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A REVIEW ON FRETTING FATIGUE CRACK INITIATION CRITERIA

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Abstract: This paper aims to provide an overview of experimental and numerical work related to fretting fatigue crack initiation criteria. The complexity of modelling fretting fatigue arises due to the presence of multi axial stress state and wear, hence, it is imperative to define the crack initiation criteria. Among many available criteria, some of the most widely used have been summarized here, to provide an insight of the topic and to demonstrate the applicability of these criteria. For convenience, different criteria have been grouped together based on the approaches used to define failure parameters. The generalize classification includes, critical plane approach, stress-invariant approach and continuum damage mechanics approach. Besides experimental work, quantum of work has been done to implement these models using finite element methods (FEM). The strength of finite element methods is demonstrated as it provides minute information about crack initiation phase, contact stresses and estimated life. At the end, conclusions are drawn to advocate the proximity and efficacy of numerical methods in comparison to experimental work.

Keywords: Fretting fatigue; crack initiation criteria; FEM.

1 INTRODUCTION

Fretting can be referred as a material damage caused by small oscillatory movements between the contacting bodies. This may lead to a deterioration of surfaces and dimensional changes. Also a remarkable decrease in service life can be observed in the presence of bulk stress [1]. The surface degradation is defined as fretting wear, whereas the development of crack can be termed as fretting fatigue [2]. For estimation of life in fretting fatigue problems the complete process is usually divided in two phases, namely, crack initiation and crack propagation. While there is still a debate on the proportion of life time taken by these two phases, most of the authors have considered both the phases for total life estimation [3,4]. Therefore, the search for selection of most suitable criteria still continues. Each criterion is suitable in certain combination of selected material, geometry, type of contact and loading conditions [5]. This article, however, presents a review on the crack initiation criteria and its applications using finite element analysis.

Fretting fatigue is a type of multi-axial fatigue having non-proportional loading [6], thus introduces multiaxial stress fields and severe stress gradients [7]. Therefore, multiaxial criteria are used to define the failure [8,9]. Different multi-axial criteria have been developed in the past that use critical damage parameters, along with various methods to define estimated life. In general, the stress or strain components combining with material constants are equated to fatigue strength limit in fully reversed tension/torsion or by Manson-Coffin and Basquin relation [10]. Hence, allowing to compute the estimated life under fretting fatigue scenario. Some researchers have used continuum damage mechanics (CDM) approach which is based on thermodynamic potential function and initiation life can be estimated using bulk material properties. Based on the approach used to model fretting fatigue crack initiation, different criteria can be classified as; critical plane approach, stress invariant approach and CDM approach.

2 CRITICAL PLANE APPROACH

According to this approach the crack initiate on specific planes known as critical planes. Therefore, the damage parameters are evaluated on these planes. Depending on the material and loading conditions, these planes are either maximum shear planes or maximum tensile stress planes. The most frequently used criteria based on this approach are presented below.

2.1 Smith-Watson-Topper (SWT) parameter

SWT parameter can be used to predict fretting fatigue crack initiation life. The parameter proposed by Smith et al. [11] can be expressed as a function of number of cycles to initiation (N_i) as:

$$\sigma_n^{max}\frac{\Delta\epsilon_1}{2}=\frac{\sigma_f'^2}{E}(2N_i)^{2b'} \quad (1)$$

Where, σ_n^{max} is the maximum normal stress in the plane of principal strain range and $\Delta\epsilon_1$ is the maximum principal strain range. σ_f' is fatigue strength coefficient and $b'$is fatigue strength exponent. This criteria is also applied to non-proportional loading by Socie [12] to predict crack initiation under multiaxial loading. The initiation is assumed to occur on the plane where, combination of range of strain normal to the plane ($\Delta\epsilon/2$) and maximum stress normal to the plane (σ_{max}), is most damaging. Lykins et al. [13] have used finite element analysis to successfully implement SWT model, where different fretting fatigue parameters have been computed to estimate the initiation life.

2.2 Fatemi-Socie (FS) parameter

Fatemi and Socie proposed a modification to Brown and Miller's critical plane formulation to predict multi axial fatigue life and to account for an additional cyclic hardening during non-proportional loading condition [14]. Furthermore, variable amplitude loading and effect of mean stress is accounted for with the proposed model. This model is applicable to the material and loading condition, where failure is produced by shear mode, thus the plane, where the shear strain range is maximum, is the critical plane. For fully reverse uniaxial strain condition the FS parameter can be defined as [14]:

$$\frac{\Delta\gamma_{max}}{2}(1+k\frac{\sigma_n^{max}}{\sigma_y})=\left[(1+\nu_e)\frac{\sigma_f'}{E}(2N_i)^{b'}+(1+\nu_p)\epsilon_f'(2N_i)^{c'}\right][1+k\frac{\sigma_f'}{2\sigma_y}(2N_i)^{b'}] \quad (2)$$

Where, $\Delta\gamma_{max}$ is the maximum shear strain range, σ_n^{max} is the maximum normal stress perpendicular to the critical plane, σ_y is the yield strength and k is a constant that is fitted from the uniaxial to torsion fatigue test data. b' and c' are fatigue strength exponent and fatigue ductility exponent respectively. ϵ_f' is fatigue ductility coefficient, ν_e and ν_p are elastic and plastic Poisson's ratio respectively. Sabsabi et al. [15] have applied FS criterion to fretting fatigue problem for comparison of experimental lives and estimated lives using extended finite element methods (X-FEM). The results showed that combining FS parameter with X-FEM provides good estimation of experimental total lives.

2.3 McDiarmid parameter

The criterion proposed by McDiarmid is a high cycle fatigue multi axial criteria. The fatigue strength is defined in terms of shear stress amplitude and maximum normal stress on critical plane of maximum shear stress amplitude [16]. The critical plane is the one where shear stress range is maximum for one cycle. The number of cycles to initiation can be related to McDiarmid parameter for the plain fatigue case as [5]:

$$\frac{\Delta\tau_{max}}{2}+\left(\frac{\tau'_{f_{-1}}}{2\sigma_u}\right)\sigma_n^{max}=\frac{1}{2}(1+\frac{\tau'_{f_{-1}}}{2\sigma_u})\sigma_f'(2N_i)^{b'} \quad (3)$$

Where, $\Delta\tau_{max}$ is the maximum increment of shear stress, σ_n^{max} is the maximum normal stress in the direction perpendicular to maximum shear stress range. $\tau'_{f_{-1}}$ is the shear fatigue limit and σ_u is the ultimate tensile stress. McDiarmid used two shear fatigue limits, one for the case where crack grows parallel to the surface and one across the surface. This criterion has been successfully implemented by Sabsabi et al. [15] and Navarro et al. [17] to estimate fretting fatigue crack initiation. Li et al. [18] has shown good agreement between experimental and estimated lives and also location and orientation of crack initiation using finite element methods.

2.4 Dang Van criterion

Dang Van proposed critical plane based initiation criteria, where the fatigue failure is dependent on shear stress and hydrostatic stress [19]. The maximum of the linear combination of both the parameters should be less than fatigue strength limit. Mathematically this criterion can be expressed as:

$$[\kappa(\tau)+3-1.5\kappa(\sigma_H)]_{max}\leq\sigma'_{f_{-1}} \quad (4)$$

Where, τ is shear stress amplitude on the examined plane, σ_H is the hydrostatic stress and $\sigma'_{f_{-1}}$is fatigue limit in fully reversed axial loading. κ is the fatigue limit ratio between fully reversed axial to torsion loading. Later, Dang Van et al. [20] proposed a three dimension criterion for fatigue limit where the results show a good agreement for industrial application. Alfredson and Cadario [6] and Nesládek et al. [10] have implemented

Dang Van criterion for evaluation of fretting fatigue crack initiation. Moreover, Nesládek et al. [10] found fatigue index error to be 0.6 and 15.3 for 5 kN and 15 kN, respectively, using finite element methods.

2.5 Papadopoulos criterion

This is a high cycle fatigue criteria which accounts for out of phase bending and torsion loading. The criteria presented by Papadopoulos et al. [21] is valid for hard metals with ratio of fatigue limit in torsion to bending in a range of 0.577 – 0.8. In this criterion a parameter called resolved shear stress is introduced for each plane. Introducing an average formula, which represents the mean average value of shear stress this criterion can be defined as:

$$\sqrt{5}.\sqrt{\frac{1}{8\pi^2}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{\chi=0}^{2\pi}(\tau(\varphi,\theta,\chi))^2 d\chi.\sin\theta.d\theta.d\varphi}+\frac{\tau'_{f_{-1}}-\sigma'_{f_{-1}}/\sqrt{3}}{\sigma'_{f_{-1}}/3}(\sigma_H)_{max}\le\tau'_{f_{-1}} \tag{5}$$

Where, $\tau'_{f_{-1}}$ is the fatigue limit under torsion and $\sigma'_{f_{-1}}$ under axial load. $(\sigma_H)_{max}$ is the maximum hydrostatic stress. Ferré et al. [22] have applied this criterion to predict crack nucleation under fretting fatigue condition, using local and non-local approaches. The error computed is found to be less than 5%. According to their research the non local approaches are more suitable to be used with finite element methods.

2.6 Findley parameter (FP)

Findley proposed the criterion as a function of maximum normal stress and amplitude of shear stress [23]. The critical plane can be referred to as a plane with maximum value of the parameter. Mathematically this criterion in expressed as

$$FP=\tau+k(\sigma_n^{max}) \tag{6}$$

Where, τ is the shear stress amplitude, σ_n^{max} is the maximum normal stress and k is the material coefficient. While applying this parameter for fretting fatigue crack initiation of spherical contact Alfredson and Cadario [6] suggested that this criteria has given better results than the others. However, the endurance limit of the criteria was higher than the experimental endurance limit. Concerning the applicability of the said criterion using finite element methods Li et al. [18] have predicted the fretting fatigue life in a scatter band of ±3N.

2.7 Shear Stress Range (SSR) parameter

This parameter is developed to predict crack location, orientation and number of cycles to fretting fatigue crack initiation and applied to Titanium alloy using different pad geometries. The parameter introduced by Lykins et al. [24] is based on shear stress range, where maximum and minimum shear stress is evaluated to find the critical plane from -90° to 90° with an increment of 0.1°. Mathematical expression is as follows

$$\Delta\tau_{crit}=(\tau_{max}-\tau_{min}) \tag{7}$$

To account for the effect of different stress ratios the formula takes the following form modified using the Walker's method.

$$SSR=\Delta\tau_{crit}=\tau_{max}(1-R_\tau)^m \tag{8}$$

Where, τ_{max} is the maximum shear stress, R_τ is the shear stress ratio and m is the fitting parameter. Apart from establishing the prediction of crack initiation and orientation, the initiation life is observed to fall in the band of ±3N from plane fatigue data. However, with different pad geometries significant variation in slip amplitude at the contact interface is observed [24].

2.8 Rolović and Tipton criteria

Rolović and Tipton presented the multi axial criteria based on combination of stresses and strains [25]. This criterion also consider damage on the critical plane with a length of 1mm crack size. The criteria has been tested against proportional and non-proportional loading conditions. The results are compared for both in phase and out-of-phase test data. Based on this criteria life estimation can be predicted with the following equation:

$$\gamma(\tau+0.3\sigma_n^{max})+\sigma_n^{max}(\epsilon_n)=\frac{\sigma_f'^2}{E}(2N_i)^{2b'}+\sigma_f'\epsilon_f'(2N_i)^{b'+c'} \tag{9}$$

Where, τ and γ are the shear stress and shear strain amplitude respectively on critical plane, σ_n^{max} is the maximum normal stress on the critical plane, ϵ_n is the normal strain amplitude, σ'_f and ϵ'_f is the fatigue strength and fatigue ductility coefficient respectively, b' and c' are fatigue strength exponent and fatigue ductility exponent respectively. This criterion is also used for evaluation of fretting fatigue problems. Li et al. [18] implemented this criterion for calculation of fatigue damage parameter and also for estimation of life. The life prediction results computed with this criterion are shown to fall in a scatter band of ±3N . Moreover, they have implemented finite element methods for calculation of normal, shear and tangential stresses at the contact interface.

3 STRESS INVAIRANT APPROACH

This approach uses stress invariants or tensor approach to define failure. Several multi axial criteria based on this approach are available in the literature, however only Crossland model in used frequently for application to fretting fatigue problems.

3.1 Crossland parameter

Crossland introduced a parameter which is based on the maximum amplitude of the second invariant of deviatoric stress tensor $\sqrt{J_2}$ and maximum hydrostatic pressure $(\sigma_H)_{max}$ [26]. Mathematically this criterion can be expressed as

$$CP = \sqrt{J_2} + \frac{{\tau'}_{f_{-1}} - \sigma'_{f_{-1}}/\sqrt{3}}{\sigma'_{f_{-1}}/3}(\sigma_H)_{max} \leq {\tau'}_{f_{-1}} \tag{10}$$

Where, ${\tau'}_{f_{-1}}$ is the fatigue limit under torsion and $\sigma'_{f_{-1}}$ fatigue limit under axial load. $(\sigma_H)_{max}$ is the maximum hydrostatic stress. This method intuitively has an advantage of shorter computation time over the critical plane approach models. This criterion has been used by many researches for computation of initiation life in fretting fatigue. Navarro et al. [5] have shown that Crossland parameter gives better standard deviation than other criteria for the case of spherical contact, however larger scatter band is observed for cylindrical contact. Nesládek et al. [10] has shown the drop of fatigue limits from plain fatigue and CP has performed well in fatigue prediction. Furthermore, finite element analysis is used to calibrate the coefficient of friction.

4 CONTINUUM DAMAGE MECHANICS APPROACH

This approach defines the failure in terms of damage, which constitutes on a damage variable. The evolution of this damage variable is calculated and failure is concluded when it reaches a critical value. This approach was first introduced by Kachanov [27], and is based on thermodynamic principle. Based on this concept, several theories have been developed and applied to predict crack nucleation in fretting fatigue problems.

4.1 Bhattacharya- Ellingwood damage model

The model presented by Bhattacharya and Ellingwood [28] is based on Continuum Damage Mechanics (CDM), which calculate cumulative fatigue damage. This model can predict crack initiation with the assumption that fatigue damage occurs before localization. The mean stress effects, loading sequence effects, stress controlled and strain controlled loading cycle effects are also accounted for. The equation for isotropic fatigue damage under uniaxial loading as function of loading cycle is given by:

$$D_n = 1 - (1 - D_0)\prod_{i=1}^{n}\left\{\begin{array}{ll} \dfrac{\dfrac{1}{1+\frac{1}{h'}}\Delta\epsilon_{0_i}^{1+\frac{1}{h'}} - \Delta\epsilon_{p1_i}^{\frac{1}{h'}}\Delta\epsilon_{0_i} + A_i}{\dfrac{1}{1+\frac{1}{h'}}\Delta\epsilon_{pm_i}^{1+\frac{1}{h'}} - \Delta\epsilon_{p1_i}^{\frac{1}{h'}}\Delta\epsilon_{pm_i} + A_i} & ; \sigma_{max_i} > \sigma_e \\ 1 & ; \text{otherwise} \end{array}\right\} \tag{11}$$

$$A_i = \frac{3\sigma_f}{4(2^{1-1/h'}H)} - \frac{\Delta\epsilon_{p0_i}^{1+\frac{1}{h'}}}{1+\frac{1}{h'}} + \Delta\epsilon_{p1_i}^{\frac{1}{h'}}\Delta\epsilon_{p0_i} \tag{12}$$

In the above set of equations (11) and (12), D_n is the damage after n cycles, D_0 is the initiation damage at fatigue cycle (for undamaged material $D_0 = 0$), H and h' are the cyclic hardening modulus and cyclic hardening exponent respectively. σ_f, σ_{max_i}, σ_e ,are the true failure stress, maximum stress for cycle i and endurance limit respectively. $\Delta\epsilon_{0_i}$, $\Delta\epsilon_{p1_i}$, $\Delta\epsilon_{pm_i}$, $\Delta\epsilon_{p0_i}$ are the threshold strain of damage increment, initial plastic strain, final plastic strain and threshold plastic strain of damage increment for cycle i respectively. When the damage value reaches the critical value $D_{N_I} \geq D_C$, the crack is initiated.

Using the same model Quraishi et al. [29] have adopted it for fretting fatigue problems. Due to fretting loading conditions, shear stresses are calculated. These shear stresses are resolved into components alternating tensile and compressive stresses. By applying CDM approach fretting fatigue initiation life is predicted by combining the damage caused by stresses in two directions. The expression relating critical damage D_c and number of cycles to initiation N_i is given by:

$$D_c = 1 - \left[\frac{\frac{1}{1+\frac{1}{h'}}\Delta\epsilon_{0_i}^{1+\frac{1}{h'}} - \Delta\epsilon_{p1_i}^{\frac{1}{h'}}\Delta\epsilon_{0_i} + A_i}{\frac{1}{1+\frac{1}{h'}}\Delta\epsilon_{pm_i}^{1+\frac{1}{h'}} - \Delta\epsilon_{p1_i}^{\frac{1}{h'}}\Delta\epsilon_{pm_i} + A_i}\right]^{2N_i} \tag{13}$$

The results shows good agreement for predicting number of cycles to failure for given maximum normal stress[29]. The S-N curves for Al 2024 and S45C are comparable with the experimental results.

4.2 Lemaitre damage model

Lemaitre introduced damage models based on CDM approach for ductile damage, creep damage and fatigue damage [30]. The damage is considered to be isotropic for predicting number of cycles to failure. Later in his work [31,32] the dissipation potential function φ was presented for the case of elastic and plastic conditions as expressed in equation (14) and (15) respectively.

$$\varphi = \frac{C}{\left(\frac{\beta}{2}\right)+1}\left(\frac{-Y}{C}\right)^{\left(\frac{\beta}{2}\right)+1}\dot{Y} \tag{14}$$

$$\varphi = \frac{C}{\left(\frac{\beta}{2}\right)+1}\left(\frac{-Y}{C}\right)^{\left(\frac{\beta}{2}\right)+1}(\dot{p}+\dot{\pi}) \tag{15}$$

Where, C and β are material dependent damage parameters, Y is elastic strain energy $\dot{p} + \dot{\pi}$ are plastic strain rate and accumulated micro-plastic strain respectively.

Based on the theory of Lemaitre [31], a promising work is contributed by Hojjati-Talemi and Wahab [33] by deriving the equations for elastic and elasto-plastic condition and developed an uncoupled damage model for application to fretting fatigue problems. They developed a predictor tool to calculate fretting fatigue crack initiation life time and initiation location by combining CDM approach with FEM. Moreover, the effect of contact geometry, axial stress, normal load and tangential load is presented. Using the dissipation potential function φ for elastic condition, the damage D and number of cycles to initiation are given by equation (16) and (17) respectively.

$$D = 1 - \left[1 - A(\beta+3)\left(\sigma_{eq,max}^{\beta+2} - \sigma_{eq,min}^{\beta+2}\right)\left\{\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_H}{\sigma_{eq}}\right)^2\right\}^{\left(\frac{\beta}{2}\right)+1} N\right]^{1/(\beta+3)} \tag{16}$$

$$N_i = \frac{1}{A(\beta+3)}\left(\sigma_{eq,max}^{\beta+2} - \sigma_{eq,min}^{\beta+2}\right)^{-1} R_v^{-\left(\frac{\beta}{2}\right)-1} \tag{17}$$

Where A and β are material dependent damage parameters and are obtained by fitting equation (17) in the experimental data. $\sigma_{eq,max}$ and $\sigma_{eq,min}$ are the maximum and minimum equivalent Von Mises stress respectively. The estimated results for initiation life time with this model are found in close approximation with the experimental data and fall in a scatter band of ±2N.

Using the function φ defined in equation (15), Hojjati-Talemi et al. [34] have derived the damage equation as a function of number of cycles. In this study CDM approach is used along with FEM to compute initiation life and XFEM is used to compute the propagation life.

$$D = 1 - \left[1 - A(m + 2\beta + 2)\left(\sigma_{eq,max}^{m+2\beta} - \sigma_{eq,min}^{m+2\beta}\right)R_v^{\beta}N\right]^{\frac{1}{(m+2\beta+2)}} \quad (18)$$

$$N_i = \frac{\left(\sigma_{eq,max}^{-m-2\beta} - \sigma_{eq,min}^{-m-2\beta}\right)R_v^{-\beta}}{A(m+2\beta+2)} \quad (19)$$

Where, m is the power constant in Ramberg-Osgood equation. The results have shown good agreement between numerical data and experimental data, also total estimated life fall in a scatter band of ±2N [34].

5 CONCLUSIONS

In this paper different approaches regarding fretting fatigue crack initiation criteria have been reviewed and the following important points can be concluded.

1. The criteria using critical plane approaches are suitable to model crack initiation and orientation. These criteria predict fretting fatigue behavior as they can also be used with non- proportional loading. Initiation life mostly fall in a scatter band of ±3N.
2. Stress invariant based approach can also be used for modeling fretting fatigue problems. This approach is intuitively more efficient than critical plane models as the parameter does not have to be computed on different planes, yet it loses the physical meaning of the problem.
3. CDM approach is recently employed to predict crack initiation/ nucleation for fretting fatigue problems. Various authors have predicted initiation life within a scatter band of ±2N using this approach. This approach can be used with coupled and uncoupled damage models for initiation and propagation phase.
4. Finite element methods can be used with various criteria for prediction of crack initiation/ nucleation location and estimated life. Almost all of the criteria can be modeled with FEM with good accuracy, also the effects of various fretting fatigue parameters can be studied. Furthermore, XFEM can be used for modeling of crack propagation and can be combined with various approaches to predict total life.
5. Studies have shown that there is no universal criteria, which gives best results for all type of materials, contact type and loading conditions. The results of the multi axial criteria also depends on the definition of crack initiation or nucleation, as crack initiation has been defined from micro level upto a length of millimeter. Various local and non-local approaches have been developed to evaluate failure parameters over specified length or volume. Therefore selection for the most suitable approach and criteria depends on various factors and vary from case to case.

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