Matrix Interpolation based Parametric Model Order Reduction for Multiconductor Transmission Lines with Delays

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Abstract—A novel parametric model order reduction (PMOR) technique based on matrix interpolation for multicondutor transmission lines with delays having design parameter variations is proposed in this paper. Matrix interpolation overcomes the oversize problem caused by input-output system level interpolation based parametric macromodels. The reduced state-space matrices are obtained using a higher-order Krylov subspace based model order reduction technique which is more efficient in comparison to the Gramian based parametric modeling where the projection matrix is computed using a Cholesky factorization. The design space is divided into cells and then the Krylov subspaces computed for each cell is merged and then truncated using an adaptive truncation algorithm with respect to their singular values to obtain a compact common projection matrix. The resulting reduced order state-space matrices and the delays are interpolated using positive interpolation schemes making it computationally cheap and accurate for repeated system evaluations under different design parameter settings. The proposed technique is successfully applied to RLC and multiconductor transmission line circuits with delays.

Index Terms—Parametric model order reduction, Krylov subspaces, delayed systems, singular values, projection matrix, interpolation.

I. INTRODUCTION

CIRCUIT analysis using electromagnetic (EM) simulation methods [1] can generate very large systems of equations. Time delays must be included during the process of modeling, when the geometric dimensions become electrically large and the frequency content of signal waveform increases [2], [3]. In such cases, comprehensive model order reduction (MOR) techniques are crucial to reduce the complexity of large scale models and the computational cost of the simulations, while retaining the important physical features of the original system.

Time-delay systems (TDSs) in the Laplace domain contains transfer function with elements of the form $e^{-s\tau}$, where τ corresponds to the time delay present in the circuit. Several techniques of MOR for TDSs have been presented during recent years, and any of the approaches based on Krylov-subspace algorithms [3]–[7] can be used as non-parametric

MOR technique. The system response of TDSs can be affected by design parameters, other than frequency, such as geometric features. Therefore, it is important to predict the response of the circuit as a function of general design parameters, such as geometric and physical features. Parametric model order reduction (PMOR) methods are well suited to efficiently perform design activities.

A number of PMOR methods have been developed in recent years for TDSs based on input-output interpolation [8]–[10]. In [8], the approach is based on a multiorder Arnoldi algorithm which is used to implicitly calculate the moments with respect to frequency and the design parameters, as well as the cross-moments. Also in [9] an input-output based interpolation technique is presented with scaling and frequency shifting, which enhances the modeling capabilities. These PMOR methods use input-output system level interpolation which are proved to be robust and accurate, but the order of the parametric macromodels may suffer from oversize due to the nature of the input-output system level interpolation. A Gramian-based PMOR for TDS is presented in [10] where an affine model is used to represent the parametric behavior, but the technique is computationally expensive as Cholesky factorization is required for the computation of the projection matrix.

This paper, proposes a matrix interpolated PMOR method for multiconductor transmission lines (MTLs) with delays. The proposed technique approximates the delays using an expansion series and uses higher-order Krylov subspace based MOR as described in Section II. Then the reduced state-space systems are interpolated as in [11]. The paper enhances the technique in [11], by implementing an adaptive truncation for the singular values of the common projection matrix for the design space considered. As the approach is based on matrix interpolation it overcomes the oversize problem in input-output system level interpolation and the technique uses higher-order Krylov MOR to compute the reduced order models (ROMs) for TDS [6], [7]. This is more efficient in comparison to the augmented MOR technique proposed for PMOR TDS in [9], as the augmentation generates an equivalent first-order system which is larger than the size of the original model, and the Gramian based MOR for PMOR of TDS [10], as the computation of Gramians are expensive due to Cholesky factorization. The proposed approach computes a set of reduced system matrices in a common subspace using higher-order Krylov MOR and interpolates these ROM and the delays in order to generate PMOR for TDSs. The design

space is first divided into cells and for each vertex model of the cell a Krylov subspace is computed. The Krylov subspaces are then merged and compacted by truncating with respect to their singular values to generate a common projection matrix using an adaptive truncation algorithm proposed in Section III A of this paper. Next, the reduced system matrices of the delayed system are interpolated using positive interpolation for PMOR as described in Section III B.

II. OVERVIEW OF MODEL ORDER REDUCTION FOR TDSs

A time-delay system of degree n with p ports having k delays τ_j, present in both the state and descriptor matrices, can be represented in general delayed state space form as:

$$\sum_{j=0}^{k} \mathbf{E_j}\dot{x}(t-\tau_j) = \sum_{j=0}^{k} \mathbf{A_j}x(t-\tau_j) + \mathbf{B}u(t)$$
$$y(t) = \mathbf{C}x(t). \qquad (1)$$

Here, $x(t) \in \mathcal{R}^n$ is the state vector; $u(t) \in \mathcal{R}^p$ is the control input with $u(t) = 0$ for $t < 0$; $y(t) \in \mathcal{R}^p$ is the output. $\mathbf{A_j}, \mathbf{E_j}, \mathbf{B}, \mathbf{C}$ are constant sparse matrices with appropriate dimensions. The time delay $\tau_0 = 0$ and $\tau_j > 0$, $j = 1, 2, \ldots, k$. From (1) we obtain the transfer function as:

$$\mathbf{H}(s) = \mathbf{C}(s\sum_{j=0}^{k} \mathbf{E}_j e^{-s\tau_j} - \sum_{j=0}^{k} \mathbf{A}_j e^{-s\tau_j})^{-1}\mathbf{B}. \qquad (2)$$

In order to calculate the moments, the exponential terms of (2) are approximated using a Taylor series [6] or Laguerre expansion [7] upto an order r. On substituting the delay expansion in (2), a r-th order transfer function is obtained of the form:

$$\mathbf{H}(s) = \mathbf{C}\left(\sigma_r s^r + \sigma_{r-1}s^{r-1} + ... + \sigma_1 s + \sigma_0\right)^{-1}\mathbf{B}. \qquad (3)$$

The r-th order Krylov subspace is defined as in [12]

$$\mathcal{K}_q(\mathbf{G}_1, \mathbf{G}_2, ..., \mathbf{G}_r, \mathbf{L}) = colspan\ [P_0, P_1, ..., P_{q-1}] \qquad (4)$$

where $\mathbf{L} = \sigma_0^{-1}\mathbf{B}$ and $\mathbf{G}_i = \sigma_0^{-1}\sigma_i$ for $i = 1, 2, ..., r$, and

$$P_0 = \mathbf{L}; \quad P_i = 0 \quad \text{for} \quad i < 0$$
$$P_j = \mathbf{G}_1 P_{j-1} + ... + \mathbf{G}_r P_{j-r}, \quad j = 1, ..., q-1 \quad (5)$$

where q is the reduced order that is estimated for the model.

This subspace is a generalization of Krylov subspaces for higher-order systems and eliminates the standard approach to model order reduction of large-scale higher-order linear dynamical systems, which is to rewrite the system as an equivalent first-order system and then employ Krylov-subspace techniques for model order reduction of first-order systems. Note that, to match the moments of an r-th order model, the matrix σ_0 should be invertible.

The column-orthogonal projection matrix $\mathbf{Q}$ for congruence transformation is found by means of the economy-size singular value decomposition (SVD):

$$\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = SVD(\mathcal{K}_q(\mathbf{G}_1, \mathbf{G}_2, ..., \mathbf{G}_r, \mathbf{L}), 0). \qquad (6)$$

In other words $\mathbf{Q}$ is equal to the left SVD factor of dimension $n \times q$ associated with the $(r+1)^{th}$ Krylov subspace.

The reduced order state-space matrices are then obtained by the following classical congruence transformations:

$$\mathbf{A_{j}}_r = \mathbf{Q}^T\mathbf{A_j}\mathbf{Q} \quad , \quad \mathbf{E_{j}}_r = \mathbf{Q}^T\mathbf{E_j}\mathbf{Q},$$
$$\mathbf{B}_r = \mathbf{Q}^T\mathbf{B} \quad , \quad \mathbf{C}_r = \mathbf{C}\mathbf{Q}. \qquad (7)$$

III. PARAMETRIC MODEL ORDER REDUCTION

Considering the effect of N design parameters $\mathbf{g} = (g^{(1)}, ..., g^{(N)})$, the descriptor state-space form (1) becomes:

$$\mathbf{E}(\mathbf{g}, \tau)\dot{x}(t, \mathbf{g}) = \mathbf{A}(\mathbf{g}, \tau)\mathbf{x}(t, \mathbf{g}) + \mathbf{B}(\mathbf{g})\mathbf{u}(t)$$
$$\mathbf{y}(t, \mathbf{g}) = \mathbf{C}(\mathbf{g})\mathbf{x}(t, \mathbf{g}). \qquad (8)$$

Two design space grids are used in the modeling process, an estimation grid and a validation grid [9]. The estimation grid is used for the construction of the PMOR while the validation grid is used to study the accuracy of the parametric model at the points that were not used during construction. Once the design space is sampled, the reduced order q has to be estimated for the samples on the estimation grid that is used for the modeling of the PMOR. For this, we adopt the double-strategy approach of [11]. The reduced order is first estimated at the corner points of the design space using a bottom-up approach or from the Hankel singular values (HSV), and afterwards any of these two strategies can be followed for the remaining samples in the estimation grid. This yields:

1) *worst-case reduced order*: the highest estimated reduced order at the corner points is extended over the entire design space. This approach can guarantee an accurate reduction over the design space.

2) *best-case reduced order*: the lowest estimated reduced order is extended over the design space. This approach can guarantee more compact models with respect to the worst-case, but the reduced order may be increased for some design space regions by a bottom-up approach to guarantee the desired accuracy.

From a practical view point for better computation and accuracy it is advisable to choose the *worst-case reduced order* strategy as the highest reduced order is used for the entire design space and the reduced order need not be computed for each sample point in the design space as in the case for *best-case reduced order*.

A. Common projection matrix computation

For each point in the estimation grid, a higher-order Krylov-based MOR method for TDSs is applied to obtain a set of projection matrices. In this paper, the Laguerre expansion method [7] is used. All the projection matrices have the same dimension in the worst-case reduced order scenario, while they may have different dimensions for the best-case reduced order scenario. Each design space cell has M vertices and for every cell the projection matrices at the vertices are merged by column stacking.

$$\mathbf{Q}_{union} = [\mathbf{Q}_1, \mathbf{Q}_2, ...\mathbf{Q}_M]. \qquad (9)$$

Next, the economy-size SVD is computed for the merged projection matrices

$$\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = SVD(\mathbf{Q}_{union}, 0). \qquad (10)$$

TABLE I
ALGORITHM 1:TRUNCATION OF THE SINGULAR VALUES FOR Q_{union}

Data: $d\Sigma = diag(\Sigma)$ from (10) and threshold
Result: q_{comm} for (11)
$ds = length(d\Sigma)$;
$q_{comm} = ds$;
$\Delta = 0$;
while $\Delta \leq$ *threshold* **do**
  $\Delta = \Delta + d\Sigma(q_{comm})^2$;
  $q_{comm} = q_{comm} - 1$;
end

A common reduced order for a cell is defined based on the first q_{comm} significant singular values of $\mathbf{Q}_{union}$ [11]. Thus a common projection matrix $\mathbf{Q}_{comm}$ is obtained

$$\mathbf{Q}_{comm} = \mathbf{U}(:, 1 : q_{comm}). \tag{11}$$

Adaptive singular values truncation: In [11] the value of the threshold is set on a trial and error base for a desired level of accuracy and compactness for the PMOR. On truncating the singular values an approximated representation $\hat{Q}_{union}$ of (9) is obtained, and can be noted that the approximation error Δ is dependent on the truncation i.e.,

$$\Delta = ||Q_{union} - \hat{Q}_{union}||_2,$$
$$\Delta = ||\sum_{i=1}^{n} U_i\Sigma_iV_i^T - \sum_{i=1}^{q_{comm}} U_i\Sigma_iV_i^T||_2,$$
$$\Delta = ||\sum_{i=q_{comm}+1}^{n} U_i\Sigma_iV_i^T||_2. \tag{12}$$

which can be written as,

$$\Delta = \sum_{i=q_{comm}+1}^{n} d\Sigma_i^2. \tag{13}$$

where, $d\Sigma$ are the diagonal elements of Σ. The adaptive algorithm for truncating the singular values of the common projection matrix $\mathbf{Q}_{union}$ is given in Algorithm 1 (see Table I). For this paper a threshold equal to 0.01 was considered to produce accurate ROMs. Then, with $\mathbf{Q}_{comm}$ the common projection matrix with dimension $n \times q_{comm}$, the congruence transformations (7) are performed to obtain the ROMs for the design space considered.

B. Multivariate Interpolation

After the computation of the reduced matrices, they are interpolated to build a PMOR. Any interpolation scheme in the class of positive interpolation operators [9] can be used, e.g., multilinear and simplicial methods [13]. Here we consider multilinear interpolation, where each interpolated matrix $\mathbf{T}(g^{(1)}, ..., g^{(N)})$ is

$$\mathbf{T}(g^{(1)}, ..., g^{(N)}) = \sum_{k_1=1}^{K_1} \cdots \sum_{k_N=1}^{K_N} \mathbf{T}_{(g_{k_1}^{(1)}, ..., g_{k_N}^{(N)})}$$
$$l_{k_1}(g^{(1)}) \cdots l_{k_N}(g^{(N)}). \tag{14}$$

TABLE II
ALGORITHM 2:PARAMETRIC MODEL ORDER REDUCTION ALGORITHM

Data: Reduced order for M samples in the estimation grid.
Result: parametric ROM
$Q_{union} = [\ ]$;
for $i \leftarrow 1$ **to** M **do**
  $\mathbf{Q}_i$;
  $Q_{union} = [Q_{union} \ \ Q_i]$;
end
$\mathbf{U}\Sigma\mathbf{V}' = SVD(\mathbf{Q}_{union}, 0)(10)$;
Q_{comm} using Algorithm 1 (Table I);
Perform congruence transformation to obtain the ROM of the TDSs ;
Matrix interpolate the ROM and τ to obtain the PMOR.
Validate the PMOR with the validation grid.

and K_1 is the number of estimation points and the interpolation kernel $l_{k_i}(g^i)$ satisfies the following constraints

$$0 \leq l_{k_i}(g^{(i)}) \leq 1,$$
$$l_{k_i}(g^{(i)}) = \delta_{k_i,i}$$
$$\sum_{i=1}^{N} l_{k_i}(g^{(i)}) = 1. \tag{15}$$

For MTLs consisting of lumped RLC components and lossless transmission line (TLs) components, the MoC [3] technique is used to model the lossless TLs. The delay for the k-th transmission line in MoC is the k-th eigenvalue of $d\sqrt{(C_{pul}L_{pul})}$ (L_{pul} and C_{pul} are the per- unit- length (P.U.L.) parameter for the inductance and capacitance respectively and d denotes the length of the TLs). The L_{pul} and C_{pul} are symmetric and positive definite. Thus, as the delays are varying linearly with respect to d of the TLs, we can obtain a good parametric reduced order delay model by interpolating all the delays using positive interpolating operators. It should be noted that the interpolation kernel functions of these methods only depend on the design space grid points and their computation does not require the prior solution of a linear system to impose an interpolatory constraint. The algorithmic steps of the proposed PMOR technique for TDSs is given in Algorithm 2 (see Table II).

C. Complexity

Concerning the complexity of the proposed PMOR technique, the most expensive step is related to the computation of the higher-order Krylov subspaces for the estimation grid. It has a complexity of $O(4n^2q)$ where q is the reduced order estimated for the model. But it can be seen that the proposed technique is much more efficient than the Gramian based PMOR for TDS [10] which has a complexity of $O(n^3)$. Then we have the computation of the singular values for the common projection matrix which uses an economy-size *SVD* to improve the computation. After obtaining the common projection matrix, congruence transformation is performed which has a complexity equivalent to that of matrix multiplication. Finally, the complexity of the last step depends on the selected interpolation scheme. Even though the most expensive step in

the proposed PMOR technique is the MOR step the PMOR makes it more efficient for repeated design evaluations under different parameter settings in comparison to the conventional analysis techniques which requires the solution of partial differential equations [2]. The complexity of the proposed PMOR increases with the number of design parameters since the number of points on the estimation grid required for modeling increases and thereby increase the dimension of the column stacked projection matrix $\mathbf{Q}_{union}$, then the *SVD* would become expensive. In order to make the algorithm more efficient it is advised to perform adaptive sampling [14] of the design space and when the number of parameters is more than 5 then a dimension reduction technique [15] can be performed.

IV. NUMERICAL EXAMPLES

A distributed system as explained in [8] is used to illustrate the efficiency of the proposed technique. The RLC networks is modeled using the conventional lumped technique [2] and the lossless TLs which cause signal propagation delays, is modeled using MoC. The general form of the modified nodal analysis (MNA) matrices using the MoC and lumped elements is described in [3].

Error criteria: The weighted RMS error between the original frequency response H_{ij} and the reduced order model $H_{r,(ij)}$ is defined as:

$$Err = \sqrt{\frac{\sum_{k=1}^{K_s} \sum_{i=1}^{P_{in}} \sum_{j=1}^{P_{out}} \frac{\left|H_{r,(ij)}(s_k) - H_{(ij)}(s_k)\right|^2}{W_{(ij)}(s_k)}}{P_{in} P_{out} K_s}}$$
$$W_{(ij)}(s_k) = |H_{(ij)}(s_k)|^2 \qquad (16)$$

Here K_s, P_{in} and P_{out} are the number of frequency samples, input and output ports of the system, respectively. The proposed PMOR technique, it is compared with the Gramian-based PMOR [10] which is also based on state-space interpolation.

A. CASE I: Variation in Length of the lossless TLs

A TDS of order 2115 is constructed using a 3 port linear interconnected network connected with lossless 3 conductor TLs. In this case the length d is varied for the range $[1\text{ cm} - 1.5\text{ cm}]$ of the TLs for a frequency range of $[1\text{ kHz} - 4\text{ GHz}]$.

The state-space matrices is computed for 5 uniformly spaced values of d, for which the estimation points are $d = \{1, 1.167, 1.33, 1.5\}$ cm and the validation points are $d = \{1.083, 1.25, 1.42\}$ cm. We opt for the *best-case scenario* and the higher-order Krylov subspaces are computed for the estimation points as described in Section II. Then a common projection matrix of dimension 360 is computed for the entire design space as described in Algorithm 1. The singular values of the merged Krylov subspaces is then truncated using Algorithm 2 to obtain a compact common projection matrix of size 148. Fig.1 and Fig.2 plots the magnitude and phase of input admittance parameter $Y_{11}(s, d)$ respectively for $d = \{1.083, 1.25, 1.42\}$ cm. As mentioned in Section III-C, the most expensive step in the PMOR technique is the MOR and as the number of estimation samples increases the computation becomes more expensive. But once an accurate
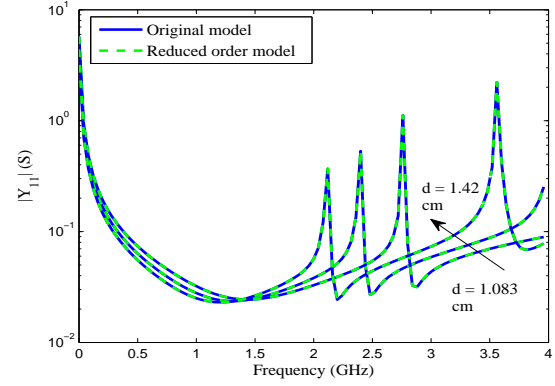

Fig. 1. Magnitude of input admittance parameter $Y_{11}(s, d)$ for $d = \{1.083, 1.25, 1.42\}$ cm.
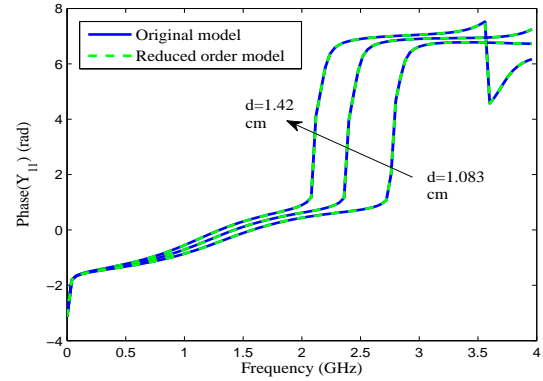

Fig. 2. Phase of input admittance parameter $Y_{11}(s, d)$ for $d = \{1.083, 1.25, 1.42\}$ cm.

PMOR is obtained, it becomes faster to predict the behavior of the system for different parameter ranges. The frequency response time for the original model is $277s$ and that for the ROM is $6.819s$, obtaining about 39 times speed up.

B. CASE II: Variation in Length of the TLs and P.U.L. parameters

For this case a TDS of order 9307 is constructed using a 4 port linear interconnected network connected with 40 lossless 4 conductor TLs. The length d and the P.U.L. parameters of the TLs are varied. The dependencies of P.U.L. parameters of the distributed network on temperature T is modeled using a first-order relation. The parameters d varies from $[1.5\text{ cm} - 2\text{ cm}]$ and T from $[-20^oC - 60^oC]$ for a frequency range of $[1\text{ kHz} - 6\text{ GHz}]$. The state-space matrices with delays (as in (8)) are computed over an uniform grid of 9×9 (d, T). A 5×5 (d, T) estimation grid, $d = \{1.5, 1.625, 1.75, 1.875, 2\}$ cm and $T = \{-20, 0, 20, 40, 60\}^oC$ is considered and a validation grid of 4×4 (d, T), $d = \{1.563, 1.687, 1.813, 1.937\}$ cm and $T = \{-10, 10, 30, 50\}^oC$ is considered. For this case the *worst-case scenario* is used and the highest reduced order estimated is 252 for the models. The higher-order Krylov subspaces are computed over the estimation grid by means of

the algorithm described in Section II. Similar to the previous case by truncating the singular values of the merged projection matrix, a common projection matrix of size 324 is obtained. The weighted RMS error (16) of the ROM with respect to the original model is 0.037. The frequency response time for the original model is $2967.1s$ and that for the ROM is $21.62s$, obtaining 138 times speed up. As in the general analysis, the TLs can be modeled by many cascaded sections of RLC components. Nonetheless, the number of sections required depends on the electrical length of TLs. TLs sometimes require many sections to meet the reasonable accuracy. Thus, lumped RLC circuits extracted from layouts usually contain large circuit matrices that make the high CPU cost in simulation [2]. The proposed PMOR technique thus helps to overcome this problem, as on obtention of an accurate PMOR, the repeated design evaluations under different parameter settings becomes more efficient.

C. Computational complexity

The computational efficiency of the proposed technique in comparison to the Gramian-based PMOR [10] is illustrated in Fig. 3. It plots the memory requirement and the CPU time of the most computational expensive steps of the respective PMOR for a 6 port TDSs with one parameter variation for systems of order= $\{915, 2115, 5715, 9307\}$. The computa-
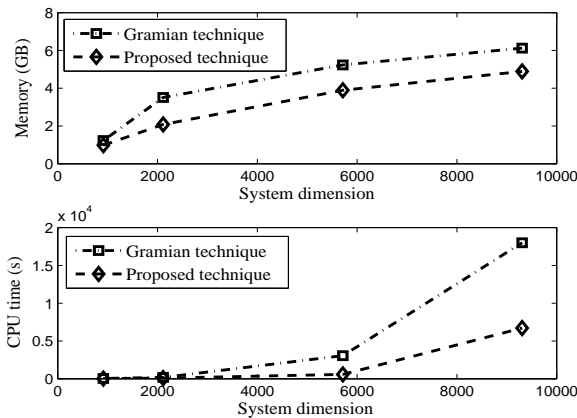

Fig. 3. Comparison of the computational efficiency.

tional cost for the Gramian-based PMOR [10] process is very high due to the Cholesky factorization and also due to the *SVD* computed on an order $n \times n$. While the computation complexity of the proposed technique is lesser for obtaining the Krylov subspace and also an economical *SVD* is performed on a matrix of size $n \times qM$ (where, $qM < n$) in order to obtain the PMOR.

V. CONCLUSION

A novel PMOR technique for MTLs with delay based on matrix interpolation is presented in this paper. Matrix interpolation preserves the same number of poles for parametric model order reduction over the design space while for input-output interpolation the order of the parametric macromodels,

suffer from oversize due to the nature of the input-output system level interpolation. The reduced order models are obtained using a higher-order Krylov subspace decomposition. First, the design space is divided into cells and for each vertex model of the cell a Krylov subspace is computed and are then merged and adaptively truncated based on the singular values to obtain a common projection matrix. The resulting reduced order models and also the delays are interpolated using positive interpolation schemes such that the parametric dependence is preserved. This PMOR approach makes multiple system evaluations under different design parameter variations computationally cheap and still accurate. The numerical examples of the RLC and MTL circuits with delays illustrates the efficiency and accuracy of the proposed PMOR technique.

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