FRACTIONAL PID CONTROL OF A POORLY DAMPED SYSTEM BASED ON A 3-PARAMETER DESIGN METHOD

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Abstract — **This paper presents the practical implementation of a fractional-order (FO) PID controller on a poorly damped system. The FOPID is tuned using a new reduced parameter model-based design method. Instead of the standard 5 tuning parameters used in the FOPID controller, the new tuning method reduces the number of tuning parameters to three based on practical tuning rules. The underlying idea is to simplify the tuning process of FOPID controllers to make them more suitable for industrial implementation. The designed controller is implemented on the real system and the performance is compared to an integer-order PID controller. The results show an improved performance for the FOPID controller. The improved performance indicates the possibility of FOPID controllers to reduce general costs in an industrial environment which will be analyzed in future research. Comparing the performance of the controllers tuned with the 3-parameter design method to those tuned with the full 5-parameter tuning method will be the next step in this research path.**

Keywords: **Fractional-order controller, reduced FOPID design, mass-spring-damper**

I – Introduction

To this day, over 95% of industrial applications are predominantly controlled by Proportional-Integral-Derivative (PID) controllers [1]. It has been stated in literature before that this PID controller while 'limited' and 'primitive' is still the preferred controller in industry due to its simplicity, ease of implementation and ample tuning rules [2].

However, as industrial applications are constantly evolving, more complex systems are introduced daily. As complexity in the control processes in unavoidable, the simple linear integer-order PID controller will not be able to cope with complex phenomenons such as non-linearities, dead bands, etc. resulting in many efforts to improve the performance of the classical linear PID controller. [3]

A controller which is similar to the integer-order PID controller but has a better performance in complex situations is the fractional-order PID controller (FOPID) [4]. This controller has in standard situations not three tuning parameters such as the classical integer-order PID, but five tuning parameters: K_p, T_i, T_d and two fractional order coefficients λ and μ. Even though, this FOPID has an improved performance compared to the integer-order PID, it is still not accepted in industry due to its increased difficulty in tuning and implementation [5, 6].

This paper presents a simplified tuning method which reduces the number of tuning parameters of the FOPID controller, making the controller easier to tune and therefore more accessible to industry. To show the performance of the controller tuned using the new design method, the FOPID is implemented on a fast but poorly damped mechatronic system: the mass-spring-damper. The mass-spring-damper system can be characterized as a classical example of an electromechanical system. Possible applications include active suspensions[7] and drives [8]. However this system is also suitable for biomedical applications such as modeling the human body [9] and applications in sound [10].

Each mass-spring construction in the mass-spring-damper system introduces a resonance peak in the frequency response of the system making this a challenging system from control point-of-view. The classical integer-order PID controller has traditionally difficulty to control these types of systems as it has only one pair of zeros to compensate the dynamics of the system. Therefore, a controller of higher order would be more suitable to control systems such as the mass-spring-damper. Advanced controllers such as fractional-order controllers can be approximated by such a high order transfer function.

The structure of the paper is as follows: the next section presents the new tuning method with a reduced number of parameters. Section three discusses the mass-spring-damper system used in this research. Section four discusses the performed measurements and the results. A conclusion is formed in the final section.

II – Proposed tuning method

Literature shows that a prominent setting for integer-order PID design is using identical zeros in the controller [11]. This idea is the starting point of the proposed 3-parameter design method.

The general structure of a fractional-order PID controller is given by

$$PI^{\lambda}D^{\mu}(s) = K_p\left(1+\frac{1}{T_i s^{\lambda}}+T_d s^{\mu}\right). \qquad (1)$$

Assuming two identical zeros z simplifies this expression as

$$PI^{\lambda}D^{\mu}(s) = K\frac{(s^{\lambda}-z)^2}{s^{\lambda}} \qquad (2)$$

with $K = K_p T_d$ and $z = \frac{-1}{2T_d}$, if

$$\lambda = \mu \qquad (3)$$

and

$$T_i = 4T_d. \qquad (4)$$

The main advantage of (2) compared to (1) is the reduced number of parameters in the fractional-order controller. The resulting expression for the FOPID expression in (2) has only three remaining tuning parameters: K, z and λ, from which the traditional parameters K_p, T_i, T_d and μ can be calculated.

Tuning of the three design parameters is done using three loop shape specifications which need to be met by the FOPID controller. These specifications are based on the definition of gain crossover frequency (ω_{gc}) and phase margin (PM).

- Phase margin specification

$$\arg[C(j\omega_{gc})G(j\omega_{gc})] = -\pi + PM, \qquad (5)$$

- Robustness to process gain variation

$$\left.\frac{\mathrm{d}(\arg(C(j\omega)G(j\omega)))}{\mathrm{d}\omega}\right|_{\omega=\omega_{gc}} = 0, \qquad (6)$$

- Gain crossover frequency specification

$$\left|C(j\omega_{gc})G(j\omega_{gc})\right|_{dB} = 0. \qquad (7)$$

Selecting suitable values for PM and ω_{gc} will define the final parameters of the FOPID controller. Imposing a robustness to gain variation should theoretically result in a controller which has the capability of handling modeling errors without a decrease in performance.

III – Mass-spring-damper system

Figure 1 depicts the mass-spring-damper (MSD) system used in this research. It consists of 3 movable masses from which the third mass is fixed for the performed experiments. The masses m_1 and m_2 are interconnected using three springs with spring constants k_1, k_2 and k_3. A damper with a damping constant c_1 is used to damp the oscillations in the system while a DC-motor gives the driving force. The input of the system is the voltage to the motor $u(t)$ while the two outputs of the system are the mass displacements y_1 and y_2 expressed in cm. The dynamics of the motor are neglected as they are much faster than those of the mass-spring-damper system resulting in a motor which can be represented by a pure static gain $F(t) = K \cdot u(t)$, with $F(t)$ the force on the first mass. The following parameters are used in the current setup: $m_1 = 1.85$ kg, $m_2 = 1.35$ kg, $k_1 = k_2 = 800$ N/m, $k_3 = 450$ N/m, $c_1 = 9$ N/(m/s) and $K = 3.5$ N/V.

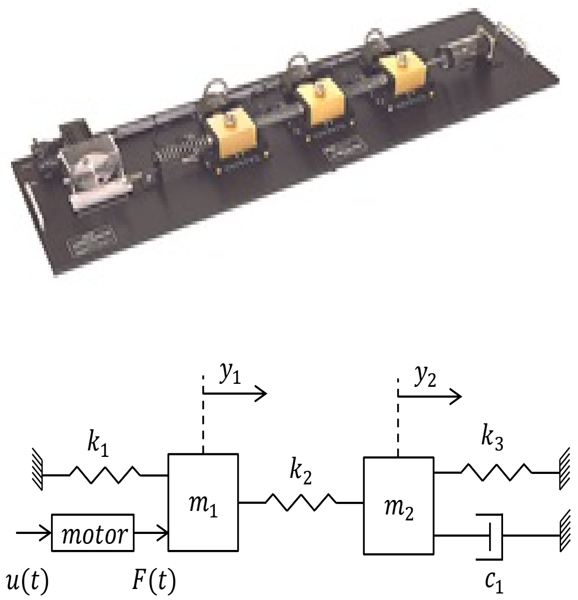

Figure 1: *Mass-spring damper system and its schematic representation.*

A complete model of the electromechanical plant describes the dynamics from $u(t)$ to $y_1(t)$ and from $u(t)$ to $y_2(t)$. The two differential equations describing the dynamics of the system are:

$$m_1\ddot{y}_1(t) + (k_1+k_2)y_1(t) = F(t) + k_2 y_2(t) \qquad (8)$$

$$m_2\ddot{y}_2(t) + c_1\dot{y}_2(t) + (k_2+k_3)y_2(t) = k_2 y_1(t) \qquad (9)$$

After taking the Laplace transforms of equations (8) and (9), the resulting transfer functions are:

$$G_1(s) = \frac{Y_1(s)}{U(s)} = \frac{K(m_2 s^2 + c_1 s + (k_2+k_3))}{den} \qquad (10)$$

$$G_2(s) = \frac{Y_2(s)}{U(s)} = \frac{Kk_2}{den} \qquad (11)$$

with

$$den = m_1 m_2 s^4 + m_1 c_1 s^3 + [m_1(k_2+k_3) + m_2(k_1+k_2)]\, s^2 + c_1(k_1+k_2)s + k_1k_2 + k_1k_3 + k_2k_3. \qquad (12)$$

Both transfer functions are expressed in m/V.

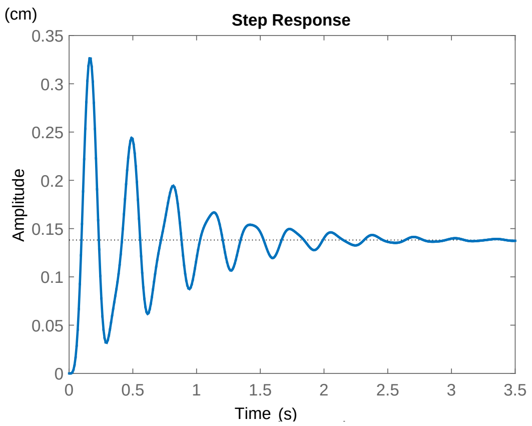

Figure 2: *Step response for the transfer function describing the displacement of the second mass.*

The goal in this experiment is to control the displacement of the second mass. Therefore, the second transfer function G_2 will be used as a model in the 3-parameter FOPID design method. Using the parameter values mentioned previously, the step response of the second transfer function is shown in Fig. 2 but expressed now in cm/V. Observe the long settling time and very high overshoot due to the low damping factor of the mass-spring-damper system.

The Bode plot of the second transfer function is plotted in Fig. 3.

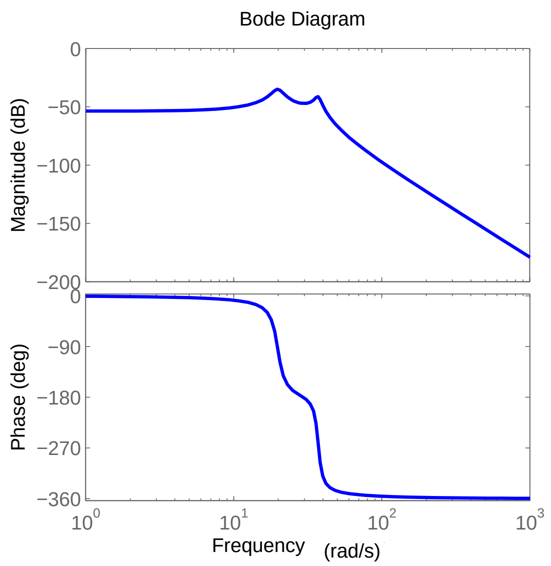

Figure 3: *Bode plot of the system describing the displacement of the second mass.*

IV – Results and Discussion

A. Controller design

Using the 3-parameter designing method presented

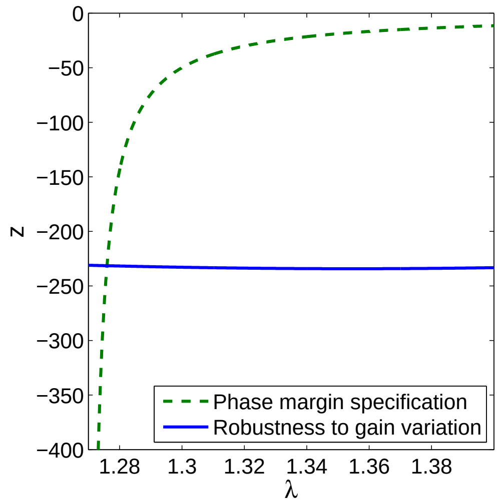

Figure 4: *Combined solution for phase margin specification and robustness specification.*

in Section II and the model derived in Section III, a FOPID controller can be designed for the mass-spring-damper system.

As design parameters a *PM* of 65° and a gain crossover frequency of 1.25 rad/s are chosen. The model of the process with the used parameter values is

$$G(s) = \frac{Y_2(s)}{U(s)} = \frac{2800}{2.498s^4 + 16.65s^3 + 4473s^2 + 1440s + 1.36 \cdot 10^6}. \tag{13}$$

To tune the fractional exponent λ in the FOPID controller, (5) and (6) are combined. The resulting value for λ will fulfill simultaneously both specifications. Both equations are analytically solved to obtain an expression of z in function of λ. Plotting both resulting expressions provides a cross-section which defines the value of λ and z which can be seen in Figure 4. Note that this is a zoom of the figure to visualize the cross-section and that the robustness to gain variation is not a constant. The value for K can be calculated after substituting the obtained values for z and λ in (7).
The resulting FOPID parameters are:

$$\lambda = 1.28,\ z = -234 \text{ and } K = 0.00018. \tag{14}$$

These values correspond with the standard FOPID parameters $K_p = 0.082$, $T_d = 0.0021$, $T_i = 0.0084$ and $\lambda = \mu = 1.28$. Figure 5 shows the open loop Bode plot for the controlled system where we can clearly see that the given *PM* and ω_{gc} are fulfilled. At a frequency of 1.25 rad/s Figure 5 shows that the magnitude of the Bode diagram is 0 dB while the phase is -115° which corresponds to a phase margin of 65°. Also notice that

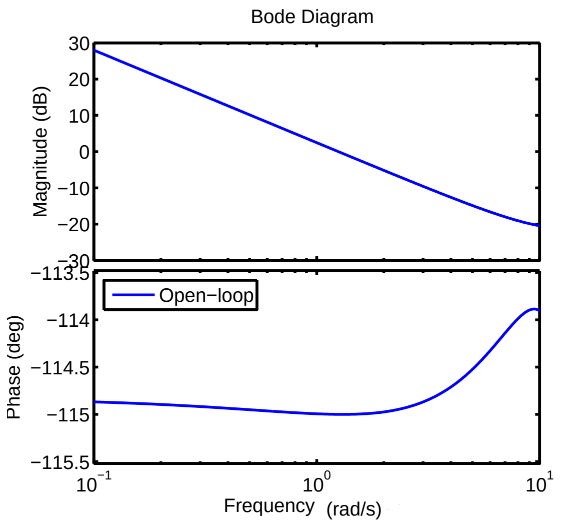

Figure 5: *Bode plot of the open loop controlled system.*

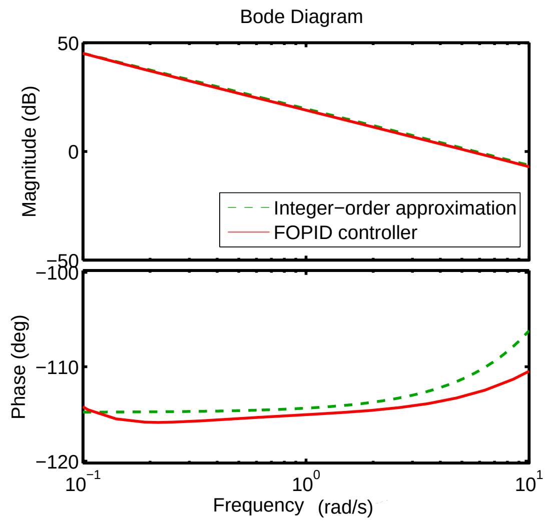

Figure 6: *Bode plots of the FOPID controller and its integer-order approximation.*

the phase around the gain cross-over frequency of 1.25 rad/s is flat which is the result of the robustness to gain variation specification.

For comparison an integer-order PID controller is also designed for the MSD system using a computer aided design tool called FRTool [12]. The FRTool is a model-based design technique which uses information from the system's frequency response. It is a graphical design method which shapes the Nichols plot by changing the place of the poles and zeros of the controller. The specification on phase margin has been used to obtain a PID controller which has the same specifications as the FOPID controller in order to compare the performance of both. The resulting PID parameters are: K_p=0.88, T_d=0.0227 and T_i=0.0909.

B. Measurements

Designing FOPID controllers is only one side of the dual problem in the theory of fractional-order controllers. Implementation is the second obstacle to tackle. Although some references discuss hardware devices for fractional-order integrators [5, 13], these devices are restricted and difficult to tune. Alternatively, literature shows methods to implement the FOPID controllers by approximating them using a finite-dimensional integer-order transfer function. The relative merits of the approximation method depend on the differentiation order and on whether an accurate frequency behavior is important.

The approximation method used in this research is the Modified Oustaloup Filter [14]. It fits the frequency response over a frequency range of interest (ω_b, ω_h). The filter is expressed by:

$$s^{\lambda} \approx \left(\frac{d\omega_h}{b}\right)^{\lambda} \left(\frac{ds^2 + b\omega_h s}{d(1-\lambda)s^2 + b\omega_h s + d\lambda}\right) \Pi_{k=-N}^{N} \frac{s+\omega_k'}{s+\omega_k} \tag{15}$$

where the filter is stable for $\lambda \in (0,1)$, $\omega_k' = \omega_b \omega_u^{(2k-1-\lambda)/N}$ and $\omega_k = \omega_b \omega_u^{(2k-1+\lambda)/N}$. The parameters used for the approximation are: $N = 6$, $\omega_b = 10^{-1}$, $\omega_h = 10^1$, $b = 10$ and $d = 9$. The Bode plots of the FOPID controller and its integer-order approximation are shown in Fig. 6. Notice that in the frequency range of interest $(10^{-1}, 10^1)$ the approximation is very good. Note that only the region of interest is plotted and that the magnitude of the FOPID is indeed a curve and not a straight line.

For the implementation of the approximated controller on the real system, this controller needs to be discretized. The sampling time used for this MSD system is 10 ms. The resulting discrete time transfer function of the FOPID controller is given by

$$C(z^{-1}) = \frac{0.88 - 7.47z^{-1} + 27.91z^{-2} - 60.77z^{-3} + 84.94z^{-4} - 79.03z^{-5} + 48.93z^{-6} - 19.43z^{-7} + 4.48z^{-8} - 0.46z^{-9}}{1 - 7.5z^{-1} + 24.62z^{-2} - 46.05z^{-3} + 53.72z^{-4} - 40.01z^{-5} + 18.58z^{-6} - 4.92z^{-7} + 0.57z^{-8}}. \tag{16}$$

Notice that the result of the approximation of the FOPID controller is indeed a higher order integer-order transfer function. Also the integer-order PID controller which is used for performance comparison is implemented in discrete time on the system with a sampling time of 10 ms.

For both controllers a measurement is performed

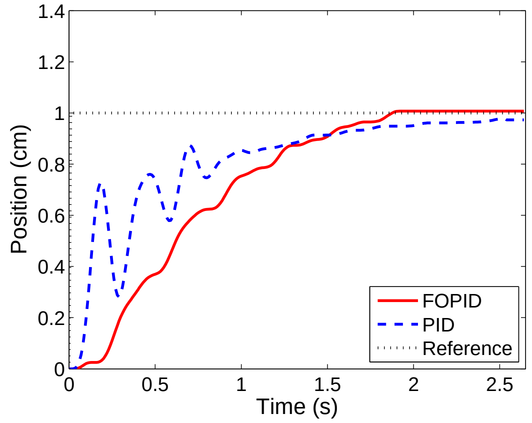

Figure 7: *Measured output for both FOPID and PID controller.*

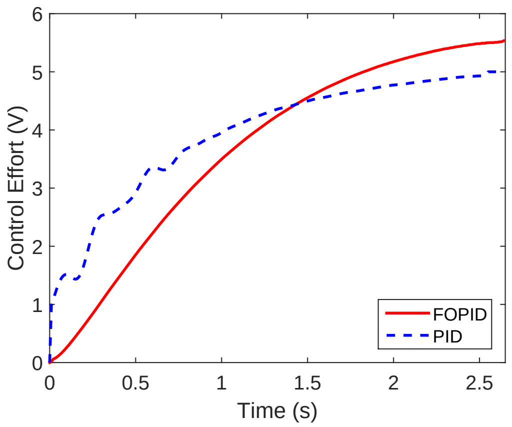

Figure 8: *Control effort for both FOPID and PID controller.*

using a constant reference signal with amplitude of 1 cm. The resulting dynamic behavior is plotted in Figure 7. The corresponding control effort of both controllers is given in Figure 8.

Notice that both controllers are able to control this poorly damped MSD system. However, from Figure 7 it is clear that the FOPID controller takes the second mass to the desired position with less oscillations than the integer-order PID controller. Also the settling time is clearly less for the FOPID controller than for the integer-order PID controller resulting in a better performance of the FOPID in comparison to the integer-order PID controller. When comparing the control effort of both controllers, it can be concluded that the FOPID controller has a similar control effort compared to the PID controller. Note that the performance of the FOPID controller can still be improved by using the full ability of fractional controllers, i.e. using 5 tuning parameters in the controller. This will give the controller more possibilities to compensate the dynamic behavior of the system. The downside is that tuning becomes more difficult and less attractive to industry. The results in Figure 7 show a clear increase in performance for the FOPID controller tuned with the 3-parameter tuning method while still maintaining simplicity in tuning. As the mass spring damper is a benchmark problem for robust controller design, future work includes a comparison between the obtained results and those of Hinf and H2 robust controllers.

V – Conclusion

This research presented a simplified 3-parameter tuning method for FOPID controllers. Reducing the number of tuning parameters for FOPID controllers combines the possibilities of increased performance characteristic in fractional controllers with the simple tuning of integer-order controllers. This new reduced tuning method may make fractional controllers more accessible to industry. The new tuning method is tested on a fast but poorly damped electromechanical system: the mass-spring-damper system. The performance of the controllers obtained using the new tuning method is compared to the performance of a classical integer-order PID controller. The results show that the 3-parameter tuning method provides a FOPID controller with an increased performance compared to the integer-order PID controller. The controlled system using the FOPID controller behaves less oscillatory and has a faster response than the system controlled with the classical PID. The presented method results in a FOPID controller with a clearly increased performance combined with a simplification in tuning. Future work will compare the performance of FOPID controllers tuned with the 3-parameter method with the FOPID controllers resulting from full 5-parameter tuning methods for several types of processes.

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