N_t + r_t^k K_t + R_{t-1}\Pi_t^{-1} B_t + D_t - T_t - C_t - I_t - B_{t+1} = 0,$$

$$\frac{\partial \mathscr{L}_t}{\partial q_t} = 0 \quad \Leftrightarrow \quad (1-\delta)K_t + I_t - \mathscr{S}\left(K_t, I_t, I_{t-1}\right) - K_{t+1} = 0.$$

Combining conditions (1.32) and (1.33) yields the labour supply equation

$$W_t = \frac{U_{1-N,t}}{U_{C,t}}. \tag{1.37}$$

Conditions (1.32) and (1.34) allow us to derive the following Euler equation for consumption

$$U_{C,t} = \beta R_t E_t^* \left(\Pi_{t+1}^{-1} U_{C,t+1} \right). \tag{1.38}$$

Optimality conditions (1.35) and (1.36) can be further simplified using condition (1.34). We get that

$$1 = Q_t \left[1 - \mathscr{S}_{I_t,t}\right] - R_t^{-1} E_t^* \left[\Pi_{t+1} Q_{t+1} \mathscr{S}_{I_{t-1},t+1}\right], \tag{1.39}$$

$$Q_t = R_t^{-1} E_t^* \left\{ \Pi_{t+1} \left[r_{t+1}^k + Q_{t+1} \left(1 - \delta - \mathscr{S}_{K_t,t+1}\right)\right]\right\}, \tag{1.40}$$

where Tobin's $Q_t \equiv q_t / \lambda_t$.

Functional Form Assumptions The following specifications of preferences and the capital adjustment cost function are considered:

$$U(C_t, 1 - N_t) = \frac{\left[C_t^{\phi} (1 - N_t)^{1-\phi}\right]^{1-\sigma} - 1}{1 - \sigma},$$

$$\mathscr{S}(\cdot) = \frac{\varsigma_I}{2} \left(\frac{I_t}{K_t} - \delta \right)^2 K_t,$$

where $\varsigma_I > 0$ is the Lucas and Prescott (1971) capital adjustment cost parameter. For these functional forms, the optimality conditions (1.37), (1.38), (1.39), and (1.40) become

$$W_t = \frac{1-\phi}{\phi} \frac{C_t}{1 - N_t},$$

$$C_t^{\phi(1-\sigma)-1} (1 - N_t)^{(1-\phi)(1-\sigma)} = \beta R_t E_t^* \left[\Pi_{t+1}^{-1} C_{t+1}^{\phi(1-\sigma)-1} (1 - N_{t+1})^{(1-\phi)(1-\sigma)} \right],$$

$$1 = Q_t \left[1 - \varsigma_I \left(\frac{I_t}{K_t} - \delta \right) \right],$$

$$Q_t = R_t^{-1} E_t^* \left\{ \Pi_{t+1} \left[r_{t+1}^k + Q_{t+1} \left(1 - \delta - \varsigma_I \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) \left(\frac{1}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta \right) - \frac{I_{t+1}}{K_{t+1}} \right) \right) \right] \right\}.$$

Written in terms of deviations from steady state, the Euler equation (1.38) becomes

$$\begin{aligned} \left[\phi(1-\sigma) - 1\right] \hat{C}_t - \frac{(1-\sigma)(1-\phi)\bar{N}}{1-\bar{N}} \hat{N}_t = \hat{R}_t - E_t^* \hat{\Pi}_{t+1} + \left[\phi(1-\sigma) - 1\right] E_t^* \hat{C}_{t+1} \\ - \frac{(1-\sigma)(1-\phi)\bar{N}}{1-\bar{N}} E_t^* \hat{N}_{t+1}. \end{aligned} \tag{1.41}$$

1.A.2 Firms' optimization problem

1.A.2.1 Final goods sector

The profit maximization problem of the final goods firm is represented as

$$\max_{\{Y_t(i)\}} P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj, \quad \forall i \in [0,1], \tag{1.42}$$

where both the final goods price P_t and the prices for the intermediate goods $P_t(j)$, $j \in [0,1]$, are taken as given. Profit maximization yields the following demand schedule for intermediate good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} Y_t. \tag{1.15}$$

The final goods producers are perfectly competitive. Thus, we have the following zero-profit condition

$$P_t \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 P_t(i) Y_t(i) di = 0. \tag{1.43}$$

This leads to the following expression for the final goods price

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon} di\right)^{\frac{1}{1-\varepsilon}}. \tag{1.44}$$

In the symmetric equilibrium all intermediate goods producers set the same price. Therefore, the aggregate price P_t and the intermediate goods prices $P_t(i)$ for all i will be the same.

1.A.2.2 Intermediate goods sector

The Lagrangian for the expenditure minimization problem for the intermediate goods producer i is given by

$$\mathscr{L} = W_t N_t(i) + r_t^k K_t(i) + \mu_t(i)\left[Y_t(i) - Z_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}\right], \tag{1.45}$$

and the corresponding first-order conditions

$$W_t = \mu_t(i)(1-\alpha) Z_t K_t(i)^{\alpha} N_t(i)^{-\alpha},$$
$$r_t^k = \mu_t(i) \alpha Z_t K_t(i)^{\alpha-1} N_t(i)^{1-\alpha}.$$

Here the Lagrange multiplier is also the real marginal cost. Therefore we will define the real marginal cost of firm i as $MC_t(i) \equiv \mu_t(i)$. In the symmetric equilibrium real marginal cost is common to all firms and given by

$$MC_t = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}\left(R_t^k\right)^{\alpha} W_t^{1-\alpha} Z_t^{-1} \tag{1.46}$$

Intermediate goods producers choose the price $P_t^*(i)$ that maximizes discounted real profits

$$E_t^* \sum_{j=0}^{\infty} (\beta\theta)^j \frac{U_{C,t+j}}{U_{C,t}} \left\{\frac{P_t^*(i)}{P_{t+j}} Y_{t+j}(i) - MC_{t+j} Y_{t+j}(i)\right\} \tag{1.47}$$

subject to

$$Y_{t+j}(i) = \left(\frac{P_t^*(i)}{P_{t+j}}\right)^{-\varepsilon} Y_{t+j}. \tag{1.48}$$

The corresponding first-order condition is

$$E_t^* \sum_{j=0}^{\infty} (\beta\theta)^j \frac{U_{C,t+j}}{U_{C,t}} P_{t+j}^{\varepsilon} Y_{t+j} \left\{ (1-\varepsilon)\left(P_t^*(i)\right)^{-\varepsilon} P_{t+j}^{-1} + \varepsilon MC_{t+j} \left(P_t^*(i)\right)^{-\varepsilon-1} \right\} = 0 \quad (1.49)$$

Given Calvo pricing, the price index (1.44) can be written as

$$P_t^{1-\varepsilon} = (1-\theta)\left(P_t^*(i)\right)^{1-\varepsilon} + \theta P_{t-1}^{1-\varepsilon}. \quad (1.50)$$

Appendix 1.B Log-linearisation

1.B.1 New Keynesian specification

For the new Keynesian model we have the following log-linearised equilibrium conditions:

$$\hat{Y}_t = \left(1 - \frac{\bar{G}}{\bar{Y}} - \frac{\bar{I}}{\bar{Y}}\right)\hat{C}_t + \frac{\bar{G}}{\bar{Y}}\hat{G}_t + \frac{\bar{I}}{\bar{Y}}\hat{I}_t, \quad (1.51)$$

$$\hat{Y}_t = \hat{Z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{N}_t, \quad (1.52)$$

$$\hat{\Pi}_t = \varphi\theta^{-1}\widehat{MC}_t + \beta\varphi\sum_{j=0}^{+\infty}(\beta\theta)^j\left[(1-\alpha)E_t^*\hat{W}_{t+j+1} + \alpha E_t^*\hat{r}_{t+j+1}^k - E_t^*\hat{Z}_{t+j+1}\right]$$
$$+ \beta(1-\theta)\sum_{j=0}^{+\infty}(\beta\theta)^j E_t^*\hat{\Pi}_{t+j+1}, \quad (1.53)$$

$$\hat{W}_t = \alpha\hat{K}_t + \hat{Z}_t + \widehat{MC}_t - \alpha\hat{N}_t, \quad (1.54)$$

$$\hat{r}_t^k = (1-\alpha)\hat{N}_t + \hat{Z}_t + \widehat{MC}_t + \hat{K}_t(\alpha-1), \quad (1.55)$$

$$\hat{K}_{t+1} = \hat{K}_t(1-\delta) + \hat{I}_t\delta, \quad (1.56)$$

$$\hat{R}_t = \rho_\Pi\hat{\Pi}_t + \hat{u}_t^R, \quad (1.57)$$

$$\hat{u}_t^R = \rho_R\hat{u}_{t-1}^R + \varepsilon_t^R, \quad (1.58)$$

$$\hat{Z}_t = \rho_Z\hat{Z}_{t-1} + \varepsilon_t^Z, \quad (1.59)$$

$$\bar{T}\hat{T}_t = \bar{G}\hat{G}_t + \beta^{-1}\bar{B}\hat{R}_{t-1} - \beta^{-1}\bar{B}\hat{\Pi}_t, \quad (1.60)$$

$$\hat{C}_t + \hat{N}_t\frac{\bar{N}}{1-\bar{N}} = \hat{W}_t, \quad (1.61)$$

$$\Gamma_1\hat{C}_t = \beta^{-1}\bar{K}\hat{K}_t + \Gamma_2\hat{W}_t + \bar{K}\bar{r}^k\hat{r}_t^k + \bar{D}\hat{D}_t - \bar{G}\hat{G}_t - \Gamma_3\hat{R}_t + SW_t^e - SR_t^e + S\Pi_t^e + Sr_t^{k,e} + SD_t^e - SG_t^e, \quad (1.62)$$

$$\hat{Q}_t = -\hat{R}_t + \sum_{j=1}^{\infty}\beta^j\left[\bar{r}^k E_t^*\hat{r}_{t+j}^k - E_t^*\hat{R}_{t+j} + \beta^{-1}E_t^*\hat{\Pi}_{t+j}\right], \quad (1.63)$$

$$\hat{Q}_t = \delta\varsigma_I\left(\hat{I}_t - \hat{K}_t\right), \quad (1.64)$$

where a circumflex denotes log-deviations from the steady state.

1.B.2 Neoclassical specification

The log-linearised equilibrium conditions characterizing the dynamics of the neoclassical specification of the model are the following:

$$\hat{Y}_t = \left(1 - \frac{\bar{G}}{\bar{Y}} - \frac{\bar{I}}{\bar{Y}}\right)\hat{C}_t + \frac{\bar{G}}{\bar{Y}}\hat{G}_t + \frac{\bar{I}}{\bar{Y}}\hat{I}_t, \tag{1.65}$$

$$\hat{Y}_t = \hat{Z}_t + \alpha\hat{K}_t + (1-\alpha)\hat{N}_t, \tag{1.66}$$

$$\hat{W}_t = \alpha\hat{K}_t + \hat{Z}_t - \alpha\hat{N}_t, \tag{1.67}$$

$$\hat{r}^k_t = (1-\alpha)\hat{N}_t + \hat{Z}_t + \hat{K}_t(\alpha - 1), \tag{1.68}$$

$$\hat{K}_{t+1} = \hat{K}_t(1-\delta) + \hat{I}_t\delta, \tag{1.69}$$

$$\hat{Z}_t = \rho_Z\hat{Z}_{t-1} + \varepsilon^Z_t, \tag{1.70}$$

$$\bar{T}\hat{T}_t = \bar{G}\hat{G}_t + \beta^{-1}\bar{B}\hat{R}_{t-1}, \tag{1.71}$$

$$\hat{C}_t + \hat{N}_t\frac{\bar{N}}{1-\bar{N}} = \hat{W}_t, \tag{1.72}$$

$$\Gamma_1\hat{C}_t = \beta^{-1}\bar{K}\hat{K}_t + \Gamma_2\hat{W}_t + \bar{K}\bar{r}^k\hat{r}^k_t + \bar{D}\hat{D}_t - \bar{G}\hat{G}_t - \Gamma_3\hat{R}_t + SW^e_t - SR^e_t + Sr^{k,e}_t + SD^e_t - SG^e_t, \tag{1.73}$$

$$\hat{Q}_t = -\hat{R}_t + \sum_{j=1}^{\infty}\beta^j\left[\bar{r}^k E^*_t\hat{r}^k_{t+j} - E^*_t\hat{R}_{t+j}\right], \tag{1.74}$$

$$\hat{Q}_t = \delta\varsigma_I\left(\hat{I}_t - \hat{K}_t\right). \tag{1.75}$$

Appendix 1.C Learning dynamics

1.C.1 Baseline model

1.C.1.1 Household

In this appendix we derive the linearised consumption function under learning. We apply the approach of Evans et al. (2009) and assume agents combine structural knowledge on the government budget constraint with expectations based on small forecasting models.

Forward iteration of the Euler equation (1.41) yields

$$-\sigma E^*_t\hat{C}_{t+j} = [\phi(1-\sigma) - 1]\hat{C}_t + (1-\phi)(1-\sigma)\left(E^*_t\hat{W}_{t+j} - \frac{\bar{N}}{1-\bar{N}}\hat{N}_t\right) - \hat{R}_t - \sum_{k=1}^{j-1}E^*_t\hat{R}_{t+k} + \sum_{k=1}^{j}E^*_t\hat{\Pi}_{t+k}, \tag{1.76}$$

where $\hat{N}_{t+j}$ is substituted out using (1.61).

Since real government debt is constant under the policy experiments considered and by (1.3) we can write the household's flow budget constraint (1.2) as

$$\beta^{-1}\bar{K}\hat{K}_t = \phi^{-1}\bar{C}\hat{C}_t + \bar{K}\hat{K}_{t+1} - \bar{W}\hat{W}_t - \bar{r}^k\bar{K}\hat{r}_t^k - \bar{B}\beta^{-1}\left(\hat{R}_{t-1} - \hat{\Pi}_t\right) - \bar{D}D_t + \bar{T}\hat{T}_t, \quad (1.77)$$

where we have used (1.61) to substitute out labour $\hat{N}_t$. Combining the government budget constraint (1.60) with the flow budget constraint (1.77) we can get the expected value inter-temporal budget constraint of the household

$$\eta\hat{C}_t + \eta\sum_{j=1}^{+\infty}\beta^j E_t^*\hat{C}_{t+j} = \beta^{-1}\bar{K}\hat{K}_t + \bar{W}\hat{W}_t + \bar{K}\bar{r}^k\hat{r}_t^k + \bar{D}\hat{D}_t - \bar{G}\hat{G}_t$$
$$+\sum_{j=1}^{+\infty}\beta^j\left[\bar{W}E_t^*\hat{W}_{t+j} + \bar{r}^k E_t^*\hat{r}_{t+j}^k + \bar{D}E_t^*\hat{D}_{t+j} - \bar{G}E_t^*\hat{G}_{t+j}\right] \quad (1.78)$$

by assuming that the transversality condition (1.11) holds. Here $\eta = \bar{C}\phi^{-1}$.

Substituting the Euler equation (1.76) into this inter-temporal budget constraint yields the consumption function

$$\Gamma_1\hat{C}_t = \beta^{-1}\bar{K}\hat{K}_t + \Gamma_2\hat{W}_t + \bar{K}\bar{r}^k\hat{r}_t^k + \bar{D}\hat{D}_t - \bar{G}\hat{G}_t - \Gamma_3\hat{R}_t + SW_t^e - SR_t^e + S\Pi_t^e + Sr_t^{k,e} + SD_t^e - SG_t^e, \quad (1.79)$$

where SW_t^e, SR_t^e, $S\Pi_t^e$, $Sr_t^{k,e}$, SD_t^e, and SG_t^e are defined by (1.5)–(1.10) and

$$\Gamma_1 \equiv \frac{\eta}{1-\beta}, \quad (1.80)$$
$$\Gamma_2 \equiv \bar{W} - \frac{\beta\eta(1-\phi)(1-\sigma)}{\sigma(1-\beta)}, \quad (1.81)$$
$$\Gamma_3 \equiv \frac{\beta\eta}{\sigma(1-\beta)}, \quad (1.82)$$
$$\Gamma_4 \equiv \bar{W} + \frac{(1-\phi)(1-\sigma)\eta}{\sigma}. \quad (1.83)$$

Since the future path of government spending is assumed to be known and given by (1.28), the term SG_t^e can be obtained as

$$SG_t^e = \bar{G}\sum_{j=1}^{+\infty}\beta^j\rho_G^j\hat{G}_t = \bar{G}\frac{\beta\rho_G}{1-\beta\rho_G}\hat{G}_t.$$

As described in the main text, the forecasts $E_t^*\hat{W}_{t+j}$, $E_t^*\hat{R}_{t+j}$, $E_t^*\hat{\Pi}_{t+j}$, $E_t^*\hat{r}_{t+j}^k$, and $E_t^*\hat{D}_{t+j}$ depend on the perceived laws of motion (1.26). In particular, for every variable y^f forecasted under learning agents use the forecast function $E_t^*\hat{y}_{t+1}^f = \psi_{y,t}X_t$ with $X_t = \left[\hat{K}_{t+1};\hat{Z}_t\right]$. Here $\psi_{y,t}$ is the vector of beliefs in the PLM for y^f. In particular, the perceived laws of motion for capital and the technology shock are given by

$$\begin{bmatrix} E_t^*\hat{K}_{t+2} \\ E_t^*Z_{t+1} \end{bmatrix} = H_tX_t, \quad (1.84)$$

with

$$H_t = \begin{bmatrix} \psi_{kk,t} & \psi_{kz,t} \\ 0 & \rho_Z \end{bmatrix}, \tag{1.85}$$

where it is assumed that the shock process (1.59) is known to the agents. From this it follows that $E_t^* \hat{y}^f_{t+j+1} = \psi_{y,t} H_t^j X_t$, for $j \geq 0$, which allows us to obtain the following expressions for the sums in (1.79):

$$SW_t^e = \Gamma_4 \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{W}_{t+j} = \Gamma_4 \psi_{w,t} \beta (I - \beta H_t)^{-1} X_t, \tag{1.86}$$

$$SR_t^e = \Gamma_3 \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{R}_{t+j} = \Gamma_3 \psi_{r,t} \beta (I - \beta H_t)^{-1} X_t, \tag{1.87}$$

$$S\Pi_t^e = \Gamma_3 \beta^{-1} \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{\Pi}_{t+j} = \Gamma_3 \psi_{\pi,t} (I - \beta H_t)^{-1} X_t, \tag{1.88}$$

$$Sr_t^{k,e} = \bar{K}\bar{r}^k \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{r}^k_{t+j} = \bar{K}\bar{r}^k \psi_{r^k,t} \beta (I - \beta H_t)^{-1} X_t, \tag{1.89}$$

$$SD_t^e = \bar{D} \sum_{j=1}^{+\infty} \beta^j E_t^* \hat{D}_{t+j} = \bar{D} \psi_{d,t} \beta (I - \beta H_t)^{-1} X_t. \tag{1.90}$$

In the neoclassical specification of the model, the consumption rule is the same as (1.79) but without the term $S\Pi_t^e$.

The sums of future expected terms in (1.13) can be handled in the same way as the sums in the consumption function. By virtue of (1.84) we obtain the following expression

$$\hat{Q}_t = -\hat{R}_t - \psi_{r,t} \beta (I - \beta H_t)^{-1} X_t + \psi_{\pi,t} (I - \beta H_t)^{-1} X_t + \beta \bar{r}^k \psi_{r^k,t} (I - \beta H_t)^{-1} X_t. \tag{1.91}$$

1.C.1.2 Firms

Log-linearisation of the first-order condition (1.49) yields

$$\hat{p}_t^*(i) = (1 - \beta\theta) \widehat{MC}_t + (1 - \beta\theta) \beta\theta \sum_{j=0}^{\infty} (\beta\theta)^j E_t^* \widehat{MC}_{t+j+1} + \beta\theta \sum_{j=0}^{+\infty} (\beta\theta)^j E_t^* \hat{\Pi}_{t+j+1}, \tag{1.92}$$

where $p_t^*(i) = P_t^*(i)/P_t$. In the symmetric equilibrium all intermediate goods producers have identical marginal costs

$$\widehat{MC}_t = (1 - \alpha) \hat{W}_t + \alpha \hat{r}_t^k - \hat{Z}_t. \tag{1.93}$$

Combining this expression with (1.92) and the log-linear approximation of the price index (1.50) we obtain condition (1.19). The sums of future expected terms in (1.19) can be handled in the same way as the sums in the consumption function. From (1.84) we have

$$\hat{\Pi}_t = \varphi\theta^{-1} \widehat{MC}_t + \beta\varphi \left[(1 - \alpha) \psi_{w,t} (I - \beta\theta H_t)^{-1} X_t + \alpha \psi_{r^k,t} (I - \beta\theta H_t)^{-1} X_t - \frac{\beta\theta\rho_Z}{1 - \beta\theta\rho_Z} Z_t \right]$$
$$+ \beta (1 - \theta) \psi_{\pi,t} (I - \beta\theta H_t)^{-1} X_t. \tag{1.94}$$

1.C.2 Alternative specification of fiscal policy

In all strategies considered, real government debt is held constant and lump-sum transfers adjust to balance the budget. Hence,

$$\bar{TR}\hat{TR}_t = \beta^{-1}\bar{B}\hat{\Pi}_t - \beta^{-1}\bar{B}\hat{R}_{t-1} + \bar{\tau}^c\bar{C}\hat{C}_t + \bar{\tau}^w\bar{W}\bar{N}\left(\hat{W}_t + \hat{N}_t\right) + \bar{\tau}^k\bar{r}^k\bar{K}\hat{K}_t + \bar{\tau}^k\bar{r}^k\bar{K}\hat{r}^k_t. \quad (1.95)$$

Strategy 2: capital tax financing In this case, the household's flow budget constraint reads

$$\beta^{-1}\bar{K}\hat{K}_t = \left[(1+\bar{\tau}^c)\bar{C} + (1-\bar{\tau}^w)\bar{W}(1-\bar{N})\right]\hat{C}_t + \bar{K}\hat{K}_{t+1} - (1-\bar{\tau}^w)\bar{W}\hat{W}_t - (1-\bar{\tau}^k)\bar{r}^k\bar{K}\hat{r}^k_t - \bar{B}\beta^{-1}\left(\hat{R}_{t-1} - \hat{\Pi}_t\right) + \bar{\tau}^k\bar{r}^k\bar{K}\hat{\tau}^k_t - \bar{D}\hat{D}_t - \bar{TR}\hat{TR}_t. \quad (1.96)$$

The reaction of the capital income tax rate is given by $\bar{\tau}^k\bar{r}^k\bar{K}\hat{\tau}^k_t = \hat{G}_t\bar{G}$. Combining this expression with (1.95) and (1.96), and following the same steps as in Appendix 1.C.1 we can derive the consumption function under learning. In particular, the inter-temporal budget constraint of the household and the consumption function can be written, respectively, as (1.78) and (1.79), with β^{-1} replaced by $\tilde{\beta}^{-1} \equiv \bar{r}^k + 1 - \delta$ and $\eta = \left[1 + (1-\phi)\phi^{-1}(1+\bar{\tau}^c)(1-\bar{\tau}^w)^{-1}\right]\bar{C}$.

Optimal investment now requires that

$$\hat{Q}_t = \beta E^*_t\hat{Q}_{t+1} - \hat{R}_t + E^*_t\hat{\Pi}_{t+1} + \beta\left(1-\bar{\tau}^k\right)\bar{r}^k E^*_t\hat{r}^k_{t+1} - \beta\bar{\tau}^k\bar{r}^k E^*_t\hat{\tau}^k_{t+1}. \quad (1.97)$$

By iterating forward and using $\bar{\tau}^k\bar{r}^k\bar{K}\hat{\tau}^k_t = \hat{G}_t\bar{G}$ we obtain the infinite-horizon optimal investment rule

$$\hat{Q}_t = -\hat{R}_t + \sum_{j=1}^{\infty}\beta^j\left[\left(1-\bar{\tau}^k\right)\bar{r}^k E^*_t\hat{r}^k_{t+j} - E^*_t\hat{R}_{t+j} + \beta^{-1}E^*_t\hat{\Pi}_{t+j} - \bar{K}^{-1}\bar{G}E^*_t\hat{G}_{t+j}\right]. \quad (1.98)$$

Strategy 3: labour tax financing In this case, the optimality condition for labour is given by

$$\hat{N}_t = \frac{1-\bar{N}}{\bar{N}}\left(\hat{W}_t - \hat{C}_t - \frac{\bar{\tau}^w}{1-\bar{\tau}^w}\hat{\tau}^w_t\right), \quad (1.99)$$

instead of (1.61). The household flow budget constraint is

$$\beta^{-1}\bar{K}K_t = \left[(1+\bar{\tau}^c)\bar{C} + (1-\bar{\tau}^w)\bar{W}(1-\bar{N})\right]\hat{C}_t + \bar{K}\hat{K}_{t+1} - (1-\bar{\tau}^w)\bar{W}\hat{W}_t + \bar{W}\bar{\tau}^w\hat{\tau}^w_t - (1-\bar{\tau}^k)\bar{r}^k\bar{K}\hat{r}^k_t - \bar{B}\beta^{-1}\left(\hat{R}_{t-1} - \hat{\Pi}_t\right) - \bar{D}\hat{D}_t - \bar{TR}\hat{TR}_t. \quad (1.100)$$

The dynamics of the labour income tax rate are determined by $\bar{\tau}^w\bar{W}\bar{N}\hat{\tau}^w_t = \hat{G}_t\bar{G}$. Combining this expression with (1.95) and iterating forward yields the inter-temporal budget constraint of the household. When (1.76) is substituted in this constraint, the following consumption function is obtained:

$$\Gamma_1 C_t = \tilde{\beta}^{-1}\bar{K}\hat{K}_t + \Gamma_2\hat{W}_t + \bar{K}\bar{r}^k r^k_t + \bar{D}\hat{D}_t - \Gamma_5\bar{G}\hat{G}_t - \Gamma_3\hat{R}_t + SW^e_t - SR^e_t + S\Pi^e_t + Sr^{k,e}_t + SD^e_t - \Gamma_6 SG^e_t. \quad (1.101)$$

where Γ_1, Γ_2, Γ_3, Γ_4, SW_t^e, SR_t^e, $S\Pi_t^e$, $Sr_t^{k,e}$, SD_t^e, and SG_t^e are defined above, again with β^{-1} replaced by $\tilde{\beta}^{-1} \equiv \bar{r}^k + 1 - \delta$ and $\eta = \left[1 + (1-\phi)\,\phi^{-1}\,(1+\bar{\tau}^c)\,(1-\bar{\tau}^w)^{-1}\right]\bar{C}$. The coefficients Γ_5 and Γ_6 are defined as

$$
\begin{aligned}
\Gamma_5 &\equiv \frac{1-\bar{\tau}^w \bar{N}}{(1-\bar{\tau}^w)\,\bar{N}} - \frac{\tilde{\beta}\,(1-\sigma)\,\phi\,(1-\bar{N})\,\eta\bar{C}^{-1}}{\sigma\left(1-\tilde{\beta}\right)(1+\bar{\tau}^c)\,\bar{N}}, \\
\Gamma_6 &\equiv \frac{(1-\sigma)\,(1-\bar{N})\,\phi\eta\bar{C}^{-1}}{\sigma\,(1+\bar{\tau}^c)\,\bar{N}} + \frac{1-\bar{\tau}^w \bar{N}}{(1-\bar{\tau}^w)\,\bar{N}}.
\end{aligned}
$$

The optimality conditions for investment are given by (1.97) and (1.98) where government spending and the capital tax rate drop out.

Appendix 1.D Alternative specification of fiscal policy

Figure 1.6 and Figure 1.7 show the impulse responses to an increase in government spending of 1% of GDP and the expectations formed by the learning mechanism for different specifications of fiscal policy.

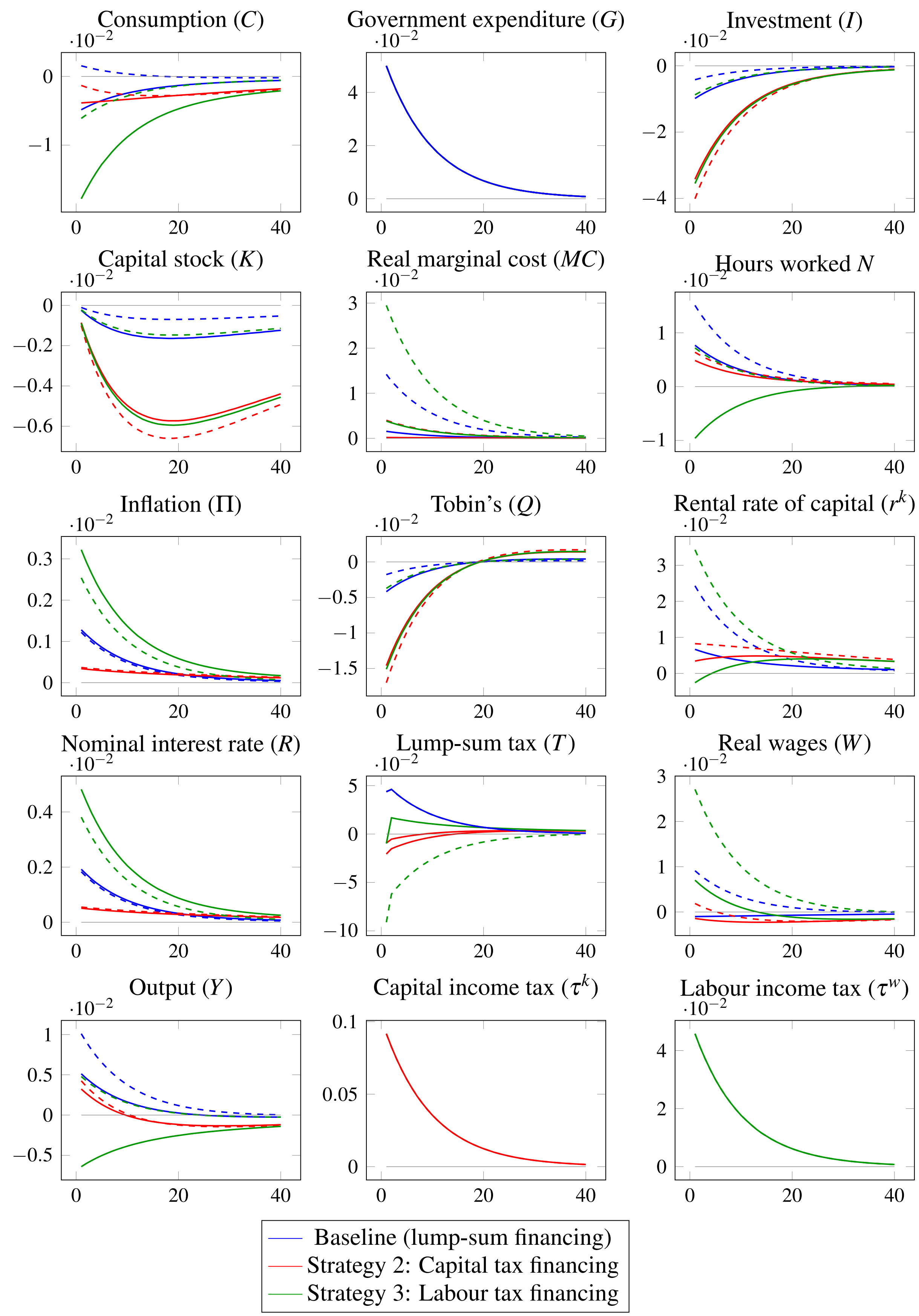

Figure 1.6: Impulse responses to an increase in government spending of 1% of GDP of the new Keynesian model for different fiscal policy specifications. The solid lines are the responses under rational expectations; the dashed lines are those under adaptive learning. The impulse response functions are measured in percentage deviations from steady-state. The horizontal axis measures quarters.

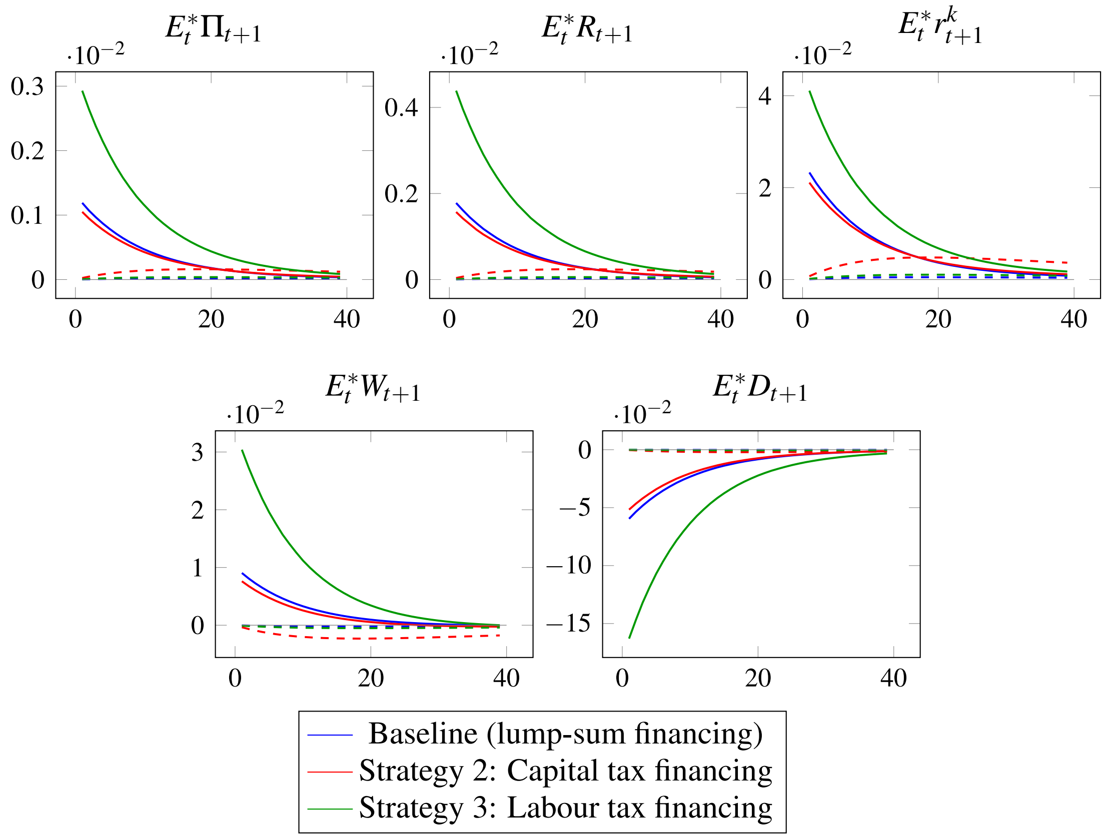

Figure 1.7: Expectations on forward-looking variables after a government spending shock of 1% of GDP of the new Keynesian model for different fiscal policy specifications. The solid lines are the responses under rational expectations; the dashed lines are those under adaptive learning. The impulse response functions are measured in percentage deviations from steady-state. The horizontal axis measures quarters.

Chapter 2

Adaptive Learning and the Transmission of Government Spending Shocks in the Euro Area

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Abstract

This paper analyses the transmission of government spending shocks in an estimated dynamic stochastic general equilibrium model for the euro area when agents use forecasting models updated by the Kalman filter to form expectations. Based on the marginal likelihood criterion, there is evidence in favour of this Kalman filter learning mechanism relative to rational expectations. Moreover, under Kalman filter learning, the transmission of government spending shocks varies over time. This variation stems from the adjustment of the beliefs of the agents on their forecasting model. Hence, this adjustment process provides an endogenous explanation for time-varying government spending multipliers. We find that, in contrast to rational expectations, the responses of private consumption to a government spending shock is positive for most periods in the sample. Moreover, the government spending multiplier for output is substantially larger under learning than under rational expectations.

1 Introduction

Recently there has been a renewed interest in the idea that changes in expectations are an important source of business cycle fluctuations — see for example Beaudry and Portier (2007), Eusepi and Preston (2011), and Jaimovich and Rebelo (2009). One important issue is the role of expectations for the transmission mechanism of government spending shocks. In particular, it is well understood that the macroeconomic effects of these shocks crucially depend on private-sector expectations. Bachmann and Sims (2012), for instance, show that the response of consumer expectations to a positive government spending shock explains the main part of the expansionary output effect in times of slack.

The analysis of government spending shocks in structural macroeconomic models is almost invariably developed under the assumption that agents hold rational expectations (see Coenen et al., 2012, for a review of the literature). This paper goes beyond rational expectations and considers agents who have limited information and must form expectations based on estimated forecasting models. In particular, we estimate a medium-scale Dynamic Stochastic General Equilibrium (DSGE) model for the euro area where agents form expectations using adaptive learning.

This paper demonstrates that the learning model, in contrast to the rational expectations model, is able to capture time variations in the macroeconomic responses to government spending shocks. Consequently, learning behaviour provides an endogenous explanation for time-varying government spending multipliers documented in several recent, more data-driven studies — see e.g. Auerbach and Gorodnichenko (2012, 2013), Kirchner et al. (2010), and Pereira and Lopes (2014). We show how variation in the belief coefficients of the agents generates time variation in the transmission of government spending shocks in the euro area. Importantly, in contrast to the time-varying parameter Vector Autoregression (VAR) studies, such as Kirchner et al. (2010), and Pereira and Lopes (2014), this variation does not stem from random changes in the structural parameters. Time variation in the transition of shocks is induced by a learning process along which agents revise their expectations about the future. In general, government spending multipliers may vary across many dimensions. A growing literature assesses the dependency of multipliers on the stance of monetary policy and the amount of slack in the economy. This paper contributes to the literature by highlighting the expectations channel as an additional driver for time variation in the spending multiplier.

Several authors have studied the relationship between adaptive learning and fiscal policy. Evans et al. (2009) investigate the effect of learning for the dynamics of anticipated and unanticipated changes in government spending in an endowment economy and the Ramsey model. Mitra et al. (2013) extend the analysis by looking at the dynamics in a Real Business Cycle (RBC) model. Hollmayr and Matthes (2015) study a permanent change in government expenditure in an RBC model with a rich fiscal sector. Gasteiger and Zhang (2014) examine the effect of adaptive learning for the impact of discretionary tax changes in a Ramsey model with variable labour supply. Giannitsarou (2006) studies the transitional dynamics of a capital tax cut under learning in a stochastic growth model.

This paper contributes to this learning literature on fiscal policy in several ways. Firstly, as explained above, this paper provides an endogenous explanation for time-varying government spending multipliers based on time-varying expectations.

Secondly, we use Bayesian estimation techniques to estimate a model for the euro area under different assumptions about how expectations are formed. The Bayesian approach allows us to test the rational expectations model against adaptive learning models, based on the marginal likelihood criterion. We find that the baseline learning model fits the data substantially better than the rational expectations benchmark.

Finally, in contrast to the aforementioned authors, this paper considers a medium-scale DSGE model similar to Christiano et al. (2005) and Smets and Wouters (2007) with a number of model features such as sticky prices and wages that are necessary to capture the persistence in the euro area data. The results presented here confirm earlier findings that these features crucially affect the impact of learning on the dynamics of government spending shocks (see Chapter 1). In particular, Chapter 1 shows that, in the first year and a half after the shock, government spending can crowd *in* private consumption when agents use the learning mechanism. By contrast, the rational expectations model predicts a substantial drop in private consumption after the shock. Moreover, on impact a co-movement between real wage and hours worked occurs.

The remainder of the paper is organised as follows. The next section discusses the log-linearised equations of the DSGE model that we estimate. Section 3 defines the rational expectations equilibrium of the model. In Section 4 we present the Kalman filter learning set-up. Section 5 discusses the estimation approach and the prior and posterior distributions of the model parameters. In this section we also show the time-varying belief coefficients of the agents' forecasting model. In Section 6 we discuss the dynamics of a government spending shock under learning and under rational expectations. The time variation in the learning mechanism, allows us to present impulse responses for each quarter in the sample. The present-value multipliers for output, private consumption, and private investment on impact and at longer horizons are presented in 7. Section 8 discusses the robustness of our results with respect to the sample period and the specification of the learning mechanism. The last section concludes.

2 The Model Economy

In this section, we describe the linearised version of the model.[1] Our model is a medium-scale DSGE model similar to that of An and Schorfheide (2007), Christiano et al. (2005), and Smets and Wouters (2007). We assume that technological progress is non-stationary. Therefore, all real trending variables are divided by the level of technology. Throughout the paper, hatted variables denote log-deviations from the steady state. Barred variables refer to steady state values.

[1] Appendix 2.A provides a detailed description of the model.

The *accounting identity* is given by

$$\hat{y}_t = \left(1 - \frac{\bar{i}}{\bar{y}} - \frac{\bar{g}}{\bar{y}}\right)\hat{c}_t + \frac{\bar{i}}{\bar{y}}\hat{i}_t + \frac{\bar{g}}{\bar{y}}\hat{g}_t, \tag{2.1}$$

where y_t, c_t, g_t, and i_t denote period t output, private consumption, government expenditure, and gross investment.

The aggregate *production function* is given by

$$\hat{y}_t = \frac{\bar{y}+\Phi}{\bar{y}}\left[\alpha\hat{k}_{t-1} + \hat{z}_t + (1-\alpha)\hat{N}_t\right], \tag{2.2}$$

where k_{t-1} is the installed capital stock, N_t is employment, and α is the elasticity of output with respect to capital. $\hat{z}_t \sim \mathcal{N}\left(0, \sigma_z^2\right)$ represents a technology shock and Φ is a fixed cost of production.[2]

The representative household maximises expected lifetime utility. Following King et al. (1988), the utility function has the following functional form:

$$U(C_t, 1-N_t) = \frac{C_t^{1-\sigma}}{1-\sigma}\exp\left(\frac{\sigma-1}{1+\phi}N_t^{1+\phi}\right), \tag{2.3}$$

with $\sigma, \phi > 0$.[3]

The *Euler equation for consumption* is given by

$$\hat{c}_t = E_t^*\hat{c}_{t+1} + c_1\left(\hat{N}_t - E_t^*\hat{N}_{t+1}\right) - c_2\left(\hat{R}_t - E_t^*\hat{\Pi}_{t+1}\right) + \hat{u}_t^b \tag{2.4}$$

with $c_1 = (\sigma-1)\bar{N}^{1+\phi}\sigma^{-1}$ and $c_2 = \sigma^{-1}$. Here Π_t is the gross inflation rate and R_t is the gross nominal interest rate. $E_t^*(\cdot)$ denotes the subjective expectations of the household at time t. The disturbance term $\hat{u}_t^b$ represents a risk premium shock à la Smets and Wouters (2007). The disturbance is assumed to obey $\hat{u}_t^b = \rho_b\hat{u}_{t-1}^b + \varepsilon_t^b$, with $\varepsilon_t^b \sim \mathcal{N}\left(0, \sigma_b^2\right)$.

Following Schmitt-Grohé and Uribe (2006), labour decisions are made by a union who represents the household and operates in a continuum of monopolistically competitive labour markets.[4] In each market the union sets the wage and supplies enough differentiated labour to satisfy demand. Nominal wages are assumed to be sticky à la Calvo (1983). In those labour markets where the union cannot re-optimise its wage, nominal wages are indexed to productivity growth (z_t) and a weighted average of target inflation (Π_t^*) and lagged inflation (Π_{t-1}). The *real wage equation* that follows from the union's wage setting decision is given by

$$\hat{w}_t = w_1\left(\phi\hat{N}_t + \hat{c}_t - \hat{w}_t\right) + w_2\hat{w}_{t-1} + w_3E_t^*\hat{w}_{t+1} + w_4\hat{\Pi}_t + w_5\hat{\Pi}_{t-1} + w_6E_t^*\hat{\Pi}_{t+1} + w_7\hat{\Pi}_t^* + \hat{u}_t^w, \tag{2.5}$$

[2] Technology A_t follows a random walk with drift in its log: $\ln(A_t) = \ln(\gamma) + \ln(A_{t-1}) + \hat{z}_t$.

[3] Here $C_t = c_tA_t$,where A_t is the level of technology and c_t is detrended private consumption. In the remainder of the paper, we work with detrended real variables.

[4] As in Schmitt-Grohé and Uribe (2006), our formulation of the labour market assures that labour supply and consumption are identical across households, even if the utility function (2.3) is non-separable in consumption and employment.

with $w_1 = (1-\theta_w)\left(1-\beta\theta_w\gamma^{1-\sigma}\right)/\left[\theta_w\left(1+\beta\gamma^{1-\sigma}\right)\right]$, $w_2 = 1/\left(1+\beta\gamma^{1-\sigma}\right)$, $w_3 = \beta\gamma^{1-\sigma}/\left(1+\beta\gamma^{1-\sigma}\right)$, $w_4 = -\left(1+\beta\gamma^{1-\sigma}\gamma_w\right)/\left(1+\beta\gamma^{1-\sigma}\right)$, $w_5 = \gamma_w/\left(1+\beta\gamma^{1-\sigma}\right)$, $w_6 = \beta\gamma^{1-\sigma}/\left(1+\beta\gamma^{1-\sigma}\right)$, and $w_7 = (1-\gamma_w)\left(1-\rho_{\pi^*}\beta\gamma^{1-\sigma}\right)/\left(1+\beta\gamma^{1-\sigma}\right)$. According to this equation, the real wage w_t gradually adjusts to the difference between the real wage and the marginal rate of substitution between consumption and leisure ($\phi\hat{N}_t + \hat{c}_t$). The adjustment depends on the degree of wage stickiness, θ_w, and the normalised discount factor, $\beta\gamma^{1-\sigma}$. Moreover, the real wage is a function of past and expected real wages, past, current, and expected inflation, as well as current target inflation. The degree of wage indexation to past inflation relative to target inflation is determined by γ_w. Finally, $\hat{u}_t^w$ represents a wage mark-up shock, which is assumed to evolve according to $\hat{u}_t^w = \rho_w\hat{u}_{t-1}^w - \mu_w\varepsilon_{t-1}^w + \varepsilon_t^w$, with $\varepsilon_t^w \sim \mathcal{N}\left(0,\sigma_w^2\right)$.

The *optimality conditions for investment and the capital stock* are given by

$$\hat{i}_t = i_1\left(\hat{i}_{t-1} - \hat{z}_t\right) + (1-i_1)E_t^*\hat{i}_{t+1} + i_2\hat{Q}_t + \hat{u}_t^i, \tag{2.6}$$

$$\hat{Q}_t = -\left(\hat{R}_t - E_t^*\hat{\Pi}_{t+1} - \sigma\hat{u}_t^b\right) + \beta\gamma^{-\sigma}\left[\bar{r}^k E_t^*\hat{r}_{t+1}^k + (1-\delta)E_t^*\hat{Q}_{t+1}\right], \tag{2.7}$$

where $i_1 = 1/\left(1+\beta\gamma^{1-\sigma}\right)$, $i_2 = 1/\left[\left(1+\beta\gamma^{1-\sigma}\right)s''\gamma^2\right]$, δ is the physical rate of depreciation and Q_t is Tobin's Q.[5] As in Christiano et al. (2005) investment is subject to adjustment costs. The cost parameter $1/s''$ is the elasticity of investment with respect to a one percent temporary increase in the current price of installed capital. The disturbance $\hat{u}_t^i$ represents an investment-specific shock and obeys $\hat{u}_t^i = \rho_i\hat{u}_{t-1}^i + \varepsilon_t^i$, with $\varepsilon_t^i \sim \mathcal{N}\left(0,\sigma_i^2\right)$.

The *stock of physical capital* evolves according to

$$\hat{k}_t = k_1(\hat{k}_{t-1} - \hat{z}_t) + (1-k_1)\hat{i}_t + k_1\hat{u}_t^i, \tag{2.8}$$

with $k_1 = (1-\delta)/\gamma$ and $k_2 = \left(1+\beta\gamma^{1-\sigma}\right)s''\gamma^2\bar{i}/\bar{k}$.

Similar to the wage setting decision of the union, intermediate goods producers set nominal prices according to a Calvo (1983) mechanism. Firms that cannot re-optimise their price, index their old price to a weighted average of target inflation (Π_t^*) and lagged inflation (Π_{t-1}). Optimal price setting gives rise to the following specification of the *new Keynesian Phillips curve*:

$$\hat{\Pi}_t = \pi_1\widehat{MC}_t + \pi_2\hat{\Pi}_{t-1} + \pi_3E_t^*\hat{\Pi}_{t+1} + \pi_4\hat{\Pi}_t^* + \hat{u}_t^\pi, \tag{2.9}$$

with $\pi_1 = (1-\theta_p)\left(1-\beta\theta_p\gamma^{1-\sigma}\right)/\left[\theta_p\left(1+\beta\gamma^{1-\sigma}\gamma_p\right)\right]$, $\pi_2 = \gamma_p/\left(1+\beta\gamma^{1-\sigma}\gamma_p\right)$, $\pi_3 = \beta\gamma^{1-\sigma}/\left(1+\beta\gamma^{1-\sigma}\gamma_p\right)$, and $\pi_4 = (1-\gamma_p)\left(1-\rho_{\pi^*}\beta\gamma^{1-\sigma}\right)/\left(1+\beta\gamma^{1-\sigma}\gamma_p\right)$. The inflation rate is a function of the real marginal cost MC_t, a price mark-up disturbance $\hat{u}_t^\pi$, past and expected future values of actual inflation, and target inflation. The coefficients of the inflation equation depend on the Calvo parameter of price stickiness, θ_p, the degree of price indexation to past inflation, γ_p, and the normalised discount factor, $\beta\gamma^{1-\sigma}$. The price mark-up disturbance follows

[5] Tobin's Q is defined as q_t/λ_t, where q_t is the Lagrangian multiplier with respect to the capital accumulation rule and λ_t the Lagrangian multiplier with respect to the household's (real) budget constraint. See the Appendix 2.A for more details.

the exogenous process $\hat{u}_t^{\pi} = \rho_{\pi}\hat{u}_{t-1}^{\pi} - \mu_{\pi}\varepsilon_{t-1}^{\pi} + \varepsilon_t^{\pi}$, with $\varepsilon_t^{\pi} \sim \mathcal{N}\left(0, \sigma_{\pi}^2\right)$. The ARMA(1,1) structure is designed to capture the high-frequency component of inflation.

Cost minimisation by the intermediate goods producers yields the following *labour demand equation* and *equation for the rental rate of capital*:

$$\hat{w}_t = \hat{MC}_t + \alpha(\hat{k}_{t-1} - \hat{N}_t) + \hat{z}_t, \tag{2.10}$$

$$\hat{r}_t^k = \hat{MC}_t + (\alpha - 1)(\hat{k}_{t-1} - \hat{N}_t) + \hat{z}_t. \tag{2.11}$$

The central bank sets the nominal interest rate according to the following *generalised Taylor rule*:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)\hat{\Pi}_t^* + \rho_R\left(\hat{\Pi}_t^* - \hat{\Pi}_{t-1}^*\right) + (1-\rho_R)\left[\phi_{\pi}(\hat{\Pi}_t - \hat{\Pi}_t^*)\right] + \phi_{\Delta y}\Delta\hat{y}_t + \hat{u}_t^r. \tag{2.12}$$

Analogous to De Graeve et al. (2009) the monetary authority gradually adjusts the interest rate in response to the "inflation gap" $\hat{\Pi}_t - \hat{\Pi}_t^*$. The coefficient ϕ_{π} controls the responsiveness of the nominal interest rate to the inflation gap. The degree of interest rate smoothing is governed by the parameter ρ_R. In addition, we allow the interest rate to react to changes in the growth rate of output, with sensitivity parameter $\phi_{\Delta y}$. Following Cogley et al. (2010), the inflation target Π_t^* is time-varying and evolves according to $\hat{\Pi}_t^* = \rho_{\pi^*}\hat{\Pi}_{t-1}^* + \varepsilon_t^{\pi^*}$, with $\varepsilon_t^{\pi^*} \sim \mathcal{N}\left(0, \sigma_{\pi^*}^2\right)$. The monetary policy shock $\hat{u}_t^r$ evolves according to $\hat{u}_t^r = \rho_r\hat{u}_{t-1}^r + \varepsilon_t^r$, with $\varepsilon_t^r \sim \mathcal{N}\left(0, \sigma_r^2\right)$.

The fiscal authority finances expenditure through lump-sum taxes. *Real government expenditure* evolves according to

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g. \tag{2.13}$$

3 Rational Expectations Equilibrium

We begin with the standard case of rational expectations as a benchmark to compare against the adaptive learning model. In the rational expectations case, agents have full knowledge of the structure of the economy. In the next section, we will relax this assumption and consider a learning mechanism where agents form expectations based on a small forecasting model.

Note that the linear model can be represented as

$$\mathbf{A}_0 \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{A}_1 \begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} + \mathbf{A}_2 E_t^* \mathbf{y}_{t+1} + B_0 \varepsilon_t = constant, \tag{2.14}$$

where $\mathbf{y}_t$ is the column vector of log-linearised endogenous variables and $\mathbf{w}_t$ is the column vector of log-linearised shocks. The vector $\mathbf{y}_t$ contains six endogenous state variables $\mathbf{y}_t^s = \left[\hat{\imath}_t; \hat{k}_t; \hat{\Pi}_t; \hat{R}_t; \hat{w}_t; \hat{y}_t\right]$ and seven forward-looking variables $\mathbf{y}_t^f = \left[\hat{c}_t; \hat{\imath}_t; \hat{N}_t; \hat{\Pi}_t; \hat{Q}_t; \hat{r}_t^k; \hat{w}_t\right]$. The vector $\mathbf{w}_t$ consists of the eight stochastic processes in the model.

When agents have rational expectations, the dynamics of the model are characterised by the following rational expectations equilibrium (REE):

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} = \mu + \mathbf{T} \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{R}\varepsilon_t. \tag{2.15}$$

4 Adaptive Learning

Following Branch and Evans (2006), Sargent (1999), Sargent and Williams (2005), Sargent et al. (2006), and Slobodyan and Wouters (2012a), agents form expectations using forecasting models updated by the Kalman filter.

For every forward-looking variable y_j^f, with $j = 1, 2, \ldots, 7$, agents use the following forecasting model

$$y_{j,t}^f = \mathbf{X}_{j,t-1}^T \beta_{j,t-1} + u_{j,t}. \tag{2.16}$$

In the learning literature this equation represents the Perceived Law of Motion (PLM). In the baseline specification of our model, the data matrix $\mathbf{X}_{j,t-1}$ contains a constant and the endogenous state variables of the model: the capital stock $\left(\hat{k}_{t-1}\right)$, the nominal interest rate $\left(\hat{R}_{t-1}\right)$, output $(\hat{y}_{t-1})$, the real wage rate $(\hat{w}_{t-1})$, investment $\left(\hat{i}_{t-1}\right)$, and the inflation rate $\left(\hat{\Pi}_{t-1}\right)$. By including all the endogenous state variables in the data matrix, our approach applies only a modest departure from rational expectations. We assume however that agents cannot access values of exogenous processes $\mathbf{w}_t$. As in Chapter 1, the exclusion of the government spending shock reflects the assumption that agents have imperfect knowledge on the general equilibrium effects of fiscal policy. Section 8 discusses the robustness of our results across alternative specifications of the forecasting models (2.16). In that section we show that our baseline specification results in the largest improvement of the marginal likelihood vis-à-vis the rational expectations model.

Let β_t denote the vector of stacked regression coefficients $\beta_{j,t}$. Following Sargent and Williams (2005), agents believe that the vector β_t evolves according to a random walk process

$$vec\left(\beta_t\right) = vec\left(\beta_{t-1}\right) + \mathbf{v}_t. \tag{2.17}$$

The shocks $\mathbf{v}_t$ are i.i.d. with covariance matrix $\mathbf{V}$. The random walk assumption is widely used in time-varying parameter Vector Autoregression (VAR) studies.

Equation (2.17) reflects the agents' view that the coefficients in their forecasting rules are not stable, but drift over time. As argued by Sargent and Williams (2005), among others, this assumption assures that the variation in the agents' beliefs does not die out. Hence, this set-up it is a natural way of accomplishing so-called "perpetual learning". A motivation for this set-up is that it allows agents to be alert to structural changes, because the Kalman filter learning algorithm discounts past data. In other words, this approach implicitly assumes that recent observations contain more accurate information about the current forecasting coefficients than past observations. Especially in a context where structural changes occur from time to time, this seems a reasonable assumption. Another justification is that the discounting of past data can be seen as a way to formalise finite-memory forecasting by the agents.

An alternative way to achieve "perpetual learning" behaviour is to consider a constant-gain variant of recursive least squares – see, for example, Milani (2007) and Orphanides and Williams (2007). In this paper, the Kalman filter is preferred over constant-gain algorithm based on the finding of Sargent and Williams (2005) that, although both algorithms have the same asymptotic behaviour, the Kalman filter converges much faster than the constant-gain algorithm.

We can write the forecasting model in the following SURE format:

$$\begin{bmatrix} y^f_{1,t} \\ y^f_{2,t} \\ \vdots \\ y^f_{m,t} \end{bmatrix} = \begin{bmatrix} \mathbf{X}_{1,t-1} & 0 & \cdots & 0 \\ 0 & \mathbf{X}_{2,t-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}_{m,t-1} \end{bmatrix} \begin{bmatrix} \beta_{1,t-1} \\ \beta_{2,t-1} \\ \vdots \\ \beta_{m,t-1} \end{bmatrix} + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{m,t} \end{bmatrix}$$

$$\Leftrightarrow \mathbf{y}^f_t = \mathbf{X}_{t-1}\beta_{t-1} + \mathbf{U}_t, \tag{2.18}$$

where we denote the (non-diagonal) covariance matrix of the regression errors $\mathbf{U}_t$ by Σ.[6]

The Kalman filter provides the optimal estimate of the belief coefficients β_t conditional on information up to period $t-1$.[7] The filter is described by the following two equations:

$$\beta_{t+1|t} = \beta_{t|t-1} + \mathbf{K}_t \left[\mathbf{y}^f_t - \mathbf{X}^T_{t-1}\beta_{t|\text{t-1}}\right], \tag{2.19}$$

$$\mathbf{P}_{t+1|t} = \left(\mathbf{I} - \mathbf{K}_t\mathbf{X}^T_{t-1}\right)\mathbf{P}_{t|t-1} + \mathbf{V}, \tag{2.20}$$

where $\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{X}_{t-1}\left[\mathbf{X}^T_{t-1}\mathbf{P}_{t|t-1}\mathbf{X}_{t-1} + \Sigma\right]^{-1}$ is the Kalman gain.

At the end of period t, the realised value of $\mathbf{y}^f_t$ is used to update the estimate of β_t based on the new information. Following equation (2.19), the Kalman gain $\mathbf{K}_t$ determines the weight assigned to the new information when forming the new estimate $\beta_{t+1|t}$.

To obtain the dynamics under Kalman filter learning, the estimate $\beta_{t|t-1}$ from equation (2.19) is substituted for β_{t-1} in equation (2.18) to generate $E^*_t\mathbf{y}_{t+1} = \mathbf{X}_t\beta_{t|t-1}$. We can insert the expression for $E^*_t\mathbf{y}_{t+1}$ in the linear approximation of the model (2.14) to obtain the following actual law of motion under learning

$$\begin{bmatrix} \mathbf{y}_t \\ \mathbf{w}_t \end{bmatrix} = \mu_t + \mathbf{T}_t \begin{bmatrix} \mathbf{y}_{t-1} \\ \mathbf{w}_{t-1} \end{bmatrix} + \mathbf{R}_t\varepsilon_t. \tag{2.21}$$

For the initial Kalman filter recursion, we need to specify the initial belief coefficients $\beta_{1|0}$, the associated covariance matrix $\mathbf{P}_{1|0}$, the parameter covariance matrix $\mathbf{V}$, and the covariance matrix of the regression errors, Σ. We follow the approach of Slobodyan and Wouters

[6]Note that the regression errors $u_{j,t}$ are linear combinations of the innovations ε_t to the stochastic processes $\mathbf{w}_t$.

[7]See Ljung and Söderström (1986) or Kim and Nelson (1999), for example.

(2012a) and use the theoretical moment matrices of the rational expectations equilibrium to derive the initial beliefs. In particular, since the OLS estimator is unbiased, we let $\beta_{1|0} = \bar{\beta} = \hat{\beta}_{OLS} = E\left(\mathbf{X}^T\mathbf{X}\right)^{-1} E\left(\mathbf{X}^T\mathbf{y}^f\right)$. It follows that the covariance matrix $\Sigma = E\left[\mathbf{U}\mathbf{U}^T\right] = E\left[\left(\mathbf{y}-\mathbf{X}\beta\right)\left(\mathbf{y}-\mathbf{X}\beta\right)^T\right]$. Furthermore, $\mathbf{P}_{1|0}$ and $\mathbf{V}$ are both taken to be proportional to the covariance matrix of the GLS estimator, which is an efficient estimator of the SURE model (2.18). Hence, $\mathbf{P}_{1|0} = \sigma_0\left(\mathbf{X}^T\Sigma^{-1}\mathbf{X}\right)^{-1}$ and $\mathbf{V} = \sigma_v\left(\mathbf{X}^T\Sigma^{-1}\mathbf{X}\right)^{-1}$.

In this paper, we consider *Euler equation* learning put forward by Evans and Honkapohja (2001) and assume that agents make one-step ahead forecasts, following e.g. Giannitsarou (2006) and Slobodyan and Wouters (2012a,b). By contrast, Evans et al. (2009), Mitra et al. (2013) and Gasteiger and Zhang (2014) consider an *infinite horizon* learning scheme developed by Preston (2005). Under this learning scheme, agents must make forecasts about forward-looking variables into the infinite future. In the *infinite horizon* approach the inter-temporal budget constraint is explicitly used when deriving agents' expectations, whereas in the *Euler equation* approach only the flow budget constraint is used. In this paper, we focus on the latter approach and leave the interesting issue of the effects of the learning type on the transmission of government spending shocks for future work.[8]

5 Bayesian Estimation

5.1 Data and Observation Equations

We estimate the model with euro area quarterly data from 1970Q2 to 2013Q4.[9] Figure 2.1 plots the time series used in the estimation. We use seven macroeconomic variables: the short-term nominal interest rate (r_t^{obs}) and the log differences of per capita real government consumption (g_t^{obs}), per capita real GDP (y_t^{obs}), per capita real consumption (c_t^{obs}), per capita real investment (i_t^{obs}), the real wage (w_t^{obs}), and the GDP deflator (Π_t^{obs}). The corresponding observation equations are

$$y_t^{obs} = \hat{y}_t - \hat{y}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \tag{2.22}$$

$$c_t^{obs} = \hat{c}_t - \hat{c}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \tag{2.23}$$

$$i_t^{obs} = \hat{i}_t - \hat{i}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \tag{2.24}$$

$$g_t^{obs} = \hat{g}_t - \hat{g}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \tag{2.25}$$

$$w_t^{obs} = \hat{w}_t - \hat{w}_{t-1} + 100(\gamma - 1) + \hat{z}_t, \tag{2.26}$$

$$\Pi_t^{obs} = \hat{\Pi}_t + 100\left(\bar{\Pi} - 1\right), \tag{2.27}$$

[8]For a discussion of these two adaptive learning approaches see Honkapohja et al. (2013).

[9]These data are extracted from the 16th update of the Area Wide Model database compiled by Fagan et al. (2005). Following Smets and Wouters (2003), the observations in the 1970s are used as a training sample and do not enter into the calculation of the marginal likelihood.

$$r_t^{obs} = \hat{R}_t + 100\,(\bar{R} - 1)\,, \tag{2.28}$$

where $100(\gamma - 1)$ is the common quarterly trend growth rate of real GDP, consumption, investment, capital, wages, and government spending, $100\left(\bar{\Pi} - 1\right)$ is the quarterly steady-state inflation rate, and $100\,(\bar{R} - 1)$ is the quarterly steady-state nominal interest rate.

5.2 Prior Distributions

The choice of the prior distributions is summarised in the left panel of Table 2.1. The prior distributions of the structural parameters are as follows. The prior standard deviation of most structural parameters is 0.1. For ϕ, σ, and s'', however, we allow for a larger standard deviation, ensuring a rather large domain for these parameters. The steady-state inflation rate is assumed to follow a gamma distribution with a mean of 2 percent on an annualised basis. The priors for the degree of indexation to past inflation, γ_p and γ_w, and for the MA parameters in the mark-up processes, μ_π and μ_w, are described by a beta distribution with mean 0.5. Following Smets and Wouters (2003), the degree of risk aversion, σ, has a normal distribution with mean 1.5 and standard deviation 0.37, and the Calvo probabilities, θ_p and θ_w, have a beta distribution with mean 0.75 and standard deviation 0.05. The Frisch elasticity of labour supply, ϕ, is assumed to follow a normal distribution with mean 2 and standard deviation 0.25. The autoregressive coefficient of the stochastic processes are all assumed to follow a beta distribution with mean 0.5, except for the coefficient of standard monetary policy shock, ρ_r. The latter has a prior mean of 0.25 to have a clear separation from the inflation target shock. Based on Smets and Wouters (2007) the prior distribution for the investment adjustment cost parameter, s'', is normal with mean 4 and standard deviation 1.5.

For the monetary policy parameters we adopt analogue priors as those used by De Graeve et al. (2009). The degree of interest rate smoothing, ρ_R, has a beta distribution with a mean of 0.75 and a standard deviation of 0.1. We adopt normal priors for the Taylor rule coefficients ϕ_π and $\phi_{\Delta y}$ with typical mean values. Following Cogley et al. (2010), we calibrate the autocorrelation of the inflation target shock to 0.985 so that it captures low-frequency movements in inflation. In consideration of the downward trend in the inflation data, the inflation target is thus designed to capture the gradual disinflation in the euro area over the past decades.

The scale parameters σ_0 and σ_v driving the learning dynamics follow a gamma distribution. It is standard to assume that the variance-covariance of shocks to the belief coefficients is smaller than the variance-covariance of the measurement errors of the forecasting models (2.18), i.e. $\mathbf{V} \ll \Sigma$ (see Sargent et al., 2006, for instance). Therefore we set the prior mean of σ_v to a small number (0.004) relative to the prior mean of σ_0 (0.04).

The standard deviations of the structural shocks are assumed to follow inverse-gamma distributions with two degrees of freedom. The prior mean for the standard deviation of the inflation target shock is taken from Smets and Wouters (2003).

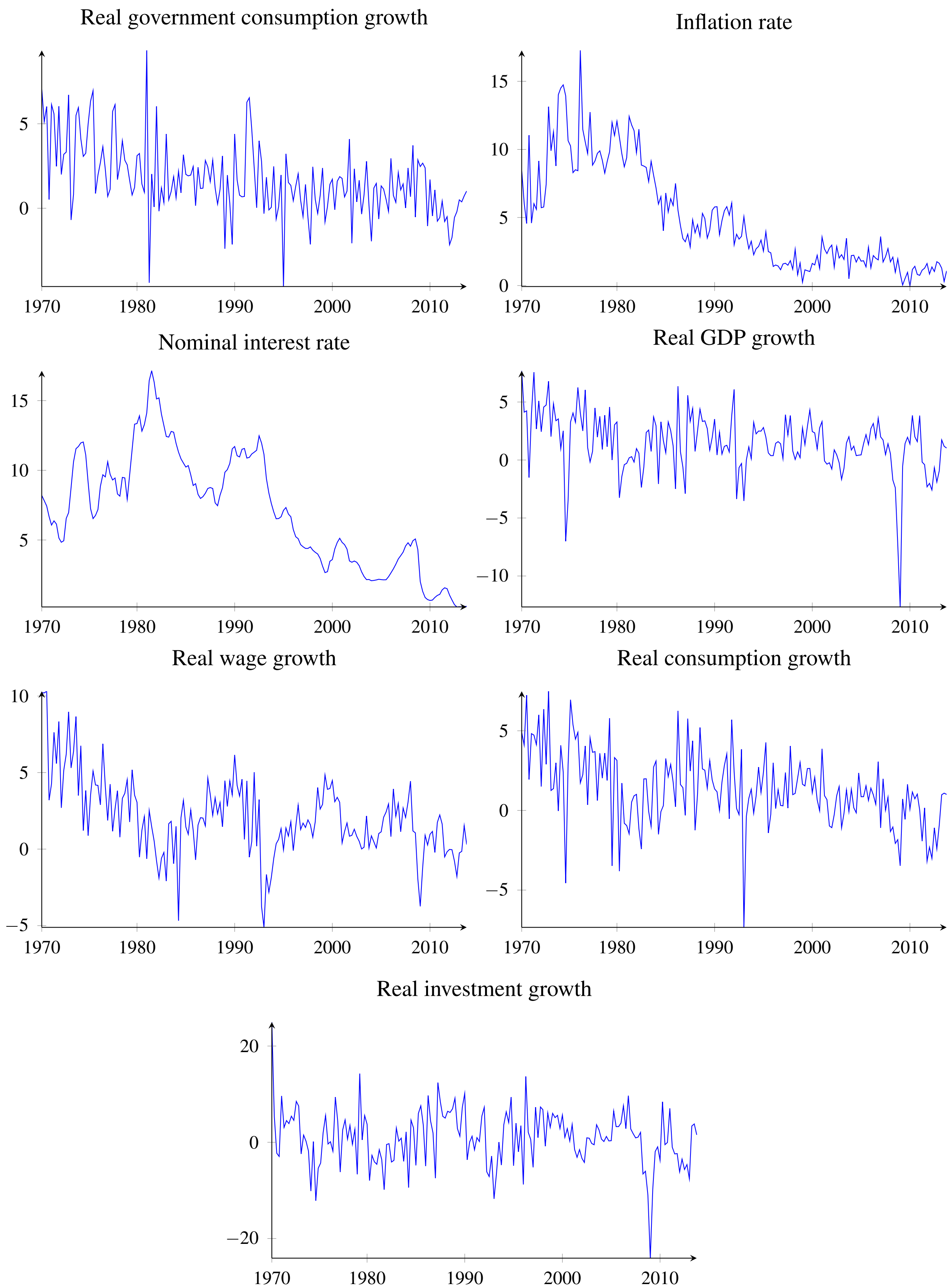

Figure 2.1: Data used in the estimation. For ease of interpretation, the variables are converted to an annual basis. In the estimation, however, we use quarterly data.

A few parameters are kept fixed in the estimation procedure. The quarterly trend growth rate of real GDP is set to the mean growth rate of GDP in the sample. The discount factor $100(\beta^{-1}-1)$ is set to 1 percent on an annualised basis. The output elasticity with respect to capital, α, is set to 0.33. The quarterly rate of physical capital depreciation, δ, is set to 0.025 so that the annual depreciation rate is 10 percent. The steady-state ratio of government spending to GDP is set to 0.20. The parameters $\bar{\varepsilon}_p$ and $\bar{\varepsilon}_w$ governing the price and wage mark-up, are clearly not identified by the data. We follow Rabanal and Rubio-Ramírez (2008) and set $\bar{\varepsilon}_p = 0.2$ and $\bar{\varepsilon}_w = 0.1$. Finally, the fixed cost in the production, Φ, is calibrated so that steady-state profits in the intermediate goods sector are zero.

5.3 Estimation Results

In this section, we present the estimation results for different assumptions regarding the formation of expectations. In particular, we compare the rational expectations results with those of the learning model.

Table 2.1 presents the posterior estimates of the model parameters and the log marginal likelihood for the rational expectations model and the learning model. Based on the marginal likelihood criterion, it is clear that there is substantial evidence in favour of the learning mechanism relative to rational expectations. This result is in line with the findings of Milani (2007) and Slobodyan and Wouters (2012a,b), who also conclude that adaptive learning significantly improves the fit of DSGE models.

Table 2.1 and Table 2.2 demonstrate that the modelling assumption of expectations affects some of the parameter estimates. The estimated price and wage stickiness and the investment adjustment costs, for instance, are lower under Kalman filter learning. This is consistent with the finding of Milani (2007). Learning introduces endogenous persistence in the model such that other sources of persistence are no longer required to match the inertia in the data. The posterior mode for the degree of interest rate smoothing increases from 0.62 under rational expectations to 0.84 under learning.

Parameter	Description	Prior distribution			Rational expectations model			Kalman filter learning model		
		Type	Mean	Std.	Mean	Mode	90% HPD interval	Mean	Mode	90% HPD interval
Structural parameters										
γ_p	Indexation of prices to past inflation	B	0.5	0.1	0.277	0.267	[0.239,0.317]	0.4268	0.4298	[0.414,0.436]
γ_w	Indexation of wages to past inflation	B	0.5	0.1	0.378	0.378	[0.298,0.459]	0.5135	0.5211	[0.505,0.525]
θ_p	Degree of nominal price rigidity	B	0.75	0.05	0.86	0.86	[0.837,0.874]	0.7612	0.761	[0.756,0.764]
θ_w	Degree of nominal wage rigidity	B	0.75	0.05	0.685	0.685	[0.658,0.723]	0.648	0.6502	[0.645,0.653]
$100(\bar{\Pi}-1)$	Quarterly steady-state inflation rate	G	0.5	0.1	0.544	0.544	[0.44,0.654]	0.696	0.6928	[0.687,0.706]
ρ_R	Degree of interest rate smoothing	B	0.75	0.1	0.618	0.618	[0.587,0.655]	0.836	0.8388	[0.831,0.84]
ϕ	Inverse Frisch elasticity of labour supply	N	2	0.25	2.275	2.275	[2.145,2.49]	1.954	1.9482	[1.945,1.967]
ϕ_π	Taylor rule inflation rate coefficient	N	1.5	0.1	1.063	1.063	[1.055,1.088]	1.519	1.5175	[1.514,1.528]
$\phi_{\Delta y}$	Taylor rule output growth coefficient	N	0.125	0.05	0.13	0.13	[0.103,0.153]	0.0728	0.0692	[0.0658,0.0789]
σ	Degree of risk aversion	G	1.5	0.37	1.181	1.181	[1.104,1.258]	1.0744	1.0847	[1.06,1.086]
s''	Investment adjustment cost parameter	N	4	1.5	2.003	2.003	[2,2.452]	5.3082	5.3135	[5.293,5.318]
ρ_b	Risk premium shock AR coefficient	B	0.5	0.2	0.633	0.633	[0.555,0.809]	0.7289	0.7262	[0.725,0.734]
ρ_g	Government expenditure AR coefficient	B	0.5	0.2	0.997	0.997	[0.992,0.999]	0.9943	0.995	[0.993,0.995]
ρ_π	Price mark-up shock AR coefficient	B	0.5	0.2	0.42	0.42	[0.305,0.53]	0.6379	0.6322	[0.629,0.647]
ρ_r	Monetary policy shock AR coefficient	B	0.25	0.1	0.489	0.489	[0.416,0.528]	0.4802	0.4816	[0.474,0.488]
ρ_i	Investment shock AR coefficient	B	0.5	0.2	0.051	0.051	[0.0157,0.0929]	0.1025	0.1071	[0.0894,0.111]
ρ_w	Wage mark-up AR coefficient	B	0.5	0.2	0.992	0.992	[0.989,0.995]	0.967	0.9606	[0.961,0.972]
μ_w	Wage mark-up shock MA coefficient	B	0.5	0.2	0.788	0.788	[0.698,0.852]	0.7073	0.7048	[0.701,0.716]
μ_π	Price mark-up shock MA coefficient	B	0.5	0.2	0.331	0.331	[0.235,0.459]	0.6368	0.635	[0.631,0.643]
σ_0	Scale of $\beta_{1\|0}$ covariance matrix matrix $\mathbf{P}_{1\|0}$	G	0.04	0.03	–	–	–	0.0124	0.012	[0.0093,0.0152]
σ_v	Scale of belief covariance matrix matrix $\mathbf{V}$	G	0.004	0.003	–	–	–	0.0109	0.0106	[0.0106,0.0114]
Marginal likelihood						−928.64			−892.57	

Note: B represents beta, G gamma, IG inverse gamma, and N normal.

Table 2.1: Prior and posterior distributions of the model parameters under Kalman filter learning and under rational expectations.

Parameter	Prior distribution			Rational expectations model			Kalman filter learning model		
	Type	Mean	Std.	Mean	Mode	90% HPD interval	Mean	Mode	90% HPD interval
σ_b	IG	0.1	2	0.046	0.046	[0.027,0.11]	0.7	0.7	[0.7,0.71]
σ_g	IG	0.1	2	0.18	0.18	[0.16,0.2]	0.19	0.19	[0.19,0.2]
σ_i	IG	0.1	2	0.98	0.98	[0.87,1.07]	1.15	1.15	[1.14,1.16]
σ_{π^*}	IG	0.02	2	0.026	0.026	[0.02,0.038]	0.061	0.061	[0.059,0.064]
σ_π	IG	0.1	2	0.15	0.15	[0.14,0.18]	0.2	0.2	[0.19,0.2]
σ_r	IG	0.1	2	0.15	0.15	[0.14,0.17]	0.1	0.1	[0.1,0.11]
σ_w	IG	0.1	2	0.31	0.31	[0.27,0.35]	0.42	0.42	[0.4,0.43]
σ_z	IG	0.1	2	0.83	0.83	[0.76,0.92]	0.84	0.84	[0.83,0.84]

Note: B represents beta, G gamma, IG inverse gamma, and N normal.

Table 2.2: Prior and posterior distributions of the standard deviations of the shocks under Kalman filter learning and under rational expectations.

In the learning model, agents update the regression coefficients (i.e. "beliefs") of their forecasting model using the Kalman filter. This updating procedure generates important variation of those coefficients over time. Figure 2.2 illustrates this. For every forward-looking variable, the figure shows the belief coefficients of the respective forecasting model. The shaded grey areas indicate euro area recession dates. As explained in Section 4, each forecasting model is a function of a constant, the nominal interest rate, inflation, investment, the capital stock, output, and the real wage rate. It is clear that most coefficients vary a lot over time, especially the coefficients with respect to the interest rate and the inflation rate. Moreover, agents seem to adjust their belief coefficients, and thus the implied expectations, quite substantially during recession periods.

6 Dynamics of a Government Spending Shock

Since the belief coefficients in the forecasting models of the agents vary over time, the transmission of shocks in the model will do so as well. Figure 2.3 plots the estimated responses of some key macroeconomic variables to a government spending shock of one percent of GDP. The law of motion under learning (cf. equation (2.21)) allows us to plot the impulse responses for each quarter. For ease of comparison, the impulse responses of the rational expectations model are plotted at the beginning of the sample.

The responses of output to the government spending shock are always positive on impact. For most periods, the learning model also generates a positive effect on private consumption, although the effect becomes slightly negative in the 1990s and the first half of the 2000s. The positive impact on consumption is in sharp contrast with the negative effect under rational expectations. Under rational expectations, agents are completely forward-looking and fully in-

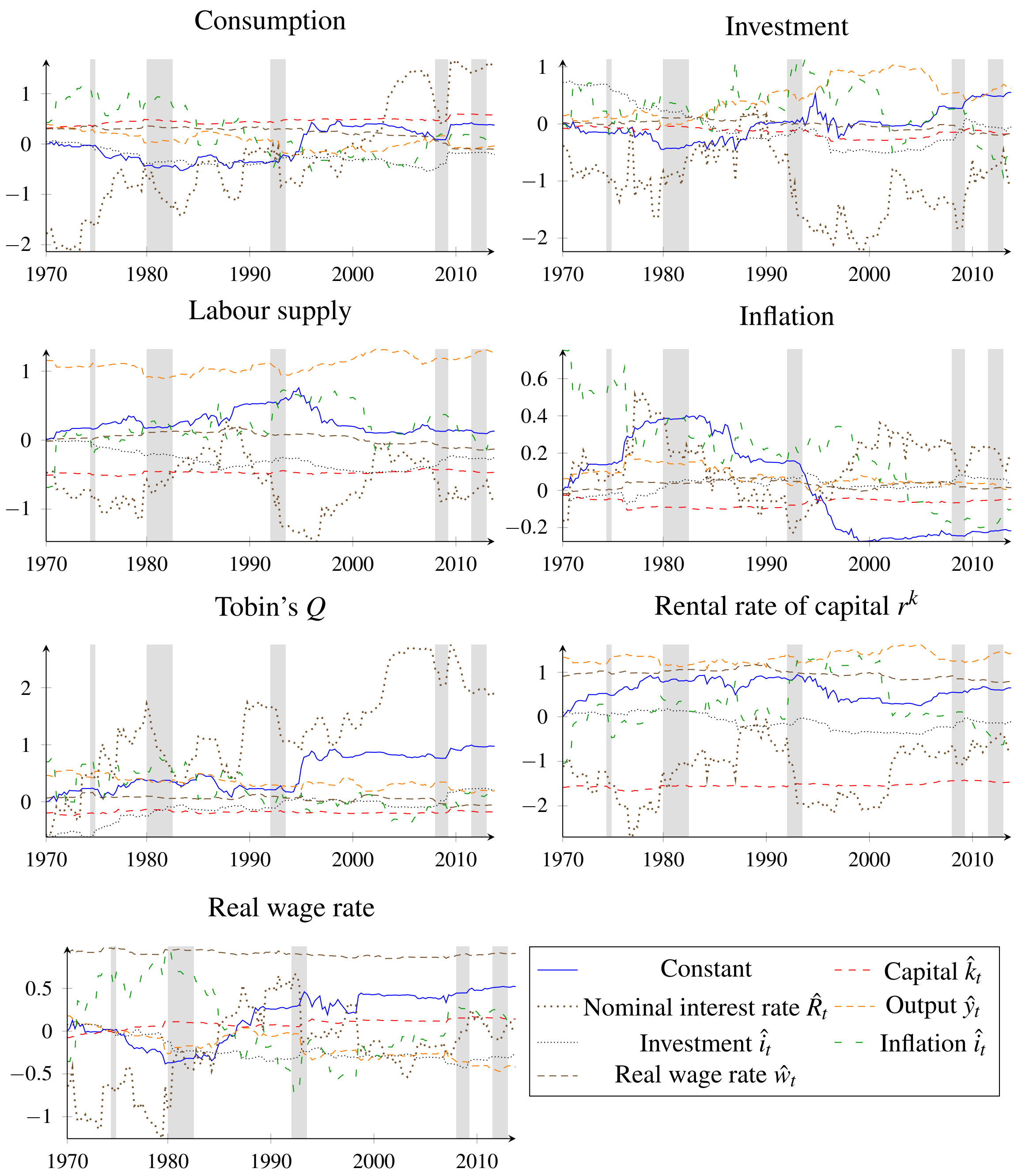

Figure 2.2: Estimated beliefs in the forecasting models of the agents. The shaded grey areas are quarters of recessions as defined by the CEPR Euro Area Business Cycle Dating Committee

corporate the negative wealth effect of higher future taxes. By contrast, under Kalman filter learning agents do not take this negative wealth effect directly into account when forming their beliefs. Hence, the learning mechanism provides an intuitive explanation for the positive response of consumption found in several empirical studies such as Burriel et al. (2010), for the euro area, and Blanchard and Perotti (2002), Fatás and Mihov (2001), Galí et al. (2007), and Perotti (2004), for the United States.[10]

The impact responses of output are high in the beginning of the sample but decline in the 1980s and 1990s. This declining trend over the sample is particularly interesting since it is in accordance with the empirical evidence found in Kirchner et al. (2010). If we examine the components of aggregate demand more closely, the downward trend is mainly driven by a decreasing impact of government spending on private consumption.

An important observation is the persistent medium- and long-term adverse effect of a government spending shock on private demand in the 1970s and – to a lesser extent – the beginning of the 1980s. In that period, time variation in private sector expectations may be driven by the oil shock in 1973 and the early 1980s recession. In response to the oil shock several European countries increased nominal short-term interest rates. The oil shock also triggered a significant drop in GDP growth over the course of 1974 (see Figure 2.1). In the model, both the rise in the nominal interest rate and the drop in output and investment, led agents to revise their expectations according to the Kalman filter learning mechanism. Figure 2.2 shows how the coefficients in the agents' forecasting models were updated in those years. Similarly, that graph illustrates how movements in the observed data series before and during the recession of the early 1980s, led to significant shifts in agents' belief coefficients. Especially, the coefficients for the nominal interest rate and the inflation rate were revised substantially during that period.

The response of private investment is always positive on impact. This is in line with the euro area estimates of Burriel et al. (2010) and Kirchner et al. (2010). On the other hand, this finding is in sharp contrast with the empirical evidence for the United States. Blanchard and Perotti (2002) and Mountford and Uhlig (2009), for instance, find a decline in investment in response to a positive government spending shock. Burriel et al. (2010) relate this difference between the euro area and the US to the reaction of the nominal interest rate. They find that in the euro area the nominal interest rate reacts more gradually to a government spending shock. This slower interest rate increase may dampen the adverse effects on private demand. Indeed, the bottom left panel of Figure 2.3 depicts a hump-shaped reaction of the nominal interest rate to the spending shock for most of the sample periods. Another observation is that, similar to Kirchner et al. (2010), the responses are in general relatively small. However, in the beginning

[10]Empirical evidence regarding the response of private consumption to a government spending shock is not conclusive, however. Studies using the structural vector autoregression method usually find that private consumption increases after a positive government spending shock. The narrative method of Ramey and Shapiro (1998), on the other hand, typically finds the opposite. Perotti (2008) provides an extensive discussion of the two methods.

of the 1980s government spending had an important crowding-out effect on private investment in the medium and long run. On the other hand, from the second half of the 1980s onwards, government spending shocks had positive effects on private investment, both in the short and long run.

Turning now to the labour market variables, the impulse response functions of Figure 2.3 show that a government spending shock has a sizeable effect on employment throughout the whole sample. Consistent with most theoretical and empirical evidence, the employment effect on impact is always positive.[11] At the long horizon, the response of employment is close to zero from the second half of the 1980s onwards.

The reaction of real wages displays a downward trend. In the first half of the sample, the wage rate goes up by 0.26 to 0.68 percent on impact. Towards the end of the sample, however, the initial response of the real wage is close to zero. Real wages adjust only gradually to the government spending shock. Especially from the second half of the 1980s onwards, the positive effects of government spending shocks on real wages are very persistent.

7 The Government Spending Multiplier

7.1 Results

Learning behaviour by the agents in our model generates endogenous variation in the government spending multipliers. Figure 2.4 shows how the present-value multipliers for output, private consumption, and investment vary over time. We calculate the multipliers on impact and at one, four, and eight years after the shock. Following Mountford and Uhlig (2009), at quarter $t = \{1970Q2, \ldots, 2013Q4\}$ the present-value multiplier for variable X over a k-period horizon is calculated as

$$\left.\frac{PV(\Delta X)}{PV(\Delta G)}\right|_t = \frac{\sum_{s=0}^{k}\left(\bar{R}/\bar{\Pi}\right)^{-s} X_{t+s}}{\sum_{s=0}^{k}\left(\bar{R}/\bar{\Pi}\right)^{-s} G_{t+s}} \frac{1}{\bar{G}/\bar{X}}, \tag{2.29}$$

where X_{t+s} is the response of variable X at period $t+s$, G_{t+s} is government spending at period $t+s$, $\bar{R}/\bar{\Pi}$ is the steady state gross real interest rate, and $\bar{G}/\bar{X}$ is the steady state government expenditure to X ratio.

The upper left panel of Figure 2.4 shows the impact multiplier of government spending for output. The multiplier under learning ranges from 0.91 to 1.19. By contrast, the multiplier under rational expectations is only 0.43.[12] In particular, the impact multiplier under learning is

[11] See Perotti (2008) for a survey of the evidence on the effects of government spending shocks on labour market outcomes.

[12] The multiplier under rational expectations refers to the multiplier evaluated at the posterior mode of the rational expectations model (see Table 2.1). An important observation is that the different size of the government spending multiplier does not stem from differences in the estimated structural parameters between the learning model and the rational expectations model. Evaluated at the posterior mode of the learning model, the output

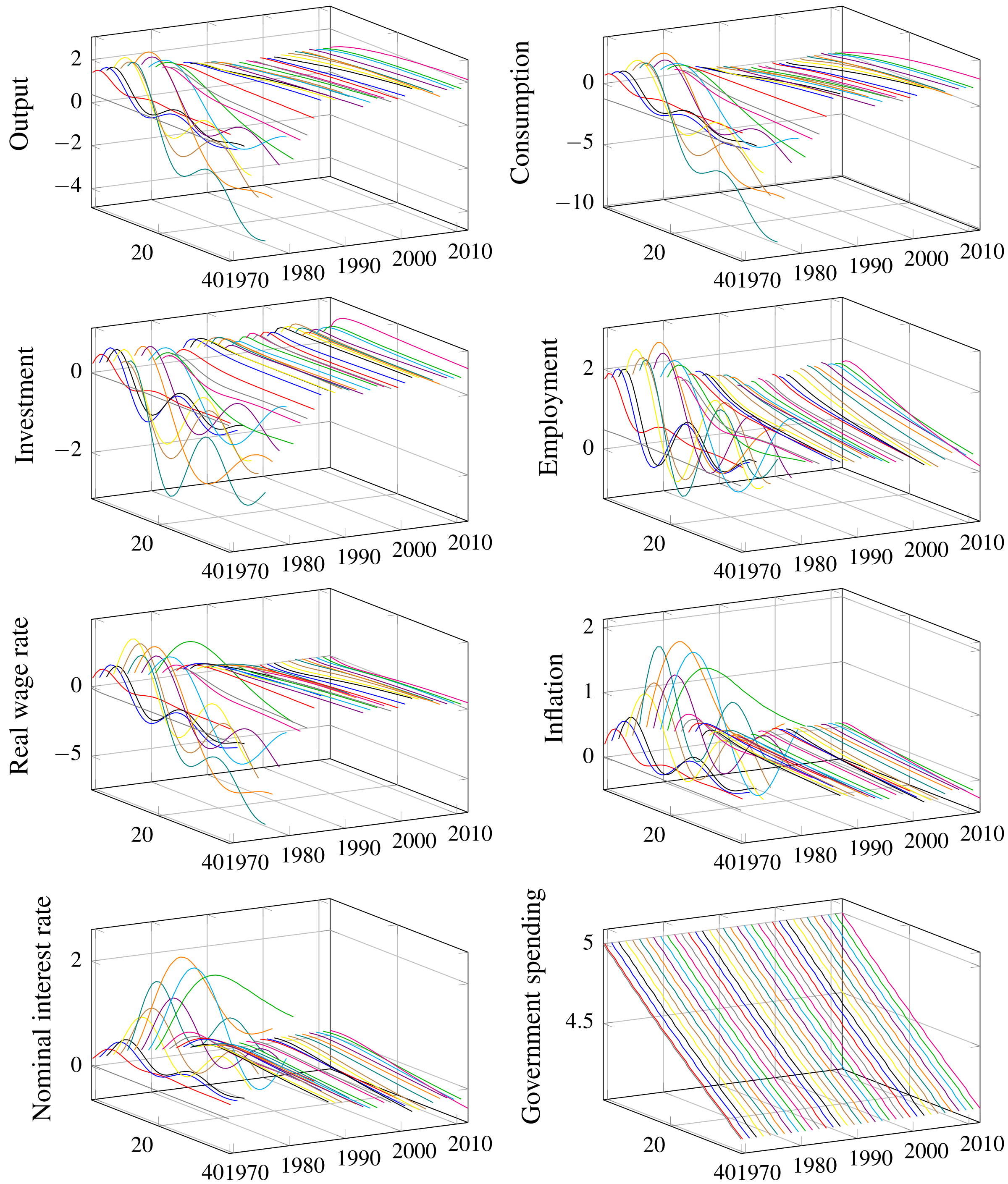

Figure 2.3: Impulse responses to an increase in government spending of 1% of GDP. Pseudo impulse responses are reported since the beliefs are held constant during the transition of the shock. The impulse responses at the beginning of the sample are those of the rational expectations model. The impulse response functions are calculated at the posterior mode and measured in percentage deviations from steady-state.

at its highest level in the first half of the sample. It peaks in the last quarter of 1977 and the second quarter of 1986, at a value of 1.31, and 1.18, respectively. During the 1980s and first half of the 1990s the multiplier declines. In the second quarter of 1994 the impact multiplier reaches an absolute minimum of 0.91. However, an important observation is the improvement in the multiplier over the past two decades. In particular, the impact multiplier on output reaches a local maximum of 1.17 in the 2008-2009 recession.

The present-value multipliers for output at medium and long horizons are shown in the upper right panel of Figure 2.4. The medium and long-run multipliers vary considerably in the 1970s. Eight years after the shock, the output multipliers becomes negative for most periods between 1975Q1 and 1979Q4. From the 1980s onwards, however, the multiplier is always positive in the first eight years after the shock. However, for most of the periods the long-run multiplier (eight years after the shock) is clearly positive and lies around 1.5 in the last two decades. Looking at the entire sample, the long-run multiplier reaches maximum values of in 1980Q1 (2.57), 1990Q1 (1.87), and 2013Q2 (1.92). On the whole, the medium- and long-term multipliers for output display significant time variation.

The second row of panels in Figure 2.4 shows the government spending multipliers for private consumption. The multiplier on impact and after one year follows the same pattern over time as the output multiplier. The multiplier is positive most of the time, but declines sharply in the 1980s and the beginning of the 1990s. The positive consumption multipliers for a considerable part of the sample are in sharp contrast to the negative rational expectations multiplier of -0.58. On impact, the consumption multiplier under learning ranges range from -0.15 in the second quarter of 1995 to 0.48 in the last quarter of 1977. The consumption multiplier generated by our learning model is on average 0.11, which is smaller than the estimate of 0.48 obtained by Burriel et al. (2010). In the most recent years of the sample (2000–2013), the impact multiplier lies around 0.03. At longer horizons, the multiplier varies a lot during the 1970s, but is always positive since the second quarter of 1984. In the last two decades of the sample, the long-run multiplier is on average 0.38.

The present-value multipliers for investment are plotted in the bottom panels of Figure 2.4. The impact multipliers are always positive and range from 0.03 in the first quarter of 1982 to 0.13 in the second quarter of 2005. The present-value multiplier one year after the shock is also always positive. At long horizons, the multiplier is negative for most of the quarters in the 1970s and beginning of the 1980s. However, from the second quarter of 1983 onwards, the present-value of the long-run multiplier (eight years after the shock) is always positive. In the recent two decades, the long-run multiplier is close to 0.16.

multiplier under rational expectations is 0.39, which is clearly below the values under adaptive learning and close to 0.43, i.e. the multiplier evaluated at the posterior mode of the rational expectations model.

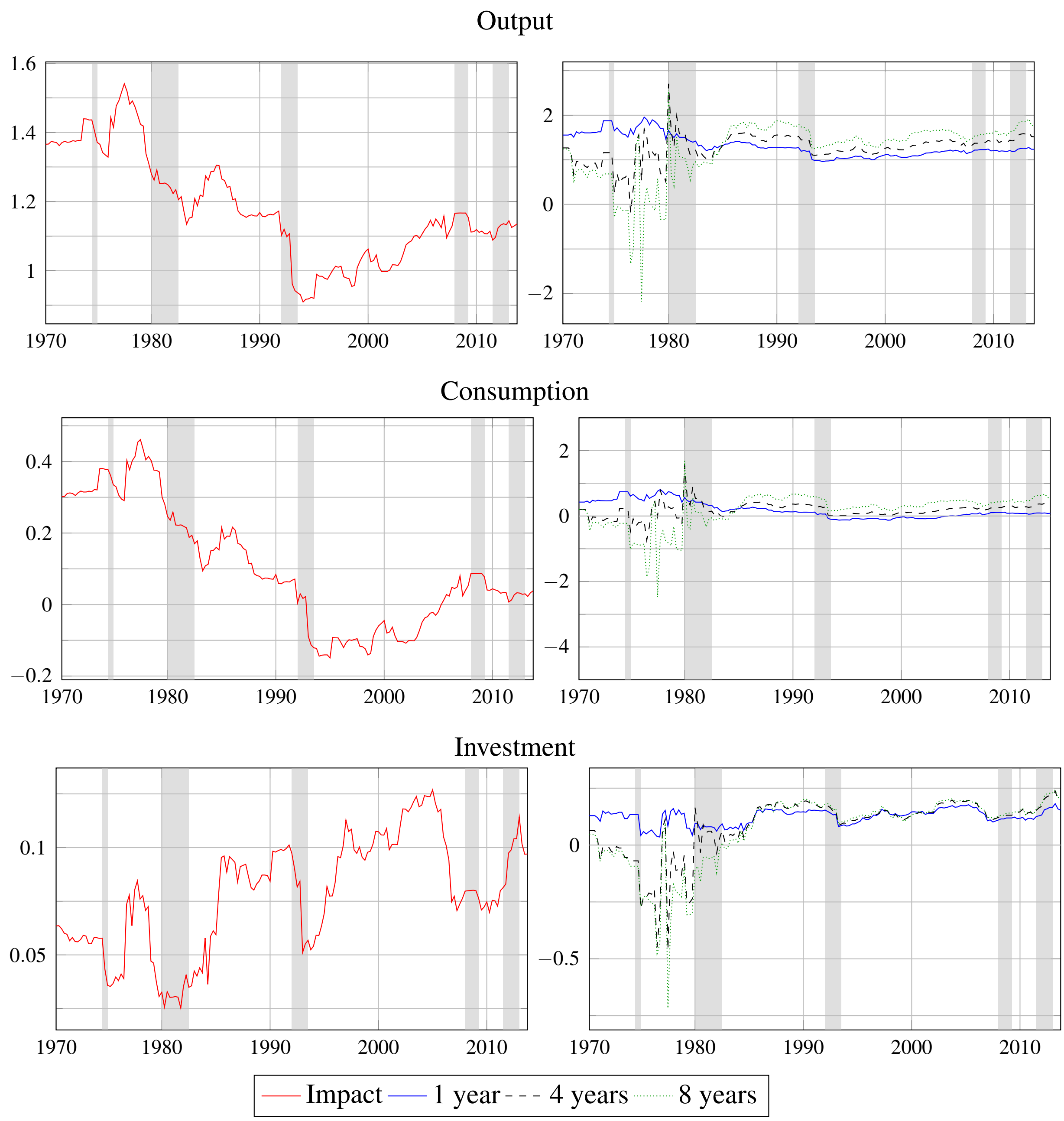

Figure 2.4: Present value government spending multipliers at selected horizons. The shaded grey areas are quarters of recessions as defined by the CEPR Euro Area Business Cycle Dating Committee.

7.2 Discussion

We now turn to discussion of the size and time variation of the government spending multipliers in the learning model. The intuition for the higher multiplier in the learning model, is that the future effects of the government spending shock are not fully anticipated. Most importantly, this leads to a different reaction of private consumption and, hence, overall economic activity. Two effects are key for understanding the difference with the rational expectations model. First, the negative wealth effect of future higher taxes is not fully anticipated. Under rational expectations, this effect almost inevitably leads to a drop in private consumption. Under adaptive learning, the consumption response may be smaller or even reversed if the forecasting model underestimates this negative wealth effect. Second, rational consumers anticipate a rise in future real interest rates. This motivates households to postpone consumption. Again, if the path of future interest rates in not correctly forecasted under learning, the consumption response may differ from rational expectations.

Generally speaking, the effects of a government spending shock heavily depend on the expectations agents have about the future effects of the shock. In the learning model, these expectations vary over time, which results in endogenous variation in the government spending multiplier. Figure 2.5 illustrates how changes in expectations contributed to the time variation of the multipliers for output and consumption. The figure focusses on the contribution of expected future consumption ($E_t^* C_{t+1}$) and expected future inflation ($E_t^* \Pi_{t+1}$) because the updating of the forecasting models for these two variables had a significant effect on the evolution of the multipliers.[13] This is not surprising, as equation (2.4) shows that the agents' consumption choice is directly based on these expectations.

The dashed red lines depict the multipliers when the belief coefficients in the forecasting model for consumption are held fixed to their 1980Q1 values. Consequently, in this scenario expected future consumption $E_t^* C_{t+1}$ after the government spending shock will not change over the period 1980Q1–2013Q4. Comparing this counterfactual scenario with the baseline leads to three findings. First, the peak in the output and consumption multiplier in the second quarter of 1986 can be attributed at higher consumption expectations after the government spending shock. Second, downward revisions of expected consumption can explain the negative consumption multipliers in the 1990s and the first half of the 2000s. If the belief coefficients in the consumption forecasting model would not have been updated, the consumption multiplier was close to zero in that period. Third, the updating of the belief coefficients can explain the (slight) increase of the multipliers in the last two decades of the sample.

The dotted brown lines show the multipliers when the beliefs in the forecasting model for inflation are held fixed. It is clear that the updating of the inflation forecasting model downplayed

[13]Figure 2.8 on page 96 of Appendix 2.B also shows the impact multipliers when the beliefs in the forecasting models of the other forward-looking variables are held constant.

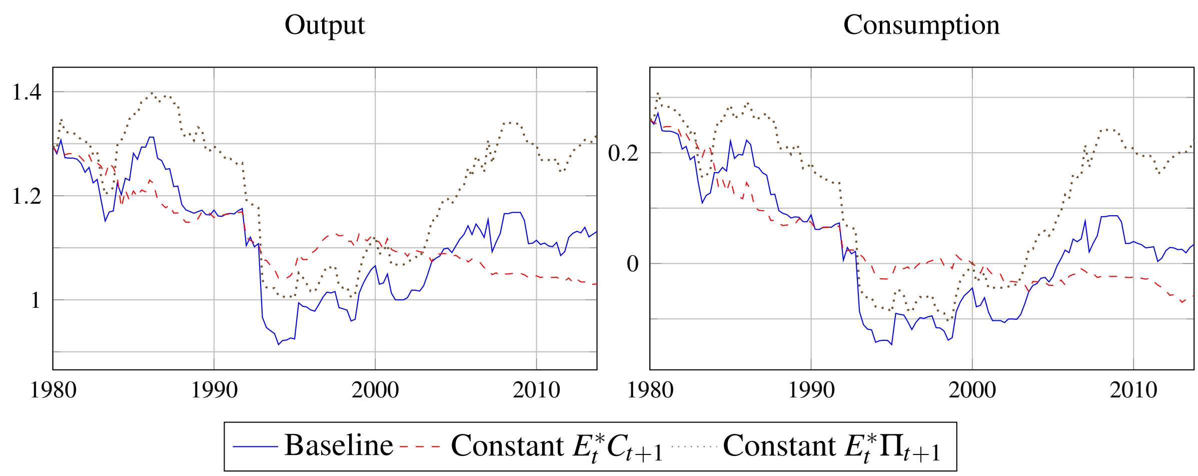

Figure 2.5: Impact multipliers for counterfactual evolutions of the belief parameters.

the impact of government spending on consumption and output. If the forecasting model would not have been updated, the output multiplier at the end of the sample (2013Q4) would have been almost identical to the multiplier in 1980Q1.

A growing literature investigates the dependency of the government spending multiplier on the stance of the business cycle. Auerbach and Gorodnichenko (2012, 2013), for example, find that the U.S. government spending multiplier on output is considerably larger in recessions than in expansions. However, Owyang et al. (2013) do not find evidence for this state dependency. Looking at the evolution of the multipliers in Figure 2.4, the learning model does not always generate higher multipliers during recession periods (grey shaded areas), although the output multiplier reaches a local maximum in the 2008-2009 recession. Moreover, agents seem to have adjusted their beliefs quite substantially during the recession of the early 1990s which has led to a smaller multiplier.

Several contributions to the literature highlight alternative driving forces behind the time variation in the government spending multipliers. Christiano et al. (2011) and Woodford (2011), for instance, show that the government spending multiplier is large when the zero lower bound on nominal interest rates is binding. Other empirical studies explain variation in spending multipliers based on other factors such as changes in private debt overhang (Bernardini and Peersman, 2015), asset market participation (Bilbiie et al., 2008), and the composition of government spending (Kirchner et al., 2010). A formal evaluation of the importance of learning behaviour relative to these other factors is an important direction for future research.

8 Robustness analysis

In this section we check the robustness of our results with respect to the choice of the agents' forecasting models and the choice of the sample period.[14]

8.1 Alternative learning schemes

Table 2.3 reports the marginal likelihood of our model for different assumptions regarding the formation of expectations. In Figure 2.6 the distribution of the impact multipliers for output, consumption, and investment under the different assumptions are summarised by box-plots.

Autoregressive model In the baseline model agents believe that the regression coefficients β_t in their forecasting models follow a random walk process. Although this is a common specification in the literature, Slobodyan and Wouters (2012a) specify the dynamics in β_t as a vector autoregressive process

$$vec\left(\beta_t - \bar{\beta}\right) = \mathbf{F} vec\left(\beta_{t-1} - \bar{\beta}\right) + \mathbf{v}_t, \tag{2.30}$$

where $\mathbf{F} = \rho \mathbf{I}$, with $\rho \leq 1$. As before, the shocks $\mathbf{v}_t$ are i.i.d. with covariance matrix $\mathbf{V}$. The Kalman filter equations (2.19) and (2.20) then become

$$\left(\beta_{t|t-1} - \bar{\beta}\right) = \mathbf{F}\left(\beta_{t-1|t-1} - \bar{\beta}\right), \tag{2.31}$$

$$\mathbf{P}_{t|t-1} = \mathbf{F} \cdot \mathbf{P}_{t-1|t-1} \cdot \mathbf{F}^T + \mathbf{V}, \tag{2.32}$$

$$\beta_{t|t} = \beta_{t|t-1} + \mathbf{K}_t \left[\mathbf{y}_t^f - \mathbf{X}_{t-1}^T \beta_{t|\text{t-1}}\right], \tag{2.33}$$

$$\mathbf{P}_{t|t} = \left(\mathbf{I} - \mathbf{K}_t \mathbf{X}_{t-1}^T\right) \mathbf{P}_{t|t-1}. \tag{2.34}$$

Since the learning parameters σ_0, σ_v and ρ are not jointly identified, Slobodyan and Wouters (2012a) fix σ_0 and σ_v to some plausible values and estimate the autoregressive parameter ρ using a uniform prior over $[0,1]$. As a robustness exercise, we follow the same approach and fix σ_0 and σ_v to the values in Slobodyan and Wouters (2012a). In contrast to the previous authors, this autoregressive specification of the belief process does not improve the marginal likelihood of the model relative to the random walk specification of the baseline model. Comparing the box-plots in the second panel of Figure 2.6 with those of the baseline model, this alternative specification does not affect the distribution of the multipliers a lot, although it slightly reduces the time variation. Moreover, the posterior estimates and impulse response functions to a government spending shock, are very similar to the baseline model.

[14] The time series of the impact multipliers for the alternative model specifications are depicted in Figure 2.9 on page 97 of Appendix 2.B.

Model specification	Marginal likelihood
Rational expectations equilibrium	−928.64
Kalman filter learning	
– Baseline model	−892.57
– Autoregressive model	−893.40
– PLM with consumption	−913.95

Table 2.3: Log marginal likelihood of different model specifications.

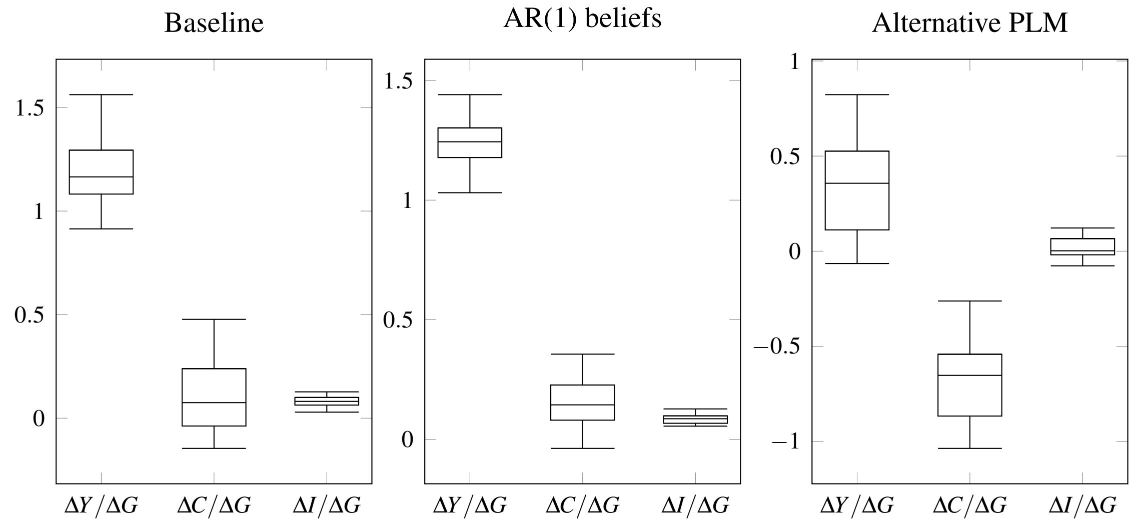

Figure 2.6: Box-plots for the impact multipliers for different model specifications. The bottom and top of the box are the first and third quartiles. The whiskers represent the minimum and maximum multipliers over the sample.

Alternative perceived law of motion We also experimented with a different specification of the forecasting model. Recall that the agents use the forecasting model $y^f_{j,t} = \mathbf{X}^T_{j,t-1}\beta_{j,t-1} + u_{j,t}$, also known as the Perceived Law of Motion (PLM). In the baseline model, the data vector $\mathbf{X}^T_{j,t-1}$ is the same for every forecasting model and consists of the capital stock, the nominal interest rate, output, the real wage rate, investment, and inflation. Hence, the PLM consists of all the endogenous state variables of the model. The last row of Table 2.3 shows the marginal likelihood of the model when private consumption is added to the PLM. We find that the baseline model outperforms this alternative specification in terms of marginal likelihood. However, this alternative PLM has an important impact on the distribution of the government spending multipliers. The right panel of Figure 2.6 shows that consumption multiplier in this model is negative, leading to a significantly lower value of the output multiplier.

Sample period		Rational expectations	Kalman filter learning
Pre-EMU	1970Q2–1998Q4	−594.3	−544.82
Post-EMU	1999Q1–2013Q4	−470.66	−451.64
Great Moderation	1984Q1–2007Q4	−715.88	−585.14
Entire sample	1970Q2–2013Q4	−928.64	−892.57

Table 2.4: Log marginal likelihood under Rational Expectations and under Kalman Filter Learning for different sample periods.

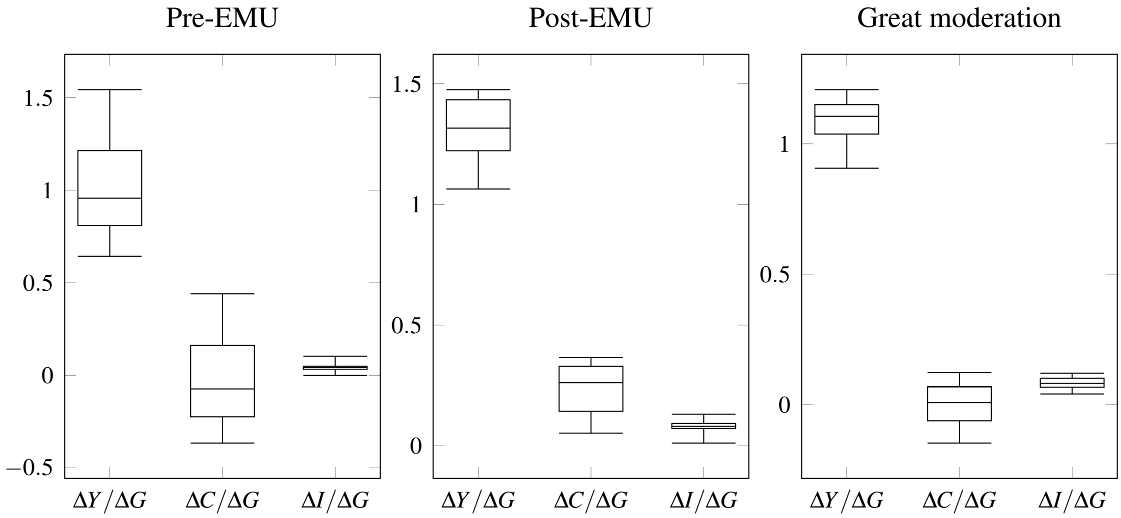

Figure 2.7: Box-plots for the impact multipliers for different sample periods. The bottom and top of the box are the first and third quartiles. The whiskers represent the minimum and maximum multipliers over the sample.

8.2 Alternative sample periods

As a robustness exercise, we also estimated the model for different sub-periods: the "Pre-EMU" period (1970Q2–1998Q4), the "post-EMU" period (1999Q1–2013Q4), and the "Great Moderation" (1984Q1–2007Q4). Table 2.4 compares the marginal likelihood under Kalman filter learning and rational expectations for four different sample periods.[15] Notice that in every sample period there is strong evidence for the learning model relative to the rational expectations benchmark in terms of marginal likelihood.

First, we compare the estimation results for the period before and after the introduction of the euro. The "Pre-EMU" and "Post-EMU" rows in Table 2.4 show that in both periods Kalman filter learning improves on the rational expectations model in terms of marginal likelihood. Second, in a supplementary estimation we restrict the sample to the "Great Moderation" period (1984Q1–2007Q4) to check if the results are not blurred by the high output and inflation volatil-

[15] See Table 2.5 and Table 2.6 on pages 98 and 99 for the posterior modes of the structural parameters over the different sub-periods.

ity in the 1970s and the non-standard monetary policy measures in the aftermath of the financial and economic crisis of 2008. Also for this sub-period Kalman filter learning significantly improves the marginal likelihood of the model. In summary, Kalman filter learning improves upon the rational expectations model in sub-periods considered.

From the box-plots in Figure 2.7 (and the time series in Figure 2.9 on page 97 of Appendix 2.B) we can see that the pre-EMU estimation leads to somewhat lower multipliers towards the end of the pre-EMU period. On the other hand, the multipliers in the post-EMU period are estimated somewhat higher than in the full sample. The estimation of the "Great Moderation" period leads to an evolution of the multipliers that is very similar to the full sample estimation. Generally speaking, the learning model also generates time variation in the multipliers within each sub-sample. This observation suggests that the time variation in the full sample does not only stem from structural differences between the sub-samples.

9 Conclusion

In this paper, we have developed a medium-scale DSGE model with adaptive learning to investigate the transmission of government spending shocks in the euro area. In particular, agents form expectations using a forecasting model with belief coefficients that are updated using the Kalman filter. We compare the model dynamics under this learning mechanism with those under rational expectations and find that the learning mechanism significantly improves the marginal likelihood of the model. Moreover, the updating of the belief coefficients generates time variation in the macroeconomic responses to a government spending shock. Hence, the expectations channel provides an endogenous explanation for time-varying government spending multipliers. In contrast to the time-varying parameter VAR approach, this variation does not stem from some random variation in the structural parameters of the model. In fact, variation in the government spending multipliers is an endogenous outcome of the model, generated by agents learning to forecast future macroeconomic variables.

We find that the responses of output to the government spending shock are always positive on impact. For most periods, the learning model also generates a positive effect on private consumption, although the effect becomes slightly negative in the 1990s and the first half of the 2000s.. The rational expectations model, on the other hand, finds a significant drop in private consumption after the shock. Hence, learning behaviour provides an explanation for the crowding-in effect of government spending on private consumption found in several empirical studies. Another difference with rational expectations, is the positive reaction of private investment to a government spending shock. This positive investment response is in line with the empirical findings for the euro area provided by Burriel et al. (2010) and Kirchner et al. (2010). In general, the responses to a government spending shock under Kalman filter learning are significantly different from those under rational expectations.

Another important observation is that Kalman filter learning generates time variation in the effects of a government spending shock, especially at the medium and long horizon. For example, although the effects on aggregate demand are always positive on impact, in the 1970s and beginning of the 1980s, government spending shocks had significant negative effects on private demand in the medium- and long-run. On the other hand, the long-term multipliers on output, consumption and investment are always positive from the second half of the 1980s onwards.

The learning approach provides an natural explanation for time variation in the transmission of government spending shocks. Obviously, this variation could also stem from other time-varying factors not considered in our analysis. The literature provides a list of factors that may explain time variation in the fiscal transmission mechanism. A formal evaluation of the importance of learning behaviour relative to these other factors is left for future research.

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Appendices

Appendix 2.A Model appendix

2.A.1 Non-linear model

2.A.1.1 Household

The representative household maximises expected discounted lifetime utility subject to its budget constraint and the the capital accumulation equation. The Lagrangian of this maximisation problem is

$$\begin{aligned}\mathscr{L} = E_0^* \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U(C_t, N_t) \\ & + \lambda_t \left[\frac{W_t}{P_t} N_t + r_t^k K_{t-1} + B_t + D_t - T_t - C_t - I_t - \frac{B_{t+1}}{\eta_t^b R_t \Pi_{t+1}^{-1}} \right] \\ & + q_t \left[(1-\delta) K_{t-1} + \eta_t^i \left(I_t - \mathscr{S}(K_{t-1}, I_t, I_{t-1}) \right) - K_t \right] \Bigg\},\end{aligned}$$

where $\beta \in [0,1)$. $E_t^*(\cdot)$ denotes the subjective expectations of the household at time t. The period utility function $U(\cdot)$ depends on consumption, C_t, and labour supply, N_t. W_t is the aggregate nominal wage. r_t^k is the real rental rate of capital. P_t is the final goods price. B_t represents the quantity of one-period bonds carried over from period $t-1$. The variable R_{t-1} denotes the gross nominal interest rate on bonds purchased in period $t-1$, and $\Pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate. D_t are the dividends from the labour union sector and the intermediate goods sector. T_t are lump-sum taxes. The stock of physical capital, K_{t-1}, depreciates at a rate δ. The function $\mathscr{S}(\cdot)$ captures the presence of adjustment costs in investment. η_t^b and η_t^i are an exogenous risk premium shock and an investment-specific shock, the dynamics of which will be specified later. λ_t and q_t are the Lagrangian multipliers associated to the budget constraint and the capital accumulation equation respectively.

The first-order conditions for consumption, bonds, investment, and capital are given by

$$\frac{\partial \mathscr{L}}{\partial C_t} = 0 \;\Leftrightarrow\; U_{C,t} = \lambda_t, \tag{2.35}$$

$$\frac{\partial \mathscr{L}}{\partial B_{t+1}} = 0 \;\Leftrightarrow\; \beta \eta_t^b R_t E_t^* \left(\lambda_{t+1} \Pi_{t+1}^{-1}\right) = \lambda_t \Leftrightarrow R_t = E_t^* \left(\frac{\lambda_t \Pi_{t+1}}{\beta \lambda_{t+1} \eta_t^b}\right), \tag{2.36}$$

$$\frac{\partial \mathscr{L}}{\partial I_t} = 0 \;\Leftrightarrow\; \lambda_t = q_t \eta_t^i (1 - \mathscr{S}_{I_t,t}) - \beta E_t^* \left(q_{t+1} \eta_{t+1}^i \mathscr{S}_{I_{t-1},t+1}\right), \tag{2.37}$$

$$\frac{\partial \mathscr{L}}{\partial K_t} = 0 \;\Leftrightarrow\; \beta E_t^* \left(\lambda_{t+1} r_{t+1}^k\right) + \beta E_t^* \left[q_{t+1} \left(1 - \delta - \eta_{t+1}^i \mathscr{S}_{K_{t-1},t+1}\right)\right] = q_t. \tag{2.38}$$

Hereafter the following functional forms are considered:

$$U(C_t, 1 - N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \exp\left(\frac{\sigma - 1}{1+\phi} N_t^{1+\phi}\right),$$
$$\mathscr{S}(\cdot) = s\left(\frac{I_t}{I_{t-1}}\right) I_t, \tag{2.39}$$

with σ, $\phi > 0$, $\mathscr{S}''(\cdot) > 0$, and in steady state $\mathscr{S}(\cdot) = \mathscr{S}'(\cdot) = 0$.

Combining these functional forms with conditions (2.35)–(2.38) we can obtain the following optimality conditions:

Euler equation for consumption

$$C_t^{-\sigma} \exp\left(\frac{\sigma - 1}{1+\phi} N_t^{1+\phi}\right) = \beta \eta_t^b R_t E_t^* \left[\Pi_{t+1}^{-1} C_{t+1}^{-\sigma} \exp\left(\frac{\sigma - 1}{1+\phi} N_{t+1}^{1+\phi}\right)\right]$$

Optimal investment

$$1 = Q_t \eta_t^i \left[1 - s\left(\frac{I_t}{I_{t-1}}\right) - s'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}}\right] + \beta E_t^* \left[Q_{t+1} \frac{U_{C,t+1}}{U_{C,t}} \eta_{t+1}^i s'\left(\frac{I_t}{I_{t-1}}\right) \left(\frac{I_t}{I_{t-1}}\right)^2\right],$$

where we have used the definition of Tobin's $Q_t = q_t / \lambda_t$.

Optimal capital stock

$$\beta E_t^* \left\{\frac{U_{C,t+1}}{U_{C,t}} \left[r_{t+1}^k + Q_{t+1}(1-\delta)\right]\right\} = Q_t$$

Capital accumulation

$$K_t = (1-\delta) K_{t-1} + \eta_t^i \left[1 - s\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

2.A.1.2 Employment agencies and labour unions

Following Erceg et al. (2000) and Schmitt-Grohé and Uribe (2006), labour decisions are made by a union who represents the household and operates in a continuum of monopolistically competitive labour markets. The union transforms the homogeneous labour supply of the household, N_t, into differentiated labour inputs and sells it to an employment agency on a continuum of labour markets indexed by $j \in [0,1]$. The representative employment agency bundles the labour supplies sold by the unions and sells it to the intermediate goods producers.

Employment agency For any intermediate goods producer i, the agency combines the differentiated labour supplies $N_t(i,j)$, $j \in [0,1]$, according to

$$N_t(i) = \left[\int_0^1 N_t(i,j)^{\frac{1}{1+\varepsilon_{w,t}}} dj \right]^{1+\varepsilon_{w,t}},$$

where $\varepsilon_{w,t}$ is a wage mark-up shock, the dynamics of which will be specified later.

The agency chooses the labour supplies as to maximise its profits:

$$\max_{\{N_t(k,l)\}} W_t N_t(i) - \int_0^1 W_t(j) N_t(i,j)\, dj, \quad \forall k,l \in [0,1],$$

where W_t is the nominal wage paid to the agency for their homogeneous labour input, and $W_t(j)$ is the wage charged by the union in labour market j.

Profit maximisation leads to the following labour demand equation:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-(1+\varepsilon_{w,t})/\varepsilon_{w,t}} N_t(i).$$

Aggregation across firms, gives the total labour demand in labour market j

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-(1+\varepsilon_{w,t})/\varepsilon_{w,t}} N_t,$$

where $N_t = \int_0^1 N_t(i) di$.

The employment agencies are perfectly competitive. Hence, we have the following zero profit condition

$$W_t \left[\int_0^1 N_t(i,j)^{\frac{1}{1+\varepsilon_{w,t}}} dj \right]^{1+\varepsilon_{w,t}} - \int_0^1 W_t(j) N_t(i,j)\, dj = 0,$$

which leads to the following expression for the aggregate nominal wage:

$$W_t = \left[\int_0^1 W_t(j)^{-\frac{1}{\varepsilon_{w,t}}} dj \right]^{-\varepsilon_{w,t}}.$$

Labour union sector In each labour market j the nominal wage $W(j)$ is set by the union. Each period in a fraction θ_w of randomly chosen markets, the union cannot re-optimise its wage. In those markets nominal wages are indexed to a weighted average of past inflation, Π_{t-1}, the inflation target, Π_t^*, and productivity growth, z_t:

$$\tilde{W}_t(j) = z_t\,(\Pi_t^*)^{1-\gamma_w}\,(\Pi_{t-1})^{\gamma_w}\,\tilde{W}_{t-1}(j),$$

with $\gamma_w \in [0,1]$ and where the dynamics of Π_t^* and z_t will be specified later.

The maximisation problem of the union is

$$\max_{\tilde{W}_t(j)} E_t^* \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{U_{C,t+k}}{U_{C,t}} \left\{ \frac{\Omega_{t+k}^w \tilde{W}_t(j)}{P_{t+k}} N_{t+k|t}(j) - MRS_{t+k} N_{t+k|t}(j) \right\}$$

with $MRS_{t+k} = -U_{N,t+k}/U_{C,t+k}$, and

$$\Omega_{t+k}^w = \begin{cases} 1 & k=0 \\ \prod_{j=1}^k z_{t+j}\left(\Pi_{t+j}^*\right)^{1-\gamma_w}\left(\Pi_{t+j-1}\right)^{\gamma_w} & k>0 \end{cases}, \tag{2.40}$$

subject to the demand for labour in market j

$$N_{t+k|t}(j) = \left(\frac{\Omega_{t+k}^w \tilde{W}_t(j)}{W_{t+k}}\right)^{-\frac{1+\varepsilon_{w,t}}{\varepsilon_{w,t}}} N_{t+k},$$

and the household flow budget constraints, for $k = 0,1,2,\ldots,+\infty$. Here $N_{t+k|t}(j)$ is the labour demand in period $t+k$ prevailing in labour market j where the wage rate was last re-optimised in period t. MRS_{t+k} is the marginal rate of substitution between consumption and leisure in period $t+k$.

The first-order condition for the optimal wage is given by

$$E_t^* \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{U_{C,t+k}}{U_{C,t}} N_{t+k|t}(j) \left\{ \frac{\Omega_{t+k}^w}{P_{t+k}} \tilde{W}_t(j) - (1+\varepsilon_w) MRS_{t+k} \right\} = 0.$$

The wage index can be written as

$$W_t^{-1/\varepsilon_{w,t}} = (1-\theta_w)\left(\tilde{W}_t(j)\right)^{-1/\varepsilon_{w,t}} + \theta_w\left(z_t\,(\Pi_t^*)^{1-\gamma_w}\,(\Pi_{t-1})^{\gamma_w}\,W_{t-1}\right)^{-1/\varepsilon_{w,t}}.$$

2.A.1.3 Final goods sector

A representative, perfectly competitive firm bundles a continuum of intermediate goods into a final good using the following CES-technology

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\varepsilon_{p,t}}}\,di\right)^{1+\varepsilon_{p,t}}, \tag{2.41}$$

where $Y_t(i)$ is the input of intermediate good $i \in [0,1]$. $\varepsilon_{p,t}$ is a price mark-up shock, the dynamics of which will be specified later.

The firm chooses the quantities of inputs so as to maximise its profit, taking as given the final goods price P_t and the intermedsiate goods prices $P_t(i)$, for all $i \in [0,1]$. The profit maximisation problem of the final good firm is represented as

$$\max_{\{Y_t(i)\}} P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj, \quad \forall i \in [0,1],$$

subject to (2.41). Profit maximisation yields the following demand schedule for intermediate good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} Y_t.$$

The final good producers are perfectly competitive. Thus, we have the following zero-profit condition

$$P_t \left(\int_0^1 Y_t(i)^{\frac{1}{1+\varepsilon_{p,t}}} di\right)^{1+\varepsilon_{p,t}} - \int_0^1 P_t(i) Y_t(i) di = 0.$$

This leads to the following expression for the final good price

$$P_t = \left(\int_0^1 P_t(i)^{-1/\varepsilon_{p,t}} di\right)^{-\varepsilon_{p,t}}.$$

In the symmetric equilibrium all intermediate good producers set the same price. Therefore, the aggregate price P_t and the intermediate good prices $P_t(i)$ for all i will be the same.

2.A.1.4 Intermediate goods sector

Cost minimisation Intermediate goods producer i rents capital, $K_{t-1}(i)$, and hires composite labour, $L_t(i)$ to produce the intermediate good i. The Lagrangian associated with the cost minimisation problem is given by

$$\mathscr{L} = \frac{W_t}{P_t} N_t(i) + r_t^k K_{t-1}(i) + \mu_t(i) \left[Y_t(i) - \left(A_t^{1-\alpha} K_{t-1}(i)^{\alpha} N_t(i)^{1-\alpha} - \Phi A_t\right)\right],$$

where Φ is a fixed cost and A_t is the level of technology. The Lagrangian multiplier $\mu_t(i)$ equals the real marginal cost MC_t. The associated first-order conditions are

$$\frac{W_t}{P_t} = MC_t(i)(1-\alpha) A_t^{1-\alpha} K_{t-1}^{\alpha}(i) N_t^{-\alpha}(i), \quad (2.42)$$

$$r_t^k = MC_t(i) \alpha A_t^{1-\alpha} K_{t-1}(i)^{\alpha-1} N_t^{1-\alpha}(i). \quad (2.43)$$

In the symmetric equilibrium all firms employ the same labour and capital inputs, so we can omit the firm-specific index i. The ratio

$$\frac{W_t/P_t}{r_t^k} = \frac{(1-\alpha) K_{t-1}}{\alpha N_t}$$

allows us to derive the following expression for the real marginal cost

$$MC_t = \frac{\left(r_t^k\right)^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \left(\frac{W_t}{A_t P_t}\right)^{1-\alpha}.$$

Profits are given by

$$Profit_t = Y_t - W_t N_t - r_t^k K_{t-1} = A_t^{1-\alpha} K_{t-1}^{\alpha} N_t^{1-\alpha} - \Phi A_t - W_t N_t - r_t^k K_{t-1}.$$

Price setting Following Calvo (1983), intermediate goods producers set nominal prices in a staggered fashion. Each period an intermediate goods producer can adjust its price with a constant probability $1-\theta_p$. The optimal price for the re-optimising firm results from the following problem. A firm i that is permitted to adjust its price in period t, sets a new price $\tilde{P}_t(i)$ so as to maximise

$$E_t^* \sum_{k=0}^{\infty} (\beta\theta_p)^k \frac{U_{C,t+k}}{U_{C,t}} \left\{ \frac{\Omega_{t+k}^p \tilde{P}_t(i)}{P_{t+k}} Y_{t+k|t}(i) - MC_{t+k} Y_{t+k|t}(i) \right\}$$

subject to

$$Y_{t+k|t}(i) = \left(\frac{\Omega_{t+k}^p \tilde{P}_t(i)}{P_{t+k}}\right)^{-\frac{1+\varepsilon_{p,t}}{\varepsilon_{p,t}}} Y_{t+k},$$

for $k = 0, 1, 2, \ldots, +\infty$, where $Y_{t+k|t}(i)$ is the supply provided in period $t+k$ by a firm i that last has reset its price in period t, and Ω_t^p is the price indexation parameter.

Assume that if a firm cannot re-optimise, its price is indexed according to

$$\tilde{P}_t(i) = (\Pi_t^*)^{1-\gamma_p} (\Pi_{t-1})^{\gamma_p} \tilde{P}_{t-1}(i)$$

where $\gamma_p \in [0,1]$. Hence, the indexation parameter Ω_t^p becomes

$$\Omega_{t+k}^p = \begin{cases} 1 & k = 0 \\ \prod_{j=1}^{k} (\Pi_{t+j}^*)^{1-\gamma_p} (\Pi_{t+j-1})^{\gamma_p} & k > 0 \end{cases}. \tag{2.44}$$

The solution to this maximisation problem satisfies the first-order condition

$$E_t^* \sum_{k=0}^{\infty} (\beta\theta_p)^k \frac{U_{C,t+k}}{U_{C,t}} Y_{t+k|t}(i) \left\{ \frac{\Omega_{t+k}^p}{P_{t+k}} - (1+\varepsilon_{p,t}) \left(\tilde{P}_t(i)\right)^{-1} MC_{t+k} \right\} = 0.$$

The price index can be written as

$$P_t^{-1/\varepsilon_{p,t}} = (1-\theta_p) \left(\tilde{P}_t(i)\right)^{-1/\varepsilon_{p,t}} + \theta_p \left((\Pi_t^*)^{1-\gamma_p} (\Pi_{t-1})^{\gamma_p} P_{t-1}\right)^{-1/\varepsilon_{p,t}}.$$

2.A.1.5 Government

Central bank Similar to De Graeve et al. (2009), the central bank follows the generalised Taylor rule

$$\frac{R_t}{\Pi_t^*} = \left(\frac{R_{t-1}}{\Pi_{t-1}^*}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\phi_\pi}\right]^{1-\rho_R} \left(\frac{Y_t/A_t}{Y_{t-1}/A_{t-1}}\right)^{\phi_{\Delta y}} u_t^r,$$

where $u_t^r = \left(u_{t-1}^r\right)^{\rho_r} \exp\left(\varepsilon_t^r\right)$, with $\varepsilon_t^r \sim \mathcal{N}(0, \sigma_r^2)$, and $\Pi_t^* = \left(\Pi_{t-1}^*\right)^{\rho_{\pi^*}} \exp\left(\varepsilon_t^{\pi^*}\right)$, with $\varepsilon_t^{\pi^*} \sim \mathcal{N}\left(0, \sigma_{\pi^*}^2\right)$.

Fiscal authority It is assumed that bonds are in zero net supply.

$$\frac{G_t}{A_t} = \left(\frac{G_{t-1}}{A_{t-1}}\right)^{\rho_g} \exp\left(\varepsilon_t^g\right), \quad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$$

2.A.1.6 Market clearing

$$Y_t = C_t + I_t + G_t$$

2.A.2 Stationary equilibrium

The level of technology $A_t$is not stationary. Its growth rate evolves according to

$$z_t = \frac{A_t}{A_{t-1}} = \gamma \exp(\varepsilon_t^Z), \tag{2.45}$$

with $\varepsilon_t^Z \sim \mathcal{N}\left(0, \sigma_z^2\right)$.

Before log-linearising, we must make all trending variables stationary. We define stationary variables

$$c_t = C_t/A_t, \quad i_t = I_t/A_t, \quad k_t = K_t/A_t, \quad w_t = W_t/\left(P_t A_t\right), \quad g_t = G_t/A_t,$$

and the stationary discount factor

$$\xi_{t+k} = U_{C,t+k} A_{t+k}^{\sigma}.$$

2.A.2.1 Household

The optimality conditions of the household in terms of stationary variables can be written as follows:

Euler equation for consumption

$$c_t^{-\sigma} \exp\left(\frac{\sigma-1}{1+\phi} N_t^{1+\phi}\right) = \beta \eta_t^b R_t E_t^* \left[\Pi_{t+1}^{-1} z_{t+1}^{-\sigma} c_{t+1}^{-\sigma} \exp\left(\frac{\sigma-1}{1+\phi} N_{t+1}^{1+\phi}\right)\right] \tag{2.46}$$

Optimal investment

$$1 = Q_t \eta_t^i \left[1 - s\left(\frac{i_t}{i_{t-1}} z_t\right) - s'\left(\frac{i_t}{i_{t-1}} z_t\right) \frac{i_t}{i_{t-1}} z_t\right] + \beta E_t^* \left[Q_{t+1} \frac{\xi_{t+1}}{\xi_t} z_{t+1}^{-\sigma} \eta_{t+1}^i s'\left(\frac{i_{t+1}}{i_t} z_{t+1}\right)\left(\frac{i_{t+1}}{i_t} z_{t+1}\right)^2\right] \quad (2.47)$$

Optimal capital stock

$$\beta E_t^* \left\{\frac{\xi_{t+1}}{\xi_t} z_{t+1}^{-\sigma} \left[r_{t+1}^k + Q_{t+1}(1-\delta)\right]\right\} = Q_t \quad (2.48)$$

Capital accumulation

$$k_t = (1-\delta) z_t^{-1} k_{t-1} + \eta_t^i \left[1 - s\left(\frac{i_t}{i_{t-1}} z_t\right)\right] i_t \quad (2.49)$$

2.A.2.2 Employment agencies and labour unions

Wage setting equation The first-order condition for the optimal wage in terms of stationary variables is given by

$$E_t^* \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{\xi_{t+k}}{\xi_t} (A_{t+k}/A_t)^{-\sigma} N_{t+k|t}(j) \left\{\frac{\Omega_{t+k}^w}{P_{t+k}} \tilde{W}_t(j) - (1+\varepsilon_{w,t}) MRS_{t+k}\right\} = 0.$$

From this it follows that the stationary reset wage, $\tilde{w}_t$, is given by

$$\tilde{w}_t = \frac{\tilde{W}_t(j)}{P_t A_t} = \frac{(1+\varepsilon_{w,t}) E_t^* \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{\xi_{t+k}}{\xi_t} (A_{t+k}/A_t)^{-\sigma} A_t^{-1} N_{t+k|t}(j) MRS_{t+k}}{E_t^* \sum_{k=0}^{\infty} (\beta\theta_w)^k \frac{\xi_{t+k}}{\xi_t} (A_{t+k}/A_t)^{-\sigma} N_{t+k|t}(j) \Omega_{t+k}^w \frac{P_t}{P_{t+k}}} \equiv \frac{\mathscr{A}_t}{\mathscr{B}_t}. \quad (2.50)$$

We can derive the following recursive expressions for $\mathscr{A}_t$ and $\mathscr{B}_t$ using the definition of Ω_{t+k}^w in (2.40):

$$\mathscr{A}_t = (1+\varepsilon_{w,t}) N_t(j) mrs_t + E_t^* \left[\beta\theta_w \frac{\xi_{t+1}}{\xi_t} (A_{t+1}/A_t)^{1-\sigma} \mathscr{A}_{t+1}\right]$$

$$\mathscr{B}_t = N_t(j) + E_t^* \left[\beta\theta_w \frac{\xi_{t+1}}{\xi_t} (A_{t+1}/A_t)^{-\sigma} z_{t+1} \left(\Pi_{t+1}^*\right)^{1-\gamma_w} (\Pi_t)^{\gamma_w} \Pi_{t+1}^{-1} \mathscr{B}_{t+1}\right]$$

where $mrs_{t+k} = MRS_{t+k}/A_{t+k} = c_{t+k} N_{t+k}^{\phi}$.

Aggregate wage index

$$w_t^{-1/\varepsilon_{w,t}} = (1-\theta_w) \tilde{w}_t^{-1/\varepsilon_{w,t}} + \theta_w \left((\Pi_t^*)^{1-\gamma_w} (\Pi_{t-1})^{\gamma_w} w_{t-1} \Pi_t^{-1}\right)^{-1/\varepsilon_{w,t}} \quad (2.51)$$

2.A.2.3 Intermediate goods sector

Cost minimisation

$$y_t = (A_t/A_{t-1})^{-\alpha} k_{t-1}^{\alpha} N_t^{1-\alpha} - \Phi \tag{2.52}$$

$$w_t = MC_t(1-\alpha)(A_t/A_{t-1})^{-\alpha} k_{t-1}^{\alpha} N_t^{-\alpha} \tag{2.53}$$

$$r_t^k = MC_t \alpha (A_t/A_{t-1})^{1-\alpha} k_{t-1}^{\alpha-1} N_t^{1-\alpha} \tag{2.54}$$

$$\frac{w_t}{r_t^k} = \frac{(1-\alpha)k_{t-1}}{\alpha N_t}\left(\frac{A_t}{A_{t-1}}\right)^{-1}$$

$$MC_t = \frac{\left(r_t^k\right)^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} w_t^{1-\alpha} \tag{2.55}$$

$$\frac{Profit_t}{A_t} = (A_t/A_{t-1})^{-\alpha} k_{t-1}^{\alpha} N_t^{1-\alpha} - \Phi - w_t N_t - r_t^k k_{t-1}\left(A_{t-1}/A_t\right)$$

Using equations (2.53) and (2.54) we have

$$\frac{Profit_t}{A_t} = \frac{w_t N_t}{MC_t(1-\alpha)} - \Phi - w_t N_t - \frac{\alpha}{1-\alpha} w_t N_t. \tag{2.56}$$

Price setting

Price setting equation

$$E_t^* \sum_{k=0}^{\infty} (\beta\theta_p)^k \frac{\xi_{t+k}}{\xi_t}\left(\frac{A_{t+k}}{A_t}\right)^{1-\sigma} y_{t+k|t}(i)\left\{\frac{\Omega_{t+k}^p}{P_{t+k}}\tilde{P}_t(i) - (1+\varepsilon_{p,t})MC_{t+k}\right\} = 0$$

It is convenient to write this condition as

$$\tilde{p}_t = \frac{E_t^* \sum_{k=0}^{\infty} (\beta\theta_p)^k \frac{\xi_{t+k}}{\xi_t}\left(\frac{A_{t+k}}{A_t}\right)^{1-\sigma} y_{t+k|t}(i)\,(1+\varepsilon_{p,t})\,MC_{t+k}}{E_t^* \sum_{k=0}^{\infty} (\beta\theta_p)^k \frac{\xi_{t+k}}{\xi_t}\left(\frac{A_{t+k}}{A_t}\right)^{1-\sigma} y_{t+k|t}(i)\Omega_{t+k}^p \frac{P_t}{P_{t+k}}} \equiv \frac{\mathscr{C}_t}{\mathscr{D}_t}, \tag{2.57}$$

where $\tilde{p}_t \equiv \tilde{P}_t(i)/P_t$ is the stationary reset price. Using the definition of Ω_{t+k}^p in (2.44) we can express $\mathscr{C}_t$ and $\mathscr{D}_t$ recursively as

$$\mathscr{C}_t = y_t(i)\,(1+\varepsilon_{p,t})\,MC_t + E_t^*\left[\beta\theta_p \frac{\xi_{t+1}}{\xi_t}\left(\frac{A_{t+1}}{A_t}\right)^{1-\sigma} \mathscr{C}_{t+1}\right]$$

$$\mathscr{D}_t = y_t(i) + E_t^*\left[\beta\theta_p \frac{\xi_{t+1}}{\xi_t}\left(\frac{A_{t+1}}{A_t}\right)^{1-\sigma} \left(\Pi_{t+1}^*\right)^{1-\gamma_p}\left(\Pi_t\right)^{\gamma_p}\Pi_{t+1}^{-1}\mathscr{D}_{t+1}\right]$$

Aggregate price index

$$1 = (1-\theta_p)\,\tilde{p}_t^{-1/\varepsilon_{p,t}} + \theta_p\left(\left(\Pi_t^*\right)^{1-\gamma_p}\left(\Pi_{t-1}\right)^{\gamma_p}\Pi_t^{-1}\right)^{-1/\varepsilon_{p,t}} \tag{2.58}$$

2.A.2.4 Government

Central bank

$$\frac{R_t}{\Pi_t^*} = \left(\frac{R_{t-1}}{\Pi_{t-1}^*}\right)^{\rho_R} \left[\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\phi_\pi}\right]^{1-\rho_R} \left(\frac{y_t}{y_{t-1}}\right)^{\phi_{\Delta y}} u_t^r,$$

Fiscal authority

$$g_t = g_{t-1}^{\rho_g} \exp\left(\varepsilon_t^g\right), \quad \varepsilon_t^g \sim \mathcal{N}(0, \sigma_g^2)$$

2.A.2.5 Market clearing

$$y_t = c_t + i_t + g_t$$

2.A.3 Steady state

Throughout the paper, barred variables refer to steady state values.

- From (2.45) we obtain $\bar{z} = \gamma$.
- From (2.46) we have: $\bar{R} = \bar{\Pi}\gamma^\sigma/\beta$.
- From (2.47) we have $\bar{Q} = 1$.
- From (2.48) it follows that $\bar{r}^k = \gamma^\sigma/\beta - 1 + \delta$.
- Equation (2.49) implies $\bar{i}/\bar{k} = 1 - (1-\delta)\gamma^{-1}$.
- From (2.57) we have $\bar{MC} = 1/(1+\bar{\varepsilon}_p)$.
- It is useful to determine the ratio $\bar{k}/\bar{N}$. From (2.54) we obtain $\bar{k}/\bar{N} = \left[\bar{r}^k/\left(\alpha \bar{MC}\right)\right]^{\frac{1}{\alpha-1}} \gamma$.
- By (2.55) we have $\bar{w} = \left[\left(\bar{MC}\alpha^\alpha(1-\alpha)^{1-\alpha}\right)/(\gamma^\sigma/\beta - 1 + \delta)^\alpha\right]^{\frac{1}{1-\alpha}}$.
- Zero profits for intermediate goods producers require that $\bar{y} - \bar{r}^k\bar{k}\gamma^{-1} - \bar{w}\bar{N} = 0 \Leftrightarrow \bar{y}/\bar{N} = \bar{r}^k\left(\bar{k}/\bar{N}\right)\gamma^{-1} + \bar{w}$.
- The ratio $\bar{c}/\bar{N}$ follows from (2.65): $\bar{c}/\bar{N} = \bar{y}/\bar{N} - \left(\bar{i}/\bar{k}\right)\left(\bar{k}/\bar{N}\right) - (\bar{g}/\bar{y})(\bar{y}/\bar{N})$.
- Condition (2.50) implies $\bar{\tilde{w}} = (1+\bar{\varepsilon}_w)\bar{mrs}$ with $\bar{mrs} = \bar{c}\bar{N}^\phi$.
- From (2.51) we see that $\bar{w} = \bar{\tilde{w}}$.
- Combining these last equalities yields $\bar{N} = \bar{w}^{1/(1+\phi)}(1+\bar{\varepsilon}_w)^{-1/(1+\phi)}(\bar{c}/\bar{N})^{-1/(1+\phi)}$.
- The fixed cost Φ is calibrated so that steady-state profits in the intermediate goods sector are zero. By (2.56) we get $\Phi = (1-\alpha)^{-1}\left(\bar{MC}^{-1} - 1\right)\bar{w}\bar{N}$.

2.A.4 Log-linearised model

Let $\hat{x}_t$ denote the log-deviation of x_t from its steady-state value, except for $\hat{\varepsilon}_p \equiv \log(1+\varepsilon_{p,t}) - \log(1+\bar{\varepsilon}_p)$, $\hat{\varepsilon}_w \equiv \log(1+\varepsilon_{w,t}) - \log(1+\bar{\varepsilon}_w)$, and $\hat{z}_t = \varepsilon_t^Z$.

2.A.4.1 Households

Euler equation for consumption

$$\hat{c}_t = E_t^* \hat{c}_{t+1} + c_1 \left(\hat{N}_t - E_t^* \hat{N}_{t+1}\right) - c_2 \left(\hat{R}_t - E_t^* \hat{\Pi}_{t+1}\right) + \hat{u}_t^b$$

with $c_1 = (\sigma - 1)\bar{N}^{1+\phi}\sigma^{-1}$, $c_2 = \sigma^{-1}$, and where the rescaled risk premium shock is defined as $\hat{u}_t^b \equiv -\sigma^{-1}\hat{\eta}_t^b$ and obeys $\hat{u}_t^b = \rho_i \hat{u}_{t-1}^b + \varepsilon_t^b$, with $\varepsilon_t^b \sim \mathcal{N}\left(0, \sigma_b^2\right)$.

Optimal investment

$$\hat{i}_t = i_1 \left(\hat{i}_{t-1} - \hat{z}_t\right) + (1 - i_1) E_t^* \hat{i}_{t+1} + i_2 \hat{Q}_t + \hat{u}_t^i,$$

with

$$\begin{aligned} i_1 &= \frac{1}{1+\beta\gamma^{1-\sigma}}, \\ i_2 &= \frac{1}{(1+\beta\gamma^{1-\sigma}) s''\gamma^2}, \end{aligned}$$

and where the rescaled investment-specific shock is defined as $\hat{u}_t^i \equiv i_2 \hat{\eta}_t^i$ and obeys $\hat{u}_t^i = \rho_i \hat{u}_{t-1}^i + \varepsilon_t^i$, with $\varepsilon_t^i \sim \mathcal{N}\left(0, \sigma_i^2\right)$.

Optimal capital stock

$$\hat{Q}_t = -\left(\hat{R}_t - E_t^* \hat{\Pi}_{t+1} - \sigma \hat{u}_t^b\right) + \beta\gamma^{-\sigma}\left[\bar{r}^k E_t^* \hat{r}_{t+1}^k + (1-\delta) E_t^* \hat{Q}_{t+1}\right] \tag{2.59}$$

using $\hat{\xi}_t = \hat{\xi}_{t+1} - \sigma E_t^* \hat{z}_{t+1} + \hat{\eta}_t^b + \hat{R}_t - E_t^* \hat{\Pi}_{t+1}$.

Capital accumulation

$$\hat{k}_t = \frac{1-\delta}{\gamma}(\hat{k}_{t-1} - \hat{z}_t) + \frac{\bar{i}}{\bar{k}}\hat{i}_t + k_1 \hat{u}_t^i, \tag{2.60}$$

with $k_1 = \left(1+\beta\gamma^{1-\sigma}\right) s''\gamma^2 \bar{i}/\bar{k}$.

2.A.4.2 Employment agencies and labour unions

Expression (2.50) for the optimal wage becomes

$$\tilde{w}_t = \left(1 - \beta\theta_w\gamma^{1-\sigma}\right)\left(\hat{mrs}_t + \hat{\varepsilon}_{w,t}\right) - \beta\theta_w\gamma^{1-\sigma}\left[(1-\gamma_w) E_t^* \hat{\Pi}_{t+1}^* + \gamma_w \hat{\Pi}_t - E_t^* \hat{\Pi}_{t+1}\right] + \beta\theta_w\gamma^{1-\sigma} E_t^* \hat{\tilde{w}}_{t+1}.$$

The aggregate wage index (2.51) can be written as

$$\hat{\tilde{w}}_t = \frac{1}{1-\theta_w}\hat{w}_t - \frac{\theta_w}{1-\theta_w}\left[(1-\gamma_w)\hat{\Pi}_t^* + \gamma_w\hat{\Pi}_{t-1} + \hat{w}_{t-1} - \hat{\Pi}_t\right].$$

Combining these two equations gives the following real wage equation:

$$\hat{w}_t = w_1\left(\phi\hat{N}_t + \hat{c}_t - \hat{w}_t\right) + w_2\hat{w}_{t-1} + w_3E_t^*\hat{w}_{t+1} + w_4\hat{\Pi}_t + w_5\hat{\Pi}_{t-1} + w_6E_t^*\hat{\Pi}_{t+1} + w_7\hat{\Pi}_t^* + \hat{u}_t^w,$$

with

$$\begin{aligned}
w_1 &= \frac{(1-\theta_w)\left(1-\beta\theta_w\gamma^{1-\sigma}\right)}{\theta_w\left(1+\beta\gamma^{1-\sigma}\right)}, \\
w_2 &= \frac{1}{1+\beta\gamma^{1-\sigma}}, \\
w_3 &= \frac{\beta\gamma^{1-\sigma}}{1+\beta\gamma^{1-\sigma}}, \\
w_4 &= -\frac{1+\beta\gamma^{1-\sigma}\gamma_w}{1+\beta\gamma^{1-\sigma}}, \\
w_5 &= \frac{\gamma_w}{1+\beta\gamma^{1-\sigma}}, \\
w_6 &= \frac{\beta\gamma^{1-\sigma}}{1+\beta\gamma^{1-\sigma}}, \\
w_7 &= \frac{(1-\gamma_w)\left(1-\rho_{\pi^*}\beta\gamma^{1-\sigma}\right)}{1+\beta\gamma^{1-\sigma}},
\end{aligned}$$

where it is assumed that $E_t^*\hat{\Pi}_{t+1}^* = \rho_{\pi^*}\hat{\Pi}_t^*$ by (2.64). The price mark-up disturbance is defined as $\hat{u}_t^w \equiv w_1\hat{\varepsilon}_{w,t}$ and follows the exogenous process $\hat{u}_t^w = \rho_w\hat{u}_{t-1}^w - \mu_w\varepsilon_{t-1}^w + \varepsilon_t^w$, with $\varepsilon_t^w \sim \mathcal{N}\left(0,\sigma_w^2\right)$.

2.A.4.3 Intermediate goods sector

Cost minimisation

Production function

$$\hat{y}_t = \frac{\bar{y}+\Phi}{\bar{y}}\left(\alpha\hat{k}_{t-1} - \alpha\hat{z}_t + (1-\alpha)\hat{N}_t\right) \tag{2.61}$$

Wage

$$\hat{w}_t = \hat{MC}_t + \alpha(\hat{k}_{t-1} - \hat{z}_t - \hat{N}_t) \tag{2.62}$$

Rental rate of capital

$$\hat{r}_t^k = \hat{MC}_t + (\alpha-1)(\hat{k}_{t-1} - \hat{z}_t - \hat{N}_t) \tag{2.63}$$

Price setting The price setting equation (2.57) becomes

$$\hat{\tilde{p}}_t(i) = \left(1-\beta\theta_p\gamma^{1-\sigma}\right)\left[\hat{\varepsilon}_{p,t}+\hat{MC}_t\right] - \beta\theta_p\gamma^{1-\sigma}\left[(1-\gamma_p)E_t^*\hat{\Pi}_{t+1}^* + \gamma_p\hat{\Pi}_t - E_t^*\hat{\Pi}_{t+1}\right] + \beta\theta_p\gamma^{1-\sigma}E_t^*\hat{\tilde{p}}_{t+1}(i),$$

and from equation (2.58) we have

$$\hat{\tilde{p}}_t(i) = \frac{\theta_p}{\theta_p-1}\left[(1-\gamma_p)\hat{\Pi}_t^* + \gamma_p\hat{\Pi}_{t-1} - \hat{\Pi}_t\right].$$

Combining these two equations yields the New Keynesian Phillips curve

$$\hat{\Pi}_t = \pi_1\hat{MC}_t + \pi_2\hat{\Pi}_{t-1} + \pi_3E_t^*\hat{\Pi}_{t+1} + \pi_4\hat{\Pi}_t^* + \hat{u}_t^\pi,$$

with

$$\begin{aligned}
\pi_1 &= \frac{(1-\theta_p)\left(1-\beta\theta_p\gamma^{1-\sigma}\right)}{\theta_p\left(1+\beta\gamma^{1-\sigma}\gamma_p\right)},\\
\pi_2 &= \frac{\gamma_p}{1+\beta\gamma^{1-\sigma}\gamma_p},\\
\pi_3 &= \frac{\beta\gamma^{1-\sigma}}{1+\beta\gamma^{1-\sigma}\gamma_p},\\
\pi_4 &= \frac{(1-\gamma_p)\left(1-\rho_{\pi^*}\beta\gamma^{1-\sigma}\right)}{1+\beta\gamma^{1-\sigma}\gamma_p},
\end{aligned}$$

where it is assumed that $E_t^*\hat{\Pi}_{t+1}^* = \rho_{\pi^*}\hat{\Pi}_t^*$ by (2.64). The price mark-up disturbance is defined as $\hat{u}_t^\pi \equiv \pi_1\hat{\varepsilon}_{p,t}$ and follows the exogenous process $\hat{u}_t^\pi = \rho_\pi\hat{u}_{t-1}^\pi - \mu_\pi\varepsilon_{t-1}^\pi + \varepsilon_t^\pi$, with $\varepsilon_t^\pi \sim \mathcal{N}\left(0,\sigma_\pi^2\right)$.

2.A.4.4 Government

Central bank

$$\hat{R}_t = \rho_R\hat{R}_{t-1} + (1-\rho_R)\hat{\Pi}_t^* + \rho_R\left(\hat{\Pi}_t^* - \hat{\Pi}_{t-1}^*\right) + (1-\rho_R)\phi_\pi\left(\hat{\Pi}_t - \hat{\Pi}_t^*\right) + \phi_{\Delta y}\left(\hat{y}_t - \hat{y}_{t-1}\right) + \hat{u}_t^r,$$

with $\hat{u}_t^r = \rho_r\hat{u}_{t-1}^r + \varepsilon_t^r$ and

$$\hat{\Pi}_t^* = (1-\rho_{\pi^*})\hat{\Pi}_{t-1}^* + \varepsilon_t^{\pi^*}. \qquad (2.64)$$

Fiscal authority

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + \varepsilon_t^g$$

2.A.4.5 Market clearing

$$\hat{y}_t = \left(1 - \frac{\bar{i}}{\bar{y}} - \frac{\bar{g}}{\bar{y}}\right)\hat{c}_t + \frac{\bar{i}}{\bar{y}}\hat{i}_t + \frac{\bar{g}}{\bar{y}}\hat{g}_t \qquad (2.65)$$

Appendix 2.B Additional results

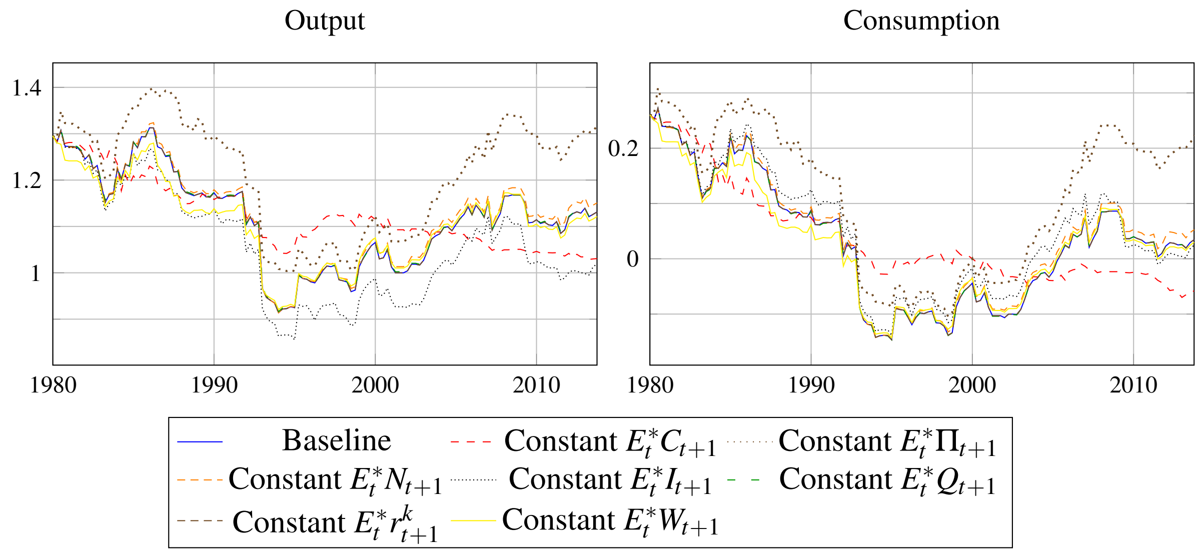

Figure 2.8: Impact multipliers for counterfactual evolutions of the belief parameters.

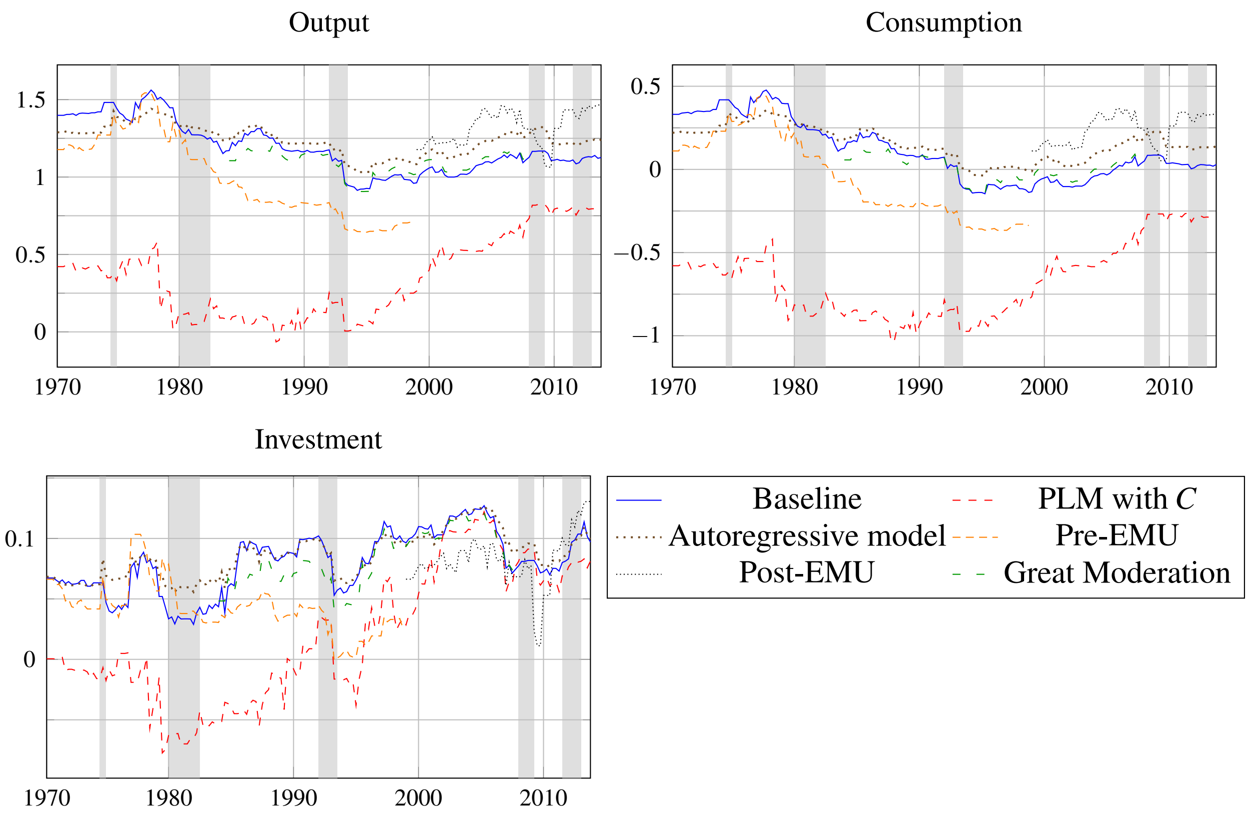

Figure 2.9: Impact multipliers for different model specifications and different sample periods. The shaded grey areas are quarters of recessions as defined by the CEPR Euro Area Business Cycle Dating Committee.

		pre-EMU 1970Q2–1998Q4		post-EMU 1999Q1–2013Q4		Great Moderation 1984Q1–2007Q4		Entire sample 1970Q2–2013Q4	
		KF	RE	KF	RE	KF	RE	KF	RE
Structural parameters									
γ_p	Indexation of prices to past inflation	0.53	0.32	0.44	0.34	0.15	0.57	0.43	0.27
γ_w	Indexation of wages to past inflation	0.61	0.43	0.41	0.65	0.48	0.49	0.52	0.38
θ_p	Degree of nominal price rigidity	0.8	0.84	0.83	0.92	0.91	0.79	0.76	0.86
θ_w	Degree of nominal wage rigidity	0.69	0.7	0.7	0.73	0.78	0.69	0.65	0.69
$100(\bar{\Pi}-1)$	Quarterly steady-state inflation rate	0.48	0.29	0.69	0.51	0.4	0.61	0.69	0.54
ρ_R	Degree of interest rate smoothing	0.75	0.58	0.84	0.66	0.72	0.86	0.84	0.62
ϕ	Inverse Frisch elasticity of labour supply	1.78	2.59	2.06	2.71	2.59	1.95	1.95	2.27
ϕ_π	Taylor rule inflation rate coefficient	1.41	1.06	1.52	1.22	1.08	1.57	1.52	1.06
$\phi_{\Delta y}$	Taylor rule output growth coefficient	0.07	0.13	0.12	0.22	0.15	0.067	0.069	0.13
σ	Degree of risk aversion	1.07	1.24	1.06	0.68	1.13	1.11	1.08	1.18
s''	Investment adjustment cost parameter	5.34	2	5.69	4.85	6.23	5.45	5.31	2
ρ_b	Risk premium shock AR coefficient	0.62	0.29	0.71	0.22	0.13	0.68	0.73	0.63
ρ_g	Government expenditure AR coefficient	1	0.97	1	0.99	0.96	0.99	1	1
ρ_π	Price mark-up shock AR coefficient	0.45	0.98	0.72	0.28	0.85	0.8	0.63	0.42
ρ_r	Monetary policy shock AR coefficient	0.44	0.41	0.46	0.19	0.19	0.3	0.48	0.49
ρ_i	Investment shock AR coefficient	0.051	0.027	0.05	0.11	0.036	0.047	0.11	0.051
ρ_w	Wage mark-up AR coefficient	0.77	0.86	0.88	1	0.81	0.74	0.96	0.99
μ_w	Wage mark-up shock MA coefficient	0.68	0.66	0.4	0.77	0.38	0.54	0.7	0.79
μ_π	Price mark-up shock MA coefficient	0.66	0.96	0.6	0.17	0.68	0.71	0.64	0.33
σ_0	Scale of $\beta_{1\|0}$ covariance matrix matrix $\mathbf{P}_{1\|0}$	0.053	–	0.02	–	0.0047	–	0.012	–
σ_v	Scale of belief covariance matrix matrix $\mathbf{V}$	0.003	–	0.019	–	0.0009	–	0.011	–
Marginal likelihood		−544.82	−594.30	−451.64	−470.66	−585.14	−715.88	−892.57	−928.64

Note: KF represents Kalman filter learning and RE represents rational expectations.

Table 2.5: Comparison of the marginal likelihoods and the posterior modes of the model parameters for different sample periods.

	pre-EMU 1970Q2–1998Q4		post-EMU 1999Q1–2013Q4		Great Moderation 1984Q1–2007Q4		Entire sample 1970Q2–2013Q4	
	KF	RE	KF	RE	KF	RE	KF	RE
σ_b	0.77	0.048	0.57	0.046	0.047	0.65	0.7	0.046
σ_g	0.19	0.18	0.2	0.18	0.16	0.16	0.19	0.18
σ_i	1.09	0.97	0.93	0.71	0.75	0.96	1.15	0.98
σ_{π^*}	0.044	0.04	0.016	0.01	0.01	0.011	0.061	0.026
σ_π	0.17	0.18	0.17	0.15	0.12	0.21	0.2	0.15
σ_r	0.12	0.17	0.082	0.15	0.16	0.1	0.1	0.15
σ_w	0.4	0.34	0.29	0.25	0.24	0.41	0.42	0.31
σ_z	0.76	0.82	0.86	0.83	0.73	0.71	0.84	0.83

Note: KF represents Kalman filter learning and RE represents rational expectations.

Table 2.6: Comparison of the posterior modes of the standard deviations of the shocks under Kalman filter learning and under rational expectations for different sample periods.

Chapter 3

Real-time Parameterized Expectations and the Effects of Government Spending

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Abstract

In this paper, we explore the effects of government spending in the real business cycle model where agents use a learning mechanism to form expectations. In contrast to most of the learning literature, we study learning behaviour in the original non-linear model. Following the learning interpretation of the parameterized expectations method, agents' forecast rules are approximations of the conditional expectations appearing in the Euler equation. We show that variation in agents' beliefs about the coefficients of these rules, generates time variation in the transmission of government spending shocks to the economy. Hence, our modelling approach provides an endogenous mechanism for time-varying government spending multipliers in the standard real business cycle model.

1 Introduction

The macroeconomic impact of fiscal policy depends crucially on the behavioural response of households to these policies. An important determinant of this behavioural response is the approach households apply to form expectations regarding the evolution of different endogenous, macroeconomic variables. The dominant paradigm used to model expectations in macroeconomics is the rational expectations (RE) hypothesis. According to this hypothesis, households have perfect knowledge about the structure of the model and understand the full complexities of the macro-economy. An alternative to the rational expectations hypothesis is provided by the learning literature (see e.g. Evans and Honkapohja, 2001).[1] In this literature, agents form expectations using a perceived law of motion. Over time, as new information becomes available, agents update the coefficients of their perceived law of motion.

This observation raises the question of whether the effects of government spending and the transmission thereof in the macro-economy are different in the basic Real Business Cycle (RBC) model using respectively rational expectations and a learning set-up. Indeed, it is well known that government spending multipliers generated by standard RBC and Dynamic Stochastic General Equilibrium (DSGE) models using rational expectations are typically constant. This theoretical finding is, however, not in accordance with the findings of several empirical studies. Auerbach and Gorodnichenko (2012) and Owyang et al. (2013), among others, show that the government spending multiplier is time-varying. In this paper, we start from these empirical findings and show that the introduction of a learning set-up in the standard RBC model can generate substantial time variation in the government spending multiplier.

Several papers have already explored the effects of fiscal policy using a learning framework within an RBC or DSGE model. Evans et al. (2009), for example, study the effects of anticipated fiscal policy changes both within an endowment economy and the Ramsey model. Their assumption is that agents fully understand and anticipate the evolution of taxes but have to forecast future factor prices using a linear learning mechanism. Building on this framework, Mitra et al. (2013) generalise the analysis of Evans et al. (2009) to a stochastic environment with elastic labour supply. Gasteiger and Zhang (2014) extend the model even further by introducing distortionary taxes. Following a similar learning approach, Benhabib et al. (2014) investigate the effects of fiscal stimulus in a new Keynesian model with a zero lower bound on the nominal interest rate.

All the aforementioned papers have enriched our knowledge on the macroeconomic effects of fiscal policy. They show that these effects can be substantially different when using the learning approach instead of the rational expectations approach. However, none of these studies explicitly focuses on the evolution and level of the government spending multiplier. Furthermore, they all study learning in the linearised counterpart of the non-linear model they are using. Indeed, it is common in the learning literature that non-linear models are first linearised around the rational expectations solution before studying their dynamics under learning. Exploring learning and the link with fiscal policy in the original non-linear model has several advantages, though. First, there is no longer the need to linearise the RE model around its steady state. Furthermore, contrary to linearised models, which necessarily lead to a local stability analysis, the context of the original non-linear system allows one to provide a more global stability analysis.

[1] In this paper we consider *Euler equation* learning as put forward by Evans and Honkapohja (2001). In this approach, agents make one-step ahead forecasts. By contrast, the *infinite horizon* approach of Preston (2005) assumes that at each date agents make forecasts about variables into the infinite future. For a discussion of these two approaches see Honkapohja et al. (2013).

Last, due to the non-linearity of the system, it is more natural to use non-linear forecasting rules compared to linearised models. This way, the usefulness of non-linear forecasting rules can be studied as well. As such, one can allow for the possibility of non-linear responses from households.

In this paper, we explore the transitional effects of government spending and the behaviour of the government spending multiplier by introducing learning in the in the original non-linear RBC model. More specifically, we adopt the learning interpretation of the parameterized expectations algorithm (PEA). This algorithm was initially developed as a solution method for non-linear, stochastic models with rational expectations (see for example den Haan and Marcet, 1990; Marcet and Lorenzoni, 1999; Marcet and Marshall, 1994). The idea behind the PEA is to replace the conditional expectations in the equilibrium conditions of the model with flexible functional forms with a finite number of arguments, e.g. polynomials. However, in Marcet and Marshall (1994), the authors also give an alternative, learning interpretation to the solution of the PEA.

We find that learning in the non-linear model leads to substantial time variation in the transmission of structural shocks in the model economy. As such, this result stands in sharp contrast with the RE and PEA solutions of the model. Our set-up thus leads to time-varying government spending multipliers. Furthermore, the time variation in our set-up itself is endogenously determined. As the economic agents update their beliefs, their response to a change in government spending changes as well, leading to a different impact on the economy.

The remainder of this paper is structured as follows. Section 2 describes the model. In Section 3, the learning mechanism is outlined in detail and compared with the rational expectations solution and the PEA solution of the model. Section 4 shows how learning behaviour in our model leads to time variation in the government spending multipliers. Finally, Section 5 concludes.

2 Model

To study the transitional dynamics of fiscal policy changes, we use the standard RBC model with elastic labour supply. In this section, we briefly introduce the different components of the model.

2.1 Households

The maximization problem of the representative household consists in maximizing

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} + b \frac{(1-n_t)^{1-\theta}}{1-\theta} \right] \tag{3.1}$$

subject to its budget constraint:

$$c_t + k_{t+1} = w_t n_t + (1 + r_t) k_t - T_t. \tag{3.2}$$

In these equations, c_t represents the household's consumption, k_{t+1} denotes the capital stock, n_t is its labour supply, and w_t and r_t are the real wage and the real interest rate. The latter is equal to the rental charge on capital after depreciation ($r_t = r_t^k - \delta$). Furthermore, b is the taste for leisure, β the discount factor, σ the coefficient of relative risk aversion, and θ the inverse of the inter-temporal elasticity of substitution in leisure. T_t is the lump sum tax in period t.

The optimality conditions of the household with respect to labour and consumption are respectively given by

$$c_t^{-\sigma} = \frac{b\,(1-n_t)^{-\theta}}{w_t} \tag{3.3}$$

and

$$\beta E_t \left\{ c_{t+1}^{-\sigma} \left[r_{t+1}^k + 1 - \delta \right] \right\} = c_t^{-\sigma}. \tag{3.4}$$

2.2 Firms

The representative firm produces the final good according to

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha} \tag{3.5}$$

where z_t evolves according to

$$z_t = z_{t-1}^{\rho_z} \exp(\varepsilon_t^Z). \tag{3.6}$$

with $\rho_z \in (0,1)$. In these equations, $\varepsilon_t^Z \sim \mathscr{N}(0, \sigma_Z^2)$ is an innovation in technology. Profits are given by

$$\mathscr{L} = z_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - r_t^k k_t \tag{3.7}$$

and the corresponding first-order conditions with respect to labour and capital respectively are:

$$w_t = (1-\alpha) z_t k_t^{\alpha} n_t^{-\alpha}, \tag{3.8}$$

$$r_t^k = \alpha z_t k_t^{\alpha-1} n_t^{1-\alpha}. \tag{3.9}$$

2.3 Government

The fiscal government finances its expenditures on goods by levying lump sum taxes. Formally, we have

$$g_t = T_t. \tag{3.10}$$

Government spending g_t evolves according to

$$g_t = g_{t-1}^{\rho_g} \exp(\varepsilon_t^G), \tag{3.11}$$

with $\rho_g \in (0,1)$ and where $\varepsilon_t^G \sim \mathscr{N}(0, \sigma_G^2)$ is a government spending shock.

3 Real-time non-linear learning

3.1 Set-up

We assume that agents, instead of having rational expectations, form forecasts by means of a non-linear learning mechanism. In particular, agents in our model approximate the conditional expectational function in the consumption Euler equation

$$E_t\phi\left(s_{t+1}\right)=E_t\left[c_{t+1}^{-\sigma}\left(1-\delta+\alpha z_{t+1}k_{t+1}^{\alpha-1}n_{t+1}^{1-\alpha}\right)\right], \tag{3.12}$$

where $s_t=[c_t,k_t,z_t]$, by a parametric function $\psi\left(x_t,\gamma_{t-1}\right)$ of the state variables $x_t=[1,k_t,z_t,g_t]$, and update the parameters γ_{t-1} using a constant-gain variant of recursive least squares. We follow the parameterized expectations literature and use an exponentiated polynomial to approximate the expectational function. More precisely, we consider the following first-order polynomial in the state variables of the model

$$E_t\phi\left(s_{t+1}\right)\simeq\psi\left(x_t,\gamma_{t-1}\right)=\exp\left[\gamma_0+\gamma_1\log k_t+\gamma_2\log z_t+\gamma_3\log g_t\right]. \tag{3.13}$$

Agents update the vector of belief parameters γ_t in real time according to this learning rule:

$$\gamma_t = \gamma_{t-1}+\kappa S_t^{-1}x_{t-1}\left[\log\left(\phi\left(s_t\right)\right)-\log\left(\psi\left(x_{t-1},\gamma_{t-2}\right)\right)\right], \tag{3.14}$$

$$S_t = S_{t-1}+\kappa\left[x_{t-1}x_{t-1}^{'}-S_{t-1}\right], \tag{3.15}$$

where S_t is the moment matrix for x_t and $\kappa\in(0,1)$ is the gain parameter. If the gain parameter κ would be equal to t^{-1}, equations (3.14)–(3.15) are the recursive formulas of the ordinary least squares (OLS) estimator for the coefficients γ in the log-linear specification of equation (3.13), i.e. $\log\left(\phi\left(s_t\right)\right)\simeq\log\left(\psi\left(x_{t-1},\gamma\right)\right)$. The update for γ in equation (3.14) uses the most recent forecast error$\log\left(\phi\left(s_t\right)\right)-\log\left(\psi\left(x_{t-1},\gamma_{t-2}\right)\right)$.

Instead of adopting the (decreasing-gain) OLS algorithm, where $\kappa=t^{-1}$, we set the gain κ to a small constant. The constant-gain case is the most relevant one for our analysis and is widely used in the adaptive learning literature (see Eusepi and Preston, 2011; Milani, 2007; Slobodyan and Wouters, 2012b, for example). As argued by, for example, Sargent (1999), Cho et al. (2002), Branch and Evans (2007), Milani (2007), and Orphanides and Williams (2007), it is a natural way of accomplishing "perpetual learning" as it places a greater weight on more recent observations. Hence, the constant gain makes sure that the variation in the agents' beliefs does not die out over time.

We are now able to formulate the dynamics of our model under non-linear learning. For some initial beliefs γ_0 and an initial state vector $x_0=[1,k_0,z_0,g_0]$, substituting the approximating function $\psi\left(x_t,\gamma_{t-1}\right)$ for the expectation function in equation (3.4) gives consumption $c_t=\beta\left[\psi\left(x_t,\gamma_{t-1}\right)\right]^{-1/\sigma}$, equation (3.3) determines labour supply n_t, and k_{t+1} follows from the resource constraint (3.2). The procedure is detailed in Appendix 3.B.

3.2 Discussion

Originally, the parameterized expectations approach was used as a method to approximate the rational expectations equilibrium of non-linear stochastic dynamic models. The original parameterized expectations algorithm (PEA) starts from a large sequence of shocks $\{z_t, g_t\}_{t=0}^{T}$, computes the corresponding endogenous variables consistent with the parameterized expectations, and iterates on the vector of parameters γ until the approximation becomes sufficiently accurate. A detailed step-by-step description of the algorithm is given in Appendix 3.A.

In this paper, we follow the suggestion of Marcet and Marshall (1994) and interpret the parameterized expectations approach as a real-time learning mechanism. Instead of holding the parameters γ constant over the stochastic simulation of the model, we let agents update them according to the learning rule above each time new information becomes available. In doing so we build on Berardi and Duffy (2015), who applied a similar learning mechanism to an optimal growth model.

3.3 Non-linear learning simulation

Before turning to the effects of government spending shocks, we compare the non-linear learning solution of the model with two conventional alternatives: the rational expectations solution and the solution provided by the original parameterized expectations algorithm (PEA).[2] Recall that in the latter case, the parameters of the approximating function are held fixed whereas in the non-linear learning model the parameters are updated over time. Given a sequence of 10,000 structural shocks, we generate a series of endogenous variables for these three different solutions.

Our assumptions on the parameter values are listed in Table 2 and are in line with the literature. The baseline value for the gain parameter, κ, is set to 0.02.[3] Given that this is a crucial parameter for the learning dynamics, we discuss the implications of different choices for κ in Section 4. The other parameters are set to values commonly used in the literature. The output elasticity of capital α is set to 1/3. According to Rogerson (2007), a reasonable range for θ in models with a macro focus is $[1,3]$. We set θ equal to 1. The coefficient of risk aversion σ equals 1 and the discount factor β is set to 0.98. The depreciation rate δ equals 0.025. The AR(1) coefficients of technology, ρ_Z, and government spending, ρ_G, are set to 0.9. The share of government spending in GDP, $\bar{g}/\bar{y}$, is set to 0.2. For the standard deviations of the technology

[2] We use the deterministic extended path method to find the rational expectations solution of the model.

[3] This is a value well within the range of estimates reported in the literature. According to Orphanides and Williams (2005, 2007) a gain parameter in the range 0.01–0.04 provides the best fit between the agents' forecasts in the model and the expectations data from the Survey of Forecasters. Using a similar strategy, Branch and Evans (2006) obtain a value of 0.0345. The estimate of Milani (2007) equals 0.0183 and hence lies within the same range. The estimated gain in Eusepi and Preston (2011) is 0.0029. The Bayesian estimation results from Slobodyan and Wouters (2012b) provide values for κ going from 0.001 to 0.06 depending on the particular learning scheme.

Parameter	Description	Value
α	Output elasticity of capital	$1/3$
β	Discount factor	0.98
σ	Coefficient of risk aversion	1
δ	Depreciation rate	0.025
b	Taste for leisure	1.44
θ	Preference parameter	1
$\bar{g}/\bar{y}$	Steady state government expenditure to output ratio	0.2
ρ_Z	Technology shock AR(1) coefficient	0.9
ρ_G	Government spending AR(1) coefficient	0.9
σ_Z	Standard deviation of the technology disturbance ε^Z	0.01
σ_G	Standard deviation of the fiscal disturbance ε^G	0.01
κ	Gain parameter in the learning mechanism	0.02

Table 3.1: Calibration.

and fiscal disturbances, σ_Z and σ_G, the value 0.01 is chosen. The parameter capturing the taste for leisure, b, is determined such that individuals work on average 1/3 of their time endowment.

It is clear from Figure 3.1 that the simulated series of both the learning model and the parameterized expectations algorithm are close to the rational expectations series. The figure shows the evolution of consumption, output, labour, and investment for 500 periods of the full sample.[4] The relatively small differences between the rational expectations and parameterized expectations solution, indicate that the latter is a reasonably good approximation of the former. This comforts us in the choice of the functional form of the approximating function. The simulation results also illustrate the local stability of the non-linear learning mechanism.[5] Over our long simulation horizon, the recursive estimation of the belief parameters does not drive the economy towards diverging paths. This stability is also reflected in the evolution of the belief parameters over time, depicted in Figure 3.2. Although the parameters can deviate from their PEA values for a sustained period of time, they remain in the neighbourhood of those values.[6]

[4]Investment is defined as $i_t = y_t - c_t - g_t$.

[5]Since our analysis is numerical, establishing analytical convergence results is beyond the scope of this paper. Stability results for constant-gain algorithms can be found in (inter alia) Evans and Honkapohja (2001). Loosely speaking, constant-gain learning mechanisms are locally stable only if the gain parameter is sufficiently small. We discuss the sensitivity of our results with respect to this parameter in Section 4.

[6]Constant gain learning implies that the parameters do not converge to a point estimate but only to a distribution around the rational expectations beliefs.

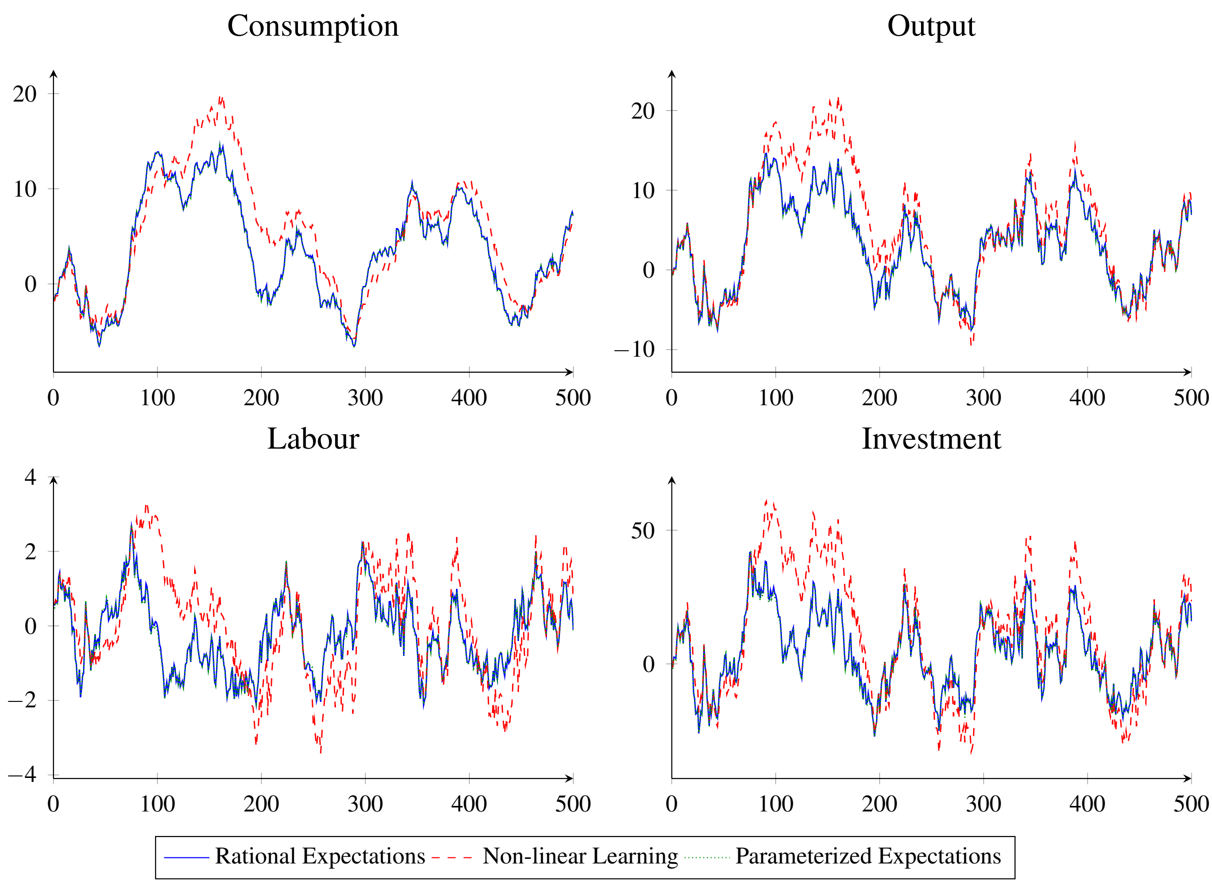

Figure 3.1: Simulated series of model variables under different assumptions regarding the formation of expectations. The series are measured in percentage deviations from the steady state. The horizontal axis measures time in quarters.

4 The effects of government spending under non-linear learning

In sharp contrast with the rational expectations and PEA solutions of the model, the non-linear learning solution generates time-variation in the transmission of structural shocks in the model economy. In this section we illustrate how this feature provides an attractive mechanism for generating variation in the government spending multipliers over time.

In standard rational expectations models, variation in the transmission mechanism of shocks is typically obtained by allowing structural parameters to vary over time. One common strategy is to estimate a fixed-parameter model on different samples and test for breaks. A downside of this strategy is that the time variation is by assumption infrequent, whereas our approach allows for time variation at a much higher frequency. Another popular strategy is to use stochastic time-varying coefficient models. In this approach, (some) model parameters are assumed to

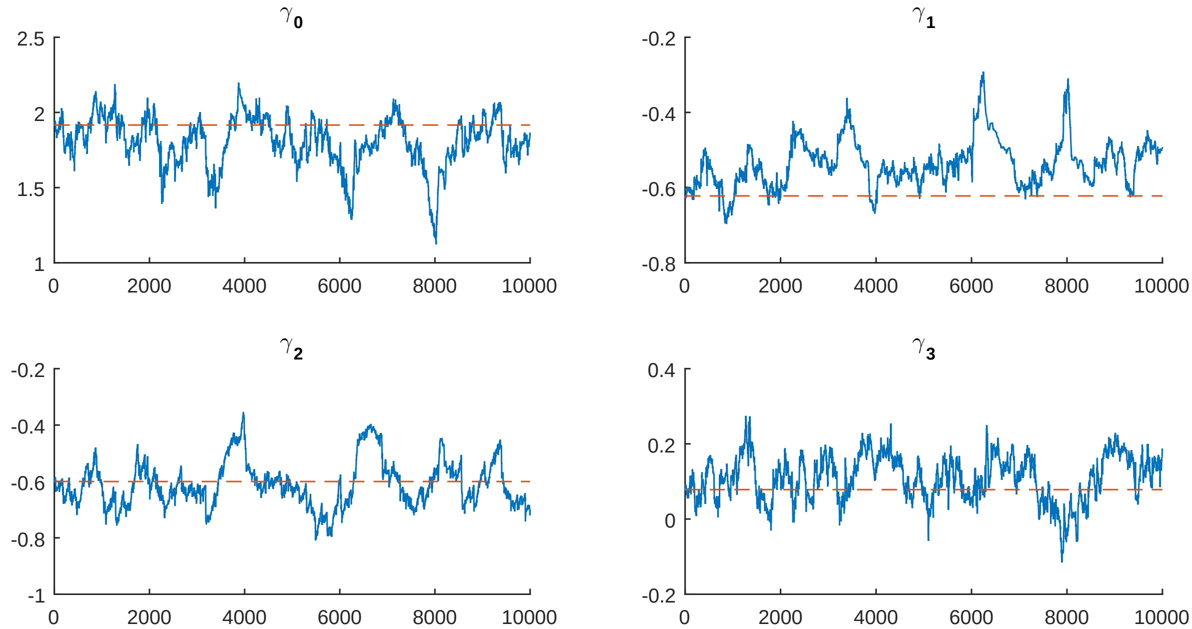

Figure 3.2: Evolution of the belief coefficients in the non-linear learning model. The series of solid lines correspond to the coefficients of the approximating function given by equation (3.13). The dashed lines represent the coefficients provided by the parameterized expectations algorithm.

follow a stochastic process.[7] Both strategies have the disadvantage that the time variation is not endogenously determined by the model.

By contrast, our approach allows for endogenous time variation of the government spending multipliers at a relatively high frequency. Figure 3.3 shows the impact multipliers that correspond to the series in Figure 3.1. The transmission mechanism that underlies the multipliers changes as agents recursively update their beliefs over time. It is well understood that expectations play a crucial role in the impact of fiscal policies, and our modelling approach highlights this expectations channel for the transmission government spending shocks.

Figure 3.4 illustrates the time variation of the government spending multipliers generated by learning behaviour in our model. It reports the distribution of the impact multipliers for output, consumption, and investment.[8] In the rational expectations equilibrium, the multipliers are time-invariant. The multipliers for output, consumption, and investment are, respectively, 0.33, -0.27 and -0.30. The relatively small output multiplier, mainly driven by a significant crowding out of private consumption, is a standard result of the real business cycle model.

[7] Examples of the first strategy are Benati (2008) and Canova (2009); examples of the second strategy include Cogley and Sbordone (2008) and Justiniano and Primiceri (2008). A more extensive overview of the existing literature goes beyond the scope of this paper.

[8] The results are based on 10 simulations of the non-linear learning model over 10,000 periods. For each simulation, the initial parameter vector is the vector provided by the parameterized expectations algorithm.

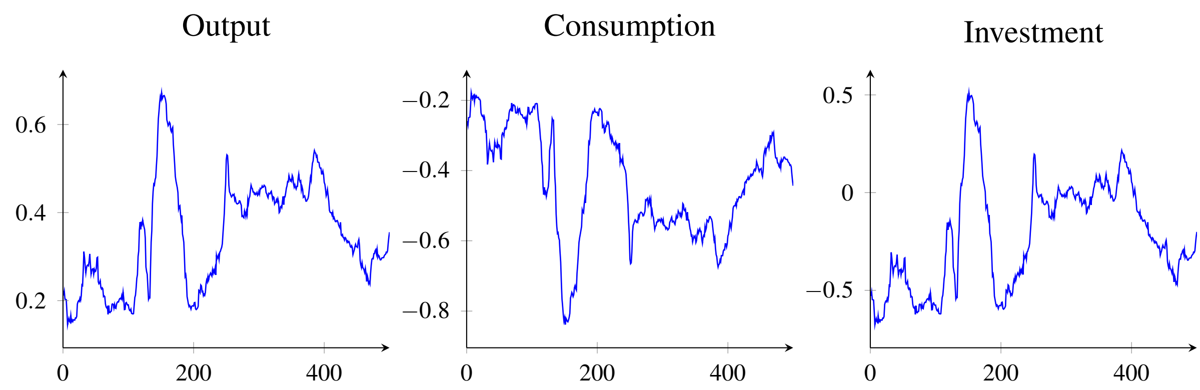

Figure 3.3: Evolution of the impact multipliers in the non-linear learning model. The series represent the impact multipliers in non-linear learning simulation for the same 500 periods as in Figure 3.1.

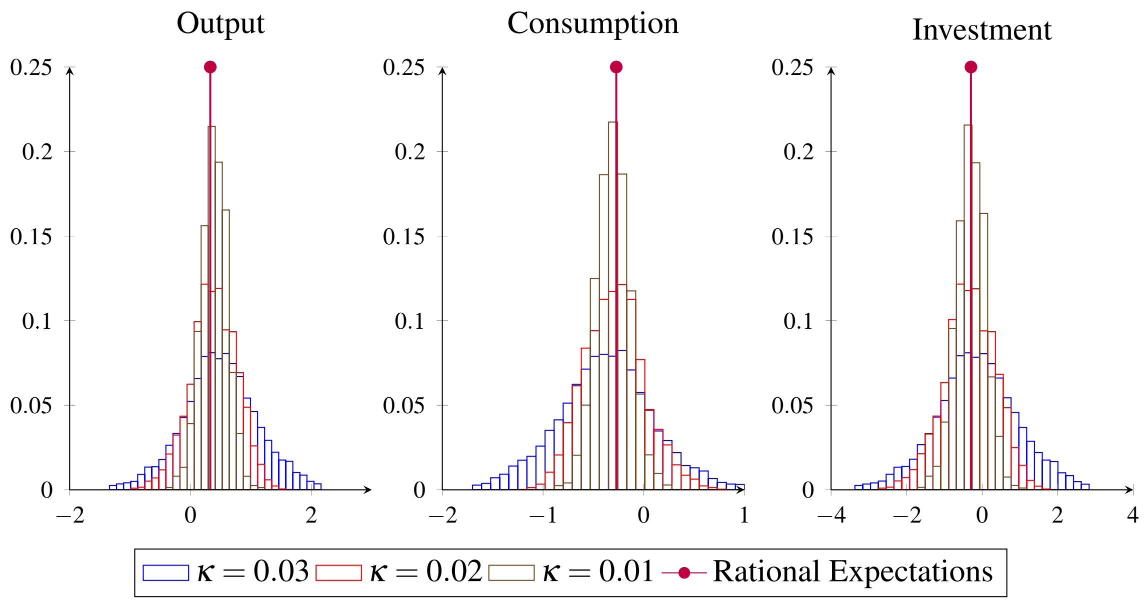

Figure 3.4: Distribution of impact multipliers based on 10 simulations of the non-linear learning model over 10,000 periods. The histograms represent the government spending multipliers for output, consumption, and investment in the non-linear learning model for different values of the gain parameter (κ). The purple lines represent the rational expectations multipliers. The vertical axis measures relative frequencies.

Under non-linear learning behaviour, however, the multipliers vary significantly around their rational expectations values. The degree of time variation is governed by the gain parameter κ in the recursive learning rule (3.14) since that parameter determines the volatility of the belief parameters. Overall, learning behaviour in our model generates substantial time variation in the multipliers and a higher gain increases this variation considerably.

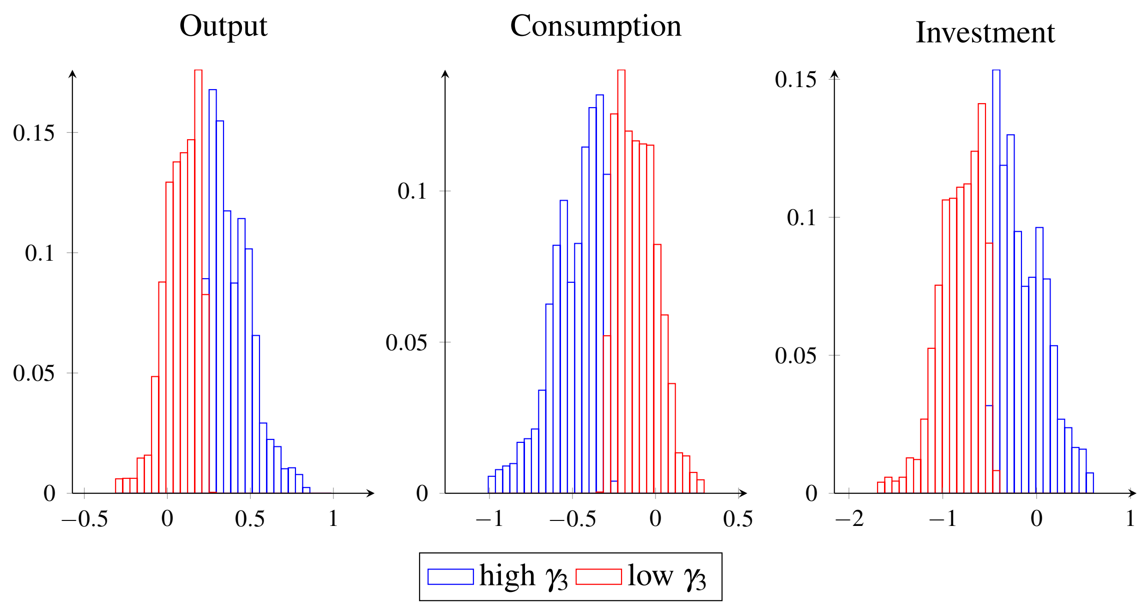

Figure 3.5: Distribution of impact multipliers for high and low values of belief parameter γ_3 . The histograms are based on a simulation of 10,000 periods of the non-linear learning model and represent the government spending multipliers for output, consumption, and investment. The high (low) γ_3 sub-sample contains the multipliers when γ_3 is above (below) its median value. The vertical axis measures relative frequencies.

4.1 Discussion

The previous paragraph showed that our set-up leads to time-variation in the government spending multiplier. This result implies that the effectiveness of an increase in government spending varies over time. For this reason, this paragraph provides some insights into the drivers of the government spending multiplier. These determinants are of particular interest for fiscal policy-makers. It allows them to assess the effectiveness of fiscal policy in the economy.

The evolution of the government spending multipliers is mainly driven by the evolution of γ_3, the coefficient for government spending in the approximating function (3.13). If γ_3 is high (low), the government spending multiplier for output is high (low) as well. Intuitively, this result makes sense. If individuals attach a higher weight to government spending in their approximating function, the impact of an increase in government spending on the decision variables will also be stronger. Figure 3.5 illustrates this relationship for the output, consumption, and investment multipliers. If γ_3 is high, the crowding out of private consumption after the government spending shock will be more severe. Hence, the consumption multiplier will be larger. Note, however, that in the context of an RBC model, a small multiplier for private consumption leads to a large multiplier for output. The output multiplier is driven by labour supply and a higher level of consumption leads, ceteris paribus, to a lower level of labour supply.

As the evolution of γ_3 is the most important driver of the government spending multiplier, the next step is to explore the drivers of γ_3. Looking at Equation (3.14), two important channels can be identified. The first one is the forecasting error $\left[\log\left(\phi\left(s_t\right)\right) - \log\left(\psi\left(x_{t-1}, \gamma_{t-2}\right)\right)\right]$. A positive (negative) forecasting error leads to an increase (decrease) in γ_3, everything else equal. The forecasting error itself is determined by the evolution of the technology shocks. More specifically, if $z_t > z_{t-1}$, the forecasting error is negative, whereas if $z_t < z_{t-1}$ it is positive. So, everything else equal, after a period of technological growth, the government spending multiplier is low and after a period of decreasing technology, the government spending multiplier is high. This result is, however, dependent on the level of government spending, which is the second channel. If government spending is above its steady state level, γ_3 will increase for a given level of technology, if it is below its steady state level, γ_3 will decrease for a given level of technology. Furthermore, the further government spending is from its steady state level, the stronger its influence on the evolution of γ_3 and thus the government spending multiplier.

Combined, these results lead to the following insights. The government spending multiplier is likely to be high if technology and government spending relative to its steady state level evolve in opposite direction. More specifically, the multiplier is high (increases) after (i) a considerable period of increasing technology and low government spending on the one hand, and (ii) after a considerable period of decreasing technology and high government spending on the other hand. The multiplier is low (i) after a considerable period of increasing technology and high government spending on the one hand and (ii) after a considerable period of decreasing technology and a low level of government spending on the other hand.[9]

We illustrate the aforementioned results using Figure 3.6. In this Figure, we highlight three different episodes during which the government spending multiplier strongly changes. The corresponding levels of technology and government spending are displayed in respectively the middle and bottom panel. Furthermore, the red line in the bottom panel denotes the steady state level of government spending. Here, we discuss the evolution of the government spending multiplier between periods 176 and 204. The other ones follow the same reasoning. At first, technology strongly increases while the level of government spending is below its steady state level. We know that this combination leads to an increase in γ_3. This increase almost directly leads to an increase in the government spending multiplier. After some time, however, technology starts to decrease. At that point, government spending is higher than the steady state level. This combination pushes the government spending multiplier upwards.

[9]Recent contributions to the literature investigate the relationship between the size of the multiplier and the state of the business cycle – see e.g. Auerbach and Gorodnichenko (2012, 2013) and Owyang et al. (2013). We have investigated this relationship using different measures for the business cycle and found that this stylized RBC model is not adequate to uncover a structural relation. Investigating this issue in a more elaborate (more demand-driven) non-linear learning model is a promising direction for future research.

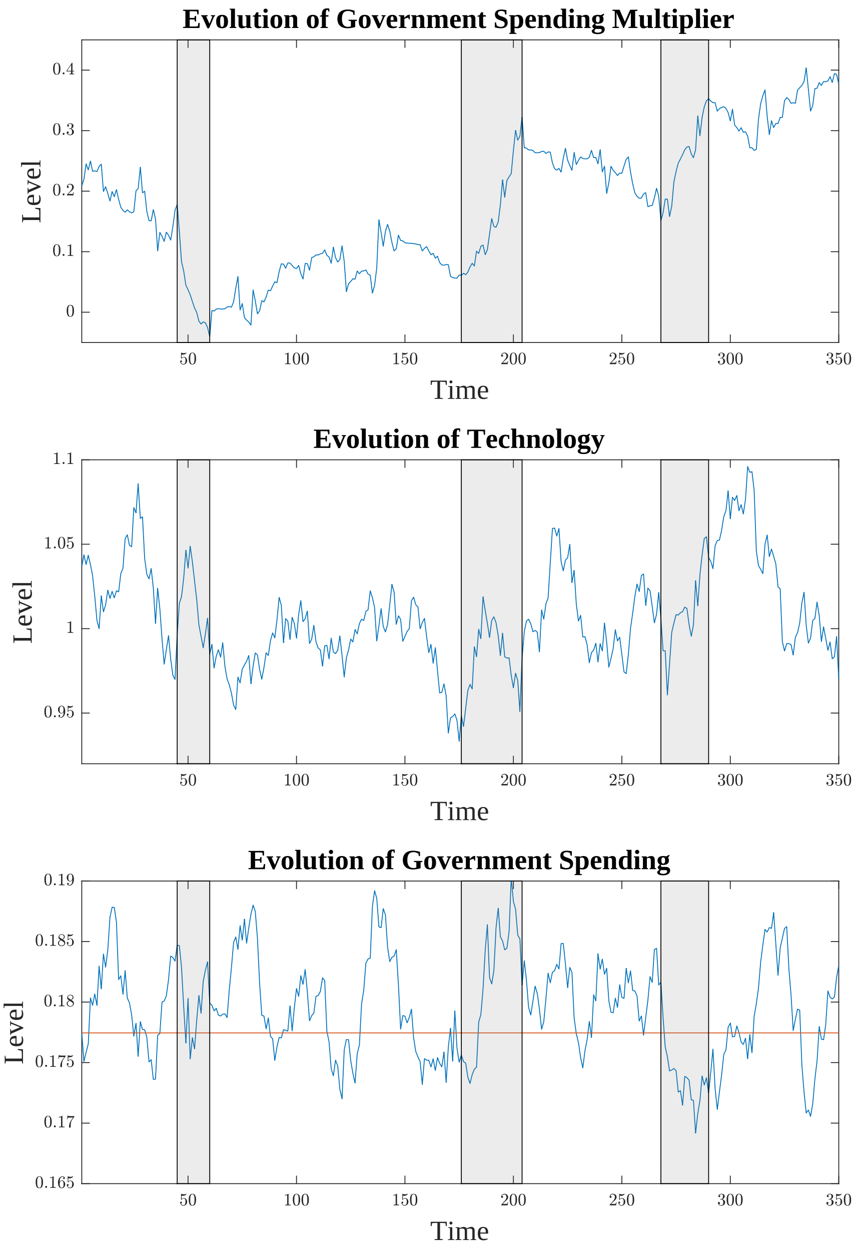

Figure 3.6: Endogenous drivers of the government spending multiplier.

4.2 Choice of the approximating function: linear versus non-linear learning

In the baseline simulation, we assume that agents approximate the expectation in the consumption Euler equation (cf. equation (3.12)) by the following non-linear function

$$\psi\left(x_t, \gamma_{t-1}\right) = \exp\left[\gamma_0 + \gamma_1 \log k_t + \gamma_2 \log z_t + \gamma_3 \log g_t\right].$$

In this section, we compare this "non-linear" learning approach with "linear learning" where agents use the following linear approximating function

$$\psi\left(x_t, \gamma_{t-1}\right) = \gamma_0 + \gamma_1 k_t + \gamma_2 z_t + \gamma_3 g_t.$$

As for the "non-linear" learning case, the parameters γ_{t-1} are updated using the constant-gain recursive least squares formulas (3.14)–(3.15) but the updating term between square brackets in equation (3.14) now becomes $\left[\phi\left(s_t\right) - \psi\left(x_{t-1}, \gamma_{t-2}\right)\right]$.

When comparing the linear with the non-linear learning approach, two interesting observations can be made. First, non-linear learning outperforms linear learning in terms of forecasting performance, especially when (large) shocks drive the economy far away from its steady state. To illustrate this, Table 3.2 reports the root-mean square error (RMSE) of the forecasts under linear and non-linear learning. In a "high volatility" simulation, when structural shocks to the economy are larger than in the baseline simulation, non-linear learning improves the forecasting performance relative to linear learning. In the baseline simulation, however – when the structural shocks are relatively small – the forecasting performance of non-linear learning is (approximately) equal to linear learning. Hence, when shocks are relatively large, non-linear learning leads to better forecasting. To provide some intuition for this result, Figure 3.7 plots the linear and non-linear approximating function for different values of the capital stock. For values close to the steady state ($k = \bar{k}$), the approximation by the linear and non-linear function is very similar. For capital (and government spending) far away from the steady state, the linear and non-linear approximation differ quite substantially. In both cases, agents can update the coefficients of the approximating function to improve their forecasting, but the results presented in Table 3.2 show that non-linear learning outperforms linear learning in terms of RMSE.

Second, the linear learning approach also generates time variation in the government spending multipliers. Figure 3.8 shows how the impact multipliers for output, consumption, and investment again fluctuate around the rational expectations multipliers, but their values are less dispersed. Under non-linear learning, the updating of the belief coefficients generate more variation in the consumption response after the government spending shock. Consequently, the effects on output and investment will also vary more.

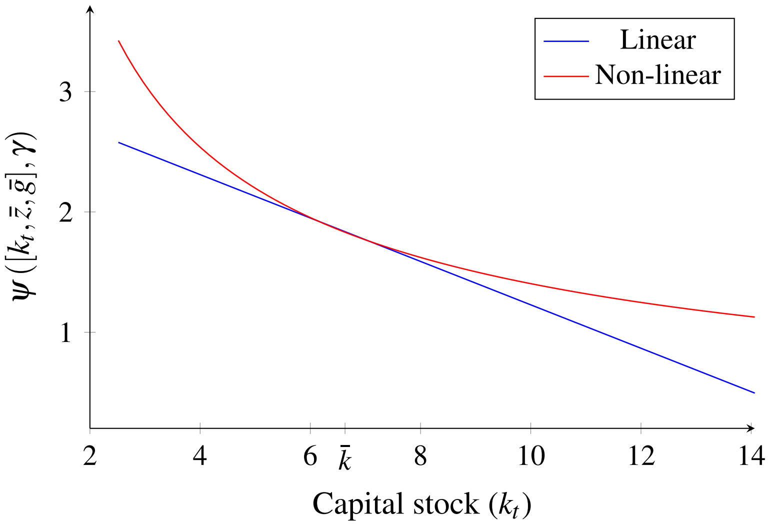

Figure 3.7: Linear and non-linear approximating function for different values of the capital stock. Government spending and technology are fixed at their steady state values and γ equals the coefficient vector of the parameterized expectations algorithm.

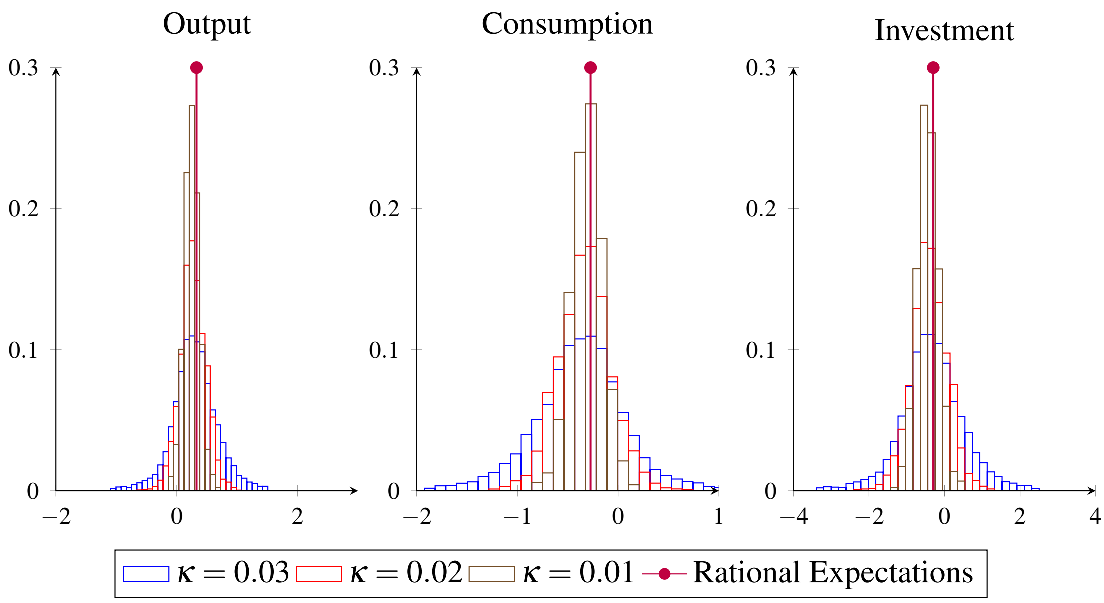

Figure 3.8: Distribution of impact multipliers based on 10 simulations over 10,000 periods of the model with a linear approximating function. The histograms represent the government spending multipliers for output, consumption, and investment for different values of the gain parameter (κ). The purple lines represent the rational expectations multipliers. The vertical axis measures relative frequencies.

Learning scheme	Root-mean-square error	
	Baseline ($\sigma_Z = \sigma_G = 0.01$)	High volatility ($\sigma_Z = \sigma_G = 0.025$)
Non-linear learning	0.010	0.026
Linear learning	0.010	0.034

Table 3.2: Forecasting performance of linear and non-linear learning. Root-mean-squared errors are calculated over 10,000 simulation periods.

5 Conclusion

In this paper, we explore the transitional dynamics following fiscal policy changes in a stochastic macroeconomic framework where agents use adaptive learning to update non-linear forecast rules to form expectations. Several papers have already studied the effect of fiscal policy using a learning framework (see e.g. Evans and Honkapohja, 2009; Gasteiger and Zhang, 2014; Benhabib et al., 2014). All of these papers, however, use linear forecast rules. In this paper, we apply a different approach. Following, inter alia, Marcet and Marshall (1994) and Berardi and Duffy (2015), we interpret the parameterized expectations algorithm (PEA) as a real-time learning mechanism used by agents to update their expectations over time in the original non-linear model.

Our main contribution is to study the dynamics of government spending in the standard RBC model where agents use this non-linear learning mechanism to form expectations. To the best of our knowledge, ours is the first study to compare the transitional dynamics resulting from this framework with the dynamics under rational expectations.

In our non-linear learning set-up, the effects of government spending shocks vary substantially over time. We have shown that this variation is endogenously driven by agents' expectations about the future. The resulting fluctuations in the government spending multipliers are in marked contrast to the time-invariant multipliers of the rational expectations solution of the model. We also show that the forecasting performance of the learning mechanism is better if agents use a non-linear approximating function instead of a linear one. In particular in the context of relatively high structural shocks which drive the economy far away from its steady state, using a non-linear approximating function to form expectations is advantageous.

Our findings provide several avenues for future research. First, it may be very fruitful to go beyond the standard RBC model. Understanding how learning affects the variability of fiscal multipliers in more elaborate models is an important topic. It may, for example, shed further light on the dependency of multipliers on the state of the business cycle and the stance of monetary policy. Second, the non-linear learning model provides a natural framework for studying the consequences of a structural change in fiscal policy. Comparing the dynamics of this learning model with the rational expectations dynamics is, in our view, a promising direction for further research.

Acknowledgements

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Appendices

Appendix 3.A Parameterized expectations algorithm

Under the parameterized expectations algorithm (PEA) the vector of coefficients γ of the approximating function $\psi(\cdot,\gamma)$ is calculated with the following iterative procedure.

Step 1 Draw a large sequence of shocks $\left\{\varepsilon_t^G, \varepsilon_t^Z\right\}_{t=0}^T$ and compute $\{g_t, z_t\}_{t=0}^T$ as defined in (3.6) and (3.11).

Step 2 Choose an initial guess γ_0 and an initial value k_0 for the capital stock.

Step 3 At iteration $i \in \{0, \ldots, i_{max}\}$, use γ_i to generate the endogenous variables $\{c_t(\gamma_i), n_t(\gamma_i), k_{t+1}(\gamma_i)\}_{t=0}^T$. In particular, (i) substituting the approximating function $\psi(x_t,\gamma_i)$ for the expectation function in equation (3.4) gives consumption $c_t = \beta\left[\psi(x_t,\gamma_i)\right]^{-1/\sigma}$, (ii) the first-order condition (3.3) determines labour supply n_t, and (iii) k_{t+1} follows from the resource constraint (3.2).

Step 4 Use the data for $t = 0,\ldots,T-1$ to run the non-linear least squares regression of $c_{t+1}^{-\sigma}\left(1-\delta+\alpha z_{t+1}k_{t+1}^{\alpha-1}n_{t+1}^{1-\alpha}\right)$ on the approximating function $\psi(x_t,\cdot)$ to obtain an estimate $\hat{\gamma}$.

Step 5 Use this estimate to update the guess for γ according to

$$\gamma_{i+1} = (1-\mu)\gamma_i + \mu\hat{\gamma}, \tag{3.16}$$

where $(0,1]$ is a damping parameter.

Step 6 Apply steps 3-5 iteratively until the convergence criterion $\sum_k \left|\gamma_{i+1}^k - \hat{\gamma}^k\right| < \tau$ is met, where k is the number of parameters in γ.

In our experiments, we set $T = 10{,}000$, $i_{max} = 200$, $\mu = 0.8$, and $\tau = 1\times10^{-5}$. The initial value for the capital stock is the steady state: $k_0 = \bar{k}$.

Appendix 3.B Learning algorithm

To simulate the learning model, we follow the following steps.

Initialisation

1. Draw a sequence of shocks $\left\{\varepsilon_t^G, \varepsilon_t^Z\right\}_{t=0}^S$ and compute $\{g_t, z_t\}_{t=0}^S$ as defined in (3.6) and (3.11).

2. Choose an initial guess γ_0 and an initial value k_0 for the capital stock.

Simulation Simulate the model S periods forward using the following scheme.

1. Approximate the conditional expectational function in the consumption Euler equation

$$E_t\phi\left(s_{t+1}\right) = E_t\left[c_{t+1}^{-\sigma}\left(1-\delta+\alpha z_{t+1}k_{t+1}^{\alpha-1}n_{t+1}^{1-\alpha}\right)\right], \tag{3.17}$$

by the parametric function $\psi\left(x_t, \gamma_{t-1}\right)$ of the state variables $x_t = [1, k_t, z_t, g_t]$.

2. Calculate the corresponding endogenous variables determined by the following system

$$c_t = \beta\left[\psi\left(x_t, \gamma_{t-1}\right)\right]^{-1/\sigma}, \tag{3.18}$$

$$c_t^{-\sigma} = \frac{b\left(1-n_t\right)^{-\theta}}{w_t}, \tag{3.19}$$

$$w_t = (1-\alpha)z_t k_t^{\alpha} n_t^{-\alpha}, \tag{3.20}$$

$$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}, \tag{3.21}$$

$$i_t = y_t - c_t - g_t, \tag{3.22}$$

$$k_{t+1} = (1-\delta)k_t + i_t. \tag{3.23}$$

3. Update the vector of parameters γ_{t-1} according to

$$\gamma_t = \gamma_{t-1} + \kappa R_t^{-1} x_{t-1}\left[\log\left(\phi\left(s_t\right)\right) - \log\left(\psi\left(x_{t-1}, \gamma_{t-2}\right)\right)\right], \tag{3.24}$$

$$R_t = R_{t-1} + \kappa\left[x_{t-1}x_{t-1}^{'} - R_{t-1}\right]. \tag{3.25}$$

Chapter 4

General Conclusion

Taken together, the findings in this dissertation show how expectations of households and firms are key for understanding the effects of fiscal policy. These findings are of key importance for academic economists and policy-makers alike. To this end, this final chapter presents two main take-away messages for policy-makers and provides directions for future research.

From a policy-maker's perspective, the presented results offer two key messages to take away. First, a better articulated role of expectations in macroeconomic models allows a significantly better analysis of fiscal policy. More precisely, modelling expectations as a learning process can substantially improve the predictions with respect to changes in government spending. This dissertation has concretely shown how this can be achieved in models that are extensively used by central banks and other policy institutions. Since policy-makers heavily rely on these models to evaluate the impact of alternative policy scenarios, improving their predictions is of crucial importance. Besides their usefulness for alternative scenario analysis, structural macroeconomic models make a valuable contribution to forecasting. Although this dissertation has focussed on the goodness-of-fit of learning models, several contributions to the literature show that learning can also improve out-of-sample forecasting performance. In any case, policy predictions of structural models crucially hinge on the treatment of expectations. Therefore, it is important that policy-makers clearly spell out their underlying assumptions about expectations formation when using structural models in their decision making.

Second, it is advisable to closely monitor consumer and business expectations when considering government spending as a tool of stabilization policy. As has been understood for a long time, shifting expectations are an important source of business cycle fluctuations. This dissertation shows how time-varying expectations condition the impact of government spending shocks. So, even though the dependency of fiscal policy effectiveness on private-sector expectations presents a big challenge, policy-makers are not completely groping in the dark. The framework presented here allows them to condition their forecasts and counterfactuals on market beliefs. Empirical measures of consumer and business confidence are extremely valuable for that purpose. From a more data-driven point of view, so-called "expectations-augmented" vector autoregressive models use these measures to improve the goodness-of-fit and forecasting performance of purely statistical models. Hence, both the structural models presented in this dissertation and their empirical counterparts are indispensable vehicles for policy-makers to assess the effectiveness of alternative fiscal policies.

The findings in this dissertation open up several avenues for future research. First, although the learning approach provides an intuitive explanation for time variation in the government spending multiplier, it is still an open question how much of the variation can be attributed to expectations and how much to other factors. Obviously, many conditions that affect the transmission of fiscal policy shocks changed over time – the stance of monetary policy, levels of private and public debt, regulation of financial markets, asset market participation, to name just a few. Evaluating the importance of learning behaviour relative to these other conditions is an important direction for future research.

A second recommendation for future research relates to the planning horizon forecasters have. The chapters in this dissertation present two rather extreme cases: infinite horizon forecasting (Chapter 1) and one-period ahead forecasting (Chapter 2). The former is at odds with the finite forecasting horizon most forecasters have, whereas the latter probably underestimates their forward-looking behaviour. Assumptions about the forecasting horizon, might be particularly important in the context of fiscal policy since many of the effects manifest themselves in the medium and long run. It would be interesting to compare the two approaches or to generalise the learning set-up to N-step ahead forecasting.

Third, uncovering structure in the variation of the effects of government spending shocks over time is of great importance. This point was brought up in Chapter 3, but more research is required to better understand which mechanisms are at play. A growing body of literature emphasises the dependency of multipliers on the state of the business cycle and the stance of monetary policy. Studying the interaction between these conditions and expectation formation would greatly enhance our understanding of the effects of fiscal policy.

Finally, the analytical framework presented in Chapter 3 is particularly suitable for analysing structural reforms in fiscal policy. As we consider the non-linearized real business cycle model, the framework can be used to investigate the transitional dynamics in a *permanent* change in government spending or a shift in taxes, for instance. Comparing the dynamics of under learning with those under rational expectations is a promising direction for further research.

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