The Mathematical Intelligencer

R´ecr´eation math´ematiques au moyen ˆage

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Récréation mathématiques au moyen âge

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by Jacques Sesiano
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REVIEWED BY ALBRECHT HEEFFER

If a compilation of medieval recreational problems needs to be published, Jacques Sesiano, a retired scholar on the history of medieval and Arabic mathematics, is very well placed to do so. Earlier this year, he published a monumental three-volume edition of the *Liber mahamaleth* [11], a medieval Latin translation of a lost Arabic text in the tradition of recreational problems. He earlier published two books on magic squares with Presses polytechniques et universitaires romandes [9], [10] and many articles on the subject, including an edition of a notebook by a medieval Italian merchant containing several recreational problems [8].

Sesiano's approach is to present a topical selection of problems with a French translation from the original sources (with the original text in footnotes), discussing their solutions and providing a mathematical commentary with generalizations of the problems. I first give an overview of these groups of problems which are treated as separate chapters, with a typical example for each type. Then I will discuss his choice of medieval sources and his mathematical commentaries.

The problems

The book contains 197 recreational problems from the Middle-Ages grouped into fifteen classes. Classifications for recreational problems have been proposed even since Johannes Tropfke wrote the first topical history of mathematics [14]. Another classification, more recent and comprehensive is a unfinished but widely circulated manuscript by David Singmaster. Sesiano uses neither of these classifications, though he occasionally refers to Tropfke. As the French terms for the problems in the overview below (given between brackets) may not be familiar to readers of the English language, they are grouped by the name as used by Singmaster [13] and followed by his section number.

1. Jugs and bottles (*transvasements*) (5.D.1)

   Best known as the problem of three full jugs with volumes a, b, c of which a has to be divided in half using b and c. The most common version is with volumes $(8, 5, 3)$.

2. Division of casks (*tonneaux*) (7.H.2)

   How to divide a, a full, b, a half full and c, an empty cask among n persons so that each gets an equal amount of the contents and of the casks. A problem from Alcuin (c. 800) uses the values $(10, 10, 10)$

3. Weights problem (*des poids*) (7.L.2.c and 7.L.3)

   Determine the least number of weights that have to be used to weigh any integral number of units on a balance from 1 to 40 inclusive, knowing that the weights can be placed on either of the scales of the balance. The solution depends on the geometric progression $1, 2, 4, 8, 16$. Variations of the problem use powers of three.

4. Monkey and coconuts problem (*distributions et partages*) (7.E)

   The English name of this problem is derived from a short story by Ben Ames Williams, published in a newspaper in 1926. It is about five sailors shipwrecked on an island. They gather coconuts all day. During the night each of the sailors takes one fifth of the remaining coconuts and gives one to the monkey. In the morning they each get one fifth, again leaving one coconut for the monkey. The question is how many coconuts they had gathered. This type of problem developed from a simpler version, popular during the Middle Ages.

5. Cistern problems (*robinets*) (7.H)

   A 'cistern' is the name for a Roman receptacle for holding liquids, usually rainwater. It can be filled by several pipes with different flow rates. What is usually given is the time that each of the pipes $a, b, c, \ldots$ needs to fill the cistern. Asked is the time in which they can fill the cistern together. Variations of the problem concern workers with different working rates that have to complete a job together.

6. Overtaking and meeting problems (*movement*) (10.A)

   Overtaking problems deal with two travellers that start from the same point and walk at rates (a, b), in which the slower one starts some time T before the faster, or they start at the same time at rates (a, b), with the slower starting some distance D ahead of the other. The question is when the faster overtakes the slower. Meeting problems are about two travellers that can cover a route in (a, b) days. They start at opposite ends at the same time and walk towards each other. The question is when they meet.

7. Large numbers (7.AB, 7.L,..)

The medieval preoccupation with large numbers is illustrated by problems on the summation of arithmetical and geometric progressions (such as the chessboard problem) and the search for perfect numbers.

8. The Josephus problem and the blind abbess and her nuns (*disposition*) (7.B, 7.Q)

   The Josephus problem is about n persons standing in a circle on a boat. By counting in steps of k, starting at some point in the circle one proceeds in a fixed direction. At each step, k people are skipped and the next person is thrown overboard. The elimination proceeds around the circle until only the last person remains. The problem asks to find the right place in the initial circle in order to survive. Another version deals with two groups of people arranged in such a way that one group survives. The blind abbess and her nuns is a problem about the rearrangement of a 3×3 square along the sides to conserve the side totals.

9. Crossing problems (*traversées*) (5.B)

   Best known as the river crossing problem with a wolf, goat, and cabbages, three jealous husbands or missionaries and cannibals on a boat.

10. Mixed problems

    Concerning the problem of the pandects (7.H.3), Fibonacci numbers (7.A), the dishonest butler (7.S.1), posthumous twins (7.G.2), the 17 camels (7.G.1), hour divination problems (7.M.4.b), and the two workers problem which is equivalent to a cistern problem (7.H).

11. Family riddles (9.E)

    Riddles about two men that marry each other's sister, mother or daughter or a father and his son who marry a mother and her daughter.

12. Magic squares (7.N)

    One of the long-time interests of Sesiano.

13. Knights tour and paths (5.F.1)

    About the well-known problem of a knight that has to cover all the squares of a chess board only once.

14. Infinite parts (*Les ensemble infinis*)

    A somewhat odd subject about division into infinite parts which is not usually considered a recreational problem.

15. Divination problems (7.AO)

    A series of problems about the divination of a number, a date or a permutation.

The sources

Sesiano lists his 19 sources with a short description in an appendix: eight Latin, six French, two Italian, one German, one Byzantine and one Arabic text. These include only seven books, others are manuscripts, some of which have not been edited before. The earliest dates from the eight century and the latest is Bachet's book of 1612, which somewhat stretches the Middle-Ages. Some of these sources have not been included in Singmaster's compilation: the Latin texts *Liber mahamaleth*, *Cautele algorismorum*, Lyon ms. 127, and the French manuscripts Tours 399, BNF 1339, 2050, the book *Le livre de chiffres et de getz*, of 1501 and the Arabic manuscript BNF 4441, which are useful additions to the known corpus of problems.

On the other hand, Sesiano does not include what is probably the most comprehensive text on recreational problems before the seventeenth century: Luca Pacioli's *Viribus Quantitatis*. Only one problem is quoted from this compilation and the book is mentioned in five footnotes. He further complains (in footnote 21) that the only edition is of poor quality, contains many errors and that the editor is incompetent. This edition does not appear in his bibliography but since he refers to an *éditrice*, this may be [3]. However, there exist a facsimile edition with a decent commentary in Italian [5]. With this and some other works from the Italian abbaco period missing, we may consider Sesiano's selection interesting but not fully representative for the Medieval period.

The commentary

Sesiano concludes each chapter with a mathematical commentary on the problems discussed. These concern reformulation in contemporary symbolism mostly including some generalizations of the problem.

Rather curiously, he never refers to contemporary treatments of recreational problems in secondary literature. Sesiano is not ignorant of this literature, he chose to completely ignore it. To name only a few: several of the problems, such as the monkey and coconut problem, were treated by Martin Gardner [2], Singmaster dedicated a separate study to the problem [12] and more recently, Jens Høyrup published an article on the related unknown heritage problem [6]. Eberhard Knobloch wrote a classic article to the weights problem and Persi Diaconis and Ron Graham dedicated a large part of their recent work on the permutation divination problem relating it to de Bruijn sequences [1]. Obviously, by ignoring the secondary literature on these subjects, Sesiano misses out on several interesting results.

What makes mathematics recreational?

The inclusion of a chapter on the division into infinite parts (*Les ensemble infinis*) made me wonder how Sesiano decided what to consider as recreational in medieval mathematics. Are these problems about what we now understand

to be recreational? Or is the criterium for inclusion that what scholars during the Middle-Ages considered to be recreational? Is there any evidence that their problems were proposed and solved for recreational purposes? If these were recreational problems for medieval scholars, did they continue to be recreational during the following centuries?

These questions not only remain unanswered in Sesiano's book, they are not even raised. Surely, the criteria for what is considered recreational in mathematics must depend on society, culture, mathematical methods and time periods. In a recent article [4] I have demonstrated that some very popular recreational problems from the Middle-Ages, such as the monkey-and-coconuts problem, disappear around the 1560's, while others like meeting and overtaking problems continue to be treated in popular works of the sixteenth and seventeenth centuries. It has been proposed that algebra, as a general problem-solving methods, spoils the recreational aspect of some types of problems, while others retain their appeal as recreational problems.

In summary, Sesiano's well-illustrated work on mathematical recreations of the Middle-Ages is a pleasant read. Its value lies in the use of the original texts from the sources and their modern mathematical commentary. For a more contextual approach on the cultural and intellectual value of mathematical recreations during that era, one has to look elsewhere.

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