Shanks-Transform Accelerated PML-Based Series Expansions for the 1-D Periodic 3-D Green's Functions of Multilayered Media

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Abstract

A Perfectly Matched Layer (PML) based formalism is proposed to derive fast converging series expansions for the 1D periodic 3D Green's functions of layered media. The Shanks transform is applied to accelerate the PML-based series.

1. Introduction

Periodic structures are of great practical use in many applications in antenna systems, microwave electronics and optics. Efficient modelling techniques rely on the Floquet-Bloch theorem to limit the simulation domain to a single unit cell. Up to now, little has been published about the 1D periodic 3D Green's function, especially when also considering the presence of a stratified dielectric background medium. In [1] the 1D periodic 3D Green's functions for a microstrip substrate are derived in the spectral domain first, and the corresponding spatial-domain quantities are obtained through an efficient sum of inverse Fourier transforms.

We propose a Perfectly Matched Layer (PML) based formalism to derive fast converging series expansions for the 1D periodic 3D Green's functions of layered media. The PMLs are used to transform the open layered medium into a closed waveguide configuration. This results in an efficient expansion for the 3D Green's function of a point source in the stratified background medium in terms of a set of discrete modes of the closed waveguide containing the PML, while the PMLs mimic the open character. As both the spectral and spatial domain series suffer from slow convergence, the Shanks transform is applied to accelerate the PML-based series.

2. Theory

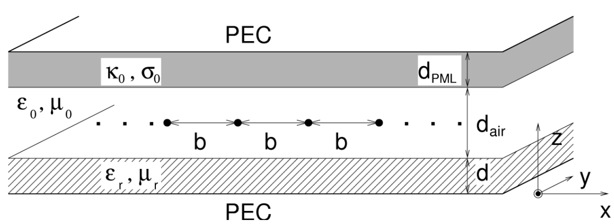

Figure 1: 1D periodic set of point sources on a PML-terminated microstrip substrate.

Consider a planar multilayered dielectric background medium in which we place a 1-D grid of point sources (Fig. 1), resulting in a periodic problem in the x-direction with the period given by b; two adjacent point source excitations differ by at most a phase factor $e^{-jk_x b}$. For a faster evaluation of the 1D periodic 3D Green's functions, we construct a parallel plate waveguide by terminating the free space with two perfect electrically conducting plates backed by a Perfectly Matched Layer (PML) with thickness d_{PML} and with material parameters κ_0 and σ_0 [2]. This results in a series expansion for the 3D Green's function for a 1D periodic grid of point sources:

$$G^{\text{per,1D}}_{A,V}(x,y,z;x',y',z') = \frac{1}{4j}\sum_{n=1}^{+\infty} A_n(\beta_n,z|z') \sum_{m=-\infty}^{+\infty} e^{-jk_x mb} H_0^{(2)}\left(\beta_n\sqrt{(x-x'-mb)^2+(y-y')^2}\right), \tag{1}$$

with β_n and the expansion coefficients $A_n(\beta_n,z|z')$ given in [3]. Applying of the Poisson transform yields following equivalent series expansion

$$G^{\text{per,1D}}_{A,V}(x,y,z;x',y',z') = \frac{1}{2jb}\sum_{n=1}^{+\infty} A_n(\beta_n,z|z') \sum_{m=-\infty}^{+\infty} \frac{1}{\beta_{n,m}} e^{-j\left(\frac{2m\pi}{b}+k_x\right)(x-x')} e^{-j\beta_{n,m}|y-y'|}, \tag{2}$$

with $\beta_{n,m} = \sqrt{\beta_n^2 - \left(\frac{2m\pi}{b}+k_x\right)^2}$.

By applying the PML formalism, we have replaced the classic time-consuming Sommerfeld integrated series over a single index m by two equivalent series (1) and (2), over double indices m and n, but for which the terms are easy to evaluate. First concentrate on the index n, which runs over the different modes in the waveguide formed by the substrate together with the PMLs. In [4] the convergence of a 2D Green's function expansion for a line source on a microstrip substrate is analyzed and it is shown that exponential convergence is obtained provided the distance $|y-y'|$ is not too small. In a similar way, series (2) converges at a rate $\sim e^{-nC_1|y-y'|}$, for n large, C_1 being constant for m fixed, thus exponentially as a function of n, yielding a rapidly converging series provided the distance $|y-y'|$ is not too small. Series (1) converges at a rate $\sim e^{-nC_2\sqrt{(x-x'-mb)^2+(y-y')^2}}$, for n large, C_2 a constant, thus exponentially as a function of n, yield a fast converging series provided that either the distance $|x-x'|$ *or* the distance $|y-y'|$ are not too small. As a function of m, on the other hand, series (2) converges at a rate $\sim e^{-m\frac{2\pi}{b}|y-y'|}$, for m large and for arbitrary but fixed n, thus exponentially as a function of m, resulting in a rapidly converging series provided the distance $|y-y'|$ is not too small. Yet, in order to obtain exponential convergence for series (1) as a function of m, it is required that the index n is sufficiently large. For small mode orders n, series (1) is slowly convergent as a function of m. In [4] the Shanks transform is proposed to accelerate convergence of the PML-based mode expansion of the 2D Green's function for a line source on a microstrip substrate. In a similar way, we apply the Shanks procedure for both series (1) and (2), in order to accelerate convergence as a function of the PML-based mode index n. Moreover, for each index n, the Shanks transform is applied to accelerate convergence as a function of the periodicity index m for both series (1) and (2).

All acceleration schemes presented up to now do not allow to calculate the 1D periodic 3D Green's function accurately and efficiently when both distances $|x-x'|$ *and* $|y-y'|$ are very small. Indeed, in [5] it is shown for the 2D case that the PML-based series does not capture the correct singular behavior of G_A at the interface of a non-magnetic ($\mu_r = 1$) microstrip substrate. Therefore, we combine part of the PML-based series (1) with one term of the Sommerfeld integrated series to capture the correct singularity. The following series is proposed in order to evaluate the 1D periodic 3D Green's function $G^{\text{per,1D}}_A$ for very small distances $|x-x'|$ *and* $|y-y'|$ at the interface:

$$G^{\text{per,1D}}_{A}(x,y,z;x',y',z') = G_A(x,y,z;x',y',z') + \frac{1}{4j}\sum_{n=1}^{+\infty} A_n(\beta_n,z|z') \sum_{\substack{m=-\infty\\ m\neq 0}}^{+\infty} e^{-jk_x mb} H_0^{(2)}\left(\beta_n\sqrt{(x-x'-mb)^2+(y-y')^2}\right). \tag{3}$$

The evaluation of the Green's function $G_A(x,y,z;x',y',z')$ for a single point source in the stratified medium is then performed by means of the classical Sommerfeld integration.

3. Examples

Consider a microstrip substrate with thickness $d = 9$ mm, $\varepsilon_r = 3$ and $\mu_r = 1$. To obtain an expansion into PML-based modes, a closed waveguide is formed by adding a perfect electrically conducting plate above the substrate, such that $d_{\text{air}} = 5$ mm, $d_{\text{PML}} = 3.5$ mm. A strongly absorbing PML is obtained for $\kappa_0 = 15$ and $\frac{\sigma_0}{\omega\varepsilon_0} = 10$. The free-space wavelength at the operating frequency is chosen to be $\lambda_0 = 2$ cm. We determine the Green's function $G^{\text{per,1D}}_V(x,y,z;x',y',z')$ for a 1D periodic set of point sources with spacing $b = 1.5$ cm (Fig. 1). Fig. 2 presents the Shanks-transform accelerated series expansions based on the spatial (1), spectral (2) and hybrid (3) PML modal series for $\left|G^{\text{per,1D}}_V(0,y,9\text{ mm};0,y',9\text{ mm})\right|$ with $k_x = 0$, as a function of $k_0|y-y'|$, which is in excellent agreement with the classic Sommerfeld integrated spectral series, accelerated following [1]. On a Pentium T7400 Centrino Duo 2.16 GHz machine with 2GB RAM, the evaluation of 200 points based on the Shanks-transform accelerated spatial series expansion (1) takes 3 s, the Shanks-transform accelerated spectral series expansion (2) takes 1 s, and the Shanks-transform accelerated hybrid series expansion (3) takes 11 s, whereas the accelerated classic Sommerfeld integrated spectral series requires 1 min 38 s of CPU time. In Fig. 3 the relative error is plotted as a function of distance, comparing the different series expansion to the hybrid series expansion, which is generally valid and thus chosen as a reference solution. All PML-based series expansions for $G_V^{\mathrm{per,1D}}$ exhibit an accuracy better than 0.004% at distances as small as $|y-y'| = \frac{\lambda_0}{60}$. The discrepancy between the hybrid series (3) and the classic Sommerfeld integrated spectral series, accelerated by the method proposed in [1], is smaller than 0.02%. Fig. 4 shows the Green's function series $\left|G_V^{\mathrm{per,1D}}(0,y,7\text{ mm};0,y',7\text{ mm})\right|$ inside the dielectric substrate, calculated as a function of $k_0|y-y'|$ for $k_x = 0$. Again, an excellent agreement is found between the spatial, spectral and hybrid Shanks-transform accelerated series expansions and the accelerated [1] classic Sommerfeld integrated spectral series, together with a significant speedup (3 s for the accelerated spatial series (1), 4 s for the accelerated spectral series (2), 11s for the accelerated hybrid series (3), versus 5 m 47 s). Finally, Fig. 5 proposes the Green's function series $\left|G_V^{\mathrm{per,1D}}(0,y,9\text{ mm};0,y',7\text{ mm})\right|$ for the excitation at the substrate-air interface and the observation point inside the dielectric substrate.

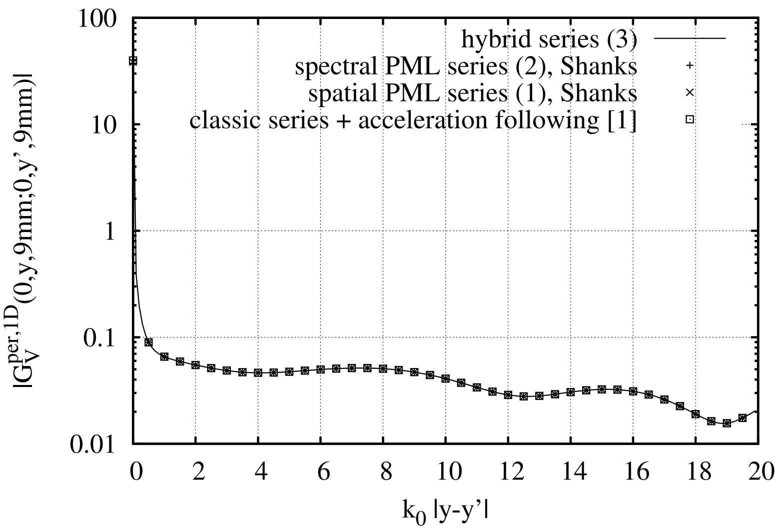

Figure 2: 1D periodic 3D Green's function $\left|G_V^{\mathrm{per,1D}}(x,0,9\text{ mm};x',0,9\text{ mm})\right|$.

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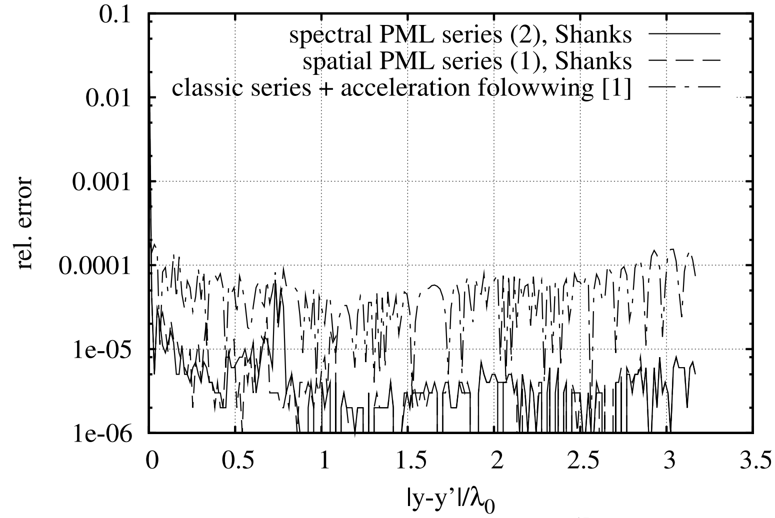

Figure 3: Relative error of the different expansions for $G_V^{\mathrm{per,1D}}(0, y, 9\mathrm{mm}; 0, y', 9\mathrm{mm})|$.

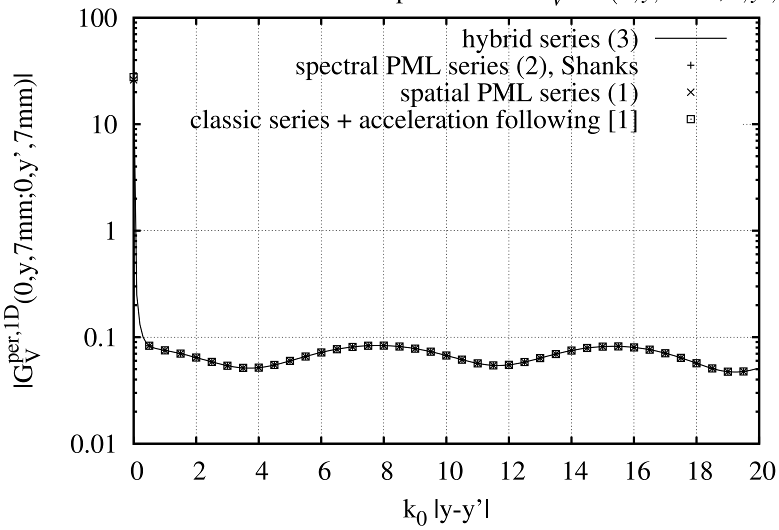

Figure 4: 1D periodic 3D Green's function $\left|G_V^{\mathrm{per,1D}}(0, y, 7\ \mathrm{mm}; 0, y', 7\ \mathrm{mm})\right|$.

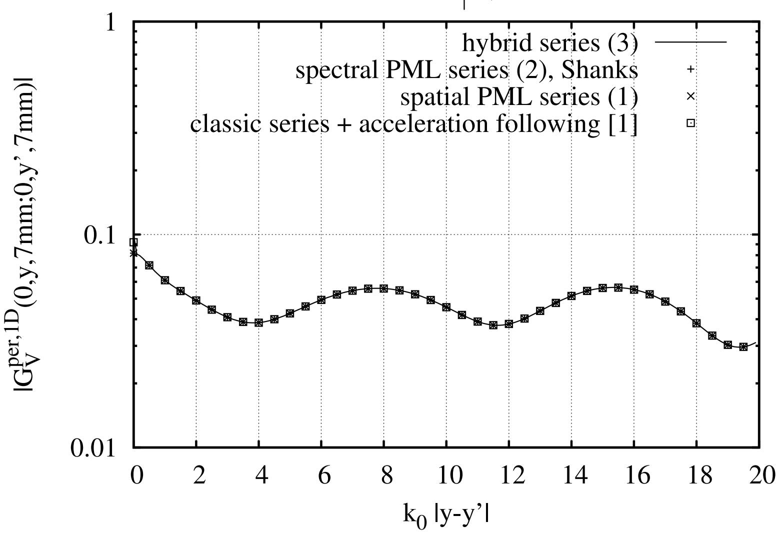

Figure 5: 1D periodic 3D Green's function $\left|G_V^{\mathrm{per,1D}}(0, y, 9\ \mathrm{mm}; 0, y', 7\ \mathrm{mm})\right|$.

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