Cylindrically-bent rectangular patch antennas: novel modeling techniques for resonance frequency variation and uncertainty

Luigi Vallozzi[1], Freek Boeykens[2], Hendrik Rogier[1]
[1]Department of Information Technology, Ghent University, Ghent, Belgium, luigi.vallozzi@ugent.be, hendrik.rogier@ugent.be
[2]Verotech BVBA, Researchpark Haasrode, Leuven, Belgium, freek.boeykens@gmail.com

***Abstract*—Wearable textile antennas are basic components in body-centric communication systems. Flexible wearable patch antennas, when integrated into a body-worn garment are subjected to bending, causing variation in the resonance frequency when compared to the flat-antenna. Bending conditions vary statistically among different human subjects. Therefore, it is very important to be able to predict performance variations due to bending. We propose novel models which allow to predict the deterministic and statistical variation in resonance frequency of rectangular wearable patch antennas. They consist of an analytical model for cylindrical-rectangular patch antennas, expressing resonance frequency as a function of the bending radius, and a novel technique based on polynomial chaos, that quantifies statistically the variations of the resonance frequency under randomly varying bending conditions. The proposed models have been experimentally and numerically verified, and proven to be much faster and computationally less expensive than traditional techniques based on EM solvers and Monte Carlo simulations, making them very advantageous tools for the design and characterization of body-worn patch antennas.**

***Index Terms*—Cavity model, microstrip antennas, random variables, textile antennas.**

I. Introduction

In the last decade, wearable textile antennas have been an important research topic, together with *wearable electronic systems* [1]. When worn on the human body, they allow to establish a wireless link between the subject and the external world. The most suitable topology for body-worn antennas, ensuring optimal integration and wearer's comfort, is the planar patch. On-body antennas are preferably made out of flexible textiles and wearable materials, able to naturally conform to the human body surface. Flexible antennas are usually designed in the ideal flat state, but when integrated into a garment and worn by a subject, they are likely to be subjected to deformations such as bending, which causes variations in the flat-antenna performance, leading to potential degradation of the wireless link quality.

An important characteristic for narrowband patch antennas is the *resonance frequency*, which is subjected to a shift when the antenna is bent, for example, around a human arm or a leg.

Bending conditions can vary in a statistical way, due to variation of body morphology of different human subjects. It is thus very important for the design engineer, to be able to predict and quantify, in a fast and inexpensive way, these resonance frequency variations, both from a deterministic and statistical point of view.

In the past years, the problem of performance variation due to bending has been solved by designing antennas with sufficiently large impedance bandwidth, as in [2]. However, in previous literature, limited efforts have been made to build models that are able to *predict* resonance frequency variations due to bending for wearable flexible patch antennas. A model for rigid rectangular patch antennas, conformed to a cylindrical surface has been proposed by Krowne [3], in which the classical rectangular cavity model of Lo [4] was extended to the cylindrical case. The model was then completed and improved by Luk [5]. Such models express analytically the resonance frequency as a function of the bending radius. However, they have the limitation of being valid only for very thin substrates. For flexible wearable antennas, bending has been exclusively studied from an experimental point of view, describing only the trend of variation of resonance frequency and not providing any physical insight into the physical mechanisms occurring in bent antennas [6]–[8]. Moreover, the previous studies do not take into account phenomena occurring in the specific case of flexible wearable antennas, such as stretching and compression.

To answer these needs, we have proposed two novel modeling techniques, allowing the description of the deterministic and statistical variation of resonance frequency of rectangular wearable patch antennas. First, an analytical model was built, based on an improved cylindrical-rectangular cavity description [9], which expresses the resonance frequency as a function of the bending radius. Next, a novel technique, based on polynomial chaos [10] has been constructed [11], based on the previous analytical model, which quantifies statistically the variations of the resonance frequency under randomly varying bending conditions. Polynomial chaos was already introduced as a faster alternative to Monte Carlo simulations, and applied to several configurations such as transmission lines [12] and multiport microwave circuits [13].

All techniques presented in this paper have been experimentally and numerically verified, and they proved to be much quicker and computationally less expensive when compared to simulation techniques based on EM solvers, for the analytical model, and Monte Carlo simulations, for the stochastic analysis method. This makes them very suitable and advantageous techniques in the design of wearable patch antennas.

The paper is organized as follows: in Chapter II the proposed novel analytical model for rectangular textile patch antennas, bent on a cylindrical surface, is described together with its experimental validation. Next, in Chapter III the proposed technique for modeling of uncertainty of resonance frequency variations of bent rectangular patch antennas is presented. The description of the experiments performed to verify the validity of the proposed polynomial chaos modeling technique is described in Section III-B, together with a new example for the numerical validation of the polynomial chaos technique, with respect to the ones already reported in [11]. The PDFs of the resonance frequency of a bent patch antenna, calculated by means of the proposed polynomial chaos technique, shows an excellent agreement with the on calculated by Monte Carlo simulations, and requires a remarkably smaller computational time (few seconds) compared to the Monte Carlo method.

Finally, the conclusions are drawn in Chapter IV.

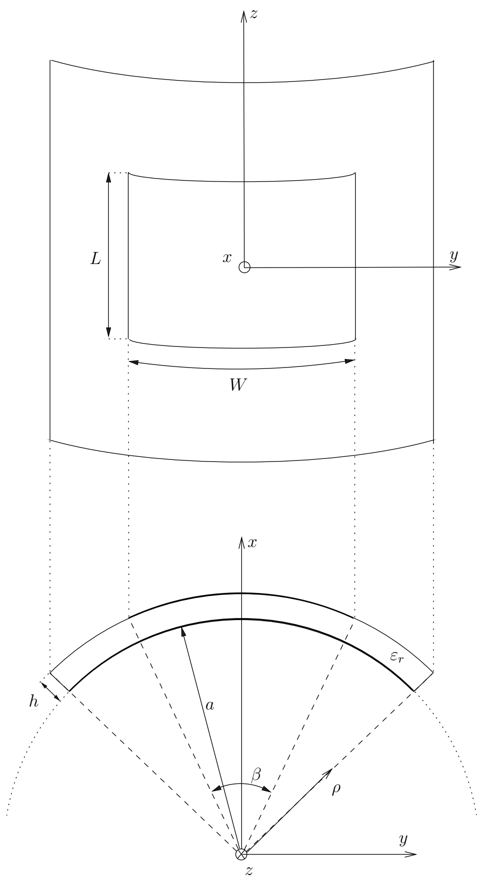

Fig. 1. Geometry of the cylindrically bent rectangular patch antenna

II. Analytical model of cylindrically-bent wearable rectangular patch antennas

A. Theory

The derivation of the proposed cavity model is based on finding the solution of the Maxwell equations inside the cylindrical rectangular cavity represented by the antenna substrate, bounded by the radiating patch and the ground plane. The fields can be written as:

$$\mathbf{E} = -\nabla \times \mathbf{F} - j\omega\mu\mathbf{A} + \frac{1}{j\omega\varepsilon}\nabla\left(\nabla \cdot \mathbf{A}\right) \tag{1a}$$

$$\mathbf{H} = -\nabla \times \mathbf{A} - j\omega\varepsilon\mathbf{F} + \frac{1}{j\omega\mu}\nabla\left(\nabla \cdot \mathbf{F}\right) \tag{1b}$$

The boundary conditions inside the cavity have to be imposed for the solution of the equations, being *electric walls* at $\rho = a$ and $\rho = a + h$ and magnetic walls at $\phi = \pm\beta/2$ and $z = \pm L/2$.

The resonance frequency as a function of bending radius is obtained as follows. By solving Equations (1), we obtain the dispersion relations, for the modes TE_z and TM_z, respectively:

$$J'_{k_\phi}\left(k_\rho a\right) Y'_{k_\phi}\left(k_\rho(a+h)\right) = J'_{k_\phi}\left(k_\rho(a+h)\right) Y'_{k_\phi}\left(k_\rho a\right) \tag{2a}$$

$$J_{k_\phi}\left(k_\rho a\right) Y_{k_\phi}\left(k_\rho(a+h)\right) = J_{k_\phi}\left(k_\rho(a+h)\right) Y_{k_\phi}\left(k_\rho a\right) \tag{2b}$$

The resonance frequencies are obtained as the zeros of these equations, which need to be solved numerically.

B. Inclusion of stretching and compression effects

1) Stretching effects: The radiating element of wearable patch antennas can be made out of materials that are stretchable. When bending occurs around its Z axis (as shown in Fig. 1) the original dimension W undergoes an elongation ΔW. In such case, the original width W is located at a certain distance ρ inside the substrate cavity between the patch and the ground plane, with $a + \frac{h}{2} \leq \rho \leq a + h$. In order to take into account this effect in our model, we introduce a factor d that determines the location of the original length inside the cavity, or equivalently the amount of stretching, into the following relationship:

$$W = \beta(a + hd) \tag{3}$$

The stretching factor d can take values in the range $0.5 \leq d \leq 1$, with 0.5 for perfectly stretchable materials and 1 for perfectly non-stretchable ones.

2) Substrate compression effects: An additional effect to which a bent wearable patch antenna is subjected to, is the compression of the substrate. Compression has a direct effect on the substrate permittivity, causing additional performance variation in terms of resonance frequency. From the performed experimental investigations, an empirical formula was derived, expressing the bent substrate permittivity as a function of the bending radius a:

$$\varepsilon_{r,\text{comp}} = \varepsilon_{r,\text{flat}} \left(1 + \eta \frac{h(d-0.5)}{a}\right) \tag{4}$$

where:

- $\varepsilon_{r,\text{flat}}$ is obtained from the measured resonance frequency of the antenna in flat state, and it captures fringing, temperature and humidity effects;
- h is the substrate thickness measured in flat state according the ISO 5084 standard [14];
- The proportionality factor η is empirically determined by a set of N measured resonance frequencies, for an antenna with non-stretchable patch material (i.e. copper foil), for N different bending radii. η is found by minimizing the error function representing the relative mean error between modeled and measured resonance frequency.

C. Results and experimental validation

The experimental validation was performed by using several wearable rectangular patch antennas. Here we show the results for two different prototypes, whose characteristics are displayed in Table I. The feed point was placed symmetrically with respect to the L_p dimension, at a distance y_f from the center, as shown in Fig. 2.

TABLE I
PARAMETERS OF THE ANTENNA PROTOTYPES

Prot.	Patch	Substr. (ε_r)	h [mm]	f [GHz]	$L_p \times W_p$ [mm]
1	Flectron (Stretch)	Aramid (1.75)	2	1.57	81.2×69.25
3	Copper foil (Non-stretch)	Cotton (1.715)	2.7	1.57	78.5×69.3
4	Copper foil (Non-stretch)	Cotton (1.715)	2.7	2.4	52.5×43.7

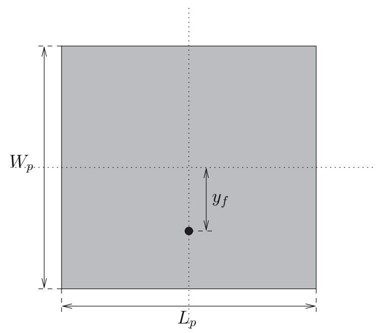

Fig. 2. Patch dimensions and feed position

During the experimental validation, the resonance frequency was measured with the antennas bent around the two main patch dimensions, for several values of the bending radius a. In particular:

- *Bending in the W_p direction*: the model of Fig.1 is used with $L = L_p$ and $W = W_p$. In this case the TE_{10} mode is excited;
- *Bending in the L_p direction*: the model is used with $L = W_p$ and $W = L_p$ and the TE_{01} mode is excited.

1) Resonance frequency: In Fig. 3 the resonance frequency results for the antenna prototypes 1 and 3, bent in the W_p direction, i.e. the resonant length, are displayed. We can make the following observations:

- For prototype 1, the measured resonance frequency almost does not vary as a function of bending radius. This is explained by the fact that this antenna has a stretchable patch, which undergoes elongation, and for which substrate compression does not occur. Bending, which produces an increase in resonance frequency, and elongation, which results in an f_{res} decrease, compensate each others' effects. The results obtained by the model reproduce very well the measured results;
- For prototype 3, having a non-stretchable patch, in contrast to prototype 1, elongation does not occur and the substrate is subjected to compression. Again, bending produces an increase of f_{res}, which is partially compensated by the effect of compression, which produces an increase of ε_r, thus an f_{res} decrease. One sees that the model reproduces the measured results in an excellent way, when the compression effects are taken into account.

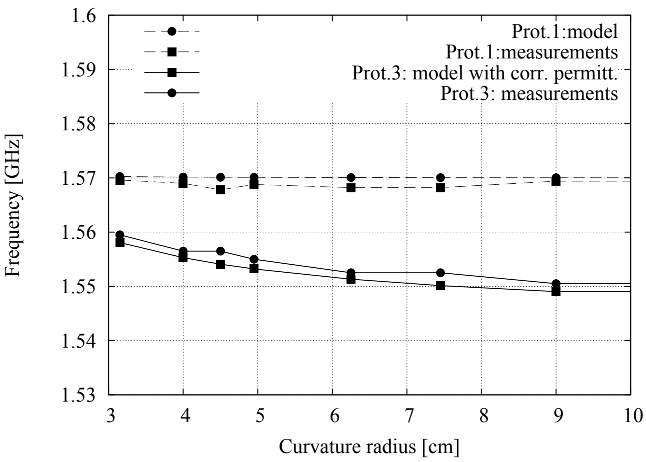

Fig. 3. Resonance frequencies as a function of bending radius, for prototypes 1 and 3, bent in the W_p direction

In the case of bending in the L_p direction, in contrast to the W_p case, bending and elongation do not produce an f_{res} shift, since L_p is not the resonant length, resulting in a nearly constant curve of $f_{res}(a)$ for prototype 1. The only phenomenon producing an f_{res} shift is the substrate compression, occurring for prototype 3, resulting in a very slight decrease of f_{res} when bending increases (i.e. a decreases), reproduced well by our model.

III. MODELING OF UNCERTAINTY IN RESONANCE FREQUENCY USING POLYNOMIAL CHAOS TECHNIQUES

The theoretical model for the cylindrical-rectangular patch antenna, described in Section II, represented the core of a novel technique that statistically quantifies the variations resonance frequency under randomly varying bending conditions. The need of such a technique originated by the fact that antenna bending can vary in a random fashion due to variations of body morphology among different users in real-life operation.

A. Polynomial chaos formalism and application to the novel cavity model

In this technique, the bending radius a of the rectangular patch antenna is treated as the *input* random variable X, characterized by its cumulative distribution function (CDF) $\mathcal{P}^X$, while the output variable Y is represented by the resonance frequency f_{res}, also described statistically by its CDF. The relationship between input and output variables is expressed by the transformation $Y = f(X)$, represented by the analytical model described in Section II. The technique allows to obtain, given the input variable X and its distribution, the statistical variation of the output variable, i.e. the complete distribution of Y. The main theory can be synthetically described as follows:

The first step consists in approximating the input-output transformation by a polynomial expansion of order P, given by

$$Y \approx f^P(X) = \sum_{k=0}^{P} y_k^X \phi_k^X(X) \tag{5}$$

where

- $\phi_k^X(X),\ k = 0,\dots,P$ are appropriate orthogonal expansion polynomials, that can be either known a priori or constructed "on the fly";
- $y_k^X,\ k = 0,\dots,P$ are unknown expansion coefficient, that are determined by the following expression, obtained by applying Galerkin weighting to Eq. (5) and using an N-point Gauss quadrature rule, obtaining:

$$y_j^X \approx \sum_{i=1}^{N} \omega_i Y(x_i)\phi_j^X(x_i), \quad j = 0,\dots,P \tag{6}$$

The quadrature points x_i are the N zeros of the polynomial $\phi_N^X(x)$ and ω_i are the corresponding weights. The right-hand side of Eq. (6) is calculated by evaluating $Y = f(X)$ for the N quadrature points of X, representing N realizations of the random variable X. The advantage of this technique lies in the fact that it can be applied non-intrusively to any input-output relationship, no matter how complicated it is.

Finally, the statistical moments of the output variable Y can be computed directly based on the knowledge of the previously evaluated $Y(x_i)$, that is:

$$E\left[g(y)\right] = \int_{\Omega} g\left(Y(x)\right) d\mathcal{P}^X(x) \approx \sum_{i=1}^{N} \omega_i g\left[Y(x_i)\right] \tag{7}$$

where $g(y)$ is an arbitrary function.

B. Experimental and numerical validation

The proposed technique was experimentally validated, using real-life distribution of the input variable X, i.e. the bending radius, that varies randomly due to the variations of body morphology of different human wearers. For the validation, we consider the particular case of antenna worn on a human arm which is assumed to have a cylindrical shape. Moreover, the antenna is assumed to be bent in the W_p direction, since this is the case giving rise to the largest variation of the output variable $Y = f_{res}$.

For the experimental validation, we made use of a prototyped textile rectangular patch antenna. First we obtained an analytical expression for the bending radius distribution, by performing a fit on statistical data of human arm radius distribution, taken from the database of the NHANES measurement campaign [15]. We obtained, as a best fit for the Probability Density Function (PDF) of the human arm radius, a truncated Gaussian function in the interval $[3, 8]$ cm, with mean $\bar{a} = 5.14$ cm and $\sigma = 0.85$ cm.

Then, a measurement campaign was performed on the antenna prototype, obtaining the resonance frequency f_{res} for several values of bending radius a in the range $[3, 8]$ cm. In order to fit the input-output relationship (i.e. the dispersion relation for the case TE_z in Eq. (2)) for the considered patch antenna under test, a least-square fit of the analytical dispersion relationship to the measured one was performed, with parameters $\varepsilon_{\text{r,flat}}$ and η. The dispersion curve resulting from this procedure, expressing the resonance frequency as a function of the radius of the arm on which the antenna is bent, is displayed in Fig. 4. Next, the CDF of the output variable

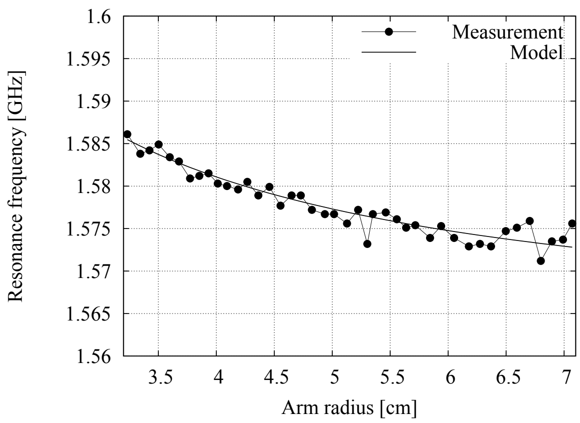

Fig. 4. Measured and fitted curve of the resonance frequency as a function of the arm radius

$Y = f_{res}$ is constructed both based on the measured data, by solving the fitted dispersion relationship, and then by using the proposed model based on polynomial chaos. This calculation required 59 s of CPU-time, plus 15 s needed to construct the CDF, which represents a remarkably short time with respect to other techniques such as Monte Carlo simulations. As shown in Fig. 5, a very good agreement is obtained between measured and modeled data.

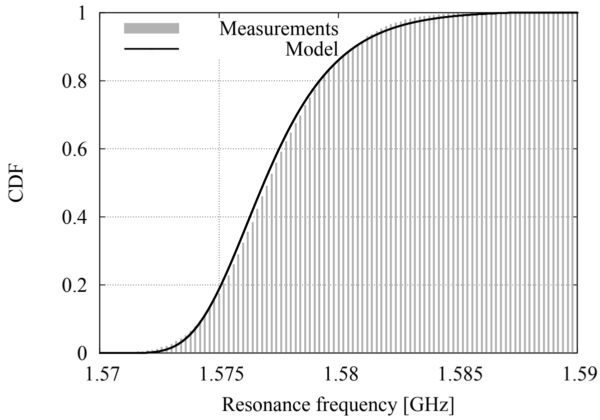

Fig. 5. Measured and modeled CDF of the resonance frequency

For the numerical validation, we considered as a test example a textile patch antenna on a cotton substrate, designed to operate around 2.4 GHz, indicated as *prototype 4*, described in Table I. We considered bending around an arm, assuming for the radius a, a normal distribution with $\bar{a} = 5$ cm and $\sigma = 7$ mm. The PDF of the resonance frequency was computed using the classical Monte Carlo approach and the proposed polynomial chaos based technique, showing an excellent mutual agreement as displayed in Fig. 6. The proposed polynomial chaos technique requires only few seconds of computational time, making it remarkably faster, and thus more advantageous, with respect to the Monte Carlo method, requiring few minutes to obtain the same results.

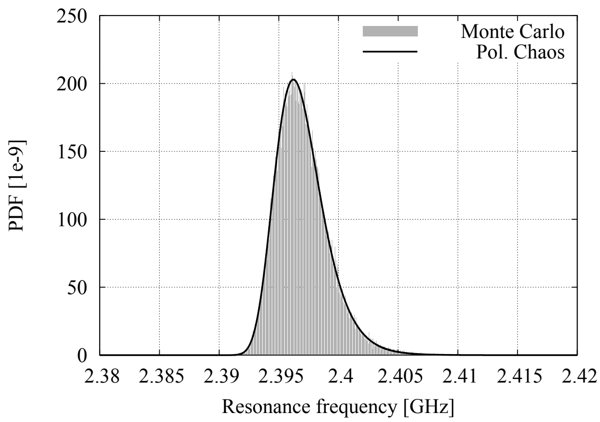

Fig. 6. PDF of the resonance frequency, for prototype 4, computed by Monte Carlo simulations and the proposed polynomial chaos technique

IV. Conclusion

Two novel techniques were developed for the design and performance prediction of wearable textile rectangular patch antennas, allowing to analytically and statistically describe the variation of resonance frequency due to varying cylindrical bending radius. The first one involves a novel analytical model describing the cylindrical-rectangular cavity formed by the substrate of a bent antenna. The formulation and solution of the dispersion relations for the cavity leads to an expression of the resonance frequency as a function of the bending radius. In contrast to existing models, the proposed one incorporates typical effects occurring in bent textile antennas, being elongation of the patch due to stretching and variation of substrate permittivity due to compression, which superimpose to the effect of bending only. Then, the analytical model was used as a core for the second techniques, based on polynomial chaos theory, which allows to compute the statistical variation, in terms of moments, of the resonance frequency, when the bending radius vary randomly, due to statistical morphological variations of different human bodies. Both techniques, representing a unique novel framework for antenna bending, were verified numerically and experimentally, on real bent antenna prototypes worn on a human arm, demonstrating their validity, and showing their remarkable advantage in terms of computation time with respect to other existing techniques such as Monte Carlo simulations.

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