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Macroeconomic effects of demographic change in an OLG model for a small open economy
- The case of Belgium -

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Macroeconomic effects of demographic change
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Abstract

In the absence of behavioural adjustments, demographic change may cut off about 0.4 to 0.5%-points on average from the annual per capita growth rate in EU countries in the next 25 years. The behavioural responses of households and firms to reduced fertility and rising life expectancy may significantly change this outcome, but the sign and the size of this change are unclear. We construct and parameterize a large-scale overlapping generations model for an open economy to quantify (the net effect of) these behavioural adjustments. Individuals in the model differ not only by age, but also by innate ability. Key endogenous variables are hours worked, investment in human and physical capital and per capita growth. Applying the model to the case of Belgium, we find that it replicates key data since about 1960 remarkably well. Simulating the model, we observe that behavioural adjustments by households and firms contribute to reverse the negative arithmetical effect of projected future demographic change on per capita growth. However, under the current policies and the current pension system, these are not strong enough. In Belgium, a net negative effect on annual per capita growth remains of almost 0.3%-points on average, mainly because of reduced fertility and a declining population at working age.

Keywords: demographic change, population ageing, economic growth, secular stagnation, overlapping generations
JEL classification: C68, D91, E17, J11, O40

1 Introduction

Population ageing poses a major challenge to the social security system in all OECD countries. While aggregate public pension and health care expenditures are expected to rise strongly, future GDP growth may fall. Belgium is no exception to the phenomenon of ageing. As in other countries, fertility has gradually declined throughout the twentieth century. This downward trend was interrupted only during the period of the baby boom in the 1950s and 1960s. At the same time life expectancy has increased enormously. Together with the retirement of the baby boom generation since about a decade and the reduced size of more recent generations, the structural increase in life expectancy will imply a drastic change in the age distribution of the Belgian population in the period 2010-2040. As we show in Figure 1, the age dependency ratio, computed as the number of people younger than 18 or older than 64 relative to the population at working age, is expected to rise from about 60% in 2010 to an unprecedented 80% by 2040. Figure 2 depicts the evolution of the size of the underlying three age groups. Changes in the number of children reflect changes in fertility. While the number of people older than 64 rises rapidly, the Belgian population at working age is expected to decline between 2020 and 2040. By 2040 there will be eight dependent persons for every ten people at working age.

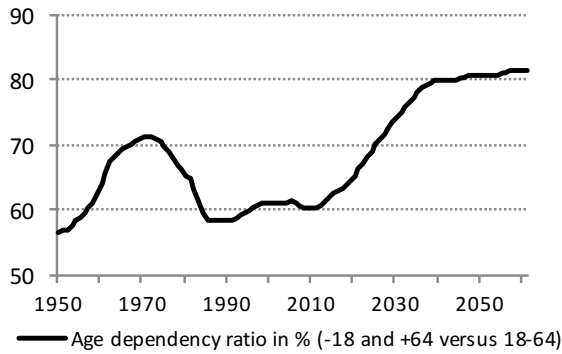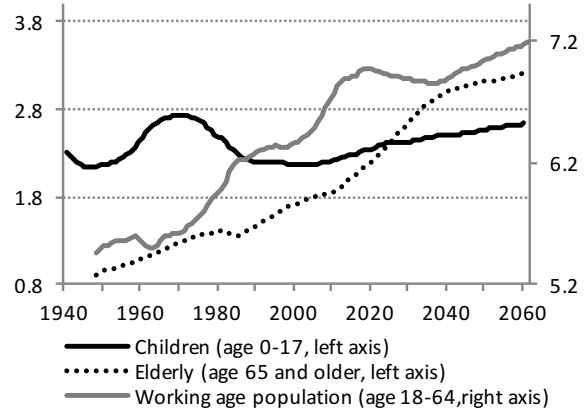**Figure 1:** Age dependency ratio in Belgium (%)   **Figure 2:** Size of three age groups (millions)

Data sources: Federal Planning Bureau, Population forecasts 2015-2060, March 2016; Belgian Federal Government (FPS Economy, Statistics Belgium).

Without behavioural changes in labour supply, education and savings, which may all affect investment in physical capital, this demographic transformation will certainly cause a significant loss in per capita output and income. As a matter of simple arithmetic, if fewer active people are available to produce output for more dependent people, and no one changes his or her behaviour, lower per capita output is unavoidable (Onder and Pestieau, 2014). Fortunately, individual behaviour can change. A large theoretical literature has demonstrated that falling fertility and increasing life expectancy (ageing) will also affect the incentives for individual households and firms to work, to save and to invest in human and physical capital. At least three major questions remain, though: (i) Will these incentives go into the right direction? The literature remains ambiguous; (ii) If they go into the right direction, will they be strong enough to reverse the negative effects of demographic transformation on per capita output? (iii) What is the impact of the underlying components of demographic change (reduced fertility versus increasing life expectancy)?

To the best of our knowledge, the existing literature does not yet provide clear and comprehensive quantitative answers to this set of questions. In a recent survey article, Lee (2016, p. 110) qualifies these or highly similar questions as to be explored further[1]. Our objective, and the main contribution of this paper, is to answer these three questions for Belgium, a small open European economy. We focus on one particular country that we know well. This also seems by far the most realistic way to proceed, given the importance of taking into account country-specific structural and institutional characteristics and the need for quantitative answers[2]. It goes without saying, though, that our approach can also be adopted to study other (small) open economies.

To obtain the answers to our questions, several steps need to be taken. First of all, a rich and realistic parameterized model of the Belgian economy will be necessary. We construct this model in the first part of this paper. It will be a 28-period overlapping generations model for an open economy, facing an exogenous world interest rate, that we calibrate to Belgium. Next to its overlapping generations setup, its main characteristics are the following. First, fertility and life expectancy are exogenous but time-varying. Second, hours worked, human capital and income are all endogenous, which implies the capacity of the model to capture and measure important behavioural responses to ageing. Third, individuals differ not only by age, but also by innate ability. We distinguish individuals with either high, medium or low ability. Individuals with higher ability enter with more human capital. They are also more productive in building additional human capital and skills when they allocate time to education. For individuals with low ability, expanding time in education is not a productive option. Furthermore, while the labour market for high and medium ability individuals is perfectly competitive and clears, above market-clearing wages imply (involuntary) unemployment among low ability individuals. The introduction of different ability types and unemployment is a distinctive element in our model, compared to recent contributions like those of Ludwig et al. (2012), Attanasio et al. (2016) and Marchiori et al. (2017). While these authors only focus on the intergenerational dimension of the impact of population ageing, our model also allows an analysis of the intragenerational dimension. We do this analysis in Devriendt and Heylen (2018a; 2018b). Last but not least, our model includes a rich specification of fiscal policy. At the revenue side, the government can change (progressive) tax rates on labour, capital and consumption. At the expenditure side, it can change non-employment benefits for involuntarily unemployed workers and government purchases of goods and services. It can also allocate resources to maintain financial balance in the public pension system, if necessary. The latter is modelled as a pay-as-you-go (PAYG) system of the defined benefit type.

As a second step, before we use the model for simulations of the future, it should convince in

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[1]Clearly, a lot of highly relevant and related work on the macroeconomic effects of demographic change has already been done, as the survey by Lee (2016) also demonstrates. Quite close to our paper are for example Krueger and Ludwig (2007), Ludwig et al. (2012), Sánchez-Romero (2013), Marchiori et al. (2017) and Attanasio et al. (2016). Future per capita growth (and its underlying determinants) are not the focus of these papers, though.

[2]One may in particular think of the impact of a country's pension system and its labour market characteristics. Bloom et al. (2007) and Dedry et al. (2017) for example have shown that different pension systems may imply very different responses in savings and capital formation. Labour market characteristics may determine to what extent changed incentives to work feed through into employment rather than unemployment.

its capacity to replicate the evolution of key data in the past. We therefore relate the calibrated model's predictions for the old-age dependency ratio, per capita growth, capital formation, employment, education, and inequality to the data in Belgium since about 1960. We find that the model performs well in this respect. Our third step is the computation of our baseline simulation. This simulation quantifies the effects of projected future demographic change on the endogenous variables in our model until 2061, under the assumption of unchanged policies and under the main rules of the current pension system. Major attention will go to our predictions for future per capita growth, and its underlying determinants (employment, physical capital formation, education and human capital formation). Counterfactual simulations will allow us to assess the impact of the separate components of demographic change.

Our predictions are not optimistic. Arithmetically, i.e. for unchanged household and firm behaviour, projected demographic change may cut off about 0.4%-points on average of the annual per capita growth rate in the next 25 years. Although we do observe sizeable (and mostly positive) behavioural adjustments by households and firms to the demographic transformation, their effects can only partially counteract the unfortunate arithmetical consequences of the rapidly increasing dependency ratio. A net negative effect on future per capita growth of 0.29%-points remains. The reasons are multiple. First, some of the behavioural responses are negative, in particular the response of private investment in physical capital. Private investment suffers mainly from the negative effect on the productivity of physical capital induced by reduced fertility and a declining population at working age. Second, the strong increase in savings that we also observe, cannot counteract this negative effect: in an open economy these savings may also be invested abroad (capital outflow). Third, some of the positive behavioural adjustments have already taken place in previous decades.

Decomposing the behavioural responses and their growth effects according to the separate components of demographic change, it is clear from our results that the main problem is reduced (low) fertility after the baby boom. This has not only resulted in a reduced population at working age, with negative effects on investment in physical capital, the retirement of the baby boom generation is now also a major factor behind the growing share of retirees. The induced behavioural effects from rising life expectancy are broadly positive, and almost strong enough to compensate the quantitative effect of longer life spans on the number of retirees.

Our results also contribute to the recent literature on secular stagnation. In line with Gordon (2014) and Cervellati et al. (2017), we provide evidence supporting the hypothesis that the observed and expected long-run slowdown in per capita growth is mainly a supply-side problem, with demographic change as one of the main "headwinds".

The structure of this paper is as follows. In Section 2 we assess the arithmetical effects on per capita growth of projected demographic change in Belgium in comparison to other European countries. We also give a brief overview of the potential behavioural responses of households and firms to demographic change, as discussed in the literature. Section 3 sets out the model. In Section 4 we describe our calibration procedure and the parameterization of the model. Section 5 demonstrates our model's capacity to replicate the evolution since about 1960 of key macro variables. In Section 6 we report our baseline and counterfactual simulation results on

the macroeconomic effects of (separate components of) demographic change until 2061. Section 7 concludes the paper.

2 Demographic change and ageing: unfortunate arithmetic and behavioural effects

In the neoclassical framework that we adopt in this paper, the long-run per capita growth rate is determined solely by the exogenous rate of technical progress. In long transition periods, however, per capita growth can be much higher or much lower than technical progress. Demographic change is a prominent possible cause of such growth deviations. Consider the following decomposition of per capita output

$$\frac{Y_t}{Pop_t} = \frac{Y_t}{Hours_t}\frac{Hours_t}{N_t^w}\frac{N_t^w}{Pop_t},$$

where $\frac{Y_t}{Hours_t}$ is real output per hour worked (or labour productivity), $\frac{Hours_t}{N_t^w}$ hours worked per person at working age and $\frac{N_t^w}{Pop_t}$ the share of people at working age in total population. Taking growth rates, it then follows that

$$\frac{\Delta Y}{Y} - n_{pop} = g_\pi + (n_{hours} - n_N^w) - (n_{pop} - n_N^w).$$

The left side of this equation is the annual per capita economic growth rate. On the right, g_π is the annual growth rate of labour productivity, n_{pop} the growth rate of total population, n_{hours} the growth rate of total hours worked, and n_N^w the growth rate of population at working age. As can easily be seen, demographic change pushes per capita growth below the growth rate of labour productivity when total population grows faster than population at working age (= rising dependency, $n_{pop} > n_N^w$) and/or when population at working age grows faster than aggregate hours worked (= falling employment rate, $n_{hours} < n_N^w$). In the very long run dependency and employment rates will be constant, and labour productivity and per capita output will grow at the same rate as technology. In long transition periods, however, things may be very different. Figure 1 showed us the evolution of the past and projected future overall dependency ratio in Belgium. Figure 3 reports the arithmetical effects of this changing dependency on per capita growth. We compare Belgium to a few other countries and the European average. Ceteris paribus, the projected unprecedented rise in the dependency ratio between 2010 and 2040 will have significant adverse effects on future per capita growth. In the next 25 years it may cut off almost 0.4%-points on average from annual per capita growth[3]. Compared to other countries, Belgium takes an intermediate position. In Germany and in the EU28 on average the negative arithmetical effects of rising dependency are larger (about 0.5%-points on average in the next 25 years). They are weaker in a country like Sweden (about 0.2%-points). In all countries, and Germany in particular, these negative arithmetical effects are the strongest in 2020-35.

The negative effects of a rising dependency ratio on per capita growth can be countered if countries succeed in raising hours worked among the smaller group of people at working age or the

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[3]As a reference for the relative importance of this effect, note that the Belgian Studiecommissie voor de Vergrijzing (2016) projected an average per capita growth rate of 1.2% in 2021-2040.

Figure 3: Arithmetical effect of changing dependency on per capita growth ($n_N^w - n_{pop}$, in %-points)
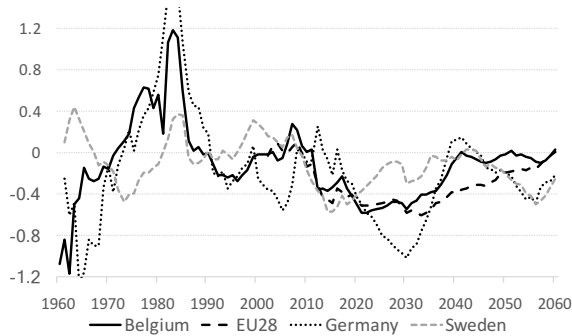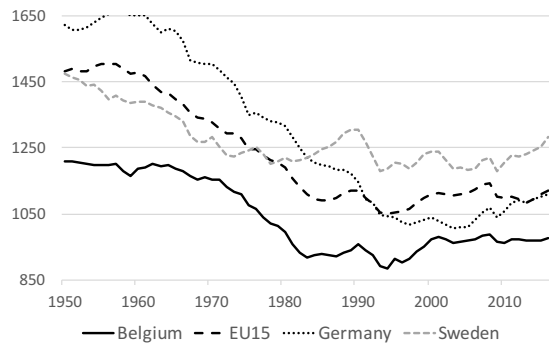
Figure 4: Average annual hours worked per person at working age

Data sources: Figure 3: see sources of Figures 1 and 2 for Belgium. EU-countries: Eurostat, population statistics and projections (baseline projections); Figure 4: OECD, 2018, Historical population data; The Conference Board Total Economy Database, 2018.

growth rate of labour productivity (output per hour). Despite some progress in the second half of the 1990s, Belgium is one of the countries with the poorest labour market performance (Figure 4). Moreover, in the last decade per capita hours worked have again stagnated. The employment gap between Belgium and other countries is particularly large for older workers and low-skilled workers (OECD, 2015). At the same time, this gap also creates huge opportunities to counter low per capita growth if Belgium manages to mobilize this enormous amount of non-employed labour. As to the growth rate of labour productivity, Belgium also lags behind other countries. In 1996-2016 the annual growth rate of real GDP per hour worked in Belgium was 1%. In the EU28 it was 1.3% (OECD.Stat, Productivity database, 2017).

The effects of demographic transformation on per capita growth in the medium to long run are not limited, however, to the arithmetic described above. Demographic change also implies a change in the structure of the population, and thus in the relative size of different age groups with different behaviour. Moreover, and maybe more important, demographic change will also affect the incentives for individual households to supply labour, to invest in human capital, and to save. Likewise, it will affect the incentives for firms to hire workers and to invest in physical capital. Changes in investment behaviour will affect labour productivity. It is unclear from the literature, however, whether the net effect of all these incentives will be positive for growth or negative. And if it is positive, whether it will be strong enough to reverse the arithmetical effect shown in Figure 3.

When it comes to aggregate savings, there are reasons that can justify a decline in response to demographic change, as well as reasons that would predict an increase. Whereas middle aged and older workers are net savers, young people and (old) retirees are generally described as borrowers or dissavers. Considering the demographic transition in Figure 2, the obvious expectation would be that aggregate savings will gradually be declining as the rising share of dependent versus active people should soon feed through in a rising fraction of dissavers (see e.g. IMF, 2014; Goodhart and Erfurth, 2014). Increasing life expectancy, however, may imply the opposite. Individuals prefer to smooth consumption over their whole lifetime. The perspective of

living longer may then force them to increase their savings during their active life (Krueger and Ludwig, 2007; Onder and Pestieau, 2014). A key element determining the extent to which this will happen, is the retirement decision and the length of working life. The ambition to maintain consumption during old age will also provide an incentive for individuals to work longer and to postpone retirement, at least if good health and productivity can be assumed (Bloom et al., 2007; Heijdra and Reijnders, 2018). The stronger the reaction in labour supply, the smaller will be the need to increase savings at middle age.

Increasing life expectancy does not only provide an incentive to work longer. It will also increase the incentive for individuals to invest in education, at least if they have the ability to do so. The reason is obvious. If people expect to live longer - starting with reduced mortality during normal working ages - and if they expect to work longer, accumulated human capital will be productive for a longer period of time. All this raises the return to investment in education (Ben-Porath, 1967; Cervellati and Sunde, 2013; Ludwig et al., 2012; Heylen and Van de Kerckhove, 2013).

The response of individuals regarding saving, working and schooling, will matter a great deal for firms' investment in physical capital. For given (constant) individual behaviour, demographic change will most likely imply disinvestment. If fertility and the size of working age population decline, this will cause an increase in the capital-labour ratio and reduce the marginal productivity of physical capital. The lower rate of return to physical capital may then lead to a fall in investment (Ludwig et al., 2012; Heylen and Van de Kerckhove, 2013)[4]. If individuals adjust their behaviour, however, the outcome may be very different. An increase in savings may push down the interest rate, and reduce the cost of investment. Increased labour supply and/or postponement of retirement, and increased accumulation of human capital will raise the marginal productivity of physical capital, and counteract the negative effect of a decline in population aged 18 to 64. An important element here is the extent to which the interest rate can indeed fall. Given our focus on Belgium, we model in this paper an open economy. A fall in the interest rate and an increase in domestic investment are then not obvious, as savings may simply be invested abroad (capital outflow). This will be the case if the rate of return to physical capital abroad is higher than at home. In the end, what matters is relative ageing in the home economy versus abroad, as well as the relative response of labour supply and schooling.

If one thing is clear from this discussion, it is that demographic change affects household and firm behaviour in many ways. Uncovering overall net effects on the macro economy will require a coherent general equilibrium model. It is our aim to construct this model in Section 3.

3   The model

We assume an open economy with an exogenous but time-varying world interest rate. Physical capital moves freely across borders. Human capital and labour however are assumed internationally immobile. Time-varying exogenous fertility and survival rates drive demographic change. Twenty-eight generations of individuals coexist. Individuals enter the model at the age of 18.

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[4]Not everyone agrees, however. According to Goodhart and Erfurth (2014), a shrinking working age population may imply the end of cheap labour. The relative cost of capital may then fall, which will encourage firms to invest in physical capital.

They live at most for 28 periods of 3 years. Within each generation one fraction of the individuals is assumed to have low innate ability, others have medium ability, a third group has high innate ability. Depending on their ability, individuals will enter the model with a different initial human capital endowment and with a different productivity of schooling. Young individuals with high or medium ability will continue education when they enter the model at 18. Individuals with low ability will not. The introduction of differences in individuals' ability and productivity of schooling is a new element compared to existing models to study the economic effects of demographic change (see e.g. Ludwig et al., 2012; Attanasio et al., 2016; Marchiori et al., 2017). Next to endogenous education and human capital, our model also has endogenous employment. Besides studying (for high and medium ability individuals) everyone optimally allocates time to labour and leisure. The labour market for high and medium ability households is assumed perfectly competitive. The labour market for low ability households is imperfectly competitive. We assume the existence of a union that sets wages for low ability workers. Above market-clearing wages may cause involuntary unemployment. As to output, domestic firms are modelled to employ physical capital and effective labour under constant returns to scale. Technology is assumed to have exogenous growth. The government in our model sets tax rates on labour (both on employees and employers), consumption and capital income. It allocates its resources to goods and services, non-employment benefits and pensions (to finance possible deficits in the public pay-as-you-go system). It may also borrow.

Concerning notation, superscript t denotes the time an individual or group of individuals (a generation) enter the model. Subscript j refers to the j-th period of life or, in other terms, the age. It goes from 1 to 28[5]. When a subscript s is used, it denotes one of three levels of innate ability: low (L), medium (M) or high (H). Last but not least, time subscripts t added to aggregate variables indicate historical time.

3.1 Demography

Demographic change in our model is captured by the evolution over time of fertility and survival rates, with the latter determining individuals' expected length of life. Equation (1) expresses the size of the youngest or 'newborn' generation at time t relative to the size of the youngest generation at t-1, following among others de la Croix et al. (2013). f_t (> 0) is the time-dependent 'fertility' rate in the model.

$$N_1^t = f_t N_1^{t-1} \tag{1}$$

Equation (2) describes the evolution of the size of a specific generation over time. We denote by sr_{j-1}^t (< 1) the probability for each individual of generation t to survive until model age j conditional on reaching age $j - 1$. This survival rate is both generation and age-dependent.

$$N_j^t = sr_{j-1}^t N_{j-1}^t, \text{ for } j = 2 - 28 \tag{2}$$

The trajectories of both f_t and sr_j^t are taken as exogenous in our model[6]. Figures 5a and 5b

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[5] Note that life starts at age 1 and not at age 0. N_3^t for example denotes the total size of the generation that entered the model at time t when this generation is at model age 3. That will be the case in time period $t + 2$.

[6] In this respect we follow Lee's (2016, p.111) recommendation. While there are clearly theories available to relate fertility and mortality to individual choices, his argument is that these theories have little proven predictive power. Any choice would then be rather arbitrary and might obscure the working of better-founded mechanisms.

show the data. Finally, the population consists of low, medium and high ability agents:

$$N_j^t = N_{j,L}^t + N_{j,M}^t + N_{j,H}^t$$
$$= (v_L + v_M + v_H)N_j^t \tag{3}$$

with v_s denoting their respective shares and $\sum_{s=L,M,H} v_s = 1$. Assuming the fertility and survival rates to be equal across ability types, these shares will be constant[7].

Figure 5: Demographic change in Belgium

(a) Fertility rate since 1948   **(b)** Age-specific conditional survival rates
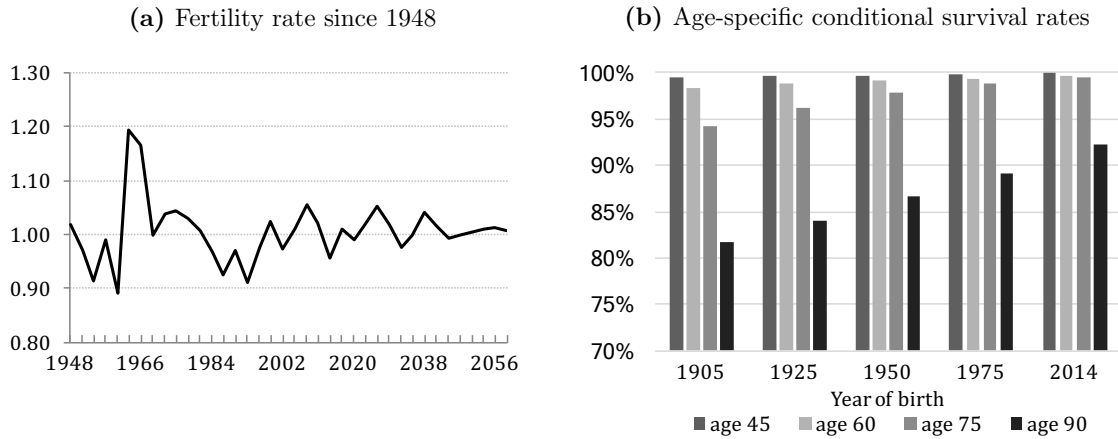
Data source: see Figures 1 and 2. Computational details are provided in Appendix D.1.

Note: the year indicated on the horizontal axis of Figure 5a is the first of the three years constituting a period in our model.

The fertility rate f_t in Figure 5a is computed as the ratio of the size of the population of actual age 18 to 20 in a particular three-year period to its size three years earlier. We observe a peak in 1963-65 and 1966-68, when the after-war baby boomers became 18. Both periods reveal a growth rate of the youngest generation by almost 20% compared to the previous period. After a few more periods with $f_t > 1$ in the 1970s, we observe a decline of the youngest generation ($f_t < 1$) in the 1980s and 1990s. Later decades show a pattern of dampened oscillation. Figure 5b shows the evolution over time of conditional survival rates, sr_j^t, for individuals born in 1905, 1925, 1950, 1975 and 2014. We observe an overall rise, with higher probabilities to survive at higher age, for people born later. The increase is particularly strong at ages 75 and 90. A child born in Belgium can now (unconditionally) expect to reach age 90 with a probability of more than 60%. As a result, a child born in 1950 that probability was less than 30%.

In line with our assumption that only physical capital is internationally mobile, we do not model migration in this paper. Our data for the exogenous fertility rate do, however, capture the impact of migration of individuals not older than 20. The data also count the children of migrants when these children reach age 18-20 (model age 1)[8].

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[7]What we have in mind, is that ability reflects individuals' IQ, the level and distribution of which are seen as constant. The assumption of constant ability levels and constant shares v_s does not exclude, however, that over time the average skill level of the population increases. This is possible in our model when the individuals of medium or high ability choose to study more and accumulate more human capital than earlier generations.

[8]See Appendix D. As we show later in this paper (Figure 9f), with this approach we are able to produce an old-age dependency ratio that matches reality very well.

3.2 Households

Individuals of the same generation and ability are grouped in households of unitary mass (Merz, 1995; Andolfatto, 1996; Boone and Heylen, 2018). As low ability individuals can be involuntarily unemployed, their household will consist both of a fraction of unemployed (u) and a fraction of employed members ($1 - u$). They pool their income, so consumption is equalized across household members. Medium and high ability households have only employed members.

3.2.1 Instantaneous household utility

The instantaneous utility function of a representative high or medium ability household of age j born at time t depends positively on consumption $c_{j,s}^t$ and leisure time $l_{j,s}^t$ (Equation 4). Preferences are logarithmic in consumption and iso-elastic in leisure. The intertemporal elasticity of substitution in leisure is $1/\theta$. Furthermore, γ_j indicates the relative utility value of leisure versus consumption. It may differ by age. The instantaneous utility function of a low ability household of age j is the same except that only the leisure time of the employed fraction of the household ($1 - u_{t+j-1}$) is taken into account (Equation 5). Implicitly, leisure of the unemployed fraction is assumed to be neutral for household utility.

$$u(c_{j,s}^t, l_{j,s}^t) = \ln c_{j,s}^t + \gamma_j \frac{(l_{j,s}^t)^{1-\theta}}{1-\theta}, \ \text{ for } s = H, M \tag{4}$$

$$u(c_{j,L}^t, l_{j,L}^t) = \ln c_{j,L}^t + \gamma_j \left(1 - u_{t+j-1}\right) \frac{(l_{j,L}^t)^{1-\theta}}{1-\theta} \tag{5}$$

with $\gamma_j > 0$ and $\theta > 0$ ($\theta \neq 1$).

3.2.2 Expected lifetime utility

A household that enters the model at time t maximizes expected lifetime utility described by Equation (6) subject to its budget and time constraints (cf. infra). In this equation β is the discount factor and π_j^t the unconditional probability to survive until age j.

$$U^t = \sum_{j=1}^{28} \beta^{j-1} \pi_j^t u(c_{j,s}^t, l_{j,s}^t) \tag{6}$$

with $0 < \beta < 1$, $\pi_1^t = 1$, $0 < \pi_j^t = \prod_{i=1}^{j-1} sr_i^t < 1$ for $1 < j < 29$, and $\pi_{29}^t = 0$.

3.2.3 Time constraints

Every period, an individual is endowed with one unit of time that can be split into hours worked while employed (n), education (e) and leisure (l) depending on age and innate ability. Equations (7) to (9) describe the age-dependent time constraints for medium and high ability individuals ($s = M, H$). Only in the first 4 periods an individual can spend time in post-secondary education next to working and enjoying leisure. From period 5 until 15, time can exclusively be allocated to labour and leisure. From period and age 16 onwards an agent is eligible for public old-age pensions. Equations (10) and (11) relate to low ability individuals. Since these individuals start working earlier than individuals of medium or high ability, they can also leave the labour market

earlier. They receive a public pension from period and age 15 onwards[9]. Unemployed low ability individuals cannot choose hours worked or leisure (for them $n_{j,L}^t = 0$). They only have 'leisure'. As mentioned before, this does not carry positive utility.

$$\text{for } j = 1 - 4 \, (age \, 18 - 29) \qquad : \qquad l_{j,s}^t = 1 - n_{j,s}^t - e_{j,s}^t \qquad (7)$$

$$\text{for } j = 5 - 15 \, (age \, 30 - 62) \qquad : \qquad l_{j,s}^t = 1 - n_{j,s}^t \qquad (8)$$

$$\text{for } j = 16 - 28 \, (age \, 63 - 101) \qquad : \qquad l_{j,s}^t = 1 \qquad (9)$$

$$\text{for } j = 1 - 14 \, (age \, 18 - 59) \qquad : \qquad l_{j,L}^t = 1 - n_{j,L}^t \qquad (10)$$

$$\text{for } j = 15 - 28 \, (age \, 60 - 101) \qquad : \qquad l_{j,L}^t = 1 \qquad (11)$$

3.2.4 Budget constraints

Households have varying budget constraints over their life cycle depending on age and innate ability. Equation (12) describes the budget constraint faced by households of high and medium ability during active life (periods 1 to 15).

$$(1 + \tau_c)c_{j,s}^t + a_{j,s}^t = (1 + r_{t+j-1})(a_{j-1,s}^t + tr_{t+j-1}) + w_{t+j-1}^s \varepsilon_j h_{j,s}^t n_{j,s}^t (1 - \tau_{w,j,s})$$
$$- T_{j,s} n_{j,s}^t + z_{t+j-1}, \text{ for } s = H, M, \, j = 1 - 15 \qquad (12)$$

Disposable income is used to consume $c_{j,s}^t$ and accumulate non-human wealth. We denote by $a_{j,s}^t$ the stock of wealth held by a type s individual at the end of the j-th period of his life. τ_c is the tax rate applied by the government on consumption goods. When individuals assign a fraction $n_{j,s}^t$ of their time to work, with productive efficiency $\varepsilon_j h_{j,s}^t$, they earn a net labour income of $w_{t+j-1}^s \varepsilon_j h_{j,s}^t n_{j,s}^t (1 - \tau_{w,j,s})$. Underlying factors are the real gross wage per unit of effective labour of ability type s (w_{t+j-1}^s), an exogenous parameter linking productivity to age (ε_j), human capital ($h_{j,s}^t$), and the average labour income tax rate ($\tau_{w,j,s}$). The contribution rate cr_1 of workers to the public pension system is included in $\tau_{w,j,s}$. Engaging in work, however, also induces costs related to child care and transportation $T_{j,s}$. Moreover, if the individuals in a household work more days a week and more weeks per year, implying higher n, these costs will rise. This explains why we have $T_{j,s} n_{j,s}^t$ in the budget constraint. One reason for T to depend on ability is for example the use of company cars, which make transportation cheaper typically for higher ability individuals. A reason for T to depend on age is the need (to pay for) for child care at low j, when households have children.

Next to labour income, disposable income consists of interest income earned on assets, $r_{t+j-1} a_{j-1,s}^t$ with r_{t+j-1} the exogenous world real interest rate, and lump sum transfers received from the government z_{t+j-1}. A final source of income are transfers from accidental bequests

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[9]This assumption also reflects reality in Belgium. Until 2013 it was possible to retire and receive public pension benefits at age 60. Moreover, many mainly lower educated workers left the labour market even sooner with unemployment-related benefits. Appendix B shows data highlighting the much earlier effective retirement of lower versus higher educated workers in Belgium. Recent Belgian governments have gradually raised the minimum age at which individuals are eligible to public pensions. In 2019 it will be 63. The statutory retirement age will be lifted from 65 now to 67 in 2030.

tr_{t+j-1} (plus interest). There are no annuity markets in our model. Transfers are uniformly distributed among the population (N_k):

$$tr_k = \frac{1}{N_k}\left[\sum_s \sum_{j=1}^{28}\left(1 - sr_j^{k-j}\right)a_{j,s}^{k-j}N_{j,s}^{k-j}\right]$$

From the eligible pension age $j = 16$ onwards individuals of high and medium ability receive public pension benefits $ppt_{j,s}^t$ (see also Section 3.3). Equation (13) presents these individuals' budget constraint.

$$(1+\tau_c)c_{j,s}^t + a_{j,s}^t = (1+r_{t+j-1})(a_{j-1,s}^t + tr_{t+j-1}) + ppt_{j,s}^t + z_{t+j-1}$$
$$\text{for } s = M, H, \ j = 16 - 28. \tag{13}$$

The budget constraint of low ability households at working age (Equation 14) looks slightly different. Here, only the employed fraction of the representative household $(1 - u_{t+j-1})$ earns a labour income while the unemployed part receives an unemployment benefit related to what they would have earned when employed. The policy parameter b indicates the gross benefit replacement rate.

$$(1+\tau_c)c_{j,L}^t + a_{j,L}^t = (1+r_{t+j-1})(a_{j-1,L}^t + tr_{t+j-1}) + bw_{t+j-1}^L\varepsilon_j h_{j,L}^t n_{j,L}^t u_{t+j-1}$$
$$+w_{t+j-1}^L\varepsilon_j h_{j,L}^t n_{j,L}^t(1-\tau_{w,j,L})\left(1 - u_{t+j-1}\right) - T_{j,L}n_{j,L}^t\left(1 - u_{t+j-1}\right) + z_{t+j-1} \tag{14}$$
$$\text{for } j = 1 - 14.$$

After retirement (from age $j = 15$ onwards) the budget constraint of low ability households is the same as the one of high or medium ability households.

$$(1+\tau_c)c_{j,L}^t + a_{j,L}^t = (1+r_{t+j-1})(a_{j-1,L}^t + tr_{t+j-1}) + ppt_{j,L}^t + z_{t+j-1}, \text{ for } j = 15 - 28. \tag{15}$$

All households in our model are born without assets. They also plan to consume all accumulated assets by the end of their life. A final assumption is that retired individuals cannot have negative assets. Algebraically, $a_{0,s}^t = a_{28,s}^t = 0$ and $a_{j,s}^t \geq 0$ for $j > 15$ (for $s = H, M$) or 14 (for $s = L$).

3.3 The pension system

Our model includes a public pay-as-you-go (PAYG) pension scheme of the defined benefit type that makes pension payments to retirees out of contributions (taxes) paid by current workers and firms. Individuals receive a pension benefit from model age $j = 16$ (for $s = H, M$) or $j = 15$ (for $s = L$) onwards, i.e. respectively actual age 63 or 60. The amount $ppt_{j,s}^t$ they receive at the time of retirement is

$$ppt_{16,s}^t = rr_s\left\{\frac{1}{15}\sum_{j=1}^{15} w_{t+j-1}^s\varepsilon_j h_{j,s}^t n_{j,s}^t(1-\tau_{w,j,s})\prod_{l=j}^{15} wg_{t+l}\right\}, \text{ for } s\text{=H,M} \tag{16a}$$

or

$$ppt_{15,L}^t = rr_L\left\{\frac{1}{14}\sum_{j=1}^{14} w_{t+j-1}^L\varepsilon_j h_{j,L}^t n_{j,L}^t(1-\tau_{w,j,L})\prod_{l=j}^{14} wg_{t+l}\right\} \tag{16b}$$

The pension benefit is related to one's own contributions during active life. More precisely, the pensioner receives a fraction of the average of revalued earlier net labour income. In Equation (16), rr_s is the net replacement rate, which can differ by ability (income), and wg is the period-wise revaluation factor applied to net labour income earned in the past. The pension will rise in the earned wage, the individual's hours of work and his productive efficiency with the latter also increasing in human capital. For retired low ability households the pension amount looks very similar, except for the lower eligibility age of 60 (model age 15). Underlying this result, is our assumption that periods of unemployment, as experienced by some household members, are considered equivalent to periods of work[10].

After the initial pension payment, the pension benefit may be revalued to adjust for a changed living standard, so $ppt_{j,s}^t$ then becomes

$$ppt_{j,s}^t = ppt_{16,s}^t \prod_{l=16}^{j-1} pg_{t+l}, \text{ for j>16 and } s=H,M \tag{17a}$$

or

$$ppt_{j,L}^t = ppt_{15,L}^t \prod_{l=15}^{j-1} pg_{t+l}, \text{ for j>15} \tag{17b}$$

with pg_k the coefficient that revalues the pension benefit of period $k-1$ to k[11].

The public pension system's budget identity is as follows:

$$\sum_{s=M,H} \sum_{j=16}^{28} N_{j,s}^{t+1-j} ppt_{j,s}^{t+1-j} + \sum_{j=15}^{28} N_{j,L}^{t+1-j} ppt_{j,L}^{t+1-j} =$$

$$cr \sum_{s=M,H} \sum_{j=1}^{15} N_{j,s}^{t+1-j} n_{j,s}^{t+1-j} w_t^s \varepsilon_j h_{j,s}^{t+1-j} + cr \sum_{j=1}^{14} N_{j,L}^{t+1-j} (1-u_t) n_{j,L}^{t+1-j} w_t^L \varepsilon_j h_{j,L}^{t+1-j} + GPP_t \tag{18}$$

with $cr = cr_1 + cr_2$.

The left side of Equation (18) indicates total pension expenditures at time t. As public pensions are organized on a PAYG basis, this amount is financed by a) the working population from taxes on their gross labour income applying contribution rate cr_1 and by b) the firms applying cr_2. In Equations (12) and (14), cr_1 is thus part of the labour tax rate $\tau_{w,j,s}$. Tailored to institutional reality in Belgium, GPP_t denotes the total resources assigned to pension payments by the government to ensure that Equation (18) holds.

3.4 Human capital production

Individuals enter the model at the age of 18 with a predetermined ability-specific endowment of human capital. In Equation (19), h_0 stands for the initial time-invariant human capital endowment of a high ability individual. Low and medium ability individuals are respectively endowed with lower human capital stocks $\omega_L h_0$ and $\omega_M h_0$ with $0 < \omega_L < \omega_M < \omega_H = 1$.

$$h_{1,s}^t = \omega_s h_0 \tag{19}$$

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[10]This assumption is in line with reality in Belgium in the previous decades.

[11]Our modelling of the revaluation factors wg and pg reflects the rules of the Belgian public pension system. We provide details on the evolution over time of these factors in Appendix D

High and medium ability individuals can engage in higher education to accumulate additional human capital in the first four periods (Equation 20a). ϕ_s is a positive ability-related efficiency parameter reflecting the productivity of schooling, and σ the elasticity of human capital growth with respect to time input. After the first four periods, human capital remains constant (Equation 20b). We assume that learning-by-doing offsets depreciation. Constant human capital, however, doesn't imply constant productive efficiency $h_j^t \varepsilon_j$, as there is still variation in the exogenous age-productivity link ε_j.

$$h_{j+1,s}^t = h_{j,s}^t \left(1 + \phi_s (e_{j,s}^t)^\sigma\right) \qquad \text{for } j = 1-4, s = H, M \qquad (20a)$$
$$= h_{j,s}^t \qquad \text{for } j \geq 5, s = H, M \qquad (20b)$$

with: $0 < \sigma < 1, \ \phi_H, \phi_M > 0$.

Individuals with low innate ability do not study at the tertiary level. Their human capital remains constant at the initial level:

$$h_{j+1,L}^t = h_{1,L}^t. \quad \forall j \qquad (21)$$

3.5 Household optimization

Low ability individuals will choose consumption and labour supply to maximize Equation (6), taking into account their instantaneous utility function (Equation 5), their time and budget constraints (Equations 10, 11, 14 and 15) and the human capital process (Equations 19 and 21). Individuals of medium and high ability will in addition choose the fraction of time they spend in education when young. They optimize Equation (6), taking into account Equation (4), and their time and budget constraints and the human capital formation process described by Equations (7)-(9), (12)-(13) and (19)-(20). For details on the optimality conditions, we refer to Appendix B.

3.6 Firms, output and factor prices

Firms act competitively on the output market. The constant returns to scale production function to produce a homogeneous good is given by

$$Y_t = K_t^\alpha (A_t H_t)^{1-\alpha} \quad with \quad 0 < \alpha < 1 \qquad (22)$$

$$A_t = (1 + g_{a,t}) A_{t-1} \qquad (23)$$

In Equation (22), K_t is the stock of physical capital at time t, while $A_t H_t$ indicates employed labour in efficiency units at that time. Technical progress is labour augmenting and occurs at an exogenous rate $g_{a,t}$. Total effective labour H_t is defined in Equation (24) as a CES-aggregate of effective labour performed by the three ability groups. In this equation λ is the elasticity of substitution between the different ability types of labour, and η_L, η_M and η_H are the input share parameters. We will impose that they sum to 1.

$$H_t = \left(\eta_H H_{H,t}^{1-(1/\lambda)} + \eta_M H_{M,t}^{1-(1/\lambda)} + \eta_L H_{L,t}^{d,1-(1/\lambda)}\right)^{\lambda/(\lambda-1)} \qquad (24)$$

Effective labour supply by the high and medium ability group is specified as

$$H_{s,t} = \sum_{j=1}^{15} N_{j,s}^{t-j+1} n_{j,s}^{t-j+1} h_{j,s}^{t-j+1} \varepsilon_j, \text{ for } s=H,M \tag{25}$$

Our assumption of a competitive labour market for high and medium ability individuals implies that the total supply of effective labour will equal demand and employment for these individuals ($H_{s,t} = H_{s,t}^d$ for $s = M, H$). For low ability households, however, effective employment is lower than supply ($H_{L,t}^d < H_{L,t}$, Equation 26). There is unemployment.

$$H_{L,t}^d = (1 - u_t) \sum_{j=1}^{14} N_{j,L}^{t-j+1} n_{j,L}^{t-j+1} h_{j,L}^{t-j+1} \varepsilon_j \tag{26}$$

$$= (1 - u_t) H_{L,t} \tag{27}$$

This brings us to wage formation and union involvement. For medium and high ability labour, the labour market and wage formation are assumed to be competitive. The total wage cost per unit of effective labour will be equal to the market-clearing marginal labour productivity (Equation 28). τ_p is the employer social contribution rate. It includes the contribution cr_2 to the public pension system.

$$(1 - \alpha) A_t \left(\frac{K_t}{A_t H_t} \right)^\alpha \eta_s \left(\frac{H_t}{H_{s,t}} \right)^{\frac{1}{\lambda}} = w_t^s (1 + \tau_p), \text{ for } s=H,M \tag{28}$$

For low ability labour, however, wages will be above the competitive level. The existence of minimum wages or union influence are obvious possible explanations. As in Sommacal (2006) and Fanti and Gori (2011) firms will therefore choose the optimizing unemployment rate among low ability individuals (Equation 29).

$$(1 - \alpha) A_t \left(\frac{K_t}{A_t H_t} \right)^\alpha \eta_L \left(\frac{H_t}{(1 - u_t) H_{L,t}} \right)^{\frac{1}{\lambda}} = w_t^L (1 + \tau_p) \tag{29}$$

In the spirit of Boone and Heylen (2018), we assume a union which sets the union wage w_t^L in Equation (30) as a markup on top of a reference wage. This reference wage is a weighted average of the competitive wage $w_t^{L,c}$, the average wage of the medium and high ability group $\left(w_t^M + w_t^H \right) / 2$ and the unemployment benefit. The competitive wage is the hypothetical wage that would occur if there were no unemployment among low ability households. The weights q_1, q_2 and q_3 sum to 1. We take their values from Boone and Heylen (2018). μ is the wage premium which we calibrate.

$$w_t^L = \left(q_1 w_t^{L,c} + q_2 \frac{w_t^M + w_t^H}{2} + q_3 b w_t^L \right) (1 + \mu) \tag{30}$$

Furthermore, firms install physical capital up to the point where the after-tax marginal product of capital net of depreciation equals the exogenous world interest rate r_t:

$$\left[\alpha \left(\frac{A_t H_t}{K_t} \right)^{1-\alpha} - \delta \right] (1 - \tau_k) = r_t \tag{31}$$

with δ the depreciation rate of physical capital, and τ_k a tax paid by firms on capital returns. If the net marginal product of capital exceeds the world interest rate, capital will flow in until equality is restored. For a given interest rate, firms will install more capital when the amount of effective labour increases or the capital tax rate falls.

3.7 Fiscal Government

Equation (32) describes the government's budget constraint. Its revenues consist of taxes on labour income paid by workers T_{nt}, taxes on capital T_{kt}, employer taxes on labour income T_{pt} and consumption taxes T_{ct}. They are allocated to interest payments on outstanding debt $r_t B_t$, spending on goods and services G_t, pension payments GPP_t, unemployment benefits UB_t and lump sum transfers Z_t. Note in Equation (33) our assumption that the government claims a fraction g of GDP for its expenditures on goods and services. Fiscal deficits explain the issuance of new government bonds $(B_{t+1} - B_t)$.

$$B_{t+1} - B_t = r_t B_t + G_t + GPP_t + UB_t + Z_t - T_{nt} - T_{kt} - T_{ct} - T_{pt} \tag{32}$$

with:

$$G_t = gY_t \tag{33}$$

$$UB_t = b \sum_{j=1}^{14} w_t^L \varepsilon_j h_{j,L}^{t+1-j} n_{j,L}^{t+1-j} u_t N_{j,L}^{t+1-j} \tag{34}$$

$$GPP_t = \text{ see Equation (18)} \tag{35}$$

$$T_{nt} = \sum_{s=M,H} \sum_{j=1}^{15} (\tau_{w,j,s} - cr_1) w_t^s \varepsilon_j h_{j,s}^{t+1-j} n_{j,s}^{t+1-j} N_{j,s}^{t+1-j} \tag{36}$$

$$+ (1 - u_t) \sum_{j=1}^{14} (\tau_{w,j,L} - cr_1) w_t^L \varepsilon_j h_{j,L}^{t+1-j} n_{j,L}^{t+1-j} N_{j,L}^{t+1-j}$$

$$T_{kt} = \tau_k K_t \left[\alpha \frac{Y_t}{K_t} - \delta \right] = \tau_k [\alpha Y_t - \delta K_t] \tag{37}$$

$$T_{ct} = \tau_c \sum_s \sum_{j=1}^{28} N_{j,s}^{t+1-j} c_{j,s}^{t+1-j} \tag{38}$$

$$T_{pt} = (\tau_p - cr_2) \sum_s w_t^s H_{s,t}^d \text{ , with } H_{s,t}^d = H_{s,t} \text{ for } s = H, M \text{ and } H_{L,t}^d = (1 - u_t) H_{L,t} \tag{39}$$

$$Z_t = z_t \sum_{j=1}^{28} N_j^{t+1-j} \tag{40}$$

Labour income taxes paid by workers $\tau_{w,j,s}$ are progressive. We model this by using a tax function

$$\tau_{w,j,s} = \Gamma \left(\frac{y_{j,s}}{\tilde{y}} \right)^\xi + cr_1 \text{ with } s = L, M, H, \xi \geq 0, 0 < \Gamma < 1 \tag{41}$$

where $y_{j,s}$ is gross labour income of a household of ability s and age j, and $\tilde{y}$ is average gross labour income. As we have mentioned before, the pension contribution rate cr_1 is a (non-progressive) part of the labour tax rate. As in Guo and Lansing (1998) and Koyuncu (2011), ξ and Γ govern the level and slope of the tax schedule. The marginal tax rate $\tau_{w^m,j,s}$ is the rate applied to the last euro earned:

$$\tau_{w^m,j,s} = \frac{\partial (\tau_{w,j,s} y_{j,s})}{\partial y_{j,s}} = (1 + \xi) \Gamma \left(\frac{y_{j,s}}{\tilde{y}} \right)^\xi + cr_1 \tag{42}$$

This means that the marginal tax rate is higher than the average tax rate when $\xi > 0$, i.e. the tax schedule is said to be progressive. Households are aware of the progressive structure of the

tax system when making decisions. Note that, consequently, in the budget constraints average tax rates are used, while in the first-order conditions (cf. Appendix B) marginal tax rates are used.

3.8 Aggregate equilibrium and the current account

Optimal behaviour by firms and households and government spending underlie aggregate domestic demand for goods and services in the economy. Our assumption that the economy is open implies that aggregate domestic demand may differ from supply and income, which generates international capital flows and imbalance on the current account. Equation (43) describes aggregate equilibrium defined for all generations living at time t. The LHS of this equation represents national income. It is the sum of domestic output Y_t and net factor income from abroad $r_t F_t$ where F_t stands for net foreign assets at the beginning of t. Aggregate accumulated private wealth is denoted by Ω_t. It can be allocated to physical capital K_t, domestic government bonds B_t and foreign assets F_t (Equation 44). At the RHS of Equation (43) CA_t stands for the current account in period t. Equation (45) denotes that a surplus on the current account translates into more foreign assets. Equation (46) is the well-known identity relating investment to the evolution of the physical capital stock.

$$Y_t + r_t F_t = C_t + I_t + G_t + CA_t \tag{43}$$

with:

$$F_t = \Omega_t - K_t - B_t \tag{44}$$
$$CA_t = F_{t+1} - F_t = \Delta\Omega_{t+1} - \Delta K_{t+1} - \Delta B_{t+1} \tag{45}$$
$$I_t = \Delta K_{t+1} + \delta K_t \tag{46}$$

4   Parameterization and calibration procedure

We have taken a first set of parameters from the literature or from existing datasets. For the discount factor β we impose a value of 0.9423, which is equivalent to a rate of time preference equal to 2% per year (see e.g. Kotlikoff et al., 2007). The value of θ, i.e. the reciprocal of the intertemporal elasticity to substitute leisure, is 2. Estimates for this parameter used in the literature, lie somewhere between 1 and 10. Micro studies often reveal very low elasticities (i.e. high θ). However, given our macro focus, these studies may not be the most relevant ones. Rogerson and Wallenius (2009) show that micro and macro elasticities may be unrelated. Rogerson (2007) also adopts a macro framework. He puts forward a reasonable range for θ from 1 to 3.

We impose a share coefficient α of physical capital of 0.375 and a depreciation rate of 4.1% per year (Feenstra et al., 2015, Penn World Table 8.1). The latter implies $\delta = 0.118$ considering that one period in the model consists of 3 years. Following Caselli and Coleman (2006), who state that the empirical labour literature consistently estimates values between 1 and 2, we set the elasticity of substitution λ between the three ability types in effective labour equal to 1.5. In the human capital production function, we choose a conservative value of 0.3 for the elasticity with respect to education time (σ). This value is within the range considered by Bouzahzah

et al. (2002), but much lower than the value imposed by Lucas (1990). The literature provides much less guidance for the calibration of the relative initial human capital of low and medium ability individuals relative to the initial human capital of high ability individuals, ω_L and ω_M. To determine these parameters we follow Buyse et al. (2017) who rely on PISA science test scores. These tests are taken from 15 year old pupils, and therefore indicative of the cognitive capacity with which individuals enter our model at age 18. We use the test scores of pupils at the 17th and the 50th percentile relative to the score of pupils at the 83rd percentile, as representative for ω_L and ω_M. This approach yields values for ω_L and ω_M of 0.653 and 0.826, while $\omega_H=1$. The parameter ξ of the tax function (cf. Equation 41) is chosen so that it generates the right amount of progressivity during the calibration period. The data with which we compare the tax function's values concern the observed differences in average personal income tax rates between three income groups in Belgium (67%, 100% and 150% of the average wage, OECD Tax Database, Table I.5). Minimizing the average root mean squared error between data and function values results in a value for ξ of 0.332. The weights used in the determination of the union's reference wage q_1, q_2 and q_3 are 0.8, 0.05 and 0.15 respectively and taken from Boone and Heylen (2018). The last parameters that we took directly from the literature are the age-specific productivity parameters ϵ_j. We follow the hump-shaped pattern imposed by Miles (1999).

A second set of 24 parameters is determined by calibration. Our calibration procedure is based on Ludwig et al. (2011). It consists of six steps.

1. We obtain an initial guess of the parameters by calibrating for a steady state as this is easy and fast. As calibration period we take 1996-2007 and impose a stationary population distribution by assuming survival rates and fertility rates to be constant at their 1996-2007 level. Parameters are determined to match observed averages of key data in the calibration period[12]. These data concern hours worked by age (averaged over the three ability types), hours worked by ability (low and medium, averaged over two large age groups), average participation in education by ability, the unemployment rate among the individuals of low ability, and wage differentials between ability groups. Our overall approach is to use data for individuals who did not finish higher secondary education as representative for low ability individuals, and data for individuals with a higher secondary degree but no tertiary degree as representative for medium ability individuals. Data for individuals with a tertiary degree are assumed representative for individuals with high ability. The middle part of Table 1 shows the 24 target values.

2. Using the parameters from step 1, we calculate an artificial initial steady state. As our demographic data are only available from 1948, we use this year as starting point. The population distribution is again assumed stationary at this point.

3. We feed the demographics, the world interest rate, the rate of technical progress and data on policy variables into the model as exogenous driving forces and calculate the transition path to the new steady state.

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[12]We report the data, and describe their construction and sources in Appendix C.

4. We calculate the simulated averages *along the transition* and compare them with the real moments in the calibration period (1996-2007). These may differ quite substantially. To take this into account, overestimation or underestimation ratios are calculated (i.e. simulated moments divided by real moments).

5. The calibration targets of step 1 are rescaled by dividing them by these ratios.

6. We repeat steps 1 to 5 until the distance between simulated and real moments (i.e. the mean squared error) in 1996-2007 is minimized. Appendix A contains the outcome of this calibration procedure, i.e. for each of the calibration targets we report the final overestimation or underestimation ratio.

An overview of all calibrated parameter values that result from this procedure is provided in Table 1.

5 Empirical relevance

To investigate the empirical relevance of our calibrated model, we first introduce time series for the exogenous variables and then check how well model simulations replicate the data for the most important endogenous variables. We compare the model's fitted values to the data for the old-age dependency ratio, per capita GDP growth, aggregate average per capita hours worked, the capital-output ratio, and the Gini coefficient in Belgium since the 1950s or 1960s. We do the same for per capita hours worked in different age groups and different ability (or education) groups, and for participation in higher education in shorter time periods.

5.1 Exogenous variables

The exogenous variables in our model concern demography (fertility and survival rates), the world real interest rate (Figure 6), the rate of technical progress (Figure 7), and a set of fiscal and pension policy parameters (Figure 8). We already showed and discussed the demographic variables in Section 3.1. Here, we focus on the other ones.

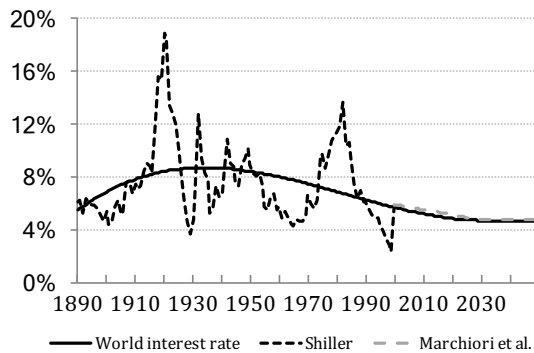**Figure 6:** Annual world interest rate

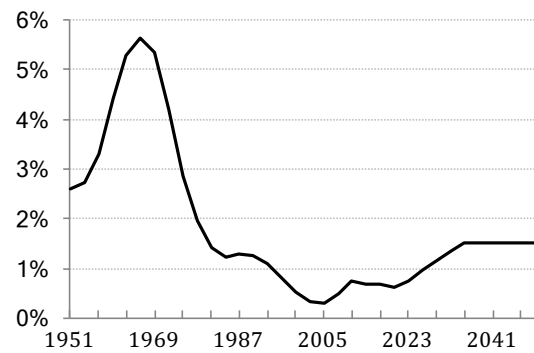**Figure 7:** Average annual rate of technical progress

Table 1: Parameterization of the model

Parameter	Description	Value
Taken from the literature or from existing datasets:		
α	Production share of capital	0.3749
λ	Elasticity of substitution between workers of different ability	1.5
θ	Inverse of the intertemporal elasticity to substitute leisure	2
δ	Depreciation rate of physical capital	0.1177
σ	Elasticity of human capital w.r.t. education time	0.3
β	Discount factor	0.9423
ε_j	Age-productivity profile by age j	$exp(0.05 age - 0.0006 age^2)$
q_1, q_2, q_3	Weights used in the determination of the union reference wage	0.8, 0.05, 0.15
ν_s	Shares of ability types in population	0.33
ω_M, ω_L	Relative initial human capital	$\omega_M = 0.826, \omega_L = 0.653$
Determined by calibration:		
η_H, η_M, η_L	Input shares of high, middle and low ability individuals	$\eta_H = 0.423, \eta_M = 0.334,$ $\eta_L = 0.244$
ϕ_H, ϕ_M	Efficiency parameters in human capital production function	$\phi_H = 0.466, \phi_M = 0.068$
μ	Union markup above the union reference wage	17%
$T_{j,L}$	Costs related to working - low (age 1 to 4; age 5 to 14)	0.004, 0.0035[1]
$T_{j,M}$	Costs related to working - medium (age 1 to 4; age 5 to 15)	0.003, 0.002
$T_{j,H}$	Costs related to working - high	0 (as reference)
γ_j	Preference for leisure	γ_1=0.7101 γ_9=0.0891 γ_2=0.2348 γ_{10}=0.0995 γ_3=0.0841 γ_{11}=0.1093 γ_4=0.0570 γ_{12}=0.1349 γ_5=0.1060 γ_{13}=0.2358 γ_6=0.0997 γ_{14}=0.2898 γ_7=0.0941 γ_{15}=0.6168 γ_8=0.0922

Target values for calibration:[2]

Average per capita hours worked by age, as a fraction of potential hours: *Education rates:*

$n_1 = 0.105$	$n_5 = 0.575$	$n_9 = 0.566$	$n_{13} = 0.367$	$e_H = 20\%$
$n_2 = 0.339$	$n_6 = 0.574$	$n_{10} = 0.544$	$n_{14} = 0.311$	$e_M = 5\%$
$n_3 = 0.502$	$n_7 = 0.574$	$n_{11} = 0.523$	$n_{15} = 0.119$	*Unemployment rate:*
$n_4 = 0.556$	$n_8 = 0.569$	$n_{12} = 0.481$		$u = 12.9\%$

Average ability-specific hours worked over two large age groups:

$\frac{1}{4}\sum_{j=1}^{4} n_{j,L}$=0.405 $\frac{1}{4}\sum_{j=1}^{4} n_{j,M}$=0.373

$\frac{1}{10}\sum_{j=5}^{14} n_{j,L}$=0.439 $\frac{1}{11}\sum_{j=5}^{15} n_{j,M}$=0.486

Pre-tax earnings ratios in Belgium: low versus medium educated: 90%, low versus high educated: 69%

Fiscal and pension policy parameters in the calibration period (averaged over 1996-2007):

$cr_1 = 0.075$	$rr_L = 0.648$	$b = 0.479$	$\tau_k = 0.254$
$cr_2 = 0.089$	$pg = 0.979$	$B/Y = 0.355$	$\tau_p = 0.284$
$rr_H = 0.577$	$g = 0.218$	$\Gamma = 0.420$	
$rr_M = 0.638$	$\tau_c = 0.197$	$\xi = 0.332$	

Notes:

(1) The reported constants are the costs related to working per capita and adjusted for technical change.

(2) For more details about the data of the target values of our calibration and the underlying exogenous variables, we refer to Appendices C and D and Section 5.1.

The assumption of an open economy with perfect capital mobility implies that the net after-tax rate of return on physical capital will always be equal to the (exogenous) world real interest rate r_t. It requires us to introduce data for this interest rate over a very long period of time. To the best of our knowledge, however, this is not readily available. Krueger and Ludwig (2007) and - more recently - Marchiori et al. (2017) have computed highly relevant series using an OLG model and taking into account projections for future demography at the world or OECD level. Their results are fairly similar, but their data do not cover the whole period since 1950. To get data also for the earliest decades, we relied on the US stock market data from Shiller (2015). Figure 6 includes his cyclically-adjusted earnings/price ratio in %. We take it as a proxy for the return to physical capital in the world in the 20th century. Combining this proxy with the simulated real interest rate series for 2000-2050 from Marchiori et al. (2017), and smoothing using a third degree polynomial, gives us our world real interest rate.

Figure 7 displays the exogenous rate of labour augmenting technical change ga_t since 1951. Our main source is Feenstra et al. (2015, Penn World Table 8.1). We used their data for TFP growth until 2011, after a double adjustment. First, a correction was necessary for the different treatment of hours worked[13]. Second, we HP-filtered the corrected data to obtain the trend rate of technical change and to exclude cyclical effects. For the years until 2021, we approximate ga_t by the growth rate of productivity per hour worked as projected by the Federal Planning Bureau (2016). As of 2022, we use productivity per worker as advanced by the Belgian Studiecommissie voor de Vergrijzing (2016) as a proxy. The projected 1.5% annual growth rate after 2034 also corresponds to the projection for the rate of technical progress of the 2015 European Commission's Working Group on Ageing.

The evolution over time of the fiscal and pension policy variables in our model is shown in Figure 8. In Appendix D.2, more detail is provided on the underlying data sources and computations. We also include the evolution of the public debt to GDP ratio in Figure 8f. We determined the lump sum transfer Z_t in Equation (32) such that our simulated model exactly replicates this evolution.
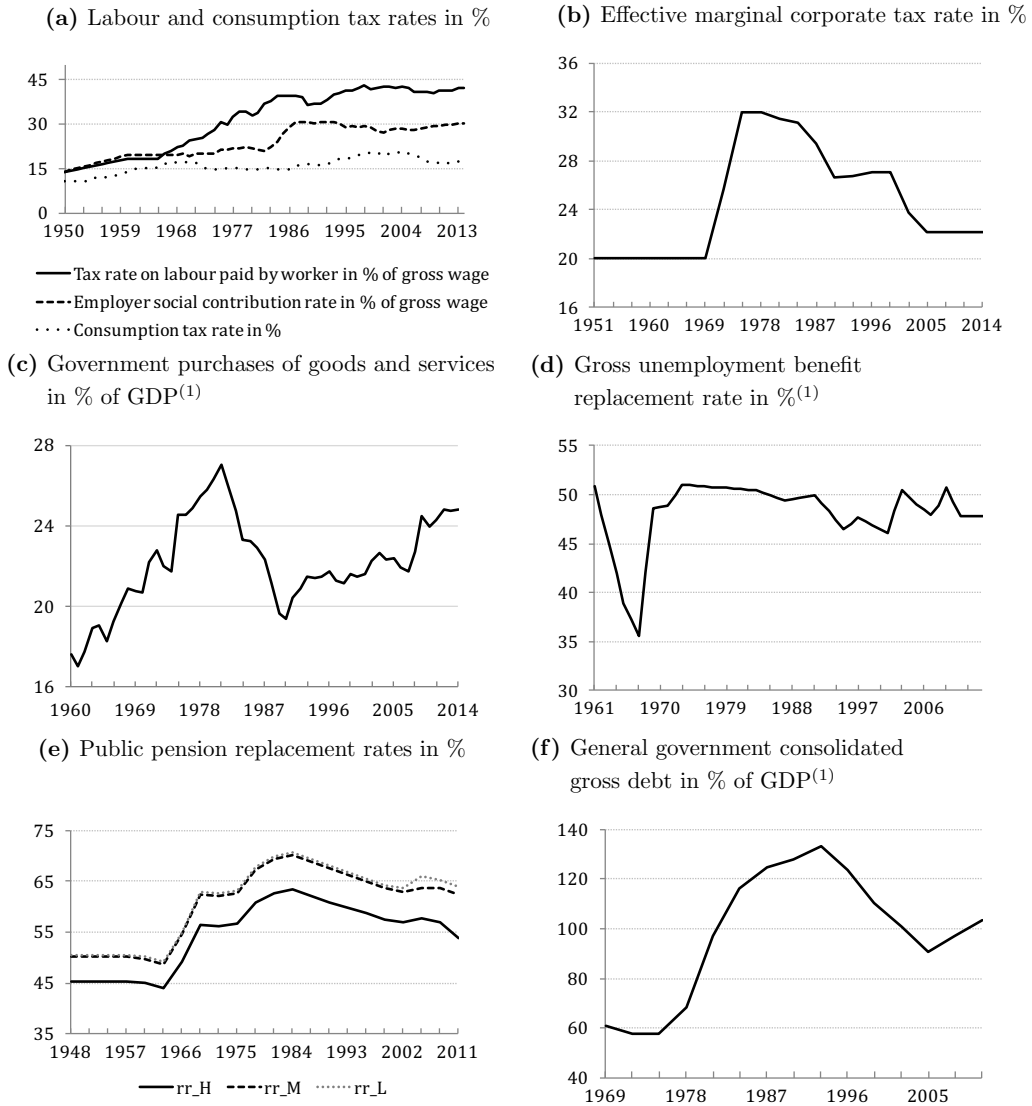
5.2 Backfitting

Our model is calibrated to match as closely as possible the data in 1996-2007. Can it also, after introducing the time-varying values of the exogenous variables, replicate the data in earlier and later periods?

Figure 9 reports the results for six endogenous macro variables since the 1950s or 1960s. The different panels reveal that our model performs very well for the evolution of per capita hours worked (panel b), the capital-output ratio (panel d) and the old-age dependency ratio (panel f). For average annual per capita GDP growth (panel a) the model series is more volatile than the data, mainly due to the assumption of perfect capital mobility, but it captures well

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[13]The Penn World Table 8.1 includes data for TFP (rtfpna) which correspond to the following production function: $Y = BK^{\alpha}(hc.L)^{1-\alpha}$, with B the level of TFP, K physical capital, hc human capital and L employment (in persons). Using comparable notation, our production function would be $Y = K^{\alpha}(A.hc.L.h)^{1-\alpha}$ with h hours worked per employed person. It then follows that $B = (A.h)^{1-\alpha}$. The relevant growth rate of A in our model can then be approximated as the growth rate of B (or rtfpna in PWT) divided by the labour share $(1-\alpha)$ minus the growth rate of hours worked per employed person.

Figure 8: Fiscal and pension policy variables

(a) Labour and consumption tax rates in %

— Tax rate on labour paid by worker in % of gross wage
---- Employer social contribution rate in % of gross wage
······ Consumption tax rate in %

(b) Effective marginal corporate tax rate in %

(c) Government purchases of goods and services in % of GDP[1]

(d) Gross unemployment benefit replacement rate in %[1]

(e) Public pension replacement rates in %

— rr_H ---- rr_M ······ rr_L

**(f)** General government consolidated gross debt in % of GDP[1]

Data sources and computation: see Appendix D.2. Notes: (1) Earlier data are assumed to be equal to their level in the earliest available year. (2) For details on policy variables not shown here (the revaluation factors wg and pg, and the contribution rates cr_1 and cr_2), see Appendix D.

the trend observed over a very long time period. In panel c the use of a different scale makes a direct comparison of the data for investment in tertiary education with the simulated model education rate among high ability individuals (e_H) impossible. Indirectly, one can see, however, the same upward trend over time, and even the slight acceleration in the major part of the 1990s. Last but not least, panel e compares the values of the pre-tax Gini coefficient generated by the model with the data. Pre-tax income includes labour income, unemployment benefits, interest income, pensions and excludes lump sum transfers. We consider the unemployed members of a low ability household as a separate group. The most important thing to note is the fairly identical stability over time in the data and in our simulated Gini.
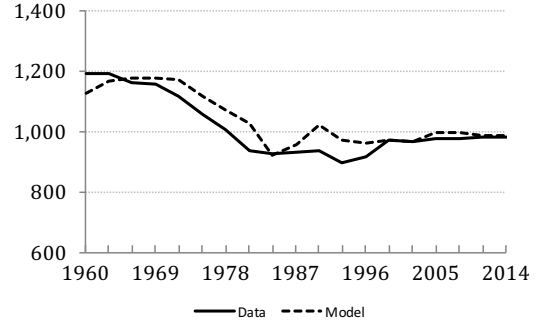
Figure 10 shows the variation in per capita hours by age and ability in 2005-2007. Panel a reports aggregated data by age. Panels b, c and d break this variable down in its constituting parts by ability. Although not perfect, the match between simulated and true data is very good.
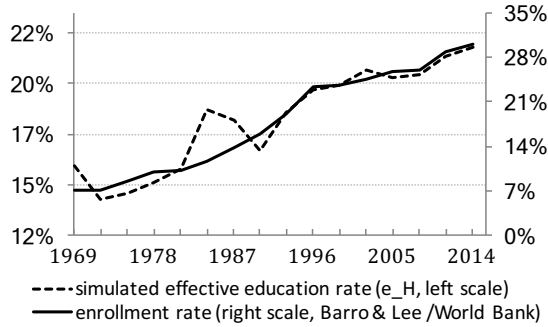
Figure 9: Backfitting

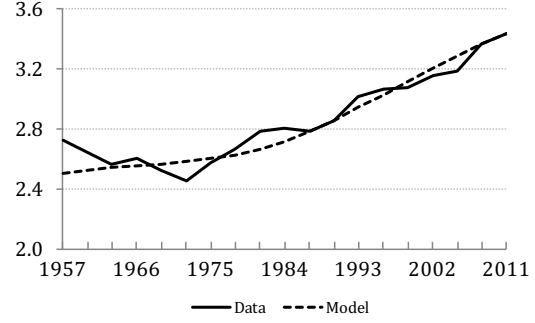**(a)** Average annual growth rate of potential GDP per capita[1]
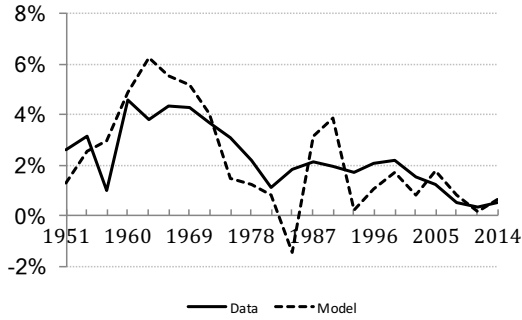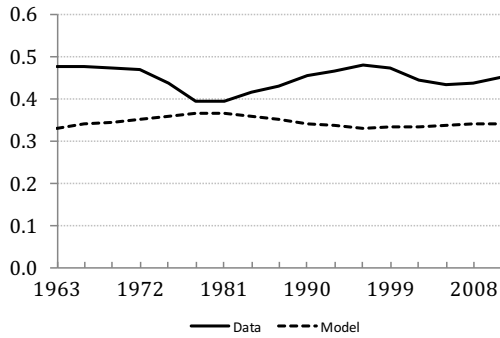

(b) Average annual per capita hours worked[2]

(c) Average education rate of the high ability group (age 18 to 29)[3]

- - - simulated effective education rate (e_H, left scale)
—— enrollment rate (right scale, Barro & Lee /World Bank)
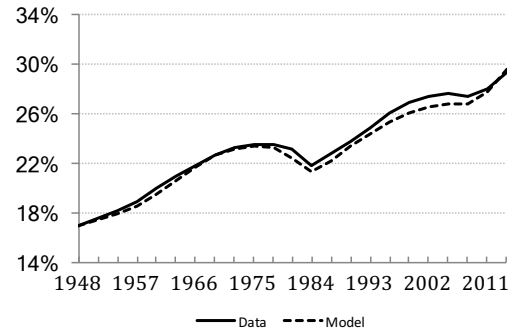
(d) Capital-output ratio

(e) Gini (pre-tax income)[4]

(f) Old-age dependency ratio

Data sources:

Figure 9a: EU Commission, AMECO, series OVGDP and population. Data points before 1960 have been computed from the Penn World Table (8.1, series rgdpna, Feenstra et al., 2015). Figure 9b: OECD, 2018, Historical population data; The Conference Board Total Economy Database, 2018. Figure 9c: Enrollment rate in tertiary education: World Bank, World Development Indicators; Barro and Lee (2013). Figure 9d: Feenstra et al. (2015): series rgdpna and rkna. Figure 9e: Gini coefficient of market income: Solt (2016), Figure 9f: the population of 65 or more divided by the population of 18 to 64: Population forecasts 2015-2060 of the Belgian Federal Planning Bureau and Statistics Belgium.

Notes:

(1) Reported growth rates are averages over consecutive periods of three years, starting in the year indicated on the horizontal axis.

(2) The reported model prediction follows from the simulated fractions of time that individuals work (n) and our assumption that potential annual hours are 2080 (see also Appendix C).

(3) Enrollment rates are multiplied by $\frac{5}{12}$ to account for the different age groups used in our model (18-29) versus the data (18-22).

(4) We computed the model Gini using Cowell's (2011) lower-bound index G_L.

Figure 10: Backfitting: annual per capita hours worked by age and ability in 2005-2007

(a) Per capita hours worked by age

(b) Per capita hours worked: high

(c) Per capita hours worked: medium

(d) Per capita hours worked: low
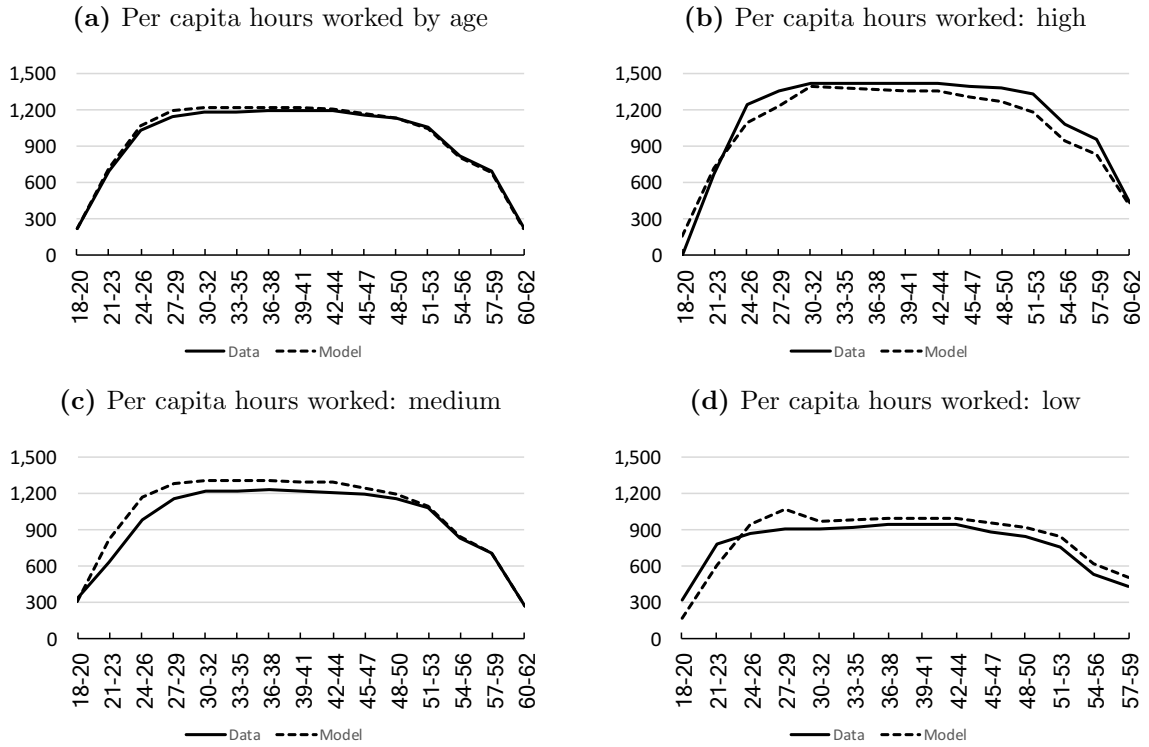
Data sources: Eurostat, Employment rates by sex, age and education (lfsa_ergaed) and Average usual weekly hours of work by professional status (lfsq_ewhuis); OECD, Labour Force Statistics, Average usual weekly hours worked on the main job. For more details on the construction of the data, see Appendix C.

6 Macroeconomic effects of demographic change

Having demonstrated the capacity of our calibrated model to replicate the level and evolution of key data in Belgium in 1960-2014, we now use the model to assess the effects of demographic changes on a wide range of endogenous macroeconomic variables. The changes we consider are the increase in life expectancy (increasing survival rates) and the strong fluctuation in the size of the youngest generation induced by changing fertility, as we have shown in Figures 5a and 5b.

6.1 Baseline simulation and counterfactual scenarios

Our considered demographic changes allow to study four scenarios: a baseline scenario and three counterfactuals. In each of them fiscal and pension policy parameters are kept constant (mostly at their level of 2014). Lump sum transfers adjust to maintain a constant public debt to GDP ratio from 2014 onwards. Furthermore, the rate of technical progress is in each scenario assumed to manifest itself as projected in Figure 7.

Our 'baseline simulation' assumes that the projections for fertility and survival rates in Figure 5, and the projection for the future world real interest rate in Figure 6, become true. The full black lines in the different panels of Figure 11 reveal the implied future size and structure of the population. They confirm for example the strong increase in the old-age dependency

ratio in panel b, and the drop in the population at working age in panel e, most dramatically so in 2010-2040, that we also observed in Figures 1 and 2. The alternative 'no demographic change' scenario counterfactually assumes that (i) the fertility rate (i.e. the growth rate of the generation of age 18 to 20) remains constant at its 1948-50 level, and (ii) the unconditional probability to survive is constant at the value that holds for the youngest generation of 1948-50. As such, the 'no demographic change' scenario imposes a constant instead of an increasing life expectancy for generations entering our model in 1948 or later. Another assumption (iii) concerns the projected evolution of the exogenous world interest rate. Existing literature is fairly unanimous in its expectation that demographic change induces a lower world real interest rate. The majority of existing studies advances a reduction in the interest rate due to demographic change of about 0.5 to 1%-point by 2025 (see e.g. Krueger and Ludwig, 2007; Ludwig et al., 2012; Marchiori et al., 2017; Attanasio et al., 2016). Among these studies, Ludwig et al. (2012) project the smallest decline. The reason is their assumption of endogenous human capital and the positive effect of demographic change on human capital accumulation. Only Attanasio et al. (2016) predict a decline of the interest rate that exceeds 1.5%-points. Building on these existing studies, and taking into account that also in our model human capital is endogenous, we impose in our 'no demographic change' counterfactual an interest rate that exceeds the baseline level by 0.5%-points from 2020 onwards. The higher interest rate gap arises gradually, starting from 1993 onwards[14,15].
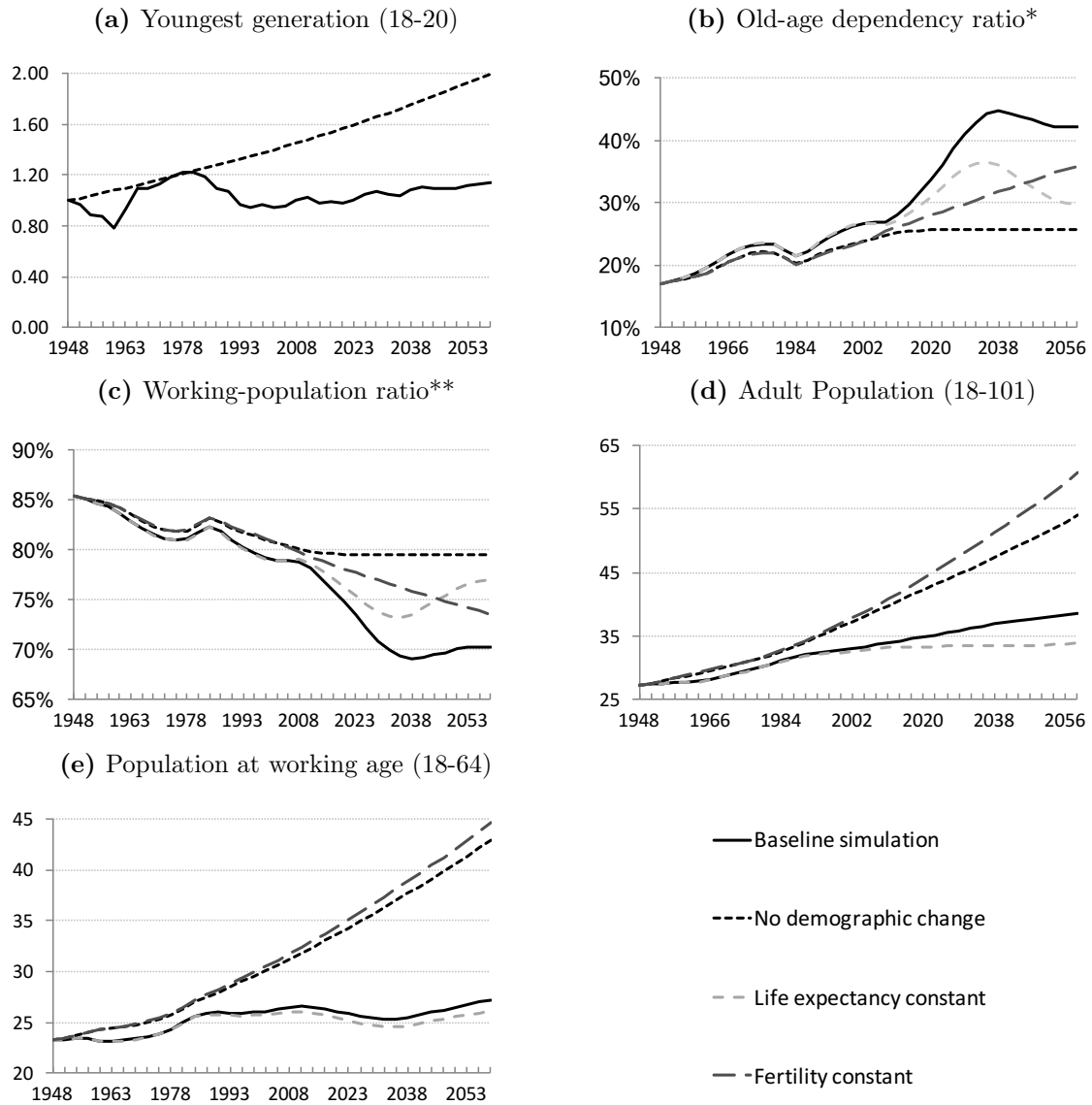
The implications of the 'no demographic change' assumption for various indicators of the size and the distribution of the population are clear from Figure 11. As one of the main (expected) counterfactual results, we observe in panel c a constant ratio of the population at working age to total population, from about 2010 onwards. Unsurprisingly, the same holds for the old-age dependency ratio in panel b. Figure 11 also reveals the projected size and distribution of the population in two other counterfactual simulations. These keep one of the demographic components constant at the level of 1948-50, while baseline projections are imposed for the other[16].

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[14]If we alternatively assume a closed economy where the interest rate adjusts to establish capital market equilibrium, we also find that demographic change leads to a lower interest rate. The difference arises in the mid 1990s, to become 0.89%-points by 2020. The results that we report further in this section are quite robust. Changes in the assumed interest rate increase in the 'no demographic change' counterfactual affects our future growth predictions only marginally (see our brief discussion at the end of Section 6).

[15]While the literature is fairly unanimous in projecting lower real interest rates due to demographic change, there is no agreement regarding the effect of demographic change and ageing on technical progress and the growth rate of TFP. Some find a negative impact (e.g. Aiyar et al., 2016), others expect positive effects (e.g Acemoglu and Restrepo, 2017). In our 'no demographic change' counterfactuals we stay in the middle. We assume no change in the imposed future growth rate of technical progress and maintain the projection of Figure 7.

[16]Note that in these two counterfactual simulations, we also adjust the assumed world interest rate. If only one demographic component changes, the interest rate drop will be smaller than the 0.5%-points imposed in the 'no demographic change' scenario, and its evolution over time will be different. We estimated the effect on the interest rate of alternatively shutting down the change in one demographic variable via a closed economy version of our model where the interest rate establishes capital market equilibrium. Details are available upon simple request. Our conclusions are not affected by the precise allocation of the 0.5%-points interest rate change to either fertility or life expectancy.

Figure 11: Demographic change: effects on the population and its distribution

(a) Youngest generation (18-20)

(b) Old-age dependency ratio*

(c) Working-population ratio**

(d) Adult Population (18-101)

(e) Population at working age (18-64)

Baseline simulation

No demographic change

Life expectancy constant

Fertility constant

*The population of 65 or older divided by the population of 18 to 64
**The population of 18 to 64 divided by the total adult population (18 to 101)

6.2 Macroeconomic effects

Arithmetically, i.e. disregarding behavioural adjustments, we found in Section 2 that projected demographic change may cut off about 0.4%-points of the annual per capita growth rate in Belgium in the next 25 years. Accounting now for behavioural adjustment, Figure 12a reveals an average per capita growth gap due to demographic change of only 0.29%-points in 2017-2040. This reduced gap clearly points to significant favourable behavioural effects of demographic change. At the same time, however, these effects are not strong enough. Accumulated over 25 years, a remaining 0.29%-points decline of the annual per capita growth rate would cost about 7% of per capita income. The different panels of Figures 13 and 14 provide more insight. Anticipating, we find that falling fertility and (especially) increasing life expectancy stimulate individual labour

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supply, private savings and human capital accumulation. Despite these favourable adjustments, future growth remains weak for at least three reasons. First, due to reduced fertility and the implied fall in population at working age, investment in physical capital declines. Second, in an open economy increased household savings may leave the country (capital outflow), which reinforces the fall in physical capital investment. Third, many of the favourable adjustments have already taken place in previous decades. A comparison of the 'fertility constant' counterfactual with the baseline simulation in Figure 12b highlights the key role of reduced fertility as the main driver of lower per capita growth. It explains almost the whole loss. Only a minor part is due to increasing life spans. The positive behavioural effects of higher life expectancy related to hours worked, education and investment in Figure 14 almost compensate its negative effect resulting from the rapidly increasing number of retirees.

Figure 12: Per capita growth effects of demographic change

(a) Average annual per capita growth rate (baseline versus 'no demographic change')

(b) Average annual per capita growth rate (baseline versus 'fertility constant' and 'life expectancy constant')
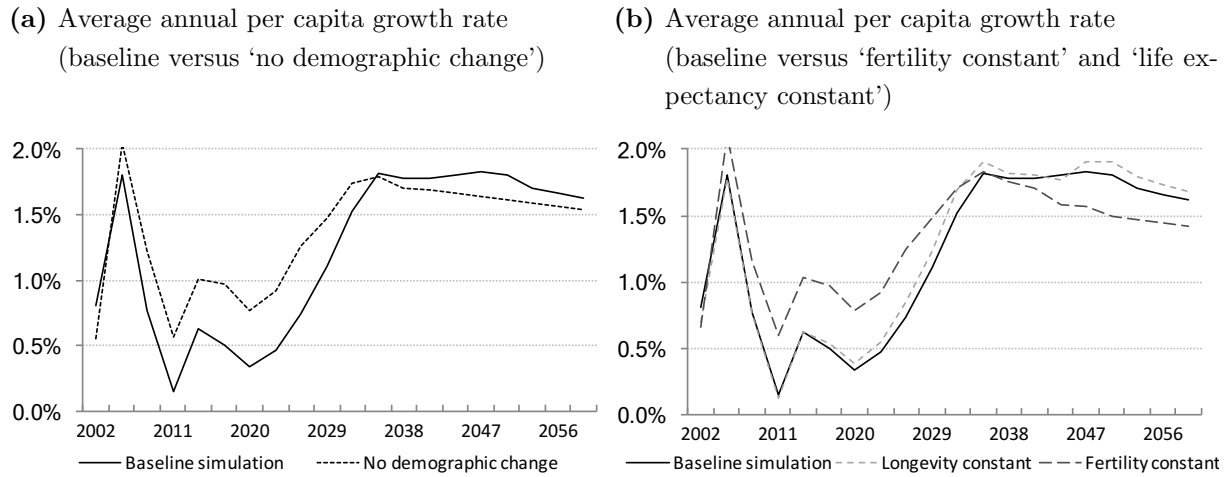

Figure 13 returns to the decomposition of per capita output growth that we presented at the beginning of Section 2. Behavioural adjustment should show up either in changes in labour productivity growth (Figure 13a), for example due to changes in investment in physical or human capital, or in changes in the growth rate of hours worked per person at working age (Figure 13b). Comparing our baseline simulation for both growth rates with their counterfactual under the assumption of no demographic change, we see that counterfactual growth has almost always been lower since the 1980s. Demographic change thus implied higher productivity growth and increasing hours worked. Since the 2000s, however, the difference has become smaller.

Figure 14 focuses on the behavioural responses themselves. We show our baseline simulations and three counterfactuals for hours worked, education (by those of high ability), gross investment in physical capital and savings (as captured by the accumulated stock of non-human wealth). We also include our results for the current account and for public pension expenditures in % of GDP. If we first compare the level and evolution of the full black line in the different panels of Figure 14 (baseline simulation) with the interrupted black line (no demographic change), our main findings are the following. Demographic change induces an increase in labour supply and per capita hours worked, an increase in schooling, an increase in savings and the stock of non-human wealth, but lower gross investment, and much higher future pension expenditures.

Figure 13: Components of per capita growth in the baseline simulation and under constant demography

(a) Growth rate of output per hour worked

(b) Growth rate of hours worked per person at working age
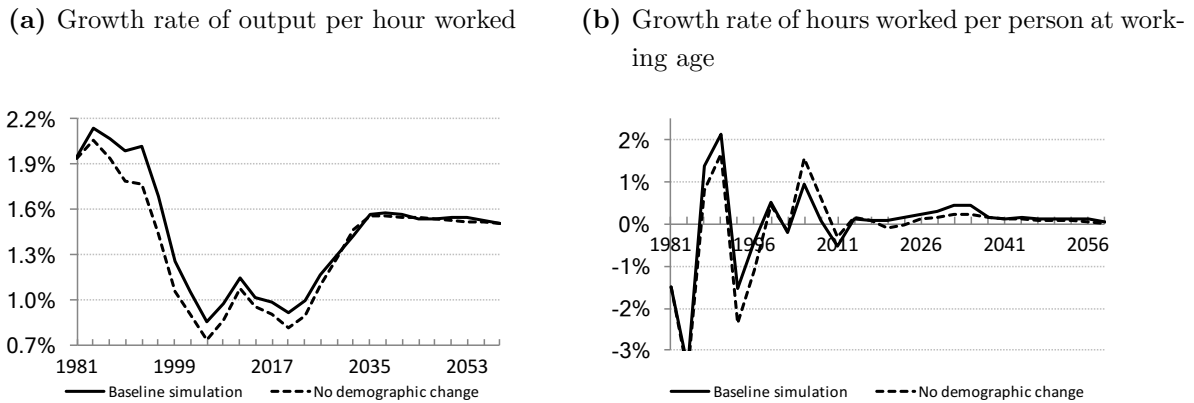
Related to the changes in savings and investment, the current account balance increases (and capital flows out).
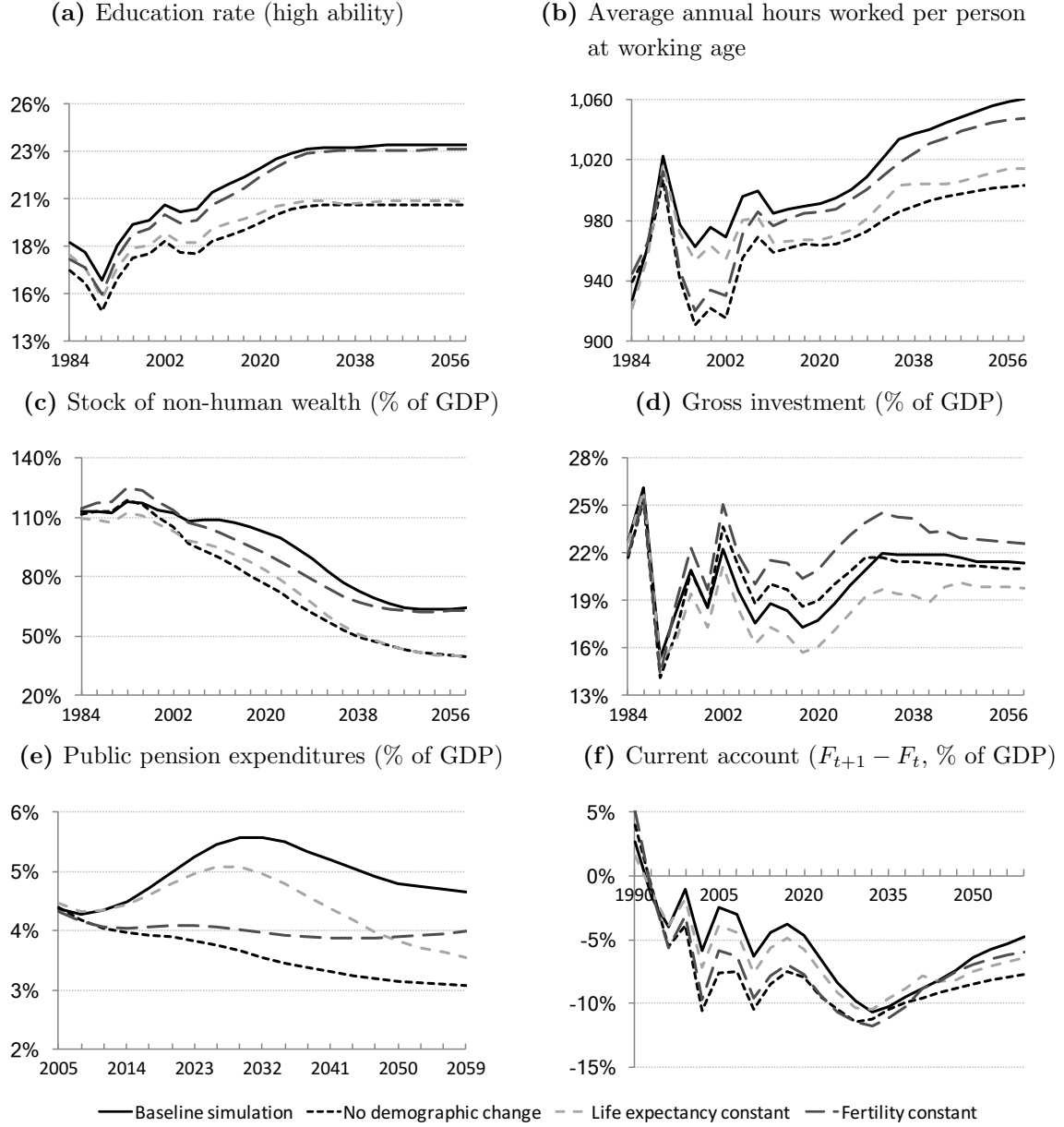
Figure 14a shows that the increased participation in higher education is mainly due to longer life expectancy. An overall increase in survival rates encourages individuals to study since it allows individuals to enjoy the returns on their investment during a (much) longer period. Increased returns will include higher labour income before the normal retirement age (which individuals will be more likely to reach) as well as higher pensions after it (which individuals will enjoy for a longer period of time). For the latter effect to materialize, however, it is important that pensions are related to the individual's own labour income and education (see the first-order condition in Appendix B).

The increase in human capital and its positive effect on the wage is one element making it attractive for individuals to work more. Another one is the perspective of longer life (including longer life in retirement for a given pension age), which implies the need for more resources at old age to maintain consumption standards. These effects will ultimately lead to increased average annual hours worked per person at working age in Figure 14b, with most of the action after 2010 coming from increased life expectancy. These positive effects will - again - be stronger if increased hours of work also feed through into higher future pensions (see Appendix B).

Next to making individuals work more, the need for more resources will also make them save more and accumulate more financial assets (and consume less when young). The dominant positive impact of increasing life expectancy on savings, is most clear in Figure 14c. This result supports the hypothesis that its positive effect on savings (due the fact that people at active age and young retirees will save more) dominates its negative effect (arising from the growing number of dissavers).

The increase in individuals' labour supply and the accumulation of more human capital are two elements that could encourage firms to invest in physical capital. Both elements raise the marginal productivity of physical capital. Figure 14d confirms this. Investment would be the highest (until about 2032) if the fertility rate was kept constant at its level of 1948-50, but life expectancy was free to vary as it did in reality. The reason for the drastic reduction of investment in the baseline simulation is clear then. It follows from the drop in the population at working age (see also Figure 11e). Although individuals optimally raise their labour supply,

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Figure 14: Demographic change: macroeconomic effects

(a) Education rate (high ability)

(b) Average annual hours worked per person at working age

(c) Stock of non-human wealth (% of GDP)

(d) Gross investment (% of GDP)

(e) Public pension expenditures (% of GDP)

(f) Current account ($F_{t+1} - F_t$, % of GDP)

——Baseline simulation ----No demographic change – – Life expectancy constant — -Fertility constant

aggregate labour input will fall due the lower number of people. The negative effect of this fall on the need for equipment and the productivity of physical capital is an important cause of capital outflow (an increase of the current account and a reduction of net capital inflow, see Figure 14f) and disinvestment.

Another determining factor of aggregate hours worked is the unemployment rate. Our simulations (not shown) reveal no noticeable effect from demographic change (or the size of labour supply) on the unemployment rate. On the one hand, a falling number of workers could provide jobs for those who are initially unemployed. On the other hand, however, the decrease in labour supply will raise competitive wages, which translates into higher wages set by the union in Equation (30). Labour demand for low ability workers will then decline also.

The strong increase in the old-age dependency ratio (Figure 11b) due to subsequent relatively smaller youngest cohorts and increasing life expectancy translates in fast growing public pension

expenditures from 2014 onwards. Over 2014-61 demographic change induces public pension expenditures to be on average 1.6% of GDP higher compared with the constant fertility and life expectancy scenario. The maximum increase is about 2% of GDP in the early and mid 2030s. Figure 14e reveals that the main cause up until 2032 is the path of fertility experienced in the past. Later, increased life expectancy is becoming more important.

The results that we showed in Figure 14 are macroeconomic results. To further substantiate our conclusions on the behavioural effects of demographic change, we include in Appendix E the results of counterfactual simulations at the level of separate cohorts (generations) of individuals over time. We distinguish cohorts that entered the model at age 18 in 1966, 1981, 1996 and 2011. We observe for most cohorts indeed higher hours worked, increased savings, and increased time allocated to education when young.

6.3 Robustness

We have tested the robustness of our results in Figures 12, 13 and 14 to variation in the projected future world real interest rate and future technical change (TFP growth). To be more specific, we simulated and compared new baselines and new 'no demographic change' counterfactuals, respectively assuming in both scenarios a higher world real interest rate (0.5%-points higher by 2030) and a lower rate of technical change (0.5%-points lower by 2035) than shown in Figures 6 and 7. The levels of all endogenous variables obviously change, but also under these alternative projections the net per capita growth effect of demographic change is very close to -0.3%-points, while the increase in pension expenditures remains about 1.6% of GDP. Also under these alternative projections, individuals increase their participation in higher education, work more and save more due to demographic change. Capital will again flow out and private investment will be lower.

In a second series of additional simulations we investigated the robustness of our results for changes in the assumed effect of demographic change on the world real interest rate. Our 'no demographic change' counterfactual simulations in Figures 12, 13 and 14 assumed a world interest rate that exceeded the baseline level by 0.5%-points from 2020 onwards. In alternative simulations we assumed smaller or larger reductions in the world interest rate due to demographic change. All in all, imposing alternative counterfactual interest rates had only limited effects. For example, imposing the same world interest rate in the 'no demographic change' counterfactual as in the baseline, the net negative per capita growth effect was 0.37%. Our main findings in Figures 13 and 14 did not change.

As a third robustness check, we used the consumption tax rate as fiscal instrument to close the public budget after 2014 in our simulations (rather than lump sum transfers). Again, this did not change our results and conclusions.

7 Conclusion

Demographic change, characterized by an increasing life expectancy, decreased fertility rates and the retirement of the baby boom generation, constitutes a major challenge to most OECD economies, including Belgium. In this paper we construct and parameterize a large-scale 28 period OLG model for an open economy to quantitatively assess the effects of this demographic

challenge. Individuals in the model do not only differ by age, but also by innate ability. The model explains within one coherent framework a wide range of variables including hours worked and employment by age and ability, unemployment among individuals with low ability and skills, investment in human and in physical capital, per capita output, the public budget, and inequality in income.

After calibration, the model replicates the fundamental evolution of important macroeconomic variables in Belgium in the previous decades quite well. This observation raises confidence that the model may produce a reliable projection of fundamental macroeconomic developments in the future. Assuming unchanged policies, our predictions are not optimistic. Demographic change may cause a significant drag on per capita growth, of on average 0.29%-points annually for the next 25 years. It also implies an increase in public pension expenditures by on average almost 1.6% of GDP annually. A key factor is the negative effect of reduced fertility and the resulting fall in the population at working age on the return to investment in physical capital. The induced positive behavioural effects from demographic change on individual labour supply and schooling can only partially counteract this drop in the size of the labour force. Moreover, rather than promoting domestic investment, the increased savings to which longer life expectancy contributes, may to an important extent flow out of the country.

Considering our expectation of a long-run slowdown in per capita growth due to demographic change, our paper tends to match with the supply-side view in the recent secular stagnation literature, as expressed by among others Gordon (2014) and Cervellati et al. (2017).

The loss of aggregate per capita growth and income is a reason for concern. Not only will it be more difficult to finance future public pension expenditures and other costs related to ageing, so it will be to cope with the rise in inequality that is also likely to follow from demographic change (Devriendt and Heylen, 2018b). Policy changes promoting aggregate hours worked and productivity are therefore of the utmost importance. In Belgium there still is an enormous potential to raise hours worked as they are currently far below the level in most other OECD countries. Policies aimed at reducing the high unemployment rate among the low educated and at increasing the low amount of hours worked among older workers are paramount, for example tax shifts targeted at promoting employment and after-tax labour income of low educated and older workers, and pension reform rewarding hours worked. Policy should also be directed towards stimulating labour productivity growth, which has been lower in Belgium than the EU average for almost two decades.

Appendix

This appendix contains details on (A) the quality of the calibration results, (B) the individuals' first-order conditions, (C, D) the construction and sources of our data on employment, education, earnings, demography, and all fiscal policy and pension policy parameters, and (E) additional simulation results for individual cohorts.

A Calibration results: model output as a fraction of the calibration targets during 1996-2007 (%)

Variable	Model output / Data	Variable	Model output / Data
n_1	100.628	n_{13}	99.028
n_2	99.884	n_{14}	98.787
n_3	99.853	n_{15}	97.753
n_4	99.862	$\frac{1}{4}\sum_{j=1}^{4} n_{j,L}$	97.753
n_5	99.915	$\frac{1}{10}\sum_{j=5}^{14} n_{j,L}$	104.648
n_6	99.961	$\frac{1}{4}\sum_{j=1}^{4} n_{j,M}$	111.573
n_7	99.970	$\frac{1}{11}\sum_{j=5}^{15} n_{j,M}$	102.987
n_8	99.905	e_M	100.276
n_9	99.793	e_H	100.415
n_{10}	99.660	rw_{LH}	100.114
n_{11}	99.527	rw_{LM}	100.033
n_{12}	99.384	u	99.957
		Mean Squared Error = 0.001372	

Notes: n_j denotes annual hours worked per individual of age j averaged over all ability groups, e_s for $s = M, H$ denotes the average fraction of time allocated to education (cf. Appendix C.2) of all individuals of ability type s in the age category 18-29, rw_{LH} and rw_{LM} stand for the relative pre-tax wage of a low ability individual versus a high ability individual and a medium ability individual respectively, u denotes the unemployment rate. The values displayed are the averages of model output as a fraction of the averages of real data (i.e. the calibration targets) during 1996-2007.

B First-order conditions of households

The law of motion of optimal consumption over time is denoted by the Euler equation (47). Adding the posibility of death ($sr_j^t < 1$) to the model implies that the consumption path becomes flatter. Since conditional survival rates drop as an individual ages, it will from a certain age turn negative. This happens when the conditional survival rate falls below $1/(\beta(1+r_t))$. Increased life expectancy, modelled as rising survival rates, will postpone this turning point.

$$\frac{c_{J+1,s}^t (1+\tau_c)}{c_{J,s}^t (1+\tau_c)} = \beta sr_J^t (1 + r_{t+J}) \tag{47}$$

The optimal labour-leisure choice for medium and high ability households is described by Equation (48). Low ability households face exactly the same choice, although they retire sooner. In each period, individuals will supply labour up to the point where the (discounted) marginal utility of leisure equals that of labour. The latter is elaborated on the right-hand side. Working one more hour in the considered period yields more resources to consume and more utility in that period itself (first term RHS), but also when retired (second and third term RHS). The gain from work in the period itself will rise when the wage, the productivity level, the probability to survive and the human capital stock are higher and when the initial consumption level, the tax levels on labour and consumption, and the transport and child care costs are lower. The second term captures the extra consumption possibilities resulting from working more during period J in the first period of retirement (model-age 16), the third term the extra consumption possibilities in all other years of retirement. These consumption possibilities depend, next to those determinants already mentioned, positively on the earnings-linked replacement rate, rr_s, the real interest rate r, the revaluation measure applied to past labour income wg, and the pension revaluation factor pg.

$$\beta^{J-1}\pi_J^t \frac{\gamma_J}{\left(l_{J,s}^t\right)^\theta} \frac{-\partial l_{J,s}^t}{\partial n_{J,s}^t} = \frac{\beta^{J-1}\pi_J^t \left(w_{t+J-1}^s \varepsilon_J h_{J,s}^t \left(1 - \tau_{w^m,J,s}^t\right) - T_{J,s}^t\right)}{c_{J,s}^t (1+\tau_c)}$$

$$+ \sum_{j=16}^{28} \frac{\beta^{j-1}\pi_j^t}{c_{j,s}^t (1+\tau_c)} \frac{\partial ppt_{j,s}^t}{\partial n_{J,s}^t}, \text{ for } J = 1 - 15, \, s = M, H \tag{48}$$

with

$$\frac{\partial ppt_{16,s}^t}{\partial n_{J,s}^t} = rr_s \frac{1}{15} w_{t+J-1}^s \varepsilon_J h_{J,s}^t \left(1 - \tau_{w^m,J,s}^t\right) \prod_{l=J}^{15} wg_{t+l}$$

and

$$\frac{\partial ppt_{j,s}^t}{\partial n_{J,s}^t} = \frac{\partial ppt_{16,s}^t}{\partial n_{J,s}^t} \prod_{l=16}^{j} pg_{t+l-1}, \text{ for } j > 16$$

Equation (49) states that the marginal utility loss from investing in education in period J for $s = M, H$ must equal the (discounted) marginal utility gain over life. The LHS makes clear that spending time in education is not rewarded in the same period: one loses leisure time. In return, however, this investment will yield extra consumption possibilities in later periods. Due to the rise in human capital, working will be rewarded more due to increased productivity and consequently labour income. This increase in labour income will -ceteris paribus- also translate into a higher pension benefit.

$$\beta^{J-1}\pi_J^t \frac{\gamma_J}{\left(l_{J,s}^t\right)^\theta} \frac{-\partial l_{J,s}^t}{\partial e_J^t} = \sum_{j>J}^{15} \beta^{j-1}\pi_j^t \left(\frac{w_{t+j-1}^s \varepsilon_j n_{j,s}^t \left(1 - \tau_{w^m,j,s}^t\right)}{c_{j,s}^t (1+\tau_c)} \frac{\partial h_{j,s}^t}{\partial e_J^t}\right)$$

$$+ \sum_{j=16}^{28} \beta^{j-1}\pi_j^t \left(\frac{1}{c_{j,s}^t (1+\tau_c)} \frac{\partial ppt_{j,s}^t}{\partial e_J^t}\right), \text{ for } J = 1 - 4, \, s = M, H \tag{49}$$

with

$$\frac{\partial h_{j,s}^t}{\partial e_J^t} = \frac{\partial \left[h_{1,s}^t \left(\prod_{i=2}^j x_{i,s}^t\right)\right]}{\partial e_J^t} = h_{1,s}^t \left(\frac{\partial x_{J+1,s}^t}{\partial e_J^t} \prod_{\substack{i=2 \\ i \neq J+1}}^j x_{i,s}^t\right), \text{ for } j = 2 - 5$$

with $x_{i+1,s}^t = \left(1 + \phi_s \left(e_{i,s}^t\right)^\sigma\right)$, and

$$\frac{\partial h_{j,s}^t}{\partial e_J^t} = \frac{\partial h_{5,s}^t}{\partial e_J^t}, \text{ for } j > 5$$

and

$$\frac{\partial ppt_{16,s}^t}{\partial e_J^t} = rr_s \frac{1}{15} \sum_{j>J}^{15} \left(w_{t+j-1}^s \varepsilon_j n_{j,s}^t \left(1 - \tau_{w^m,j,s}^t\right) \frac{\partial h_{j,s}^t}{\partial e_J^t} \prod_{l=j}^{15} wg_{t+l} \right)$$

and

$$\frac{\partial ppt_{j,s}^t}{\partial e_J^t} = \frac{\partial ppt_{16,s}^t}{\partial e_J^t} \prod_{l=16}^{j} pg_{t+l-1}, \text{for } j > 16$$

C Employment, education rates and earnings ratios in Belgium

C.1 Annual hours worked per capita

For each ability group annual hours worked per capita are computed as the employment rate in persons multiplied by annual hours worked per employed person. In the description of the model in Section 3, the supply of labour hours is expressed as a fraction of potential hours. This fraction is denoted by $n_{j,s}^t$. Our proxy for potential hours per person is 40 hours per week during 52 weeks per year. For individuals of high and medium ability $n_{j,s}^t$ will also be the fraction of hours actually worked. For individuals of low ability, who can be unemployed, the fraction of hours worked is $n_{j,L}^t(1 - u_t)$. Reported hours will be higher if a larger fraction of those in an age or skill group are employed, and if those employed work more hours. We report the data by age and ability averaged over 1996-2007 in Figure 15.

Data sources and computation:

Employment rate in persons by age and ability (education): As proxy for the high ability group, we use data for individuals with a tertiary degree. The representative individual of medium ability will have a higher secondary degree, but no tertiary degree. The representative lower ability individual will obtain a lower secondary degree at best. We show the data in Figure 16. Data source: EUROSTAT, Employment rates by sex, age and highest level of education attained, lfsa_ergaed.

Annual hours worked per employed person: The data are computed as weekly hours worked multiplied by the average number of weeks worked per year. The former differ by age and by ability (see Figure 18, and our description below). Due to lack of data, the latter are assumed to be the same for all workers.

Hours worked per week by age: OECD Stat (Labour Force Statistics, Average usual weekly hours worked on the main job).

Hours worked per week by ability: We combine data from ILO, which links professions to education levels, with data from Eurostat on weekly hours worked by profession. ILO links professions categorized in ISCO-08 groups to ISCED-97 levels of education. As the low ability group corresponds to ISCED-97 groups 1 to 2 (primary and lower secondary education), the medium group to groups 3 and 4 (upper secondary or post-secondary, non-tertiary education), and the high ability group to ISCED-97 groups 5 and 6 (tertiary education), the professions of low, medium and high ability are composed as follows. Low ability individuals can be found among clerical support workers, service and sales workers, craft and related trades workers, plant and machine operators and assemblers, and workers in elementary occupations. Medium ability workers can be found in the same occupations, but not in elementary occupations. We expect high ability individuals to be ISCO-08 technicians and associate professionals, professionals, and managers. Data sources: ILO (2012), International standard classification of occupations, structure, group definitions and correspondence tables. EUROSTAT, Average number of usual weekly hours of work in the main job, by sex, professional status, full-time/part-time and occupation, lfsq_ewhuis). The data concern dependent employees.

Average number of weeks worked per year: average annual hours actually worked per worker (dependent employment) divided by average usual weekly hours worked on the main job by all workers (dependent employment). OECD Stat, Labour Force Statistics, Hours worked.

Unemployment rate among individuals of low ability: As a proxy we use data on the unemployment rate among individuals without higher secondary education. EUROSTAT, Unemployment rates by sex, age and educational attainment, lfsa_urgaed. Figure 17.

Figure 15: Annual per capita hours worked by ability (education) and age (averaged over 1996-2007)
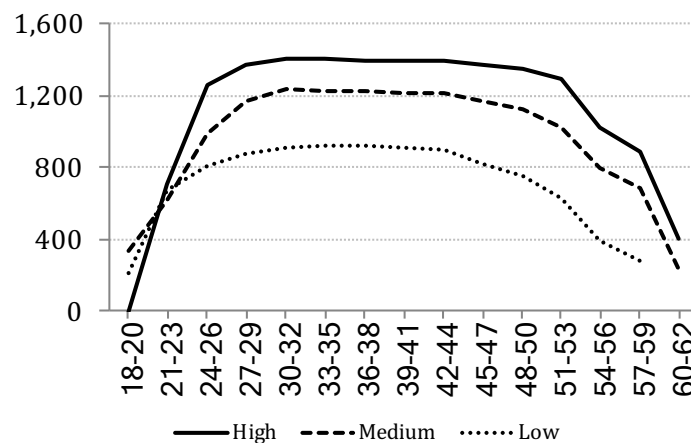
C.2 Education rate: the participation rate in education in hours

The participation rate in education in hours (e) follows from multiplying the participation rate in persons by actual to potential annual hours studied per student, and the so-called completion

Figure 16: Employment rate in persons by ability and age (in %, avg 1996-2007)
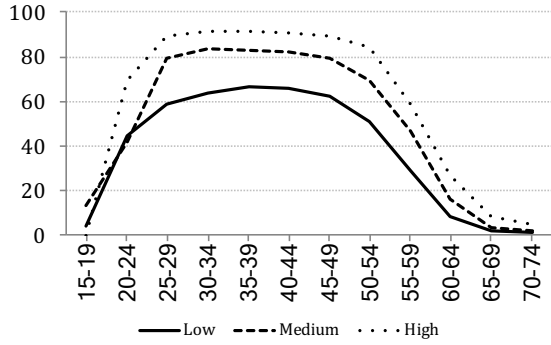
Figure 17: Unemployment rate among individuals of age 15 to 64 without higher secondary degree (in %)
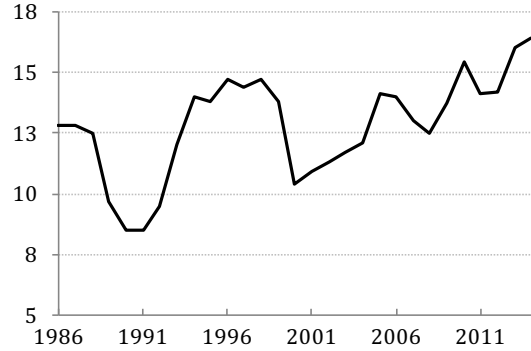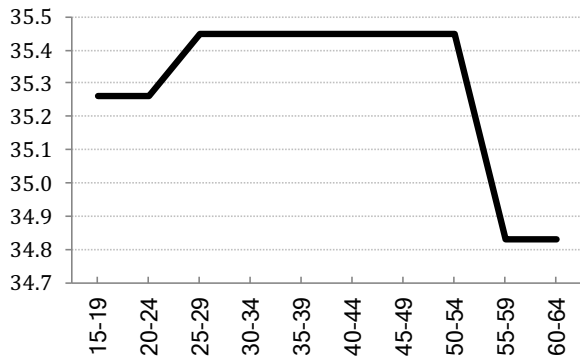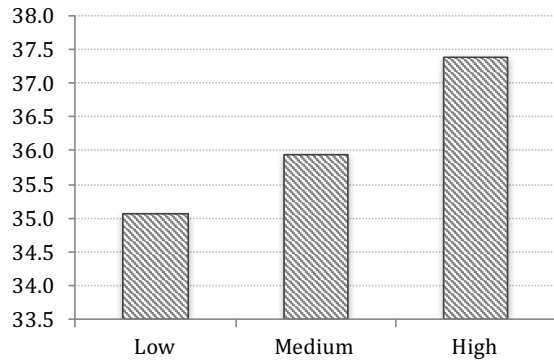

Figure 18: Hours worked per week

(a) per person by age (dependent employment, avg 1996-2007)

(b) by ability-type

rate. Equation (50) and our explanation below illustrate the computation of the data that set the calibration target for the education rate among high ability individuals in the model.

$$e_H = \frac{\text{persons in tertiary education (fte)}}{\left(\dfrac{1}{3}\right)\text{population of 18-29}} \cdot \frac{\text{annual hours studied per student}}{\text{potential annual hours}} \cdot \text{completion rate} \tag{50}$$

The calculation of the participation rate in persons reflects the assumption that the typical high ability individual has the talent to study at the tertiary level. We divide the total number of students in tertiary education (in full-time equivalents, fte) by the assumed total number of young individuals of high ability, i.e. one third of the population aged 18 to 29. If the number of students (in fte) in tertiary education in the data exceeds 33% of the population, we allot this surplus to medium ability individuals (e_M) who obtained a higher secondary degree and can also study at the tertiary level. Actual to potential annual hours follows from multiplying the ratio of actual to potential hours of study per week and the ratio of actual to potential weeks of study per year. The first part we set to 0.7. For the second part, we rely on the assumption that individuals study 42 out of 52 weeks. The completion rate is the fraction of students that is expected to graduate. It is set to 0.7, the average observed in Belgium during 1996-2007 (OECD, Edu-

cation at a Glance 2000, 2002, 2004, 2006, 2007, 2009). By accounting for the fact that not all students also obtain their diploma, we try to better measure *effective* hours studied, i.e. time in education that - as in the model - effectively leads to the accumulation of relevant human capital.

Figure 19 shows the evolution of our measure for e_H in the data in 1996-2007. Our measure for e_M is computed in the same way. But then the participation rate in persons counts all students in post-secondary non-tertiary education and the excess of students in tertiary education above 33% of the population of 18 to 29.

Data source: Participation rate in education in persons by age : OECD, Education and Skills, Students enrolled by age.

Figure 19: Participation rate in education in hours (%, 1996-2007)
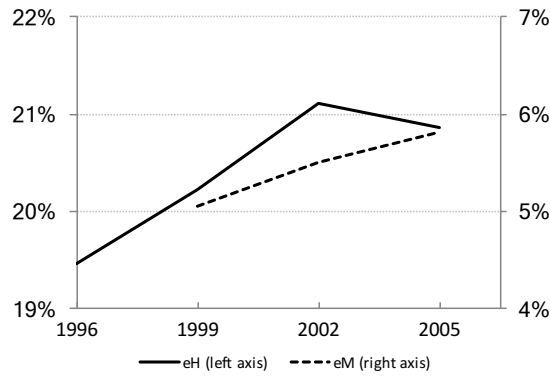
C.3   Pre-tax earnings ratios in Belgium

Data on relative earnings for Belgium are available from OECD Education at a Glance. We use the data for workers of age 25 to 64 during the calibration period (1996-2007). In line with our overall approach, we consider earnings of a worker without higher secondary education as representative for low ability individuals, and earnings of a worker with a higher secondary degree but no tertiary degree as representative for workers with medium ability. The earnings of a worker with a tertiary degree are assumed representative for workers with high ability. The relative wages of low versus high ability workers is 69%. The relative wage of low versus medium ability workers is 90%.

D   Data on demography, fiscal policy and pensions

D.1   Demography

f_t: fertility rates
Data source: population by age since 1948 (Bevolkingsvooruitzichten 2015-2060 of the Belgian Federal Planning Bureau and Statistics Belgium)
Computation: We divided the population of age 18 to 20 during three years by the population of age 18 to 20 in the previous three years. The fertility rates are displayed in Figure 5a. As

to the impact of migration, both natives and immigrants of age 18 to 20 are included in the youngest generation. They affect population dynamics in our model. People who enter or leave the country after the age of 20 do not. Children of immigrants are included in the fertility rate when they become 18.

sr_j^t: conditional survival rates
Data sources: Statistics Belgium, Mortality rates before 1998 are by age category (sometimes 4 years, sometimes 5) and start from 1946. As of 1998 data are annual. Prospects were provided by the Belgian Federal Planning Bureau and Statistics Belgium (Bevolkingsvoorzichten 2015-2060).
Computation: Survival rates were calculated by substracting the mortality rate from 1. Some conditional survival rates are shown in Figure 5b.

D.2 Fiscal policy and pensions

Γ_t: overall average household tax rate on gross labour income (% of gross wage)
Data sources: OECD Government Revenue Statistics, Details of tax revenue - Belgium, and OECD Economic Outlook (available via OECD.Stat).
Computation: Total tax revenues of individuals on income and profits (code 1110) plus social security contributions (code 2100) are divided by the gross wage bill.

τ_p: employer social contribution rate (% of gross wage)
Data sources: OECD Government Revenue Statistics, Details of tax revenue - Belgium, and OECD Economic Outlook (available via OECD.Stat).
Computation: we divide the social contributions paid by employers (code 2200) by private gross wage bill (the gross wage bill minus government wages).

τ_c: Consumption tax rate (in %)
Data source: McDaniel (2007, updated 2014).

τ_k: Tax rate on capital returns
Data sources: after 1982: effective marginal corporate tax rates (Devereux et al., 2002). 1970-1981: extrapolated based on the evolution of Belgium's statutory corporate income tax rates.

g: government spending on goods and services as a fraction of GDP
Data sources: The data include government consumption and fixed capital formation (OECD Economic Outlook No 98)

rr_L, rr_M, rr_H: net own-earnings related pension replacement rates
Data sources and description: OECD Pensions at a Glance (2005,2007,2009,2013) presents net pension replacement rates for individuals at various multiples of average individual earnings in the economy. Taking into account that relative to average earnings, earnings of the low (no upper secondary degree), medium (upper secondary degree) and high ability group (tertiary degree) are 86%, 95% and 122% (OECD Education at a Glance, 2011), we consider the data for individuals at 87,5% of average earnings as representative for the low ability group, individuals

with average earnings as representative for the medium ability group, and individuals with 125% of average earnings as representative for the high ability group. Country studies show the composition (sources) of this net replacement rate. Our proxy for rr_s includes all earnings-related pensions and mandatory occupational pensions when they depend on wages or hours worked. Data before 2002 are extrapolated using Scruggs (2007), Ebbinghaus and Gronwald (2009), and Cantillon et al. (1987).

Other pension parameters: Both the revaluation factor applied to past labour income in the determination of the pension benefit of new retirees wg, and the revaluation factor applied to adapt pension benefits of existing retirees to increased living standards pg follow the Belgian reality. In Belgium, only labour income earned between 1955 and 1974 underwent real revaluations according to wg^n with $n = 1$ in 1974, $n = 2$ in 1973, ..., $n = 20$ in 1955 and $wg = 1.036$ in 1974-1996, $wg = 1.032$ in 1997, $wg = 1.028$ in 1998, ..., $wg = 1$ as of 2005 (Festjens, 1997). pg is set to 1 before 1969, 1.023 annually between 1969 and 1992, 0.993 between 1993 and 2013, 1.003 for 2014-15, 1.005 for 2016-21 and 1.002 afterwards. Data before 1984 are from Festjens (1997). Observations until 2015 and future values were taken from Studiecommissie voor de Vergrijzing (2016). The contribution rates of individuals and firms to the public pension scheme cr_1 and cr_2 are 7.5% and 8.9% respectively (OECD Pensions at a Glance, 2013).

b: Gross unemployment benefit replacement rate for the low ability group
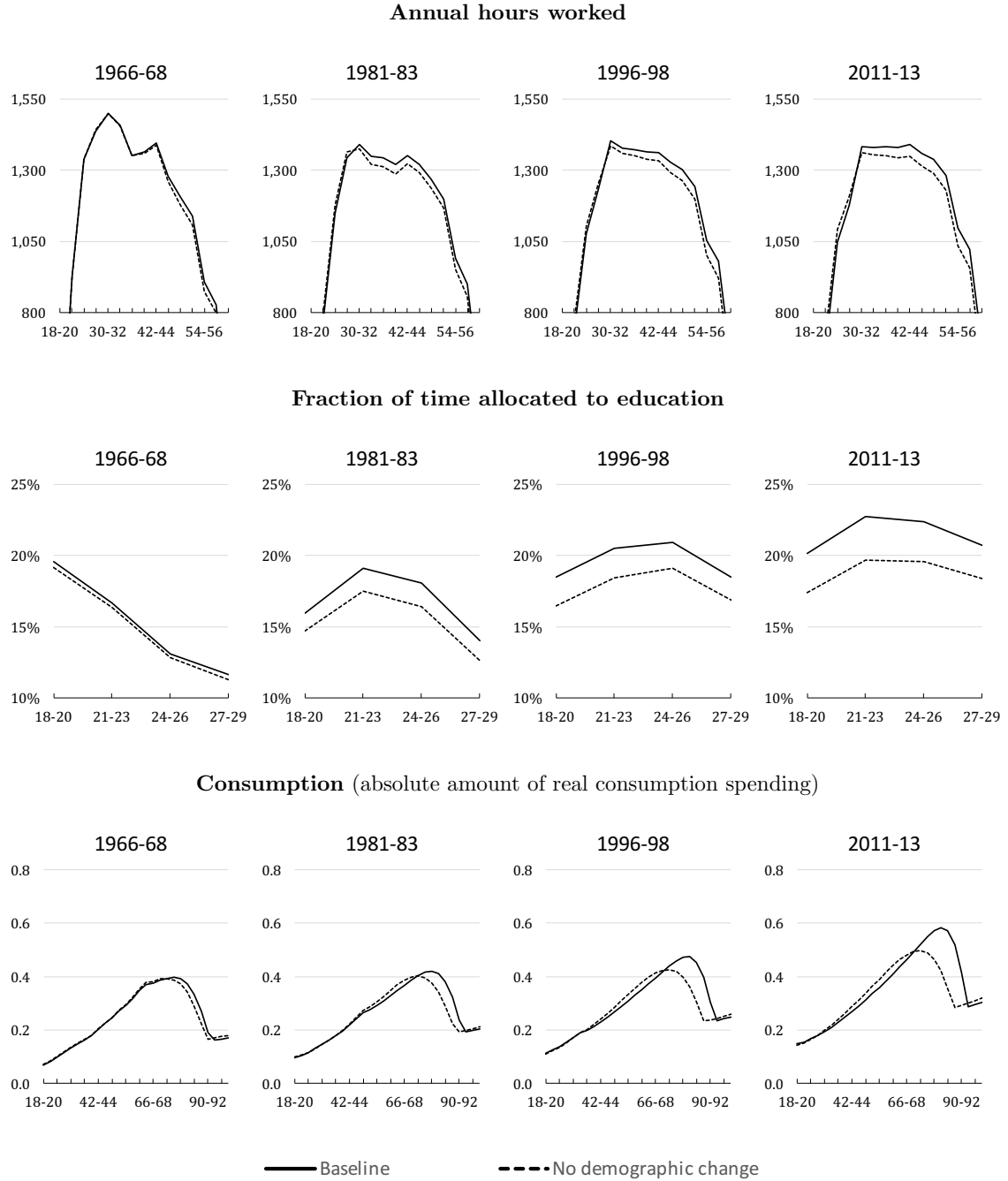*Data sources and description*: OECD Database on Benefit Entitlements and Gross Replacement Rates for data going from 1961 to 2007. The reference earnings are 67% of average earnings. For 2008-2014, we extrapolate this data series with the trend observed in the gross replacement rates for an individual that has average earnings (OECD Benefits and Wages, Gross Replacement Rates). In model period 14, $(1 - \tau_{w,L} - b)/2$ is added as a bonus to the benefit replacement rate as to account for the Belgian redundancy pay system (stelsel van werkloosheid met bedrijfstoeslag or SWT).

B: General government consolidated gross debt in % of GDP
Data source: European Commission, AMECO, series UDGGL.

E   Effects of demographic change at the level of individual cohorts

Figure 20: Life-cycle profiles of the youngest generation (of high ability) in 1966, 1981, 1996 and 2011

Annual hours worked

Fraction of time allocated to education

Consumption (absolute amount of real consumption spending)

——— Baseline   ‑‑‑‑ No demographic change

Note: each panel of this figure shows the evolution of annual hours worked, education time or consumption over a representative individual's life. The horizontal axis represents the individual's age. We consider individuals who enter our model (at age 18) in 1966-68, 1981-83, 1996-98 and 2011-13. The full black line is our baseline simulation, the dotted line the 'no demographic change' counterfactual.

References

Acemoglu, D. and Restrepo, S. (2017). Secular stagnation? The effect of aging on economic growth. *NBER Working Paper*. N° 23077.

Aiyar, S., Ebeke, C., and Shao, X. (2016). The impact of workforce aging on European productivity. *IMF Working Paper Series*. N° 16/238.

Andolfatto, D. (1996). Business cycles and labor-market search. *The American Economic Review*, 86:112–132.

Attanasio, O., Bonfatti, A., Kitao, S., and Weber, G. (2016). Global demographic trends: consumption, saving and international capital flows. In Piggott, J. and Woodland, A., editors, *Handbook of the Economics of Population Aging*, pages 179–235. Elsevier North-Holland.

Barro, R. and Lee, J. (2013). A new data set of educational attainment in the world, 1950-2010. *Journal of Development Economics*, 104:184–198.

Ben-Porath, Y. (1967). The production of human capital and the life-cycle of earnings. *Journal of Political Economy*, 75:352–365.

Bloom, E., Canning, D., Mansfield, R., and Moore, M. (2007). Demographic change, social security systems and savings. *Journal of Monetary Economics*, 54:92–114.

Boone, B. and Heylen, F. (2018). Cross-country differences in unemployment: fiscal policy, unions and households preferences in general equilibrium. *Scandinavian Journal of Economics*. forthcoming.

Bouzahzah, M., de la Croix, D., and Docquier, F. (2002). Policy reforms and growth in computational OLG economies. *Journal of Economic Dynamics and Control*, 19:2093–2113.

Buyse, T., Heylen, F., and Van de Kerckhove, R. (2017). Pension reform in an OLG model with heterogeneous abilities. *Journal of Pension Economics and Finance*, 16:144–172.

Cantillon, B., Peeters, J., and Ridder, E. D. (1987). *Atlas van de Sociale Zekerheid*. Acco.

Caselli, F. and Coleman, W. (2006). The world technology frontier. *American Economic Review*, 96:499–522.

Cervellati, M. and Sunde, U. (2013). Life expectancy, schooling, and lifetime labor supply: theory and evidence revisited. *Econometrica*, 81:2055–2086.

Cervellati, M., Sunde, U., and Zimmermann, K. (2017). Demographic dynamics and long-run development: insights for the secular stagnation debate. *Journal of Population Economics*, 30:401–432.

Cowell, F. (2011). *Measuring Inequality*. Oxford University Press.

de la Croix, D., Pierrard, O., and Sneessens, H. (2013). Aging and pensions in general equilibrium: Labor market imperfections matter. *Journal of Economic Dynamics and Control*, 37:104–124.

Dedry, A., Onder, H., and Pestieau, P. (2017). Aging, social security design, and capital accumulation. *Journal of the Economics of Ageing*, 9:145–155.

Devereux, M., Griffith, R., and Klemm, A. (2002). Corporate income tax reforms and international tax competition. *Economic Policy*, 35:451–495.

Devriendt, W. and Heylen, F. (2018a). Coping with demographic change: Macroeconomic performance and welfare inequality effects of public pension reform. *Working Paper, Faculty of Economics and Business Administration, Ghent University.* N° 2018/948.

Devriendt, W. and Heylen, F. (2018b). Macroeconomic and distributional effects of demographic change in an open economy - The case of Belgium. *Paper revised and resubmitted to Journal of Demographic Economics.*

Ebbinghaus, B. and Gronwald, M. (2009). International policy diffusion or path dependent adaptation? The changing public-private pension mix in europe. In Ebbinghaus, B., editor, *Varieties of Pension Governance. The Privatization of Pensions in Europe.* Oxford Univ. Press. 2011.

Fanti, L. and Gori, L. (2011). On economic growth and minimum wages. *Journal of Economics*, 103:59–82.

Federal Planning Bureau (2016). *Economische vooruitzichten 2016-21.*

Feenstra, R. C., Inklaar, R., and Timmer, M. P. (2015). The next generation of the Penn World Table. *American Economic Review*, 105:3150–3182. Available for download at http://www.ggdc.net/pwt.

Festjens, M. (1997). De pensioenhervorming. Een nieuwe generatie en een nieuw contract. *Federal Planning Bureau Planning Paper.* N° 82.

Goodhart, C. and Erfurth, P. (2014). Demography and economics: Look past the past. *VoxEU.org.* 4 November.

Gordon, R. J. (2014). The turtle's progress: Secular stagnation meets the headwinds. In Teulings, C. and Baldwin, R., editors, *Secular Stagnation: Facts, Causes and Cures*, pages 47–59. CEPR Press and VoxEU.org, London.

Guo, J. and Lansing, K. (1998). Indeterminacy and stabilization policy. *Journal of Economic Theory*, 82:482–490.

Heijdra, B. J. and Reijnders, L. S. (2018). Longevity shocks with age-dependent productivity growth. *Journal of Pension Economics and Finance*, 17:200–230.

Heylen, F. and Van de Kerckhove, R. (2013). Employment by age, education, and economic growth: effects of fiscal policy composition in general equilibrium. *The B.E. Journal of Macroeconomics*, 13:49–103.

IMF (2014). Perspectives on global real interest rates. *World Economic Outlook.* April, chapter 3.

Kotlikoff, L. J., Smetters, K., and Walliser, J. (2007). Mitigating America's demographic dilemma by pre-funding social security. *Journal of Monetary Economics*, 54:247–266.

Koyuncu, M. (2011). Can progressive taxation account for cross-country variation in labor supply. *Journal of Economic Dynamics and Control*, 35:1474–1488.

Krueger, D. and Ludwig, A. (2007). On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. *Journal of Monetary Economics*, 54:49–87.

Lee, R. (2016). Macroeconomics, aging and growth. In Piggott, J. and Woodland, A., editors, *Handbook of the Economics of Population Aging*, pages 59–118. Elsevier North-Holland.

Lucas, R. (1990). Supply-side economics: an analytical review. *Oxford Economic Papers*, 42:292–316.

Ludwig, A., Schelkle, T., and Vogel, E. (2011). Online appendix to "demographic change, human capital and welfare". *Review of Economic Dynamics*. Technical Appendices 08-168.

Ludwig, A., Schelkle, T., and Vogel, E. (2012). Demographic change, human capital and welfare. *Review of Economic Dynamics*, 15:94–107.

Marchiori, L., Pierrard, O., and Sneessens, H. (2017). The EU-US unemployment puzzle revisited: institutions, demography, and capital flows. *Journal of Demographic Economics*, 83:259–305.

McDaniel, C. (2007). Average tax rates on consumption, investment, labor and capital in the OECD 1950-2003. *Arizona State University*. Updated 2014 (http://www.caramcdaniel.com/researchpapers).

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36:269–300.

Miles, D. (1999). Modelling the impact of demographic change upon the economy. *The Economic Journal*, 109:1–36.

OECD (2015). Statistical annex. *Employment Outlook*. Paris.

Onder, H. and Pestieau, P. (2014). Is aging bad for the economy? Maybe. *Economic Premise*, 144. The World Bank.

Rogerson, R. (2007). Taxation and market work: is Scandinavia an outlier? *Economic Theory*, 32:59–85.

Rogerson, R. and Wallenius, J. (2009). Micro and macro elasticities in a life cycle model with taxes. *Journal of Economic Theory*, 144:2277–2292.

Sánchez-Romero, M. (2013). The role of demography on per capita output growth and savings rates. *Journal of Population Economics*, 26:1347–1377.

Scruggs, L. (2007). Investigating welfare state change. In Clasen, J. and Siegel, N. A., editors, *Welfare state generosity across space and time*, pages 133–165. Edward Elgar.

Shiller, R. (2015). *Irrational Exuberance*. Princeton University Press, Third edition, online data. (http://www.econ.yale.edu/ shiller/data.htm).

Solt, F. (2016). The standardized world income inequality database. *Social Science Quarterly*, 97:1267–1281. (http://myweb.uiowa.edu/fsolt/swiid/swiid.html).

Sommacal, A. (2006). Pension systems and intragenerational redistribution when labor supply is endogenous. *Oxford Economic Papers*, 58:379–406.

Studiecommissie voor de Vergrijzing (2016). *Jaarlijks verslag 2016*.