Efficient outdoor sound propagation modeling with the finite-difference time-domain (FDTD) method: a review

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ABSTRACT

The finite-difference time-domain (FDTD) method, solving the inhomogeneous, moving medium sound propagation equations, also referred to as the Linearized Euler(ian) Equations (LEE), has become a mature reference outdoor sound propagation model during the last two decades. It combines the ability to account for complex wave-related effects like reflection, scattering and diffraction near or in between arbitrary objects, and complex medium-related effects like convection, refraction and (turbulent) scattering. In addition, it has the general advantages of a time-domain method. It is indicated that the numerical discretisation scheme should be chosen depending on the flow speed of the background medium. Perfectly matched layers, applicable to cases in presence of (non-)uniform flow, are state-of-the-art perfectly absorbing boundary conditions that are key in outdoor sound propagation applications, where only a small part of the unbounded atmosphere can be numerically described. Various ways to include frequency-dependent outdoor soils are summarized, like time-domain impedance plane boundary conditions and explicitly including the upper part of the soil in the simulation domain. Approaches for long-distance sound propagation, including moving calculation frames and hybrid modeling are discussed. This review deals with linear sound propagation only.

1.INTRODUCTION

During the last two decades, time-domain modelling has received a lot of interest as it was shown to have great potential. One of the major advantages is that the response over a broad frequency range can be obtained with a single simulation run only, on condition that a short acoustic pulse is excited at the source position. Clearly, the spatial discretisation will limit the range of frequencies that can be sufficiently resolved. As analysis of a system's response is often more convenient in frequency domain, a Fourier transform can still provide the necessary information in a post-processing step. Time-domain models further allow including non-linear effects that appear near high amplitude sources. A time-domain approach directly models the waveform; therefore, its distortion can be captured, corresponding to a transfer of sound energy in between sound frequencies. The latter is less trivial in a frequency-domain technique, focusing on a single frequency at a time. In time domain, moving sources and related doppler-shifts, as well as transient behavior can be simulated directly. In addition, source localization using time-reversal techniques clearly need a time-domain approach. While traditionally sound propagation is treated in the frequency domain, time-domain approaches emerged in the last two decades mainly due to the increased access to computing power.

Moving and inhomogeneous media may strongly effect sound propagation in the near field of realistic sources, as well as in the far field. Near airplanes, e.g., sound is emitted in a flowing medium, and the

radiation pattern is strongly influenced by the shape of structures like wings or outlet ducts. Similarly, engine noise exiting the outlet duct of a car results in a specific radiation pattern influenced by strong temperature gradients at the interface between the jet and surroundings, in combination with high outflow speeds. A model capable of taking into account both diffraction and scattering by arbitrary shapes in combination with complex flow effects is required in such applications.

Such initial, near field propagation effects, after generation of sound, are relevant for engineering radiation of sound at the source. But also in the far field, sound propagation is influenced considerably by flow and medium inhomogenities, often referred to as atmospheric effects. Especially gradients in the wind velocity and air temperature affect sound propagation in the atmospheric boundary layer over large distances [1][2]. Downwind or under temperature inversion conditions, wave-guiding is observed, strongly increasing sound pressure levels. Upwind, extended shadow regions are formed, where only turbulent scattering or creeping sound waves could lead to some sound penetration from a specific source [3]. Furthermore, atmospheric absorption processes are strongly influenced by relative humidity and air temperature, and to a lesser extent, by atmospheric pressure [4].

A volume-discretisation time-domain technique is well suited to study propagation of broadband sound in arbitrary moving media and temperature fields. This paper reviews specifically the finite-difference time-domain method as an approach to model sound propagation in the atmospheric boundary layer. Topics of concern are defining a suitable set of sound propagation equations and their numerical discretisation; the treatment of infinitely and finitely absorbing boundary conditions; and approaches for long-range sound propagation. Linear acoustics are considered.

2.GOVERNING SOUND PROPAGATION EQUATIONS

Starting equations for time-domain implementations of sound propagation in the atmosphere have been scrutinized by Ostashev et al. [5]. Departing from the linearized equations of fluid dynamics, simplifications can be made when the atmosphere is assumed to be an ideal gas, when the flow speed vector v_0 is smaller than the speed of sound c, when spatial variations in ambient air pressure are neglected, and when internal gravity waves are of no concern. This leads to the following closed set of coupled partial differential equations in the particle velocity vector v and acoustic pressure p :

$$\frac{\partial p}{\partial t} + \mathbf{v_0} \cdot \nabla p + \rho_0 c^2 \nabla \cdot \mathbf{v} = 0 , \qquad (1)$$

$$\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v_0} \cdot \nabla \right) \mathbf{v} + \left(\mathbf{v} \cdot \nabla \right) \mathbf{v_0} + \frac{1}{\rho_0} \nabla p = \mathbf{0} . \qquad (2)$$

In these equations t denotes time and ρ_0 is the mass density of air.

This set of equations [6][7][8][5] has been commonly proposed to study in detail sound propagation in the atmosphere where there is interest in the effect of a combination of wave phenomena as observed in a motionless medium (like reflection, diffraction and scattering by arbitrary objects) and flow effects

like refraction, convection and (turbulent) scattering. These equations are often called linear(ized) Euler(ian) equations (LEE).

Equations (1) and (2) only describe sound propagation. Consequently, sound generation terms are absent in contrast to common aeroacoustic equations. The flow does not generate sound here (but, e.g., deforms the wavefronts) and the acoustics do not influence the (non-acoustic) macro fluid flow. Therefore, these equations were referred to as "sound propagation in background flow" in Ref. [7]. In contrast to reference frequency-domain outdoor sound propagation techniques like the Parabolic Equation (PE) method (see e.g. [1] for an overview) or the Fast Field Program (FFP) (see e.g. [1] for an overview), arbitrary flow fields can be accounted for. Including an inhomogeneous medium does not induce difficulties in a volume discretisation technique as will be discussed further in the text.

In the equations above, atmospheric absorption is not included. Although such terms can be relatively easily added to Eq. (2) (e.g. as a general diffusion term as proposed in Ref. [6]), this is not optimal from an efficiency point of view. A more efficient approach is performing appropriate filtering on the time-domain signal afterwards, avoiding additional simulations for the wide range of air temperature and relative humidity combinations that are commonly observed in the atmosphere.

3.NUMERICAL DISCRETISATION

In contrast to frequency domain techniques, both the temporal and spatial discretisation of the governing sound propagation equations are of concern, strongly influencing numerical accuracy, numerical efficiency and numerical stability. The specific choice could depend on the application of interest and the magnitude of the background flow.

A primary choice is whether to only use pressures to discretise the sound propagation domain (called p-FDTD), or using both particle velocity components and pressures (p-v FDTD) as depicted in Fig. 1. While only using pressures strongly reduces the number of unknowns in the calculation grid, already in its simplest form second order derivatives are necessary. Consequently, additional fields need to be stored in memory to prevent mixing old and new fields during time-stepping, partly mitigating this memory-related benefit. p-FDTD has a certain popularity in room acoustics and audio applications [9][10][11], where flow is of no concern. p-v FDTD is the obvious way to implement the coupled set of partial differential equations Eqs. (1) and (2). A single equation with only the acoustic pressure as unknown cannot be derived, unless simplifications are made to the flow field.

In case of p-v FDTD, a choice between a staggered and collocated spatial discretisation has to be made (see Fig. 1). In a staggered grid, the pressures and velocities do not appear at the same physical locations in the grid, but are shifted in place. Typically, the pressures are located in the centre of each grid, while the particle velocity components are located on the faces that border each computational cell. In a collocated grid, pressures and all components of the velocities appear e.g. in the centre of each computational cell.
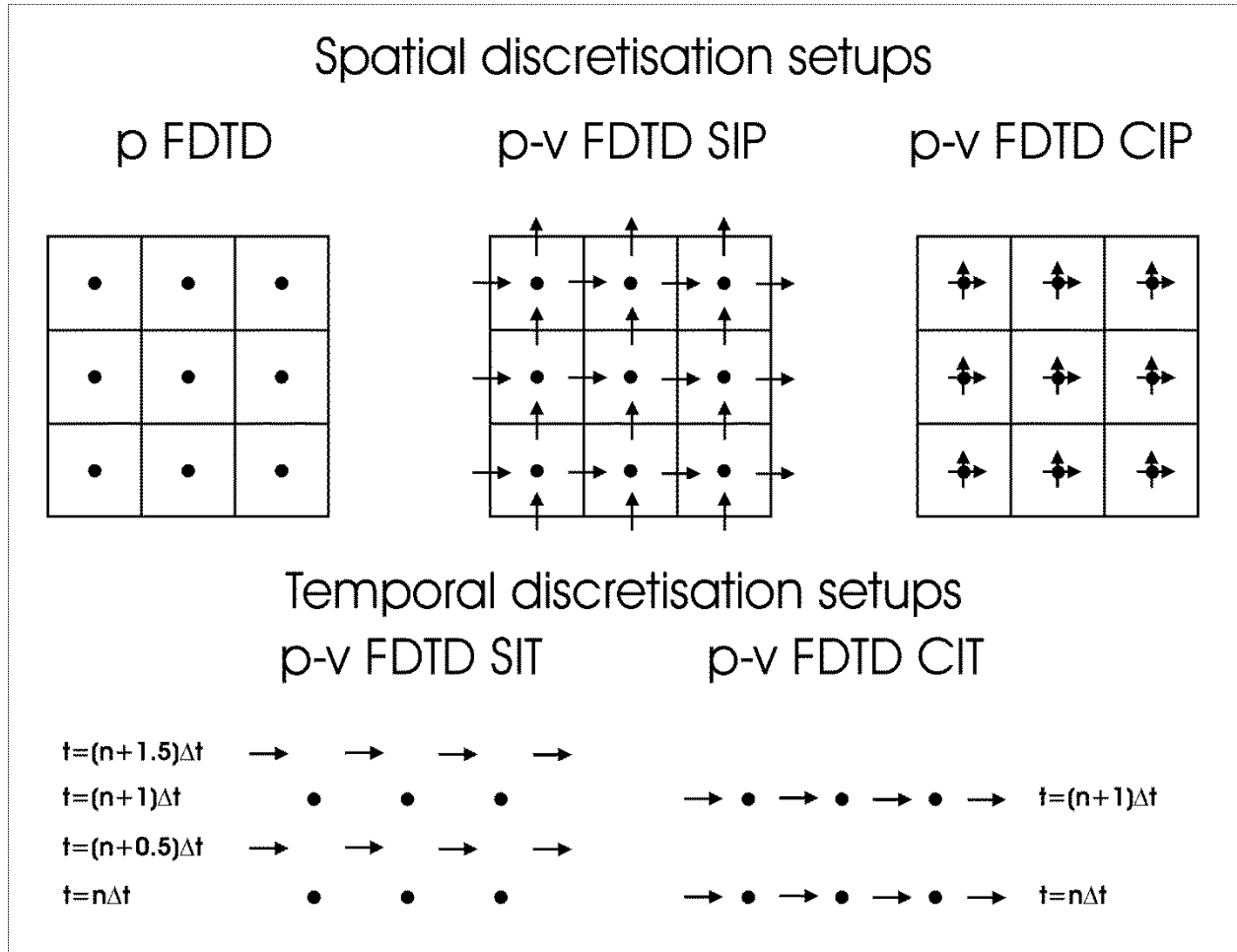
Figure 1: The different basic spatial and temporal discretisation setups in FDTD. Acoustic pressures are indicated with dots, the components of the particle velocities with arrays. (SIP=staggered-in-place, CIP= collocated-in-place, SIT=staggered-in-time, CIT=collocated-in-time).

3.1. Homogeneous still medium

In absence of flow, so when $v_0=0$, such a staggered-in-place grid has remarkable properties. This specific scheme originates from the so-called Yee cell [12], developed to solve the Maxwell equations with the FDTD technique in electromagnetic applications. At a lowest possible order, it can be shown that the error term from the Taylor expansion is reduced with a factor 4 relative to the collocated spatial representation of velocities and pressures. It can be seen as if the discretisation is virtually halved by taking the spatial gradients closer to the location where they are actually needed. The staggered-temporal approach (where the pressures and velocities are not updated at the same discrete times, but in a leap-frog manner) has similar benefits. A major advantage of such a staggered temporal grid is the possibility for in-place-computation : the old values are replaced by the new ones in computer memory during time-integration, which is highly efficient.

The courant number governs both numerical stability and accuracy. Following its definition, it actually demands that within one time step, a sound wave may travel at maximum one grid cell further. Detailed

analysis of the FDTD scheme in absence of flow shows that amplitude errors are absent at all Courant numbers [13]. The phase error depends on the propagation direction and is therefore asymmetric. There are no phase errors for sound propagation along the diagonal of square cells when the Courant number is equal to one [13]. Exactly along a coordinate axis, the phase error is largest. A Courant number of 1 is also most interesting along this propagation direction. Not only from the viewpoint of accuracy, but also for numerical efficiency this is optimal : a minimum number of time steps are needed for sound to reach a given distance.

Given this inherent phase error, a rather fine spatial discretisation is needed to reduce this numerical dispersion and 10 computational cells per wavelength are usually advised in a lowest-order discretisation. When accurate phase predictions are required for a specific application, an even finer spatial discretisation should be chosen.

Higher-order schemes enhance phase accuracy at a given spatial discretisation. Higher order spatial discretisation seems most optimal when applying so-called dispersion-relation preserving schemes [14], where the Taylor coefficients are slightly adapted, depending on the specific scheme, to further reduce dispersion errors. Although higher-order schemes are clearly interesting, time steps need to be reduced to keep simulations stable; relaxing the need for a finer spatial discretisation should be weighed against the increase in calculation time. Furthermore, this could lead to a much more complicated boundary treatment than in compact schemes. This is a drawback, as FDTD is especially interesting in cases with many objects and interfaces at close distance, including object with concave and convex parts and including complicated object-induced gradients in the propagation medium.

Higher-order temporal discretisation approaches including more terms from the Taylor series expansion to better approach the continuous derivatives is not interesting as for each increase in order, additional pressure and velocity fields have to be kept in memory during time-stepping to avoid overwriting. This strongly increases the computational cost which is often a bottleneck in FDTD applications. More advanced low-storage techniques have been developed like the one proposed by Bogey and Bailly [15] based on the Runge-Kutta approach, still explicitly solving these equations.

3.2. Inhomogeneous medium

Inhomogeneous mediums are rather easily modeled in a volume-discretisation technique. Each grid cell can be assigned a different temperature, leading to a (local) variation in sound speed. Clearly, the highest temperature will determine the time step, meaning that in other parts of the propagation domain the phase error cannot be minimized anymore.

3.3. Moving medium

In case of moving media, staggered-in-place is still an appropriate choice for spatial derivatives. The temporal discretisation needs more care as instability issues might appear. The (forward-difference) staggered-in-time (SIT) approach, as used in a motionless medium, was shown to be both unstable and inaccurate [16], albeit computationally fast. Although this method has been employed successfully in specific applications involving flow [6][17], its use should be discouraged certainly in case of higher flow

speeds and long-distance sound propagation. The accuracy for this specific scheme was quantified in Ref. [16], for sound propagation in a uniform (very) low-Mach number flow. In Ref. [5], a similar comparison with an analytical solution at higher flow speeds further confirmed its inaccuracy.

The theoretically correct and numerically stable approach for temporal derivatives in flow is the collocated-in-time (CIT) scheme [5][16], involving centered finite-differences. In contrast to the staggered-in-time approach, pressures and velocities are now updated at the same discrete times. Consequently, an additional particle velocity field and acoustic pressure field need to be kept in memory to perform time stepping. As a result, the memory cost is doubled, while time steps are halved. The increase in computational cost, relative to sound propagation in a still medium, is therefore large. This discretisation scheme shows [5] to be capable of accurately solving Eqs. (1) and (2), up to Mach numbers equal to 1. Similarly as in absence of flow, there are no amplitude errors. The phase error is hardly affected by the presence of flow [16].

In a way to conciliate the accuracy and stability of the CIT scheme and the calculation efficiency of SIT, the so-called prediction step staggered-in-time (PSIT) approach was proposed in Refs. [7] and [8] and scrutinized in Ref. [16]. The staggered-in-time updating of acoustic pressures and particle velocity components is retained, but an intermediate step is introduced, updating the fields deliberately neglecting flow. These field are then subsequently used in the equations including the flow terms. This approach allows keeping an explicit updating scheme. In contrast to the CIT scheme, only a single additional field needs to be stored, whose memory allocation can be used both for the acoustic pressures and the components of the particle velocities. The phase error is not affected relative to sound propagation in a still medium, while the amplitude error is strongly reduced relative to the SIT scheme [16]. For the rather low wind speeds as typically observed in the atmospheric surface layer, this method shows to be sufficiently accurate (let say, for Mach numbers smaller than 0.1). This scheme shows to be marginally unstable. However, with increasing flow speeds, stability and accuracy issues appear as well with PSIT and the use of CIT is then advised.

3.4.Turbulent medium

Although detailed computational fluid dynamics (CFD) models like direct numerical simulation (DNS) or large-eddy simulations (LES) could produce momentary turbulence fields (that are directly employable in a finite-difference time-domain model, see e.g. Ref. [18]), the huge computational cost involved is still an important barrier for their wide use. In addition, the desired spatial resolution in a finite-difference time-domain technique is often too demanding.

A common approach to account for turbulent scattering is the so-called frozen turbulence approach [1]. The sound propagation medium is perturbed by local variations in the flow speed and temperature in such a way that the statistics of the turbulent realizations are close to those of specific turbulence models or correspond to experiments. Next, sound propagation is calculated through various realizations, allowing to calculate statistics on e.g. the sound pressure. Such turbulent field realizations can be constructed in various ways, like e.g. with the turbule or quasi-wavelet approach as proposed by Goedecke et al. [19][20] or by projecting fluctuation spectra, with random phase, to the physical space

[21][22]. These approaches have been applied to homogeneous isotropic turbulence above a flat ground. Examples of such realisations are shown in Fig. 2.

In Van Renterghem [7], the turbulent kinetic energy field ("k-field"), as calculated with a standard k-ε turbulence closure CFD model, was used to adapt the local turbulence strength of such turbules. To some extent, the inhomogeneous turbulence as observed in the urban environment near roof level is accounted for. Heimann and Blumrich [23] used a somewhat related technique, and added the concept of "transient turbulence" to avoid calculating through many realizations of a turbulent atmosphere.

It was argued in Ref. [21] that the FDTD method has high potential to account for turbulent scattering at all angles and to include multiple scattering, in combination with wave phenomenons present in a non-turbulent (moving and and inhomogeneous) atmosphere.
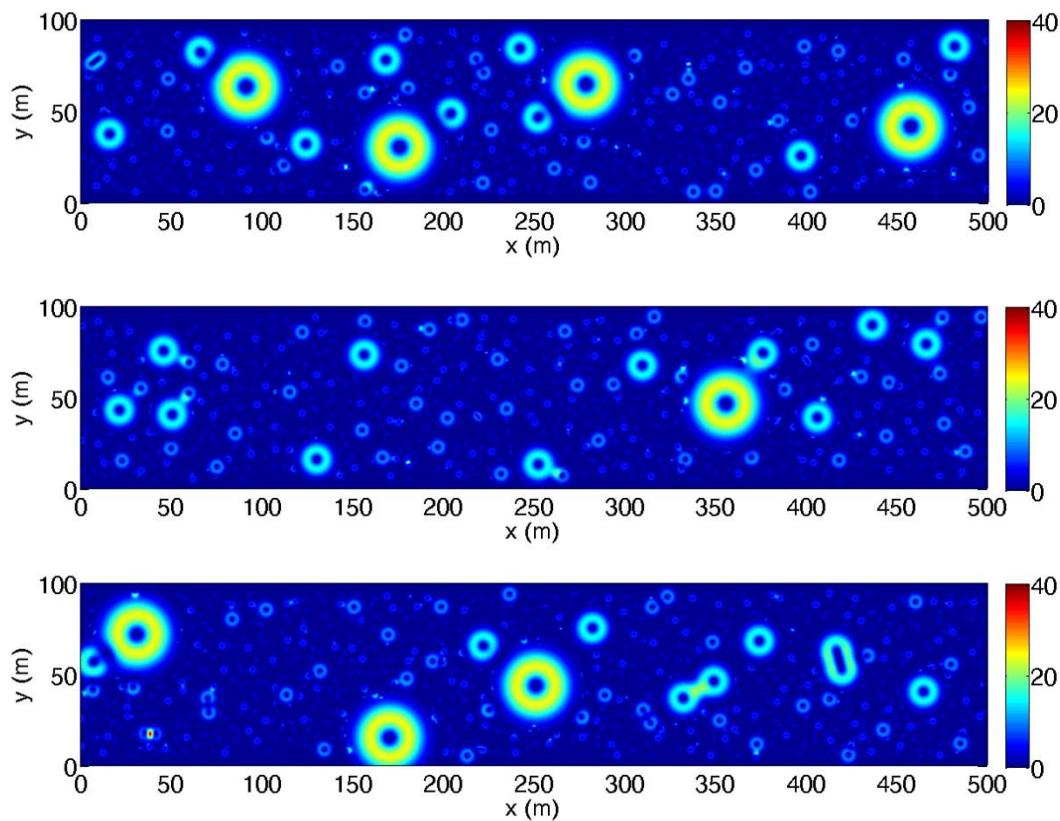

Figure 2 : Three turbulent field realisations employing the turbule [20] theory for flow velocity turbulence (homogeneous and isotropic turbulence; solenoidal eddies). A Kolmogorov spectrum is used, with $Cv^2=0.035 \ m^{4/3}/s^2$. Eight length scales have been considered, with radii of the vortices ranging from 1.5 m to 0.11 m. Turbulent kinetic energy fields are depicted (in m^2/s^2).

4.INFINITE BOUNDARY CONDITIONS

Two types of infinite boundary conditions are generally of concern, namely a perfectly reflecting and a perfectly absorbing boundary condition. Full reflection is easily modeled in a staggered spatial grid since on the faces bordering a grid cell, the normal particle velocity components are defined. Setting this component to zero models a rigid boundary condition.

Implementing perfectly absorbing boundary (PAB) conditions is challenging in a full-wave numerical technique. However, PABs are essential in outdoor sound propagation : the unbounded sound propagation region (i.e. the atmosphere) has to be truncated to a finite simulation domain. An impedance plane boundary condition, on which the same impedance as in the propagation medium is imposed, is a sound approach from a theoretical point of view, but shows to have a too limited accuracy in FDTD implementations. Numerical experiments show that for normally incident sound waves, typical reductions in reflection are lower than 40 dB relative to the incident sound energy [7].

Therefore, so-called "zonal techniques" are needed; various approaches were reviewed by Hu [24]. The perfectly matched layer (PML) theory, originally proposed by Berenger for electromagnetics FDTD applications [25], is a highly accurate PAB and is state-of-the-art in acoustic FDTD applications [26]. The basic idea is that a layer of cells is needed near the grid borders having a gradual increase of the damping inside this layer. This ensures that there is sufficient acoustic energy loss when the sound waves reach the borders of the computational domain, while the change in impedance near the interface between the bulk grid and the absorbing layers is minimal. In absence of flow, a damping term is introduced in both the equation updating the velocity and pressure over time. Imposing perfect absorption at all angles of incidence and at all sound frequencies leads to surprisingly simple "matching" conditions: the ratio of the damping coefficient in both equations is the mass density of the bulk propagation medium. An essential step, in analogy to the original formulation by Berenger [25], is splitting the acoustic pressure - although it is a scalar - in an orthogonal and parallel component relative to the interface to introduce an additional degree of freedom. However, PMLs in unsplit physical variables have been developed as well [27] as theoretical analysis showed that the split-set of equations is not strongly well-posed [28].

With increasing number of cells to constitute the absorbing layer, the reflection at the interface decreases. Based on the numerical experiment described in Ref. [7], a thickness of about 20 cells seems most efficient, leading to a reduction in acoustic energy upon reflection of more than 100 dB.

PMLs have been developed for a moving medium as well [26][27][28][29][30]. However, in the implementation proposed in Refs. [7] and [8], the matching conditions were shown not to depend on the (uniform) flow speed. Still using uniform-flow PMLs in presence of (strong) flow gradients only leads to a slightly worsened performance [7], meaning that the PMLs remain highly accurate. However, dedicated approaches are possible [31].

5. FINITE-IMPEDANCE BOUNDARY CONDITIONS

Materials, from a historical point of view, have been acoustically characterized mainly in the frequency domain. A frequency-independent impedance is easily implemented and accurate in a p-v FDTD grid. However, most materials behave strongly different in function of sound frequency. Using a fixed

impedance and repeating calculations to obtain the response at various frequencies of interest is therefore highly inefficient: One of the main assets of a time-domain technique, namely the possibility to acquire a broadband response with a single simulation, would be lost then.

There are two main approaches, either using an impedance plane as a material boundary condition, or including (a part of) the material in the simulation domain.

5.1. Impedance plane approach

In a brute force approach, direct convolution is a possibility, yielding the time-domain analogy of any frequency-domain function. However, such an operation is unattractive as it is computationally highly demanding. In theory, the full time history has to be kept in memory at each boundary point for accurate representation of the interaction between the sound wave and the material.

Two types of impedance plane approaches have been considered in time-domain simulations. A first option is considering a physical model. A second option is using specific functions, allowing a (relatively) easy translation to the time-domain. Next, the parameters of such formulations (sometimes referred to as "template functions") are found by curve fitting to obtain a specific frequency-impedance behavior. The physical sound propagation equations are not explicited in the latter. Therefore, only forms obeying a number of conditions could lead to stable and accurate time-domain implementations: Causality (meaning that only previous values are needed to perform time-updating), passivity (meaning that the real part of the impedance is larger than or equal to zero for all sound frequencies), and reality (the particle velocity and pressure in time-domain are real values; consequently, also the time-domain representation of the frequency-impedance model must be real) are needed [32].

This section summarizes some impedance plane boundary conditions that were shown to be applicable to model reflections from typical outdoor soils. The methods discussed are expected to have a wider applicability, though.

5.1.1. Expansion around jw

An attractive model, with a clear physical meaning, is a mass-damper-spring system [13][33], characterized by the following impedance equation :

$$Z(\omega) = a_{-1} \frac{1}{j\omega} + a_0 + a_1 j\omega \qquad (3)$$

with j the imaginary unit and ω the pulsation, a_{-1} is the spring constant, a_0 a material damping constant and a_1 the mass; all these material parameters are expressed per unit area. Interestingly, each term can be easily translated to the time domain : the $1/j\omega$–term corresponds to a simple time integration, demanding only a single additional variable to be kept in memory at each boundary point. The constant term is solved by linear interpolation over time, while the mass term $j\omega$ corresponds to a time-derivative. However, the unbounded air propagation time step should be reduced with a factor $\sqrt{3/2}$ to ensure numerical stability [13]. It was shown in Ref. [34] that such a system obeys the necessary

conditions of causality and passivity on condition that realistic physical values are used for the coefficients. Putting such a system in series allows introducing more complex behavior of materials or increases the degree of freedom in curve fitting [35]. In order to capture the typical steep descent in impedance in the low frequency range for outdoor soils, Heutschi et al. [36] proposed to add a $(1/j\omega)^2$ term to approach e.g. the Delany and Bazley model [37], widely used to represent reflection on grass-covered soil in outdoor sound propagation. However, the double integration needed to express this additional term in the time-domain was found to lead to a positive feedback loop and instable results [36]. The latter was solved, in a practical way, by slightly damping this term and did not induce a significant loss in numerical accuracy for typical cases of sound propagation outdoors for source and receiver at low heights. This is consistent with the analysis performed by Dragna [34], showing that the Delany and Bazley model [37] is actually not suited as a time-domain boundary conditions since it is theoretically not possible to simultaneously obey the 3 conditions of reality, causality and passivity.

5.1.2. Direct convolution

The following sum of terms can be efficiently implemented in time domain :

$$Z(\omega) = \sum_{i=0}^{n} \frac{a_i}{b_i + j\omega}, \tag{4}$$

as the inverse Fourier Transform is a sum of exponentially decreasing functions [38]. Consequently, this allows calculating a direct convolution not entailing an excessive computational cost. Furthermore, each term can be implemented in a recursive way, only needing a single additional variable per term considered necessary to approach a particular frequency-impedance curve. This method was shown to be applicable to the Miki model [39] to approach typical outdoor soils like grass-covered ground and snow [40]; rigid backing was used for the latter. This direct convolution can be considered as a generalization of the mass-spring-damper approach following analysis in Ref. [41].

5.1.3. Rational functions

Another possibility is describing a frequency dependent impedance by a rational function [42][43][44]:

$$Z(x) = \frac{\sum_{i=0}^{n} a_i x^i}{\sum_{j=0}^{m} b_j x^j} \text{ with } x = f(j\omega) \tag{5}$$

Ostashev et al. [43] used Pade-approximants for the Zwikker and Kosten [45] impedance boundary condition and for the Attenborough 4-parameter model [46]. It was shown in Ref. [43] that when using fractional derivatives, a causal time-domain boundary condition can be formulated. The fractional derivatives can be efficiently calculated using exponentially decaying functions in a recursive approach.

The use of rational functions to approach frequency-dependent impedances is related to infinite impulse response (IIR) filters, that are commonly designed in the z-domain. Such an approach is known

to be able to provide the required real coefficients. Design in the z-domain rapidly leads to discretised FDTD equations. This methodology was shown to be accurate and efficient in typical room acoustics applications [47].

5.2. Modeling sound propagation through (a layer of) soil

The aforementioned time-domain impedance plane boundary conditions assume locally reacting materials. Non-locally reacting materials can be modeled by including (part of) the porous material inside the simulation domain, in contrast to only modeling the interface between two media. In the first option, sound propagation inside the material is explicitly resolved and the physical sound propagation equations in the material should be known. Also when there is only interest in the reflected waves at the interface, such an approach can be interesting.

5.2.1. Poro-rigid frame model

A popular model to account for soil reflections in finite-difference time-domain simulations is the one by Zwikker and Kosten [45]. This rigid-porous medium model assumes that the constituting part of the material, i.e. the frame, does not vibrate with the incident sound wave. This is a reasonable assumption when the density of the material matrix and its stiffness are significantly higher than those in air. For many outdoor soils, this condition is fulfilled. The model employs three material parameters namely the flow resistivity σ, the porosity φ and the so-called structure constant k_s, which is linked to the tortuosity:

$$\nabla \cdot p + \rho' \frac{\partial \boldsymbol{v}}{\partial t} + \sigma \boldsymbol{v} = \boldsymbol{0} \tag{6}$$

$$\frac{\partial p}{\partial t} + \rho' c'^2 \nabla \cdot \boldsymbol{v} = 0 \tag{7}$$

$$\rho' = \frac{\rho_0 k_s}{\varphi} \tag{8}$$

$$c' = \frac{c}{\sqrt{k_s}} \tag{9}$$

The main reason for its popularity is its ease in implementation and the fact that additional numerical constraints are limited compared to sound propagation in unbounded air. A general damping term is introduced in the velocity equation (Eq. (6)), which is proportional to the flow resistivity of the soil, and the (scalar) mass density ρ' and speed of sound c' in the material are adapted based on the porosity and structure constant, as illustrated by Eqs. (6)-(9).

The implementation of Eqs. (6)-(9) has been analyzed in detail. Some care is needed when using this type of model in case of higher flow resistivities. To ensure accuracy, a finer grid than needed in bulk air must be applied to accurately capture the rapid decrease in sound pressure near the soil interface. A

sudden grid refinement with a factor 4 is proposed in Ref. [17] near the interface air-ground. A gradual grid refinement, reducing spurious reflections in the refinement area, was proposed in Ref. [7].

Numerical stability of the FDTD implementation of the Zwikker and Kosten model [45] was scrutinized in Ref. [48]. With increasing product of flow resistivity and medium porosity, time steps should be slightly decreased to obtain stability. Increasing the structure factor seems to stabilize simulations. The rigid-porous medium seems to be less restrictive as for the time step in case of an already fine spatial grid in unbounded air. However, smaller time steps are needed in case large spatial steps are used in unbounded air.

The rather simple approach by Zwikker and Kosten does not capture all relevant physics of sound interacting with outdoor soils, especially for propagation through soils at high frequencies and low flow resistivities [49]. In its basic form, sound reflection is rather accurately modeled. Making the parameters weakly frequency-dependent extends its range of applicability [49], however, departing from the rather straightforward implementation as discussed above.

5.2.2.Poro-elastic frame models

Poro-elastic soil models have been implemented as well in the time-domain. The coupled movement of the frame and air inside the porous medium then needs to be accounted for. Two approaches, inspired by Biot's theory [50][51], have been reported for time-domain modeling of outdoor sound propagation. Dong et al. [52] only discretise the pore air particle velocity and frame velocity inside the porous medium, while above the soil only acoustic pressures are used in representing the sound field. Ding et al. [53] consistently follow the p-v FDTD approach in both the unbounded air and the poro-elastic medium. This means that inside the soil, not only the pore air pressure and particle velocity, but also the frame pressure and frame velocity are resolved.

Compared to a rigid-porous medium, additional medium parameters are the bulk modulus of the frame and frame density. In Biot's frequency domain approach, some medium parameters were assigned an imaginary part, in order to model frame damping. An additional frame damping coefficient has been used instead in Ref. [53], while Dong et al. [52] directly model friction in the air-filled pores. No additional numerical stability issues have been reported in these references.

6.LONG DISTANCE SOUND PROPAGATION

A volume discretisation technique like FDTD, even when fully optimized, is not well suited to calculate sound propagation up to large distances. In this section, some alternatives are discussed to tackle this problem and summarized in Fig. 3.

6.1.Moving frame FDTD

On condition that a short, acoustic pulse is excited at the source position, the moving frame FDTD allows performing calculations at minimum computational cost. Instead of updating the fields throughout the full spatial grid, calculations are only made in a limited zone of the grid, centered around the propagating pulse. This calculation frame then moves with the speed of sound in the desired direction. A

moving frame approach prevents updating acoustic variables in zones where hardly any acoustic energy is present. The moving calculation frame should include the full vertical extent of the grid, however.

This method has been applied to sound propagation from a coherent line source over a rigid ground plane in a refractive atmosphere [17], and was shown not to decrease numerical accuracy. A fixed calculation frame has been used in the latter. When using programming languages allowing dynamic memory allocation, even more efficient implementations are possible. In Ref. [54], multiple narrow frames are allowed by threshold-based memory allocation, allowing to follow distinct reflections as well. Clearly, if the sound propagation problem becomes too reverberant, no gain is observed anymore by the use of a moving frame.

6.2. Hybrid modelling

Many noise control engineering cases need highly detailed modelling in a source zones, while at the same time, sound propagation up to a large distance is required. Clearly, it is very hard to conciliate these two aspects within a single numerical technique.

Various spatial domain decomposition methods have been proposed in the field of outdoor acoustics, typically combing a full-wave technique near the source, with the parabolic equation method [55] or a geometrical acoustics approach to reach far away receivers [56].

Especially the combination between FDTD and PE [57] is interesting as the effect of flow is accounted for in both the source region and outside, while wave-related effects are preserved. A PE method assumes one-way sound propagation in a limited angle around the source and the effective sound speed approach [1]. These assumptions allow efficient yet accurate predictions of long-distance sound propagation in the atmosphere. Although a combination between a full wave technique and geometrical acoustics has its merits, an essential problem is that the latter relies on a high-frequency approach outside the source region. Note that usually the low-frequency content of a source becomes dominant after long distances propagation as high frequencies are easily absorbed in the atmosphere [4].

The hybrid method between FDTD and PE has been analysed in detail in Ref. [57]. The Green's Function Parabolic Equation (GFPE) [1] has been used, given its high efficiency (e.g. the possibility for stepping at various wavelenghts in the propagation direction) and the possibility to a capture most relevant outdoor sound propagation aspects (like range-dependent arbitrary sound speed profiles, range-dependent ground impedance, accounting for terrain undulations [1][58][59], turbulent scattering, etc.). A one-way coupling was proposed, where FDTD constitutes a vertical array of pressures at a frequency of interest at close distance from the source region, forming the starting function for GFPE. This vertical array is then subsequently propagated up to the receiver distance as in a standard GFPE calculation. Accuracy is hardly affected by this coupling. The mismatch in grid height between FDTD and PE is solved in Ref. [57] by employing the decrease of sound pressure in the PML towards the top of the FDTD grid, forming the basis for the extrapolation of the pressure towards the top of the PE grid, which is typically much higher. The gain in computational efficiency is huge, relative to applying FDTD to the full sound propagation region, if even possible [57].
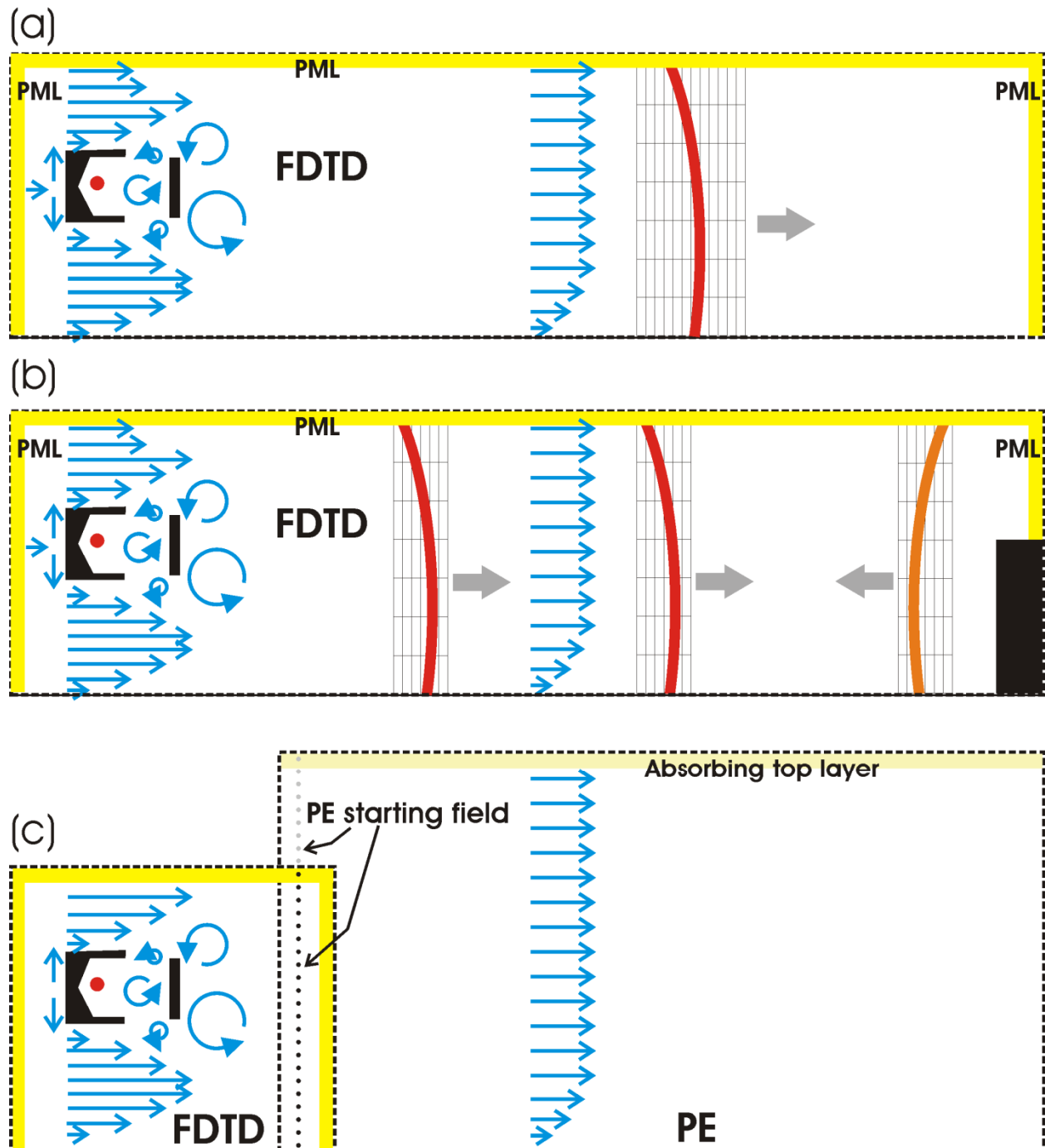
Figure 3 : Schematic representation of long-distance sound propagation approaches based on finite-difference time-domain modeling, employing (a) moving frame FDTD (fixed-width calculation frame), (b) dynamic moving frame FDTD, and (c) a hybrid FDTD-PE method.

6.3.Undulating terrain

Long-distance sound propagation typically involves ground undulations that might significantly affect sound propagation. Transformation from the traditional structured Cartesian grid to a curvilinear

coordinate system, following the terrain undulations, is a popular and efficient technique [54][60][61]. This method allows time-domain impedance plane boundary conditions to be applied to the uneven terrain, while the same numerical approaches as in a rectangular coordinate system can still be used.

7.CONCLUSIONS AND DISCUSSION

The finite-difference time-domain method has become a mature reference sound propagation model over the last two decades. It has the advantages of a time-domain method namely the possibility to obtain a broadband frequency response with a single simulation, the possibility to include non-linear effects and the treatment of moving and realistic sources. In an arbitrary moving medium, finite-difference time-domain implementations are needed that resolve both the particle velocity and the acoustic pressure.

The numerical discretisation scheme strongly influences numerical stability, numerical accuracy, and numerical efficiency. Since FDTD calculations are typically computationally highly demanding, well-though choices should be made. Analysis shows that staggered spatial grids are interesting both in presence and absence of flow. In flow, however, collocated temporal grids are accurate and stable up to Mach numbers of 1 when solving the LEE, but lead to a strong increase in computational cost. The prediction-step staggered-in-time (PSIT) approach shows to be a compromise between accuracy, stability and computational efficiency at low Mach number flows.

Perfectly matched layers efficiently model perfectly absorbing boundaries when truncating the atmosphere to a finite calculation domain. The presence of flow does not deteriorate its behavior.

The treatment of finite absorbing boundaries has long been considered an important drawback of time-domain approaches. However, various techniques have been proposed to model impedance plane boundary conditions as well as explicitly modeling sound propagation inside a porous medium.

Long-distance sound propagation is either possible with fixed or dynamic moving calculation frames or by hybrid modeling. A coupled FDTD-PE method is especially interesting as it allows including moving and inhomogeneous medium effects in both the source region and during propagation towards receivers far away. Curvilinear coordinate transformations allow the inclusion of undulating terrains that might especially be relevant in case of long-distance sound propagation.

An interesting numerical technique closely related to the finite-difference time-domain technique is the pseudo-spectral time-domain method [62]. Essentially, spatial derivatives are solved in the wavenumber-domain, needing a multiplication only. The efficiency of this method relies heavily on the availability of efficient forward (and backward) Fourier transforms. In addition, phase errors are absent on condition that the Nyquist sampling criterion is obeyed, leading to the need of only 2 cells per wavelength in theory. This method corresponds to a spatial stencil equal to the full extent of the simulation domain along a particular coordinate axis and can therefore be considered as a high order finite-difference approach. This method has been applied to Eqs. (1) and (2) by Hornikx et al. [63] for applications in outdoor sound propagation. Although its large potential, approaches to implement frequency-dependent boundary conditions are lacking. Up till now, only frequency-independent

impedances have been implemented, either by modeling a second medium with a different density [63], or with a constant and real impedance plane [64].

REFERENCES

1. Salomons, E., Computational atmospheric acoustics, Kluwer, Dordrecht, The Netherlands, 2001.
2. Attenborough, K., Li, K. and Horoshenkov, K., Predicting outdoor sound, Taylor and Francis, London, UK, 2007.
3. Emleton, T., Tutorial on sound propagation outdoors, Journal of the Acoustical Society of America, 2001, 100, 31–48.
4. ISO 9613-1:1996 Acoustics – attenuation of sound during propagation outdoors - Part 1. International Organisation for Standardisation, Geneva, Switzerland, 1996.
5. Ostashev, V., Wilson, D., Liu, L., Aldridge, D., Symons, N. and Marlin, D., Equations for finite-difference, time-domain simulation of sound propagation in moving inhomogeneous media and numerical implementation, Journal of the Acoustical Society of America, 2005, 117, 503-517.
6. Blumrich, R. and Heimann, D., A linearized Eulerian sound propagation model for studies of complex meteorological effects, Journal of the Acoustical Society of America, 2002, 112, 446–455.
7. Van Renterghem, T., The finite-difference time-domain method for sound propagation in a moving medium, PhD Thesis, Ghent University, 2003.
8. Van Renterghem, T. and Botteldooren, D., Numerical simulation of the effect of trees on downwind noise barrier performance, Acta Acustica united with Acustica, 2003, 89, 764-778.
9. Kowalczyk, K. and van Walstijn, M., Formulation of Locally Reacting Surfaces in FDTD/K-DWM Modelling of Acoustic Spaces, Acta Acustica united with Acustica, 2008, 94, 891-906.
10. Lokki, T., Southern, A. and Savioja, L., Studies on seat dip effect with 3D FDTD modeling, Proceedings of Forum Acusticum 2011, Aalborg, Denmark.
11. Webb, C. and Bilbao, S., Computing room acoustics with CUDA - 3D FDTD schemes with boundary losses and viscosity, IEEE International Conference on Acoustics, Speech, and Signal Processing, 2011, 317-320.
12. Yee, K., Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media, IEEE Transactions on Antennas and Propagation, 1966, 14, 302–307.
13. Botteldooren, D., Finite-difference time-domain simulation of low frequency room acoustic problems, Journal of the Acoustical Society of America, 1995, 98, 3302–3308.
14. Tam, C. and Webb, J., Dispersion-relation-preserving finite difference schemes for computational acoustics, Journal of Computational physics, 1993, 107, 262-281.
15. Bogey, C. and Bailly, C., A family of low dispersive and low dissipative explicit schemes for flow and noise computation, Journal of Computational physics, 2004, 194, 194-214.
16. Van Renterghem, T. and Botteldooren, D., Prediction-step staggered-in-time FDTD: an efficient numerical scheme to solve the linearised equations of fluid dynamics in outdoor sound propagation, Applied Acoustics, 2007, 68, 201-216.
17. Salomons, E., Blumrich, R. and Heimann, D., Eulerian Time-Domain Model for Sound Propagation over a Finite-Impedance Ground Surface. Comparison with Frequency-Domain Models, Acta Acustica united with Acustica, 2002, 88, 483-492.
18. Symons, N., Aldridge, D., Marlin, D., Wilson, D., Patton, E., Sullivan, P., Collier, S., Ostashev, V. and Drob, D., 3D Staggered-Grid Finite-Difference Simulation of Sound Refraction and Scattering

in Moving Media, <u>Proceedings of the 11th International Symposium on Long Range Sound Propagation</u>, 2004, Fairlee, VT, USA.

19. Goedecke, G. and Auvermann, H., Acoustic scattering by atmospheric turbules, <u>Journal of the Acoustical Society of America,</u> 1997, 102, 759–771.

20. Goedecke, G., Wood, R., Auvermann, H., Ostashev, V., Havelock, D. and Ting, C., Spectral broadening of sound scattered by advecting atmospheric turbulence, <u>Journal of the Acoustical Society of America</u>, 2001, 109, 1923-1934.

21. Cheinet, S., Ehrhardt, L., Juve, D. and Blanc-Benon, P., Unified modeling of turbulence effects on sound propagation, <u>Journal of the Acoustical Society of America</u>, 2012, 132, 2198-2209.

22. Ehrhardt, L., Cheinet, S., Juve, D. and Blanc-Benon, P., Evaluating a linearized Euler equations model for strong turbulence effects on sound propagation, <u>Journal of the Acoustical Society of America</u>, 2013, 133, 1922-1933.

23. Heimann, D. and Blumrich, R., Time-domain simulations of sound propagation through screen-induced turbulence, <u>Applied Acoustics</u>, 2004, 65, 561–582.

24. Hu, F., Absorbing Boundary Conditions, <u>International Journal of Computational Fluid Dynamics</u>, 2004, 18, 513-522.

25. Berenger, J., A perfectly matched layer for the absorption of electromagnetic waves, <u>Journal of Computational Physics</u>, 1994, 114, 185-200.

26. Hu, F., On Absorbing Boundary Conditions for Linearized Euler Equations by a Perfectly Matched Layer, <u>Journal of Computational Physics</u>, 1996, 129, 201-219.

27. Hu, F., A stable, perfectly matched layer for linearized Euler equations in unsplit physical variables, <u>Journal of Computational Physics</u>, 2001, 173, 455-480.

28. Abarbanel, S. and Gottlieb, D., A Mathematical Analysis of the PML Method, <u>Journal of Computational Physics</u>, 1997, 134, 357–363.

29. Tam, C., Auriault, L. and Cambuli, F., Perfectly matched layers as an absorbing boundary condition for the linearized Euler equations in open and ducted domains, <u>Journal of Computational Physics</u>, 1998, 144, 213-234.

30. Abarbanel, S., Gottlieb, D. and Hesthaven, J., Well-posed Perfectly Matched Layers for Advective Acoustics, <u>Journal of Computational Physics</u>, 1999, 154, 266-283.

31. Hu, F., A Perfectly Matched Layer absorbing boundary condition for linearized Euler equations with a non-uniform mean flow, <u>Journal of Computational Physics</u>, 2005, 208, 469–492.

32. Rienstra, S., Impedance models in time domain including the Helmholtz resonator model, <u>Proceedings of Twelfth AIAA/CEAS Aeroacoustics Conference</u>, Cambridge, MA, USA, 2006.

33. Tam, C. and Auriault, L., Time-Domain Impedance Boundary Conditions for Computational Aeroacoustics, <u>American Institute of Aeronautics and Astronautics Journal</u>, 1996, 34, 917–923.

34. Dragna, D. and Blanc-Benon, P., Physically admissible impedance models for time-domain computations of outdoor sound propagation, <u>Acta Acustica united with Acustica</u>, under review.

35. Sakamoto, S., Nagatomo, H. and Asakura, T., Treatment of absorbing boundary condition for FDTD analysis in room acoustics, <u>Proceeding of the International Congress on Acoustics (ICA)</u>, Madrid, Spain, 2007.

36. Heutschi, K., Horvath, M. and Hofmann, J., Simulation of ground impedance in finite difference time domain calculations of outdoor sound propagation, <u>Acta Acustica united with Acustica</u>, 2005, 91, 35–40.

37. Delaney, M. and Bazley, E., Acoustical properties of fibrous absorbent materials, <u>Applied Acoustics</u>, 1970, 3, 105-116.

38. Fung, K.-Y. and Ju, H., Time-domain Impedance Boundary Conditions for Computational Acoustics and Aeroacoustics, International Journal of Computational Fluid Dynamics, 2004, 18, 503–511.

39. Miki, Y., Acoustical properties of porous materials - Modifications of Delany-Bazley models, Journal of the Acoustical Society of Japan, 1990, 11, 19–24.

40. Dragna, D., Cotté, B., Blanc-Benon, P. and Poisson, F., Time-Domain Simulations of Outdoor Sound Propagation with Suitable Impedance Boundary Conditions, American Institute of Aeronautics and Astronautics Journal, 2011, 49, 1420-1428.

41. Reymen, Y., Baelmans, M., and Desmet, W., Time-Domain Impedance Formulation Suited for Broadband Simulations, Thirteenth AIAA/CEAS Aeroacoustics Conference, Rome, Italy, 2007.

42. Chandler-Wilde, N. and Horoshenkov, K., Padé approximants for the acoustical characteristics of rigid porous media, Journal of the Acoustical Society of America, 1995, 98, 1119-1129.

43. Ostashev, V., Collier, S., Wilson, D., Aldridge, D., Symons, N. and Marlin, D., Padé approximation in time-domain boundary conditions of porous surfaces, Journal of the Acoustical Society of America, 2007, 122, 107-112.

44. Bin, J., Yousuff Hussaini, M. and Lee, S., Broadband impedance boundary conditions for the simulation of sound propagation in the time domain, Journal of the Acoustical Society of America, 2009, 125, 664-675.

45. Zwikker, C. and Kosten, C., Sound absorbing materials, Elsevier, New York, 1949.

46. Attenborough, K., On the acoustic slow wave in air-filled granular media, Journal of the Acoustical Society of America, 1987, 81, 93-102.

47. Jeong, H., Drumm, I., Horner, B. and Lam, Y., The modelling of frequency dependent boundary conditions in FDTD simulation of concert hall acoustics, Proceedings of 19th international congress on acoustics (ICA), 2007, Madrid, Spain.

48. Picó, R., Roig, B. and Redondo, J., Stability Analysis of the FDTD Scheme in Porous Media, Acta Acustica united with Acustica, 2010, 96, 306-316.

49. Wilson, D., Ostashev, V., Collier, S., Symons, N., Aldridge, D., and Marlin, D., Time-domain calculations of sound interactions with outdoor ground surfaces, Applied Acoustics, 2007, 68, 175-200.

50. Biot, M., Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range, Journal of the Acoustical Society of America, 1956, 28, 168–178.

51. Biot, M., Theory of propagation of elastic waves in a fluid-saturated porous solid. II. High-frequency range, Journal of the Acoustical Society of America, 1956, 28, 179–191.

52. Dong, H., Kaynia, A., Madshus, C. and Hovem, J., Sound propagation over layered poro-elastic ground using a finite-difference model, Journal of the Acoustical Society of America, 2000, 108, 494-502.

53. Ding, L., Van Renterghem, T., Botteldooren, D., Horoshenkov, K. and Khan A., Sound absorption of porous substrates covered by foliage: experimental results and numerical predictions, Journal of the Acoustical Society of America, 2013, 134, 4599-4609.

54. de Greve, B., Van Renterghem, T. and Botteldooren, D., Long range FDTD over undulating terrain, Proceedings of Forum Acusticum, Budapest, Hungary, 2005.

55. Premat, E., Defrance, J. , Priour, M., and Aballea, F., Coupling BEM and GFPE for complex outdoor sound propagation, Proceedings of Euronoise 2003, 2003, Naples, Italy.

56. Hampel, S., Langer, S., Cisilino, A., Coupling boundary elements to a ray tracing procedure, International Journal for numerical methods in engineering, 2008, 73, 427-445.

57. Van Renterghem, T., Salomons, E. and Botteldooren, D., Efficient FDTD-PE model for sound propagation in situations with complex obstacles and wind profiles, Acta Acustica united with Acustica, 2005, 91, 671-679.

58. Bérengier, M., Gauvreau, B., Blanc-Benon, P. and Juvé, D., Outdoor Sound Propagation: A Short Review on Analytical and Numerical Approaches, Acta Acustica united with Acustica, 2003, 89, 980-991.

59. Van Renterghem, T., Botteldooren, D., and Lercher, P., Comparison of measurements and predictions of sound propagation in a valley-slope configuration in an inhomogeneous atmosphere, Journal of the Acoustical Society of America, 2007, 121, 2522-2533.

60. Heimann, D. and Karle, R., A Linearized Euler Finite-Difference Time-Domain Sound Propagation Model with Terrain-Following Coordinates, Journal of the Acoustical Society of America, 2006, 119, 3813-3821.

61. Dragna, D., Blanc-Benon, P. and Poisson, F., Time-domain solver in curvilinear coordinates for outdoor sound propagation over complex terrain, Journal of the Acoustical Society of America, 2013, 133, 3751-3763.

62. Hesthaven, J., Gottlieb, S. and Gottlieb, D., Spectral methods for time-dependent problems, monographs on Applied and Computational Mathematics, Cambridge, UK, 2007.

63. Hornikx, M., Waxler, R. and Forssén, J., The extended Fourier pseudospectral time-domain method for atmospheric sound propagation, Journal of the Acoustical Society of America, 2010, 128, 1632–1646.

64. Spa, C., Garriga, A. and Escolano, J., Impedance boundary conditions for pseudo-spectral time-domain methods in room acoustics, Applied Acoustics, 2010, 71, 402–410.