Step-wise numerical procedure for the time-dependent modelling of concrete beams taking into account creep and creep recovery

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Abstract

The time-dependent behaviour of concrete structures can be modelled by viscoelastic integral equations, which in most cases cannot be solved in a closed form. As an alternative, an algebraic method, the Age Adjusted Effective Modulus method (AAEM) was introduced. However, in cases were the load history on the structural element exhibits a significant amount of load changes, the use of numerical step-by-step methods, in which the stiffness matrix is multiplied with such viscoelastic integral equations, poses an advantage. Since the concrete strain is a function of the load history, this load history needs to be stored for each structural element. However this poses no longer a problem with computers.

The load history results from the construction and the in-service stages of a structure. The loads during the construction period or the early service life invoke time-dependent deformations and can compromise the later serviceability of the structure. Due to the time effects, unacceptable deformations may be latent for several years. Commonly the construction sequence is neglected in the design of concrete structures with respect to creep and shrinkage.

Furthermore, it is recognized that only part of the creep deformations can be recovered, even if loads are applied during a short period of time. This can have a significant effect on the later deformation occurring at a higher age. The fact that only a partial recovery is observed means that the viscoelastic behaviour of concrete is different for unloading. In this paper an existing numerical approach is extended in order to handle the viscoelastic behaviour for both the loading and unloading of concrete, taking into account the modelling of non-recoverable creep. Hereby, constitutive viscoelastic laws for loading and unloading suggested in the literature are used.

Measurements of T-shaped concrete beams that are subjected to load changes are used to verify the suggested calculation method.

Keywords: **C**oncrete creep, Creep recovery, Load history, Numerical methods

1. Introduction

This paper expands an integral-type computational approach for the analysis of time-dependent effects of concrete structures (Sassone & Casalegno, 2012). In this approach the finite element method is coupled with a numerical solution of the Volterra integral equation used to describe an aging linear viscoelastic material. This method is justified based on the principle of superposition. It is suggested that this approach is suitable for simple and complex structures (such as non-homogenous structures and sequential construction procedures). However, the principle of superposition only yields accurate results if some conditions are met. First, the stresses in the structures must stay within service limits, i.e. less than 40% of the compressive strength of the concrete. Second, there cannot be significant changes in the environmental conditions (Z. P. Bažant & L'Hermite, 1988). If it is assumed that the stresses in the concrete are limited, like prescribed in design codes and no significant changes in the environmental condition occur the only bottleneck is to deal with the possible unloading of the concrete elements. Relocating of removing stacks of building materials, the use of particular equipment or the construction procedure can cause both a load increase and a load decrease (Gilbert & Ranzi, 2010). Hence, it is the authors' opinion that the existing approach has to be extended to the case of unloading since the creep response of unloading can show significant differences compared to the creep response of loading.

2. Concrete creep and creep recovery

Concrete creep is a time-dependent phenomenon that causes the deformations of concrete to increase under a sustained stress. Creep recovery is the reverse effect, meaning the decrease of concrete deformations after a sustained stress is removed. It is generally accepted that only part of the initial creep deformation can be recovered while another part stays irrecoverable (Yue & Taerwe, 1992, 1993). This behaviour is illustrated in Fig. 1.

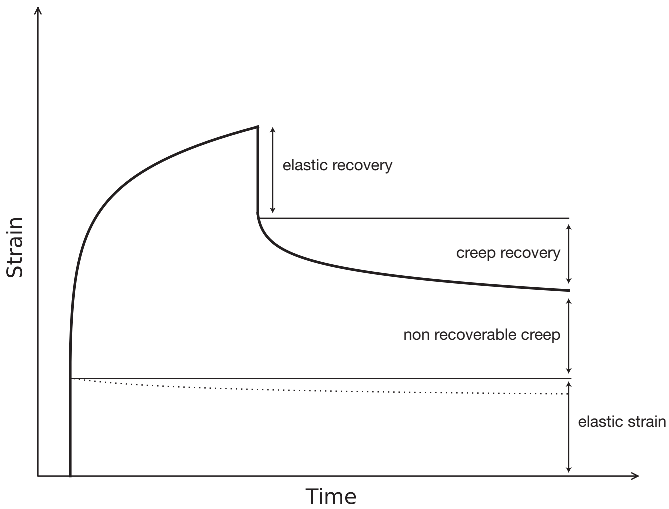


Fig. 1 Creep and creep recovery

The creep properties of concrete are usually determined by measuring the creep coefficient. This results in the most accurate way of determining the creep strains. However, creep measurements typically cost time, which is usually unjustifiable for common engineering projects. In absence of test-results many authors suggested mathematical models to predict the creep coefficient in function of time to be used in structural calculations (e.g. Eurocode 2, CEB-fip Model Code 90, fib Model Code 2010, model B3 by Bazant-Baweja, Gardner-Lockman model, ACI209, …)

The creep compliance function is given by:

$$J(t,\tau)=\frac{1}{E_c(t)}+\frac{\varphi(t,\tau)}{E_{28}} \tag{1}$$

where $\varphi(t,\tau)$ is a creep function, $E_c(t)$ is the time-dependent modulus of elasticity and E_{28} is the modulus of elasticity at 28 days. (1) cannot be used for every prediction model recommended in design codes. For example, the creep function suggested in the ACI209 and the B3 prediction models (Z. Bažant & Baweja, 1995) are determined based on the time-dependent modulus of elasticity. For those prediction models (1) must be rewritten as:

$$J(t,\tau)=\frac{1+\varphi(t,\tau)}{E_c(t)} \tag{2}$$

Common design codes do not mention creep recovery. However, creep recovery can be associated with models that separate the creep strain into delayed elastic and flow, where the flow part is an irreversible strain. Mathematical models for delayed elasticity are suggested (e.g. MC78, IMC78). In these two models, delayed elasticity and creep recovery are assumed identical (Yue, 1992). The creep recovery function should be monotonously increasing with a negative rate of increment. Further the finale value of the creep recovery function must be finite (Yue & Taerwe, 1992). The creep recovery compliance function is given by:

$$J(t,\tau)=\frac{1}{E_c(t)}+\frac{\varphi_{cr}(t,\tau)\beta_{cr}(t,\tau,t')}{E_{28}} \tag{3}$$

where $\varphi_{cr}(t,\tau)$ is the ultimate value of the creep recovery which may, depending on the model, be associated with the loading history. $\beta_{cr}(t,\tau,t')$ is the development of creep recovery with time which may be associated with the loading history. $E_c(t)$ is the time-dependent modulus of elasticity and E_{28} is the modulus of elasticity at 28 days. $\beta_{cr}(t,\tau,t')$ incorporates an additional parameter t', which is the age of the concrete at unloading. The interval $[\tau,t']$ is the duration of the load. The expression for $\varphi_{cr}(t,\tau)$ and $\beta_{cr}(t,\tau,t')$ depends on the used model.

3. Numerical procedure for beam elements in FEM analyses

The relation between the nodal forces $\{f\}$ and the nodal displacements $\{s\}$ for a 2D beam element is given by (4)

$$\{f\}=[K]\{s\} \tag{4}$$

where $[K]$ is the stiffness matrix of a beam element with 6 degrees of freedom.

The viscoelastic constitutive law expressed by the creep compliance function is then introduced as follows (Sassone & Casalegno, 2012):

$$\{s(t)\}=[K]^{-1}E_{c,ref}\int_0^t J(t,\tau)\{\mathrm{d}f(\tau)\} \tag{5}$$

where $E_{c,ref}$ is the reference modulus of elasticity of concrete used to compose the stiffness matrix $[K]$.

To return to the fundamental relation given in (4) it is necessary to invert (5). Since the integration of most creep compliance functions is not invertible, (5) is first approximated based on the trapezoidal rule. Hereby the time t is divided in k steps. These steps are not required to have the same length. The error term is in the order of Δt^3 (Zdeněk P. Bažant, 1975).

$$\begin{aligned}\{\Delta s\}_{t_k}&=[K]^{-1}E_{c,ref}\frac{1}{2}\left[J(t_k,t_k)+J(t_k,t_{k-1})\right]\{\Delta f\}_{t_k}\\&+[K]^{-1}E_{c,ref}\sum_{i=2}^{k-1}\frac{1}{2}\left[J(t_k,t_i)+J(t_k,t_{i-1})-J(t_{k-1},t_i)-J(t_{k-1},t_{i-1})\right]\{\Delta f\}_{t_i}\end{aligned} \tag{6}$$

The nodal displacement increments $\{\Delta s\}$ at time t_k depend on the stress increments $\{\Delta f\}$ at time t_k and the stress increments of the previous steps t_i. Appropriate time discretisation should be adopted, e.g. it has been recommended that the time steps are spread uniformly on a logarithmic scale (Z. P. Bažant & L'Hermite, 1988; Jendele & Phillips, 1992). (6) can be rewritten as follows in function of $\{\Delta s\}_{t_k}$:

$$\{\Delta f\}_{t_k} = \frac{2}{\left(J\left(t_k,t_k\right)+J\left(t_k,t_{k-1}\right)\right)E_{c,ref}}[K]\{\Delta s\}_{t_k}$$
$$-\frac{1}{J\left(t_k,t_k\right)+J\left(t_k,t_{k-1}\right)}\sum_{i=2}^{k-1}\left[J\left(t_k,t_i\right)+J\left(t_k,t_{i-1}\right)-J\left(t_{k-1},t_i\right)-J\left(t_{k-1},t_{i-1}\right)\right]\{\Delta f\}_{t_i} \tag{7}$$

where the relation between the load increments $\{\Delta f\}_{t_k}$ and the additional deformations $\{\Delta s\}_{t_k}$ is given. This equation can be regarded as a viscoelastic extension of the relation given in (4).

Thus, the viscoelastic problem is converted to a sequence of elastic problems where the effects of concrete creep are taken into account as additional nodal loads, calculated based on the load history.

As mentioned above, this numerical procedure is justified by assuming that the principle of superposition is valid. Considering the same compliance function as for a load increase the principle of superposition treats the removal of a load as a negative load, which induces the same time-dependent strains equal and opposite to those induced by a positive load. Hence, an alternative compliance function for the removal of a load is necessary.

The numerical procedure can be extended for simple geometric cases based on the two-function method introduced by Yue and Taerwe (1993). Hence, the viscoelastic behaviour is modelled using a creep compliance function $J(t,\tau)$ for loading and a creep recovery compliance function $J_{cr}(t,\tau,t')$ for unloading. (6) can now be rewritten, taking into account this two-function method.

$$\begin{aligned}\{\Delta s\}_{t_k} &= \mathcal{H}\left(\{\Delta f\}_{t_k}\right)[K]^{-1}E_{c,ref}\frac{1}{2}\left[J\left(t_k,t_k\right)+J\left(t_k,t_{k-1}\right)\right]\{\Delta f\}_{t_k} \\ &+\mathcal{H}\left(-\{\Delta f\}_{t_k}\right)[K]^{-1}E_{c,ref}\frac{1}{2}\left[J_{cr}\left(t_k,t_k\right)+J_{cr}\left(t_k,t_{k-1}\right)\right]\{\Delta f\}_{t_k} \\ &+[K]^{-1}E_{c,ref}\sum_{i=2}^{k-1}\mathcal{H}\left(\{\Delta f\}_{t_i}\right)\frac{1}{2}\left[J\left(t_k,t_i\right)+J\left(t_k,t_{i-1}\right)-J\left(t_{k-1},t_i\right)-J\left(t_{k-1},t_{i-1}\right)\right]\{\Delta f\}_{t_i} \\ &+[K]^{-1}E_{c,ref}\sum_{i=2}^{k-1}\mathcal{H}\left(-\{\Delta f\}_{t_i}\right)\frac{1}{2}\left[J_{cr}\left(t_k,t_i\right)+J_{cr}\left(t_k,t_{i-1}\right)-J_{cr}\left(t_{k-1},t_i\right)-J_{cr}\left(t_{k-1},t_{i-1}\right)\right]\{\Delta f\}_{t_i} \\ &+\mathcal{H}\left(-\{\Delta f\}_{t_k}\right)[K]^{-1}E_{c,ref}\frac{1}{2}\left[J_{cr}\left(t_k,t_{k-1}\right)+J_{cr}\left(t_k,t_{k-2}\right)-J_{cr}\left(t_{k-1},t_{k-1}\right)-J_{cr}\left(t_{k-1},t_{k-2}\right)\right]\{\Delta f\}_{t_k}\end{aligned} \tag{8}$$

where $\mathcal{H}$ is the Heaviside function.

In (8) it is assumed that a certain load increment Δf is either positive of negative. Hence, change in deformations during a time interval t_k is divided in a part due to loading and a part due to unloading. An additional term is added to take into account the removed loads, i.e. a removed load cannot be part of the loading history at a later age than the age of removal. In order to validate this extended numerical procedure, it was implemented in MATLAB and validated using the experimental results described in the following section.

4. Application to a simply supported concrete beam

The test setup for the reinforced concrete beam, subjected to a load history under four-point bending, is given in Fig. 2. The beams have a total length of 5.3m with a span of 5m.

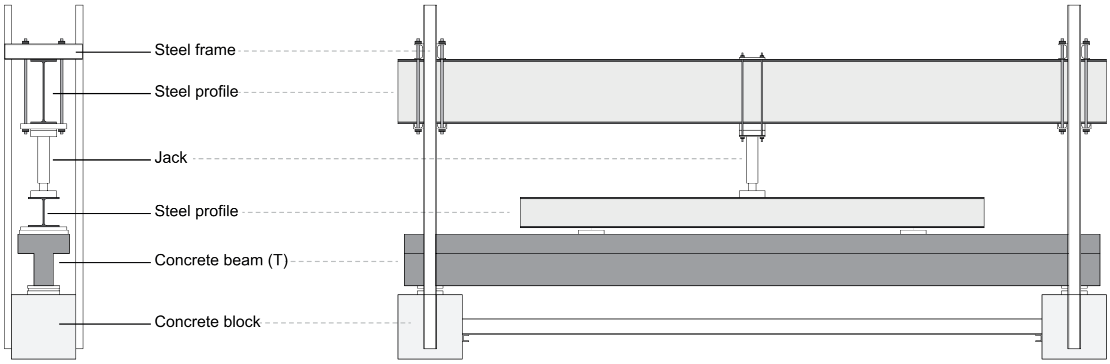


Fig. 2 T-shaped concrete beam, experimental setup

Three such beams were cast in the Magnel Laboratory for Concrete Research. The concrete and cross-section properties of one of these beams, which is used for the validation further in this contribution, is summarised in Table 1.

Table 1 Geometrical, mechanical and environmental properties of the T-shaped concrete beams

Width b	400 mm
Height h	400 mm
Width web b_w	150 mm
Height flanges h_f	150 mm
Area of the cross-section A_c	96470 mm^2
Area of the reinforcement steel A_s	1030 mm^2
Concrete strength f_c	C35/45
R.H.	60%
Temperature	20°C
Type of cement	CEM I 52.5 N

The longitudinal reinforcement consists of 2 bars Ø20mm and 2 bars Ø16mm at the bottom for the beams of 5.3m. The stirrups (Ø8) are placed every 250mm near the midspan and every 200mm near the supports. The concrete cover is 25mm. A detailed illustration is given in Fig. 3.

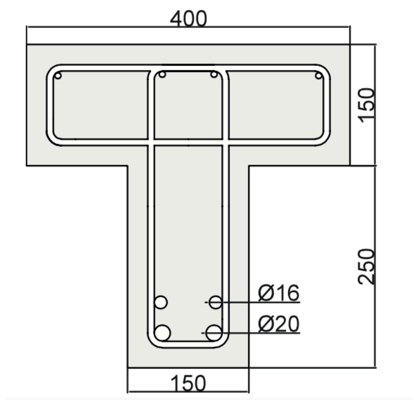


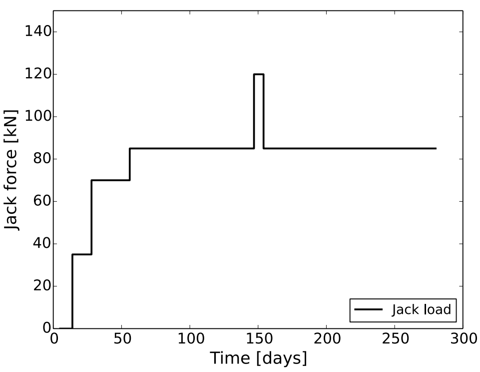
Fig. 3 Cross-section of the T-shaped beam.

Five days after casting the concrete beam was placed on its supports. Starting from this moment the midspan deflection was monitored with a linear variable differential transformer (LVDT).

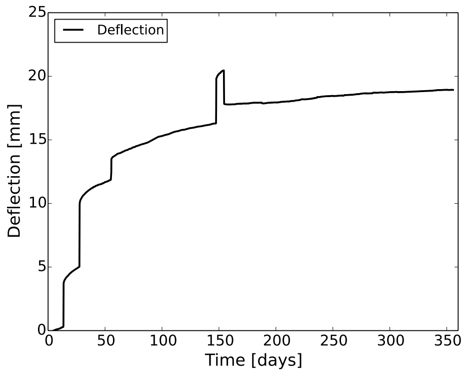
The load on the beam is applied in different steps to simulate the effects of different construction phases. Three loads are defined, the dead load G1, a permanent load G2 (14 kN/m) and a life load Q (10 kN/m). At an age of 147 days the frequent load combination (G1 + G2 + Q) is applied for a period of 7 days; afterwards the quasi-permanent load combination (G1 + G2 + 0.3Q) is applied on the beam. A period of 7 days was chosen to represent singular cases of the frequent load combination. An overview of the load history is given in Table 2 and Fig. 4.

Table 2 Load history

Load history	Duration [days]	Age [days]
G_1	14	14
$G_1 + 0.5G_2$	14	28
$G_1 + G_2$	28	56
$G_1 + G_2 + 0.3Q$	91	147
$G_1 + G_2 + Q$	7	154
$G_1 + G_2 + 0.3Q$	4 years	



(a)



(b)

Fig. 4 **(a)** Jack load in function of time **(b)** measured midspan deflection using an LVDT

Consequently, the beam illustrated in Fig. 2 is modelled with 20 beam elements (21 nodes). Point loads are placed on nodes 6 and 16 at the location of the jack forces. An illustration of the model is given in Fig. 4.

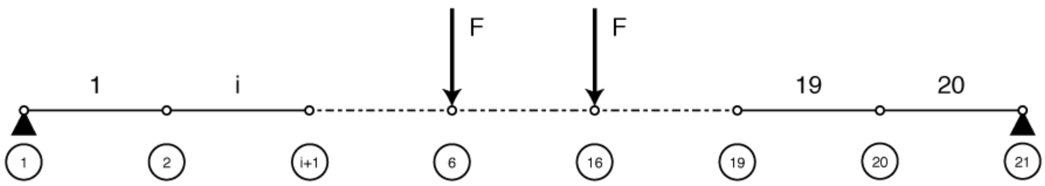


Fig. 5 Model of the beams. The point loads are place on node 6 and 16.

The beam is uncracked during the first 14 days when there is no load applied other than the dead load. The load applied at 14 days exceeds the cracking load. Starting from this age the stiffness of all but the first and last element was adjusted to account for concrete cracking. An effective stiffness was calculated based on the work of Branson (1963) interpolating the uncracked and cracked bending stiffness as followed:

$$E_{ef} = \left(\frac{M_{cr}}{M} \right)^m I_{uncr} + \left(1 - \left(\frac{M_{cr}}{M} \right)^m \right) I_{cr} \tag{9}$$

If was found for these T-shaped beams that m=1 gives good predictions of the time-dependent deflections.

The result of the calculation for the initial model and the extended model are given in Fig. 6. For the creep compliance function the model suggested by CEB-FIP model code 90 is used (CEB-FIP, 1993). For the creep recovery compliance function the model for the delayed elasticity from CEB-FIP model code 78 is used. As mentioned above, it is assumed that the delayed elasticity is identical to creep recovery, (Yue, 1992). In this model the development of creep recovery with time and its ultimate value can be estimated by:

$$\beta_{cr}(t,\tau,t') = \left(\frac{t-t'}{t-t'+328} \right)^{\frac{1}{4.2}} \tag{10}$$

$$\varphi_{cr}(t,t') = 0.4 \tag{11}$$

Substituting (10) and (11) into (3) yields:

$$J_{cr}(t,\tau,t') = \frac{1}{E_c(t)} + \frac{0.4\left(\frac{t-t'}{t-t'+328} \right)^{\frac{1}{4.2}}}{E_{28}} \tag{12}$$

The initial numerical procedure overestimates the recovered creep immediately after unloading. The extended model predicts the unloading behaviour better. The recovery is not overestimated and the development rate of the recovery leans more towards the measurement date. Nevertheless it is noted that the difference between the two procedures is less distinct sufficiently long after unloading.

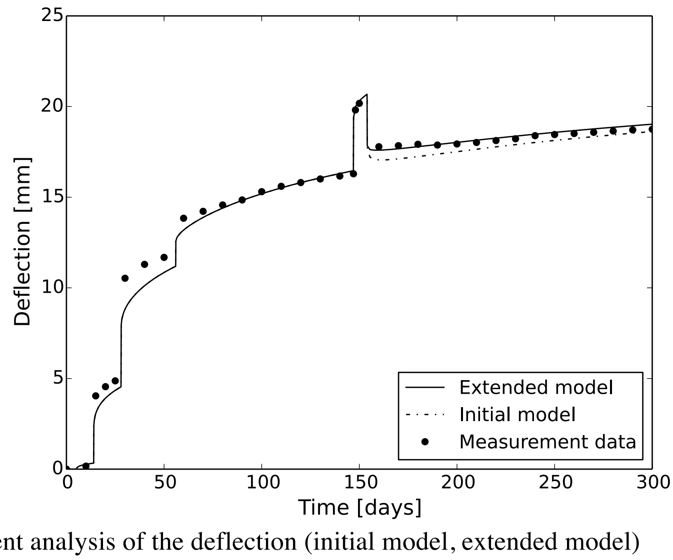


Fig. 6 Time-dependent analysis of the deflection (initial model, extended model)

5. Conclusion and further research

An extension towards creep recovery was implemented in a general numerical procedure that combines the viscoelastic Volterra equation with finite elements. Generally, a separate compliance function was introduced to deal with creep recovery specifically. It was assumed that the creep recovery is identical to the delayed elasticity, as suggested in literature.

The extended procedure was validated on deflection measurements from T-shaped reinforced concrete beams subjected to a load history under four-point bending. The midspan deflection was calculated both with the original model and the extended model. It is observed that the extended model enables to predict the irrecoverable creep after unloading, compared to the original model. The extended model also predicts the creep rate better immediately after unloading. However, it is noted that the differences between the two approaches is less distinct sufficiently long after unloading. It should be noted that in the considered case the irrecoverable creep is less significant than in the case of pure compression members.

This calculation approach still requires the storage of the load history of every element, which is not eligible in structural analysis of complex structures since it requires significant memory space and extends the time needed to finish a calculation. Rate of creep methods can be used, but these methods assumed that the creep behaviour could be characterised with one single creep curve for any stress history. This, of course, is not true. Another approach is to first approximate the creep and creep recovery compliance function with a Dirichlet series. This has the advantage that the integration of the compliance function in this form can be solved analytically. The solution only depends on the result of the previous step. This, of course, benefits the memory use during analysis.

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References

Bažant, Z.P. & Baweja, S. (1995), Justification and refinements of model B3 for concrete creep and shrinkage. Materials and Structures, Vol. 28, No. 7, pp. 415-430.

Bažant, Z.P. (1975), Theory of creep and shrinkage in concrete structures: A precis of recent developments. Mechanics Today, Vol. 2, pp. 1-93.

Bažant, Z.P. & L'Hermite, R. (1988), Mathematical modeling of creep and shrinkage of concrete. Wiley, New York, USA

Branson, D.E. & Metz, G.A. (1963), Instantaneous and Time-dependent Deflections of Simple and Continuous Reinforced Concrete Beams. Department of Civil Engineering and Auburn Research Foundation, Auburn University

CEB Bulletin 124/125. (1978), International System of Unified Standard Codes of Practice for Structures, Vol. II, CEB-FIP Model Code for Concrete Structures. Comité Eurointernational du Béton and Fédération International de la Précontrainte, Lausanne, Switserland.

CEB Bulletin 213/214 (1993), CEB-FIP Model Code 1990. Comité Euro-International du Béton, Lausanne, Switserland.

Gardner, N.J., & Lockman, M.J. (2002), Design provisions for drying shrinkage and creep of normal-strength concrete. ACI Materials Journal, Vol. 98, No. 2, pp. 159-167.

Gilbert, R.I., & Ranzi, G. (2010), Time-Dependent Behaviour of Concrete Structures. Taylor & Francis, New York, USA.

Jendele, L., & Phillips, D.V. (1992), Finite element software for creep and shrinkage in concrete. Computers & Structures, Vol. 45, No. 1, pp. 113-126.

Sassone, M., & Casalegno, C. (2012), Evaluation of the structural response to the time-dependent behaviour of concrete: Part 1 - An internationally harmonized format. The Indian Concrete Journal, Vol. 86, No. 12, pp. 39-51.
Yue, L.L. (1992), Creep Recovery of Plain Concrete under Uniaxial Compression. (Doctor in applied sciences), Ghent University, Belgium.
Yue, L.L. & Taerwe, L. (1992), Creep recovery of plain concrete and its mathematical modelling. Magazine of Concrete Research, Vol. 44, No. 161, pp. 281-290.
Yue, L.L. & Taerwe, L. (1993), Two-function method for the prediction of concrete creep under decreasing stress. Materials and Structures, Vol. 26, No. 5, pp. 268-273.