Robustness of Structures
COST Action TU0601
1st Workshop, February 4-5, 2008, ETH Zurich, Zurich, Switzerland

Inherent Robustness of RC Slabs and Beams

Geoffrey Decan
PhD student, Department of Structural Engineering, Ghent University
Magnel Laboratory for Concrete Research, Technologiepark 904, B-9052 Ghent, Belgium

and

Luc Taerwe
Professor, Department of Structural Engineering, Ghent University
Magnel Laboratory for Concrete Research, Technologiepark 904, B-9052 Ghent, Belgium

Abstract

The paper starts with a brief review of provisions in Eurocode 2 concerning minimum reinforcement. Next, two cases of *inherent robustness* of concrete structures are investigated i.e. membrane action in RC slabs and the effect of minimum reinforcement on the structural behaviour of continuous beams.

Models for membrane action in rectangular RC slabs have been developed in the early 1960's and recently gained attention again in the framework of accidental loading situations. Both compressive membrane action (small displacements) and tensile membrane action (large displacements) are discussed.

The effect of minimum secondary reinforcement in a two span RC beam when the central support is removed, was analysed through a full 3D FE analysis. Both compressive arching and tensile catenary action could be observed. When the central support was removed the beam was able to sustain the design load through catenary action.

1. Introduction

When robustness is explicitly considered in structural design, this often implies a larger cost. Hence, it can be useful to study the effect of minimum and secondary reinforcement in statically indeterminate RC structures. These types of reinforcement can increase the possibilities for redistribution of internal forces, and consequently increase structural robustness without extra cost.

The loss of a vertical support is often considered to assess robustness of a RC structure. In this accidental case, concrete slabs display large deformations and membrane action is activated which results in a load transfer to the remaining supports. Also in the case of blast or impact loading, membrane action in RC slabs occurs.

In the following section code provisions concerning minimum reinforcement are discussed. In section 3, a review of membrane action in RC slabs is given. Section 4 deals with a nonlinear FE analysis is presented of a two span continuous beam with axial restraint provided by longitudinal tie reinforcement.

2. Code provisions

2.1 Reinforcement requirements

In current design practice the main reinforcement in RC beams and slabs is calculated for the critical sections. Part of this reinforcement is curtailed based on the envelope of the acting tensile force. Design codes of RC structures also provide

- rules for minimum amounts of reinforcing steel e.g. to avoid brittle failures and excessive cracking
- detailing rules e.g. reinforcement arrangements at intermediate and end supports of beams and slabs, in columns and walls, etc.
- ductility conditions
- reinforcement requirements for fire design

It is useful to investigate to which extent these code provisions contribute to robustness in providing additional load paths or increasing the ultimate load (and deflection) with respect to the value aimed at by applying current design methods. If this contribution would turn out to be significant, this would mean that by complying with the above mentioned rules, a basic minimum robustness is obtained with no, or only a marginal, extra cost. Traditionally, most RC structures have not explicitly been designed for robustness, and yet it can be observed that, in case of unexpected accidental loading, some degree of *inherent robustness* is available.

2.2 Tying systems

In Section 9.10 of EN 1992-1-1 (Eurocode 2) it is stated as a principle that concrete structures which are not explicitly designed to withstand accidental actions, shall have a suitable tying system to prevent progressive collapse by providing alternate load paths after local damage. The following types of ties have to be provided:

- peripheral ties (also along internal edges);
- internal ties, connected to the peripheral ties, in each floor and roof level in two directions at approximately perpendicular angles;
- horizontal ties at edge columns and walls;
- vertical ties in columns and/or walls in panel buildings of five storeys or more to limit the damage of collapse of a floor in the case of accidental loss of the column or wall below.

In the latter case, continuous vertical ties should be provided from the lowest to the highest level, capable of carrying the load in the accidental design situation, acting on the floor above the column/wall accidentally lost. Other solutions e.g. based on the diaphragm action of remaining wall elements and/or membrane actions in floors, may be used if equilibrium and sufficient deformation capacity can be verified. For all cases, recommended numerical values for the design tie forces are specified in EC2.

3. Membrane action in RC slabs

3.1 Horizontally restrained slabs

The first systematic analytical approach for membrane action in slabs is apparently due to (Wood, 1961). Interior panels of continuous RC slabs have edges which are restrained against lateral displacement by the stiffness of the surrounding panels. Fig. 1 schematically shows the load – central deflection curve of a uniformly loaded two-way rectangular reinforced concrete slab with laterally restrained edges (Park, 1964).

In the early part of the loading range, compressive membrane action is induced as a result of the restraint against outward movement. As can be derived from typical moment – axial force diagrams, this results in ultimate flexural loads which may be much higher than the failure loads predicted by the yield-line theory (point B in fig. 1). After the local maximum reached in B, the supported load decreases rapidly with further deflection owing to the reduction in the compressive membrane forces. Eventually a stage is reached (near C) where the membrane forces in the central region of the slab change from compression to tension and the slab boundary restraints begin to resist inward movement of the edges. At this stage, because of the large elongation of the slab surface, the cracks in the central region penetrate the whole thickness of the slab and yielding spreads throughout this region (Park, 1964). Beyond C, the load is carried by the reinforcement acting as a tensile membrane and, with further deflection, the load carried increases until the reinforcement starts to fracture at D. In the practical case of gravity loading, which magnitude remains unchanged as the slab deflects, "snap through" occurs from B to a point at the same load level on the branch CD.

In (Park, 1964) it is shown that under the assumption that the slab acts as a tensile membrane, the deflection z satisfies the following differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{T_y}{T_x} . \frac{\partial^2 z}{\partial y^2} = -\frac{p}{T_x} \tag{3.1}$$

where

x, y	rectangular coordinates parallel to the slab edges and in the plane of the slab
p	uniformly distributed load per unit area of the slab
T_x, T_y	yield forces, per unit with, of the reinforcement in the x and y directions which is placed over the whole area of the slab

By solving Eq.(3.1) it can be shown that the ratio

$$\frac{p l_x^2}{T_x \Delta} = f\left(\frac{l_x}{l_y}, \frac{T_x}{T_y}\right) \tag{3.2}$$

with Δ the central deflection, is proportional to the inverse of an infinite series with the terms depending on the parameters T_x/T_y and l_x/l_y. For a given slab geometry and reinforcement arrangement, Eq.(3.2) gives a linear relation between p and Δ and defines the portion of the load-deflection curve between C and D in fig. 1. The limiting point D will depend upon the ductility of the steel. In Eurocode 2, ductility of reinforcing steel is defined by the ratio of the tensile strength to the yield stress f_t/f_y and the elongation at maximum force ε_u. Values of $(f_t/f_y)_k$ and ε_{uk} for reinforcement classes A, B and C are given in annex C of Eurocode 2.

A typical test result obtained by Park (1964) is shown in fig. 2. The straight line corresponds to the solution given by Eq. (3.2) and shows that the model based on pure membrane action is conservative. Moreover, strain-hardening of the reinforcement was not considered.

Many other test programs demonstrate that the ultimate load of laterally restrained single panels may be significantly higher than that given by Johansen's yield-line theory, particularly if the boundary restraint is stiff, the span to depth ratio of the slab panel is high and the reinforcing steel ratio is small.

3.2 Horizontally unrestrained slabs

Although perfectly unrestrained slabs are rare, a number of intermediate states can be imagined, e.g. a corner panel of a continuous RC plate where two edges only have very limited horizontal restraints. Unlike horizontally restrained slabs, these slabs develop no compressive membrane action. At large deflections tensile membrane action occurs in the central span region and a compressive ring develops around the slab's perimeter (fig. 3).

An analytical approach, based on rigid plastic behaviour with a change of geometry, was developed in (Hayes, 1968) and refined in (Bailey, 2001) and finally in (Bailey and Toh, 2007). The method allows to calculate the ultimate load as an enhancement above the yield-line load. It is based on a predefined yield-line pattern and considers self-equilibrated in-plane forces, which increase with increasing deflection (fig. 4). Compressive membrane action develops around the perimeter and tensile membrane action in the center of the slab as observed during testing (Bailey *et al.*, 2008). Two modes of failure are considered, comprising fracture of reinforcement across the shorter span of the slab and compression failure of concrete in the corners of the slabs.

3.3 Inherent robustness

In fig. 5, typical test results as reported in (Park, 1964) and (Bailey et al., 2008) are shown. The horizontal line represents the collapse load calculated with Johansen's yield-line theory. Taking into account membrane action, much higher load carrying capacities can be obtained as demonstrated by the experimental results. The ratio

$$R_{inherent} = \frac{p_{u,\mathrm{exp,\,membrane}}}{p_{u,\mathrm{design\ model}}} \tag{3.3}$$

could be termed "inherent robustness". In this formula $p_{u,\mathrm{exp.\,membrane}}$ represents the experimental ultimate load accounting for membrane action and $p_{u,\mathrm{design\ model}}$ stands for the ultimate load obtained from a current design model, in this case typically yield-line theory. This inherent robustness can be defined in a similar way for other structural components and parts. The margin $p_{u,\mathrm{exp.\,membrane}} - p_{u,\mathrm{design\ model}}$ is not exploited in practical design. Several authors point to the fact that, in case one would take advantage of this margin, deflections under service conditions would be unacceptably high.

Compressive membrane action at small displacements has been accounted for in the design of bridge decks and FRP reinforced concrete slabs (Taylor and Mullin, 2006). Research into membrane action at large displacements was almost curtailed after the pioneering work in the 1960's due to the above mentioned deflection limitations. Interest into large displacement behaviour however, has been revitalized recently for application to the practical design of structures subjected to fire (Bailey, 2003 and Foster *et al.*, 2004). It is clear that for other accidental loading situations, this theory could turn out to be extremely useful to determine the inherent robustness of RC concrete slabs.

4. Modelling of a two span RC beam with longitudinal tie reinforcement

Creating a finite element model to study the large deformation behaviour of concrete beams involves a lot of care in the definition of material models, mesh definition and boundary conditions. In this study, advanced finite element software (TNO Diana, 2007) was used to perform the analysis. The beam model was generated with an application developed by the first author for this purpose.

4.1 Modelling details

Material models

For the concrete, a material model has been applied that allows for cracking. A parabola-rectangular diagram has been assumed in compression up to a compressive strain of 3.5‰. For the tension diagram a Hordijk diagram was assumed because the exponential softening proved beneficial for the convergence of the calculations (fig. 6 – left). For the reinforcing steel, a Von Mises yield criterion with strain hardening was applied (fig. 6 – right). Bond-slip behaviour between reinforcement and concrete could not be modelled in this 3D analysis.

Geometry and meshing

The RC beams have a rectangular cross section (400 mm x 198 mm) and are symmetric with respect to the central support in both geometry and loading. Both spans are 5 m long. Due to the symmetry and the fact that no lateral instability phenomena are taken into account only one quarter of the two span RC beam had to be modelled (fig. 7). Due to the large difference in strain over the depth of the beam in the plastic hinge regions, a mesh refinement was necessary in these zones. To ensure that the numerical solution would be sufficiently stable, a rather dense mesh was chosen. The elements adopted were twenty-node isoparametric solid brick elements (CHX60) and a 3x3x3 integration scheme was used.

The necessary reinforcement was calculated according to Eurocode 2. A bar was also placed in the centre of the beam to simulate the behaviour of longitudinal tying bars. The area of the longitudinal reinforcement was slightly varied to investigate the effect of the amount of reinforcement on the structural behaviour.

Loading and boundary conditions

The beam is subjected to its dead weight and in each span a point load is applied at 2 m from the exterior supports. At the central support axial displacements are prohibited due to symmetry. For the same reason all lateral displacements are constrained along the other symmetry plane. At the end supports and the central support no vertical displacements are allowed along a transverse line over the beam width. To prevent local splitting effects a perfectly elastic-plastic steel plate was inserted between the support and concrete elements. When catenary effects are investigated some type of horizontal restraint is necessary. Therefore no horizontal displacement is allowed at the nodes at mid depth of the end sections, which simulates the presence of tying reinforcement.

4.2 Results

Standard beam

In the standard beam the minimum reinforcement consists of four corner bars with a diameter of 12 mm, which run over the full length of the beam. This case has been taken as a reference to compare other reinforcement arrangements. First, a loading test was simulated for the reference case where the central support is present. Both the applied load and normal force in the beam are plotted as a function of the applied displacements in fig. 8.a.

Up until formation of the first cracks, the behaviour is linear elastic and no normal forces occur in the beam. When the first cracks appear, the length of the beam tends to increase. Because the beam is constrained this results in a compressive force in the beam. The load increases until a first plastic hinge forms at the central support resulting in a loss of stiffness. When the second plastic hinge forms, the initial maximum load is reached and a softening branch follows. Because only very limited parts of the reinforcement over the central support were in the strain-hardening branch, a second maximum can be observed when hardening initiates in the plastic hinges. When deformations continue to increase a tensile force develops in the longitudinal reinforcement. Due to catenary action these tensile forces can transfer loading well beyond the first load maximum until the ultimate tensile strain of the reinforcement is reached, which in this case equals 10 %.

To assess the loss of the central support, a static nonlinear analysis was performed on the same beam with the central support removed. Although the loss of a support is a dynamic process, a static analysis is deemed to be sufficient when each section is ductile enough to absorb most of the energy that is released upon failure of the support (Val and Val, 2006). Because only minimum reinforcement is present over the central support to resist the sagging moment that is applied, the load carrying capacity of this new configuration appears to be very limited. Very large displacements are observed and at a deflection of almost 350 mm, the service load level of 100 kN is reached (fig. 8.b).

Influence of minimum reinforcement
In a second stage the diameter of the minimum reinforcement was varied to study the effect of minimum reinforcement on the load carrying capacity and in particular the catenary effect. Intuitively it can be argued that increasing the amount of minimum reinforcement to be used will increase the amount of reinforcement in the least reinforced section. As a consequence; the maximum tensile force that can be generated by the reinforcement will increase as will the catenary effect. This is clearly illustrated in fig. 9.a which shows three different simulations corresponding to corner bars with diameter 10, 12 and 16mm respectively.

Influence of bottom reinforcement at the central support
When the central support fails, the maximum sagging moment occurs in a section which was not designed for it. It could be argued that putting extra bottom reinforcement will significantly improve the structural behaviour of the beam. One way of realizing more bottom reinforcement is by not curtailing the span reinforcement towards the central support. In the standard case, only 50 % of the span reinforcement continues over the central support. Fig. 9.b shows the result for different ratios of bottom reinforcement continuing over the central support i.e. 50%, 72% and 100%. Although the initial load carrying capacity and ductility of the beam can be significantly improved, the ultimate carrying capacity due to catenary action is the same in all cases. This is mainly due to the fact that catenary action is defined by the total amount of longitudinal reinforcement in a section. At the central support there is a significant amount of top reinforcement which allows the catenary tensile stresses to be only slightly influenced by the amount of bottom reinforcement that is present there.

5. Conclusions

- Minimum reinforcement requirements and detailing rules provide a basic contribution to the inherent robustness of RC structures.
- In both restrained and unrestrained RC slabs, membrane action can significantly increase the failure load above the value obtained from classical yield-line theory. In this case *inherent robustness* may be defined as the ratio of the real ultimate load, taking into account membrane action, to the ultimate load obtained from yield-line theory.
- To illustrate the use of catenary action in accidental situations, a typical two span RC beam was modelled. The central support was removed to study the effect of the destruction of a column on the structural behaviour of the beam. In the case of horizontal restraint, a significant increase in load carrying capacity could be observed due to catenary action. The effect of reinforcement bars continuous over the full length of the beam and the effect of not curtailing the bottom reinforcement near the central support were investigated. It was found that the load carrying capacity at large deflections is governed by the reinforcement in the least reinforced section, but that continuous bottom reinforcement over the central span is important to provide sufficient ductility.

Acknowledgements

The authors are grateful for the support received from the Research Foundation Flanders (FWO) as part of the research program "Robustness of concrete structures and its influence on the global safety level".

References

Bailey, C.G. (2001) *Membrane action of unrestrained lightly reinforced concrete slabs at large displacements*, Engineering Structures, vol. 23, no. 5, May 2001, pp. 470-483.

Bailey, C.G. (2003) *Efficient arrangement of reinforcement for membrane behaviour of composite floor slabs in fire conditions*, Journal of Constructional Steel Research, vol. 59, pp. 931-949.

Bailey, C.G.; Toh, W.S. (2007) *Behaviour of concrete floor slabs at ambient and elevated temperatures*, Fire Safety Journal, vol. 42, pp. 425-436

Bailey, C.G.; Toh, W.S. and Chan, B.M. (2008) *Simplified and Advanced Analysis of Membrane Action of Concrete Slabs*, ACI Structural Journal, Jan-Feb 2008, pp.30-40.

EN1992-1-1 (2004) *Eurocode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings*, CEN.

Foster, S.J.; Bailey, C.G.; Burgess, I.W. and Plank, R.J. (2004) *Experimental behaviour of concrete floor slabs at large displacements*, Engineering Structures, vol. 26, 2004, pp. 1231-1247.

Hayes, B. (1968) *Allowing for membrane action in the plastic analysis of rectangular reinforced concrete slabs*, Magazine of Concrete Research, vol. 20, no. 65, December 1968, pp. 205-212

Park, R. (1964) *Tensile membrane behaviour of uniformly loaded rectangular reinforced concrete slabs with fully restrained edges*, Magazine of Concrete Research, vol. 16, no. 46, March 1964, pp.39-44.

Taylor, R. (1965) *A note on a possible basis for a new method of ultimate load design of reinforced concrete slab*, Magazine of Concrete Research, vol. 17, no. 53, December 1965, pp. 183-186.

Taylor, S.; Mullin, B. (2006) *Arching action in FRP reinforced concrete slabs*, Construction and building materials 20, pp. 71-80.

TNO Diana (2007) *Diana 9.2 Finite element analysis: User's manual*, TNO Diana BV, Delft, The Netherlands.

Val, D.V. and Val, E.G. (2006) *Robustness of frame structures*, Structural Engineering International, no. 2, pp.108-112.

Wood R.H. (1961) *Plastic and elastic design of slabs and plates*, Thames and Hudson, London.

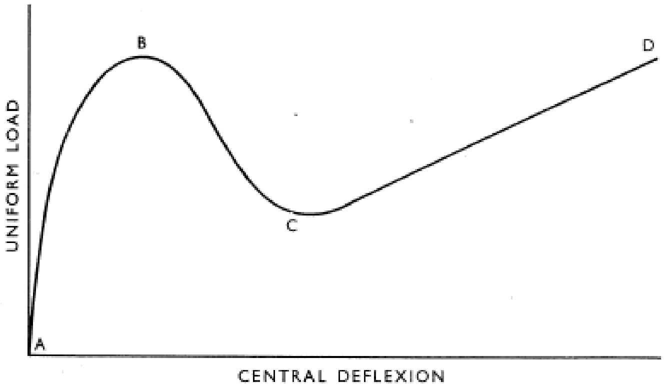


Figure 1: Schematic load-deflection curve of a slab with full edge restraint (Park, 1964).

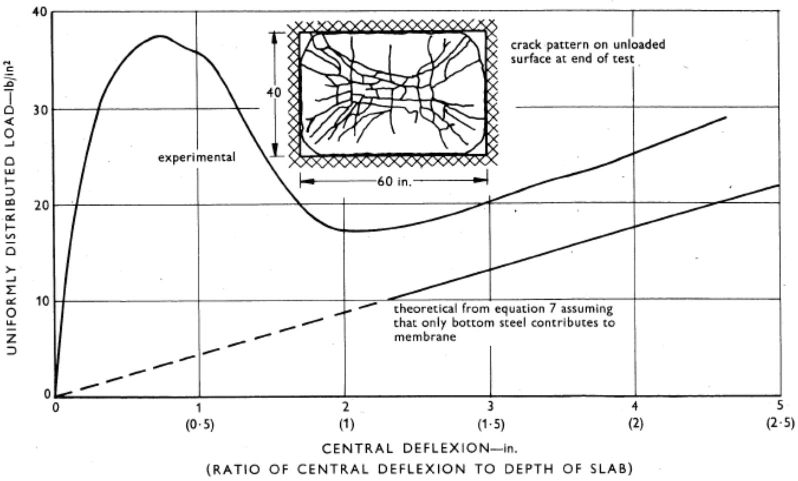


Figure 2: Load-deflection curve and crack pattern of restrained rectangular slab (Park, 1964)

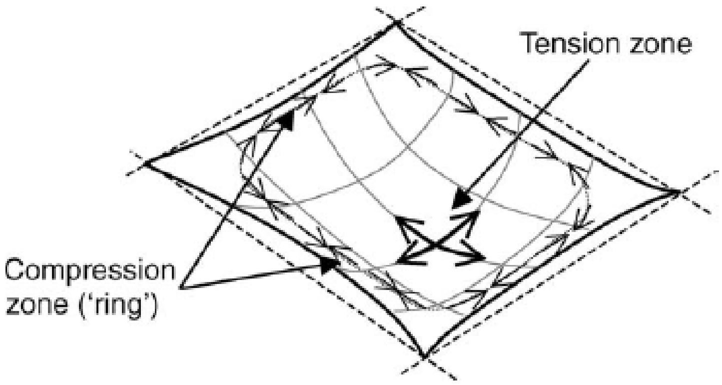


Figure 3: Membrane action of horizontally unrestrained slabs (Foster *et al.*, 2004)

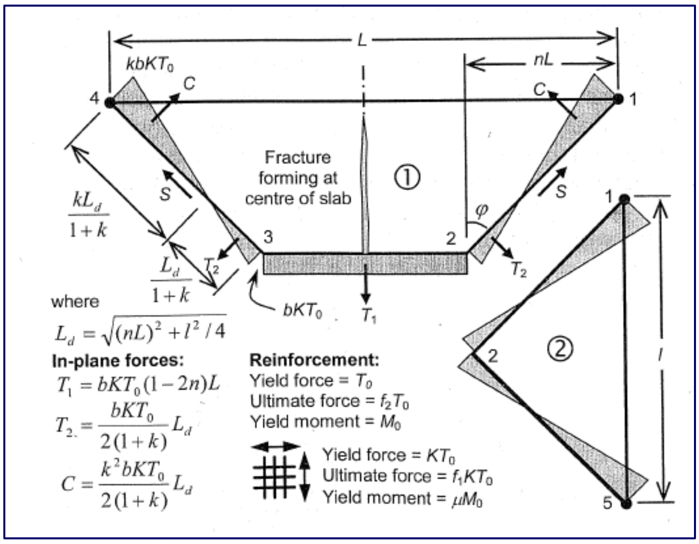


Figure 4: In-plane stress distribution patterns along assumed yield lines
(Bailey and Toh, 2007).

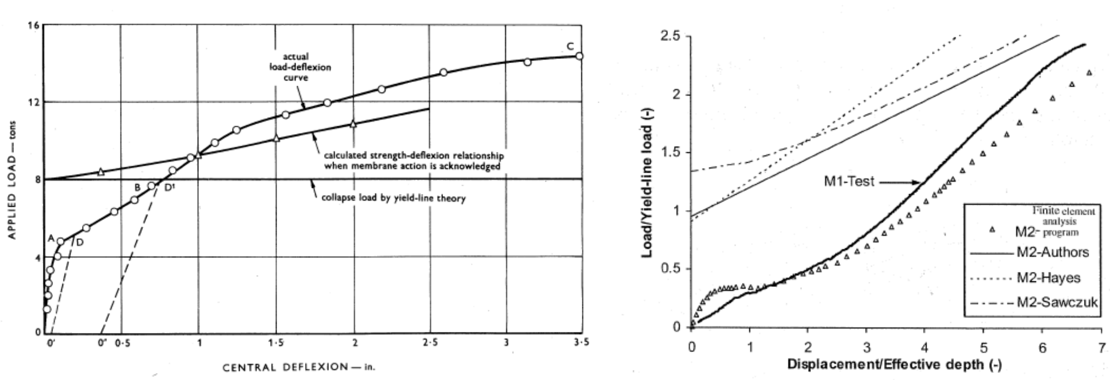


Figure 5: Load-deflection curves for typical restrained (Park, 1964) and unrestrained slabs
(Bailey *et al.*, 2008).

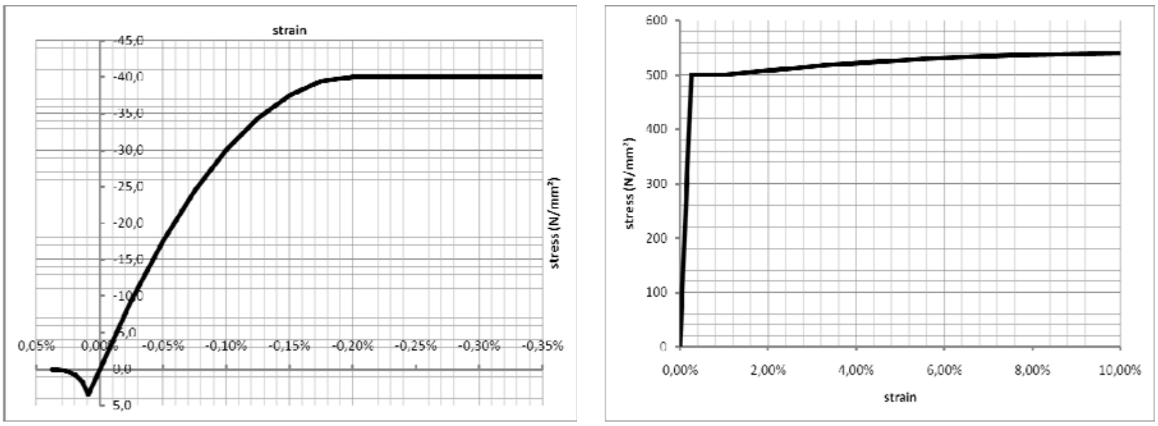


Figure 6: Material models (left: concrete; right: reinforcing steel).

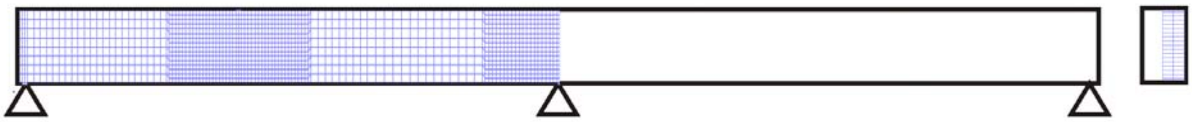
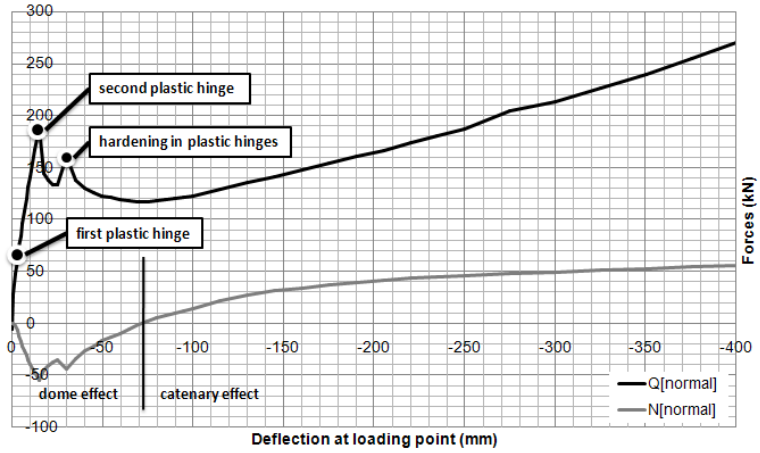
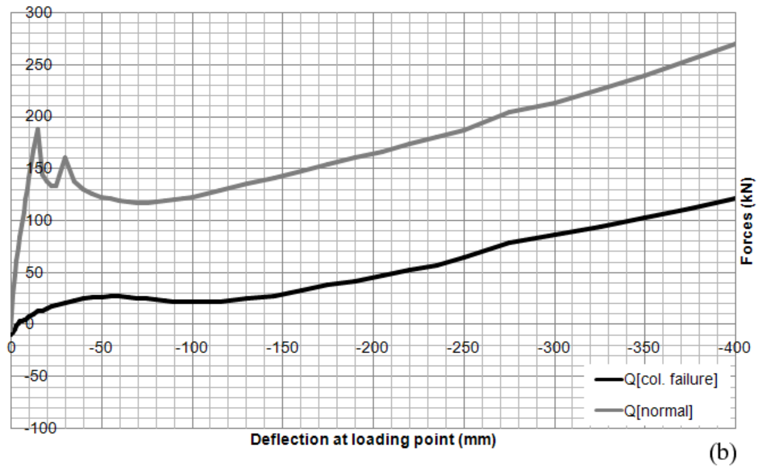


Figure 7: FE model of the RC beam.

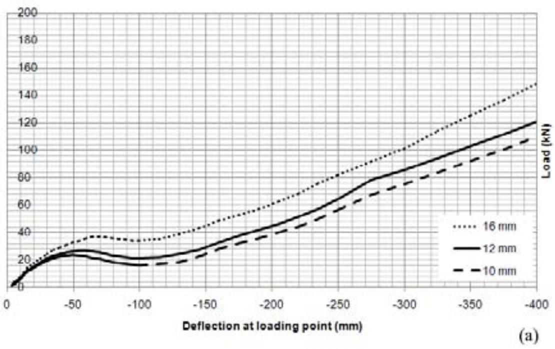


(a)

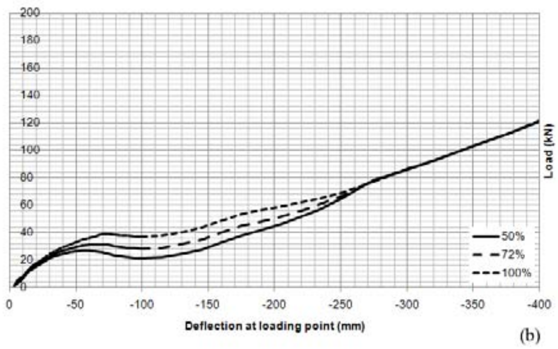


(b)

Figure 8: Results of analysis of the standard beam – (a) applied load and normal force; (b) with and without central support.



(a)



(b)

Figure 9: Results of analysis – (a) influence of minimum diameter and (b) influence of bottom reinforcement at the central support.