

Correction to Jin-Ting Zhang’s “Approximate and Asymptotic Distributions of Chi-Squared–Type Mixtures With Applications”

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Abstract

Zhang derives approximations for the distribution of a mixture of chi-squared distributions. The two derived approximation bounds in Theorem 1 both contain an arithmetic error. These are corrected here.

Keywords: Corrections; Chi-squared approximation; Chi-squared–type mixtures.

1 Introduction

Zhang (2005) studies approximate distributions of the chi-squared–type mixture

$$T = \sum_{r=1}^q c_r A_r, \quad A_r \sim \chi_1^2(u_r^2), \quad \text{independent}, \quad (1)$$

where $c_r, r = 1, 2, \dots, q$ are nonzero real coefficients and $u_r^2, r = 1, 2, \dots, q$ are the non-central parameters of the chi-squared variates $A_r, r = 1, 2, \dots, q$. The normalized version

T^* of this mixture T is approximated, both by a standard normal distribution Z , and by a single chi-squared distribution R^* (Zhang, 2005, Equation (6)). Denoting the density function of a random variable X by $f_X(x)$, and denoting the standard normal case $f_Z(x)$ by $\phi(x)$, Zhang (2005) derives the following approximation bounds in his Theorem 1:

Theorem 1. (a) *For the normal approximation of T , when $\Delta < 1/8$,*

$$\sup_x |f_{T^*}(x) - \phi(x)| < .1323 \left\{ 4 + \frac{.2803}{(1 - 8\Delta)^2} \right\} d^{*-1/2}. \quad (2)$$

(b) *For the chi-squared approximation of T , when $\Delta < 1/10$,*

$$\sup_x |f_{T^*}(x) - f_{R^*}(x)| < .1403 \left[\left\{ 3 + \frac{.1572}{(1 - 10\Delta)^{5/2}} \right\} M + \left\{ 3 + \frac{.1572}{(1 - 10/d)^{5/2}} \right\} / d \right]. \quad (3)$$

Here, Equation (2) corresponds to (Zhang, 2005, Equation (12)) and Equation (3) corresponds to (Zhang, 2005, Equation (13)). These equations contain variables that are undefined in this particular correction; their definitions are irrelevant to the discussion at hand.

2 Arithmetic Errors

The floating point constants that are present in Theorem 1 are derived in the Proof of Theorem 1, in (Zhang, 2005, Appendix B). Both the factors directly following the $<$ sign are correct, but the factors in the numerator of the fractions are erroneously derived. Page 283 of Zhang (2005) features the line “The assertion (a) follows by noting that [...] $8!^{1/4}/6 = .2803$ ”. In fact, $2.3617 < 8!^{1/4}/6 < 2.3618$. Page 284 of Zhang (2005) features the line “The assertion (b) follows by noting that [...] $10!^{1/4}/(2^{7/2}) = .1572$ ”. In fact, $3.8577 < 10!^{1/4}/(2^{7/2}) < 3.8578$.

3 Correction

Rounding the numbers up to the fourth decimal, the first half of (Zhang, 2005, Theorem 1) should read:

Theorem 1. (a) *For the normal approximation of T , when $\Delta < 1/8$,*

$$\sup_x |f_{T^*}(x) - \phi(x)| < .1323 \left\{ 4 + \frac{2.3618}{(1 - 8\Delta)^2} \right\} d^{*-1/2}.$$

(b) *For the chi-squared approximation of T , when $\Delta < 1/10$,*

$$\sup_x |f_{T^*}(x) - f_{R^*}(x)| < .1403 \left[\left\{ 3 + \frac{3.8578}{(1 - 10\Delta)^{5/2}} \right\} M + \left\{ 3 + \frac{3.8578}{(1 - 10/d)^{5/2}} \right\} /d \right].$$

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References

Zhang, J.-T. (2005). Approximate and Asymptotic Distributions of Chi-Squared-Type Mixtures With Applications *Journal of the American Statistical Association* 100(469), 273–285.