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A Statistical Comparison of Two Safety Stock Replenishment Mechanisms in a Cyclic Stochastic IRP

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Abstract: Preventing stock-out in a replenishment system, in which customer demand rates are stochastic with constant averages, can be accomplished via safety stocks hold at each customer. These safety stocks should be replenished back to their initial levels each time they are used. However, sometimes it occurs that a truck does not have enough capacity to carry the amount of the product the visited customers need to restore their stocks and safety stocks to their required levels. Therefore, the carried extra amount of the product, intended to replenish safety stocks, should be divided amongst the customers in some optimal or fair manner. To achieve this fair allocation, we propose and analyze two policies, the first called Fair-share and the second Ratio methods. Details on how these two methods are implemented, to achieve the level of service expected at each customer, are discussed and illustrated. In addition a simulation model is developed and used to compare the performance of each policy in the long run.

Keywords: Fair-Share, Inventory Routing Problem, Simulation, Service Level, Safety Stock, Backlog.

1. INTRODUCTION

The idea of integrating different planning processes in supply chain is gaining popularity amongst both researchers and supply chain planners. The Inventory routing problem (IRP) is a critical issue in supply chain management, and is being investigated for over three decades. Researchers have investigated and implemented IRP from different points of view in many industries (Campbell et al., 1998, Bertazzi et al., 2008, Özener et al., 2013, Aghezzaf, 2007). These models consider a fleet of vehicles planned to visit every customer in a certain period of time from a central depot (or several decentralized warehouses). The main goal is to find the best feasible delivery route in a way that costs for inventory, backlog, and transportation are optimized under a certain service level.

Coelho et al. (2014) and Raa and Dullaert (2008) state there are two main approaches to plan IRP system known as short term and long term. In short term planning, due to the uncertainty of demands and flexibility of system to change the routes, in each cycle (day, week, or even month) the routes are changed and replenishment will occur in a new routing system. It prevents stock out and minimizes the inventory level in a more flexible way. In long term planning which is suitable for more stable demand rates, the customers are following constant or stochastic but stationary demand rates which makes the decision maker capable of planning for a long term horizon. During this period the main goal is to find optimum level for the inventory to satisfy the demand and prevent backlog.

Although it improves customers' reliability to prevent lost orders/backlog, in some cases it is unavoidable experiencing stock-out/backlogs. Due to the periodic replenishment, every customer needs to keep an inventory buffer to cover demand uncertainty during the replenishment cycle. This inventory buffer includes a safety stock which is assigned based on cycle time, replenishment lead time, and standard deviation of demand rates (Raa and Dullaert, 2008).

In this paper a replenishment evaluation system is considered for a distribution system with one depot and multiple scattered customers. Customer demands are assumed to be Normally distributed. We assume the external supplier has unlimited stock and thus can always fill up a warehouse order immediately. All lead times are considered constant. Shortages at the customers are fully backlogged.

Section 2 reviews in depth some articles to find out more about the history of IRP. The methodology is described in section 3. To clarify, an example of an IRP model will be presented in section 4 and the results and discussions come afterward to display the evaluations.

2. A BRIEF LITERATURE REVIEW

Federgruen and Zipkin (1984), are the first to tackle the integration of routing and inventory management. Their model considers retailers with random periodic demand rates. A buffer inventory is available for each retailer to prevent stock-out in each cycle. Minimization of inventory level, total transportation, and backorder costs while allocating an optimized level of inventory in the warehouse are their main objectives. They applied Traveling Salesman Problem to reduce inventory and shortage costs as well as transportation costs.

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Long term IRP planning was presented first by Larson (1988). He assigned customers to the defined groups by a heuristic method to deliver the products in a single route. Anily and Federgruen (1993) introduced “fixed partition policies” and tried to minimize the average costs of transportation and inventory. They considered customers in separated partitions which are replenished by one truck. Applying an index based sales-point evaluation, Golden et al., (1984) developed a heuristic method of prioritizing customers to be replenished in a cycle. The highest ratio in this method is in urgency to receive the demanded material. They have defined certain levels as boundaries to simplify making the decisions. From another point of view Aghezzaf (2007) developed a robust cyclic replenishment policy to optimize inventory level and cycle time while there are stochastic but stationary demand rates and travel times. Safety stocks are provided for each customer to cover their demand uncertainty during lead time. In addition the safety stock level in truck is considered in the optimization model to include the changes in demand into capacity of the vehicle. Monte Carlo simulation is applied to illustrate the effects of this model to a simplified real-life case. Coelho et al. (2014), investigate stochastic inventory routing problem in a replenishment system with probabilistic demand rates which allows customers to experience shortages in inventory. They provide a comprehensive review of articles related to IRP with different structures from several researchers in this field.

Keeping backlog in control is one of the main responsibilities of a supply chain manager. Although lots of heuristic methods have been developed to prevent backlogs, in some cases backlog occurrence is unavoidable.Gruen et al. (2002), state according to a worldwide survey about the behaviour of customers during backlog, only 15% of all customers decide to wait for the product, other 85% will substitute another product from another sale point or even not buying it at all. In all these cases demand for the preferred product–store combination is lost. To take backlog/lost-sales into account, researchers have applied penalties in their optimization models for backlog occurrence to calculate backlog costs as well as inventory and transportation costs (Raa and Dullaert, 2008, Özener et al., 2013, Coelho et al., 2014). Nevertheless there are some authors Dong and Tomlin (2012) who developed models for backlog prevention and risk management. Safety stock, emergency sourcing, dual sourcing, demand management and process improvement are some of the solutions provided by researchers. Furthermore, they have advised business interruption insurance and operational measures as two extra tools to manage backlog. Considering backlog together with inventory level at a certain level of service, affects routing system in optimization models. Nowadays it is a challenging topic for all the supply chain planners to reach their customers with highest reliability.

Although the safety stock is supposed to reduce backlog occurrence, there is still lost sales/backlog happening. Due to the limited capacity of vehicles, and the necessity to cover all the customers’ demand (inventory and backlog) in a tour, it is critical to find the best method for allocating materials in truck (when all demands plus backlogs in one tour exceed the truck’s capacity) to customers in a way that all of them are replenished fairly.

The problem tackled in this paper is concerned with the best way of allocating materials in the vehicle to cover the stochastic demand of customers when there is not enough capacity to cover the total demands and backlogs in a tour. Two methods are evaluated, the first entitled Fair-share and the second Ratio. To analyse the effectiveness of these policies, an illustrative case in an IRP model is simulated to evaluate the safety stock replenishment policies statistically.

3. SAFETY STOCK REPLENISHMENT POLICIES

3.1 Fair-share vs. Ratio Mechanism

The fair-share mechanism means in general a method to share benefits or costs in a manner that none of the parties loses and everyone gets a fair portion of the total benefit or cost. A prerequisite in the situation we consider is the availability of information on the stock at all locations, including the in-transit quantities. Lot sizing is done at a central decision point (depot) and the portions are transferred by a truck to the retailers. Rather than parcelling out inventory according to individually planned orders, it allocates on-hand inventory according to aggregate net requirements for the network. The quantity delivered is a fair-share of what is available in the truck. If more than sufficient stock is available to meet total system requirements, the extra stock will be returned to the depot for the next tour (Aghezzaf et al., 2011, Van den broecke et al., 2008).

Whereas the demand rate is the only factor that is taken into account in the Fair-share method, the Ratio method calculates a ratio of backlog and demand in each cycle for each customer and uses this ratio as an index of importance. It means the truck follows the customers’ rate to decide about replenishing available materials. In other words, if one customer experienced higher level of backlog than other customers in the tour, he deserves more products to be delivered relatively.

In this paper we will attempt to demonstrate the following proposition:

Proposition: The Ratio mechanism performs better than the Fair-share mechanism from inventory and service levels’ point of view in cyclic replenishment.

This proposition states that the Ratio mechanism replenishes the material in a fairer way. It means either backlog or inventory levels would be reduced by implementing Ratio policy for stock replenishment while service level is covered. In the rest of the paper, the equations to calculate the replenishment mechanisms are explained. In continue the illustrative case will present a statistical comparison of the two policies.

3.2 The safety stock policy models

Consider several retailers $R_i, i \in \{0, 1, \dots, n\}$ with demand rates of d_i, scattered around one depot R_0 with infinite capacity. They are replenished by vehicle v with K as current capacity in the cycle period CP_j, $j \in \{0,1, \dots, m\}$. Each customer has a current inventory level during period t which is shown by I_{it} . It is also allowed to have backlog BL_{it}. SS_{it} indicates the level of safety stock for each customer during each period of t and it is calculated based on CP_j and number of customers. Therefore $I_{max_i t}$ and $I_{it} = d_{it} * CP_j$ are maximum and current level of inventory at customer i during time period t.

The quantity of products to be delivered for customer i (dl_{it}) in Fair-share method is calculated as follow:

$$(1). dl_{it} = d_{it} * \frac{K}{\sum_{i=1}^{n} d_{it}}$$

It would be a fair portion of products that is available in the truck and will be unloaded to the retailer R_i.

Ratio method works differently. To involve backlog level into delivery quantity, the equation 1 should change as follow:

$$(2). dl_{it} = (d_{it} - I_{it-1}) * \frac{K}{\sum_{i=1}^{n}(d_{it} - I_{it-1})}$$

Equation 2 calculates the needed quantity of products to be delivered to retailer i during time period of t. The inventory level during the last time period for retailer i is subtracted from the requested demand. Due to the normally distributed demand rates, the level of delivered products in Ratio method would compensate some shortages/abundance of products at customers in the tour. To see the behavior of the two presented policies, in next section an illustrative example will be modeled and simulated In order to evaluate the methods.

4. ILLUSTRATIVE TEST CASE

In this section a replenishment system is presented to clarify the effects of these two policies on inventory and backlog levels. There are twenty customers in the model served from a central depot by one truck. Customers are already assigned to the several tours.

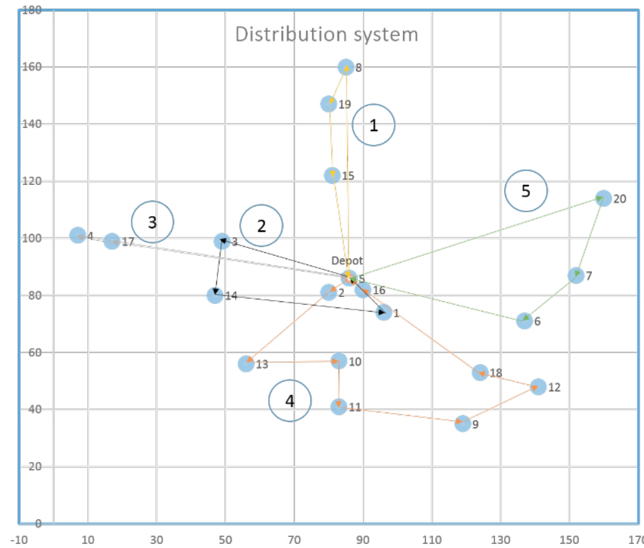

Fig 1. Example's Scheme in Simulation model

As it is displayed in (fig. 1) in this case, the truck always uses the coloured path for each tour. The average demand rates are shown in table 1. Replenishment cycle times are constant and take 3 and 6 days which is also presented in table 1. Demand rates are Normally distributed with 5 percent standard deviation. Truck's capacity is 80 tons. To have a better idea about the two presented policies, the customers in one of the tours will be evaluated according to the defined indicators. Tour number 4 in fig. 1 shows the under consideration customers for this example.

4.1 Indicators

- Average inventory level in one period for both Fair-share and Ratio mechanism

The level of stock at the end of planning horizon is measured to calculate the inventory costs and evaluate the considered policies.

- Inventory rates based on periodic average demand

To avoid the effects of large demand rates, the level of stock are divided by demand rates to involve the effects on inventory level.

- Achieved service level

There are several methods to calculate service level. Here you see the method used for service level calculation presented by (Silver et al., 1998).

S is the specified probability of no stock out per replenishment cycle.

$$(3). S = \frac{Number\ of\ replenishment\ that\ I_{it} > 0\ at\ the\ end\ of\ period}{Total\ number\ of\ replenishment\ cycle}$$

Where I_{it} is the level of inventory at the customer i.

In this paper we calculate S that measures the number of replenishments without stock-out. In other words, the percentage of replenishments with completely satisfied demand divided by the number of all replenishments cycles, is counted as service level.

The idea is to measure the indicators in order to discover the difference between two defined policies and evaluate them in terms of inventory and service level reduction/enlargement. In addition the variance of the achieved results would be analysed to check the fairness between all the customers in the tour.

We run the simulation model for 6 days planning horizon (one period) and repeat it for 200 times. The results for both Fair-share and Ratio mechanism are presented in diagrams. The average values of all indicators are collected and visualized in different figures to make it easier comparing the replenishment policies.

Table 1 Basic information about the distribution model

Customers Numbers	Transferred products in each cycle	Demand Rate	Standard Deviation 5 percent	Cycle time
8	28,8	9,6	2,49	3
19	21,6	7,2	1,87	3
15	22,2	7,4	1,92	3
5	5,4	1,8	0,47	3
3	14,4	4,8	1,27	3
14	28,8	9,6	2,49	3
1	30	10	2,60	3
17	34,8	5,8	4,26	6
4	43,2	7,2	5,29	6
2	4,8	0,8	0,59	6
13	3,6	0,6	0,44	6
10	14,4	2,4	1,76	6
11	7,2	1,2	0,88	6
9	9,6	1,6	1,18	6
12	21,6	3,6	2,64	6
18	3,6	0,6	0,44	6
16	10,8	1,8	1,32	6
20	28,8	4,8	3,53	6
7	15,6	2,6	1,91	6
6	6	1	0,73	6

5. RESULTS AND DISCUSSIONS

Fig. 2 shows the average values of the inventory level at the end of planning horizon for each customer in the tour (red tour fig. 1). These values are presented in table 2. As it is clear, the level of inventory in both policies are very close to each other with small differences. In general, Ratio mechanism has received more moderated values than Fair-share, since the trend is less wavered. Obviously, the demand rate for each customer has a critical rule in assigning the average level of inventory. Therefore, figure 3 and table 3 are presented to clarify the effect of demand rates on inventory levels.

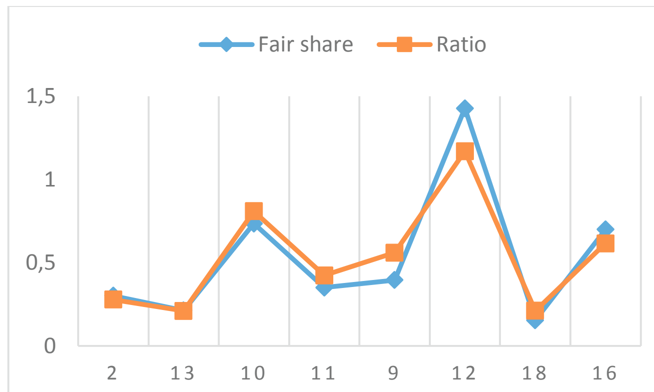

Fig. 2 Average inventory level

Table 2 presents the average values achieved from the simulation for 8 considered customers on the replenishment tour. The values for the level of inventory and service are shown in this table.

Table 2 results of the considered tour

Customer Numbers	Fair-share Mechanism		Ratio Mechanism	
	Ave inventory	Average service level	Ave inventory	Average service level
2	0,302	0,663	0,278	0,849
13	0,213	0,658	0,210	0,849
10	0,735	0,663	0,809	0,849
11	0,350	0,654	0,423	0,849
9	0,395	0,619	0,560	0,849
12	1,428	0,688	1,170	0,849
18	0,154	0,644	0,211	0,849
16	0,699	0,702	0,616	0,849

In table 3, the ratios of inventory based on demand rates for each customer are provided. These values show the effectiveness of two policies without considering the level of demand rate for the customers.

Table 3. Inventory level based on demand rate

Customer Numbers	Fair-share Mechanism	Ratio Mechanism
2	0,062827	0,057947
13	0,059237	0,058325
10	0,051034	0,056175
11	0,048644	0,058745
9	0,041169	0,058317
12	0,066101	0,054158
18	0,042945	0,058611
16	0,064745	0,057049

Figure 3 visualizes the values from table 3. Horizontal axis shows the considered customers and vertical axis stands for the rate of average inventory level based on customers' demand.

The trend for Ratio mechanism shows almost a flat line, while Fair-share trend has fluctuations among the customers in the tour. In other words, according to the demand rates, Ratio policy has shown a fair behaviour in replenishing products between customers in the tour than Fair-share. Although in Fair-share policy the idea is to distribute a fair portion of demanded product to the customers, but considering backlog level history for the customers caused a more moderated inventory level among the customers in the tour.

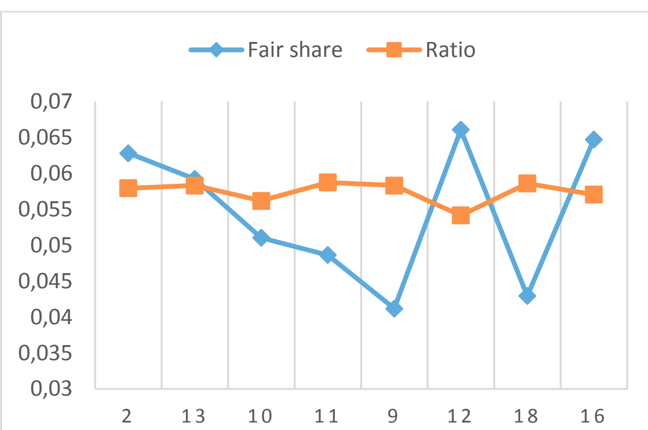

Fig. 3 Inventory level based on demand rate

Figure 4 illustrates the level of service during the planning horizon according to equation 3. The service level is measured based on the number of periods which the level of inventory at the end is positive. It shows the capability of the system to cover the requested demand or in other words, the assurance the supply chain planner makes to retailers in order to provide the demanded materials on time and in right quantity. As it is displayed in figure 4, the level of service for Fair-share is around 65% which is 20% less than level of service for Ratio mechanism. This diagram simply shows the higher confidence of Ratio policy compare to Fair-share in demand satisfaction.

The other point is about the variance of the values for customers in this figure. As it is shown in figure 4, the average level of service for all the customers in the tour is the same for Ratio mechanism which shows the equality in products distribution, while in Fair-share there are some differences among the customers in the tour in terms of service level.

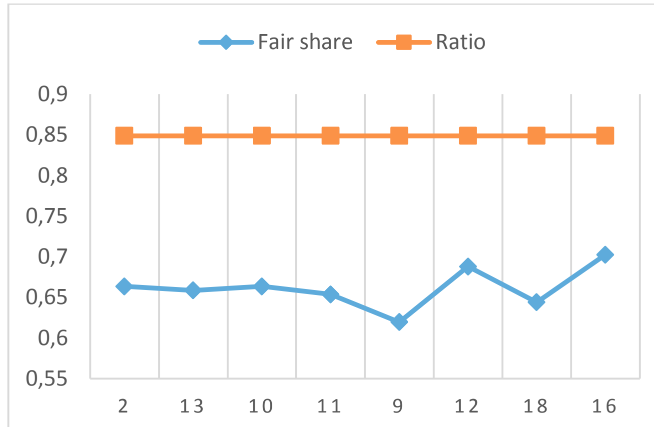

Fig. 4 Service level

To have a comprehensive view of the whole system's service level (with 20 customers), figure 5 is provided. Horizontal axis denotes the number of tours (indicated in figure 1) and vertical axis shows the average level of service for each group of retailers. Two considered mechanisms for products distribution are behaving differently in different tours. The lowest service level is achieved in tour number 3 which is around 58% and the highest belongs to tour 5 with 100% demand coverage.

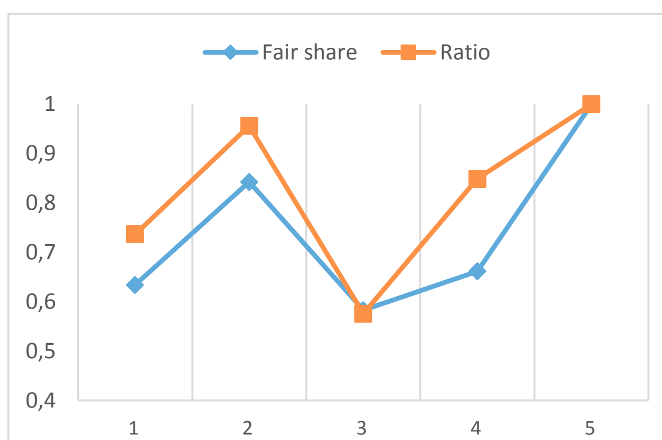

Fig. 5 The average service level for each tour

The trend for Ratio policy shows higher level of assurance in demand satisfaction compare to Fair-share mechanism except tours 3 and 5 which indicates equality for both of them. Although both presented mechanisms are implemented in the same routing system, the Ratio policy offers better outcome than Fair-share in both service level and inventory. Higher demand coverage certainty and less stock at the customers at the end of planning horizon together with lower fluctuations in level of service and inventory for customers in a tour, prove that Ratio mechanism is a prefered policy to be applied in products distribution.

6. CONCLUSION

This paper presents and discusses the effects of two policies on safety stock replenishment in cyclic inventory routing problem with stochastic demand rates for the cases in which the products quantity carried in the truck is not enough to cover both average demand, backlogged quantity, and safety stock. Fair-share and Ratio are two mechanisms proposed and considered in this study as two safety stock distribution policies. Fair-share distributes the extra products based on customer's demand rate, while Ratio uses a rate of backlog level during the last period for each customer.

According to the results of the illustrative case, all the indicators have shown the advantage of Ratio mechanism compare to Fair-share. It improves the replenishment system by diminishing the level of inventory and enhancing the service level. The results indicate the fairness of Ratio in allocating materials in truck to the customers in each tour. Based on this policy, all the customers in a tour have the same chance to experience backlog. In addition, the level of stock is optimized and minimized for the considered planning horizon. Fluctuations for different customers in a tour for the level of inventory has reduced by using Ratio mechanism instead of Fair-share. It improves the level of fairness between all the customers in a tour that helps avoiding them making a collusion with other parties from other distribution centers.

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