DYNAMIC MODELS FOR A HEAT-LED ORGANIC RANKINE CYCLE

T. Erhart[a,*], M. van den Broek[b], U. Eicker[a]

[a]University of Applied Sciences Stuttgart
Schellingstrasse 24, 70190 Stuttgart,Germany
[b]Department of Industrial System and Product Design, Ghent University,
Graaf Karel de Goedelaan 5, 8500 Kortrijk, Belgium
[*]e-mail: tobias.erhart@hft-stuttgart.de
web page: www.zafh.net

EXTENDED ABSTRACT

KEYWORDS: ORC, heat-led, dynamic modelling, impulse turbine

INTRODUCTION

Drawn by the benefits of decentralised and renewable power supply, over 150 Organic Rankine Cycles (ORC), in a range from $400\,\mathrm{kW_{el}}$ to $2\,\mathrm{MW_{el}}$, have been installed in Central Europe. The majority of modules are biomass fired and heat-led by district heating networks. With rising fuel prices however, the economic situation has become critical for many of these facilities and improvements in efficiency are indispensable.

The research reported here, provides models to simulate units of that type in order to achieve higher cycle efficiencies. An operating power plant with a design power of $1\,\mathrm{MW_{el}}$ serves as validation.
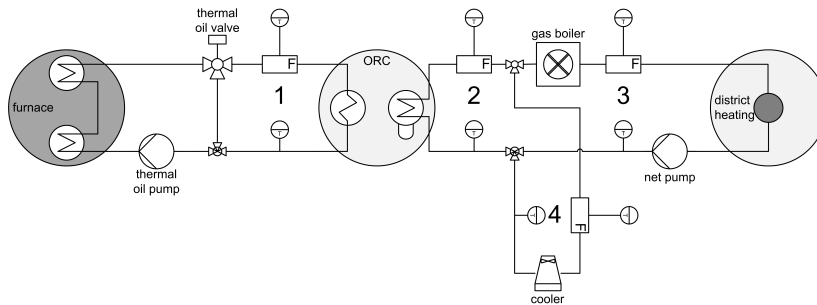
Figure 1: Entire power plant system: furnace, OR-cycle, district
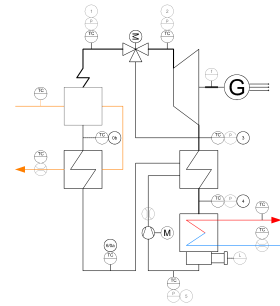

Figure 2: ORC scheme

AIM OF WORK

A heat-led ORC has mainly two degrees of freedom: the heat source temperature level and the heat sink cooling ability. Within the above mentioned constraints for the operation of such a power plant, there is still room for optimisation. Optimal operation would be achieved by a combination of low super-heating in the evaporator, high feed vapour pressure, and low condensing pressures. As a consequence, the mass flow through the turbine reaches its maximum. To achieve low condenser pressures, the sink temperature level must be kept as low as possible, while maintaining the capability to respond to a varying heat demand with smooth adaptation. In order to find an optimisation based on models, the interaction of the turbine and alternator with the system is observed hereinafter.

METHODOLOGY

Over several years, data have been monitored, unified and filtered to obtain data sets for modelling and validation. As depicted in Fig. 1 and Fig. 2, sensors throughout the entire plant and the OR-cycle provide data. The data is collected via the PROFIBUS node of the Siemens S7 PLC and provided by OPC-Server to a client application, writing the data into a database. In case of the turbine and alternator this includes: vapour input pressure and temperature, output pressure and temperature, the electric feed-in, turbine rotational frequency and overall mass flow in the system.

MODELLING

Turbine: the vapour expanding unit is a single-stage, axial impulse type turbine with a steady rotational speed of 3000 RPM. The pressure difference is converted by 24 De Laval Nozzles with an outflow angle α of 19°. Under design conditions the isentropic outflow velocity is 360 m/s. The nozzle efficiency of 92% is given by the manufacturer. This leads to a flow coefficient of 0.958 and resulting exit velocities around 300 m/s. Under design conditions, Mach 1 in the critical cross section is approximately 130 m/s. With this geometry (Fig. 4) the maximum blade efficiency could reach 89.4%. The design isentropic efficiency of the unit is 78% [manufacturer]. The turbine is modelled similar to
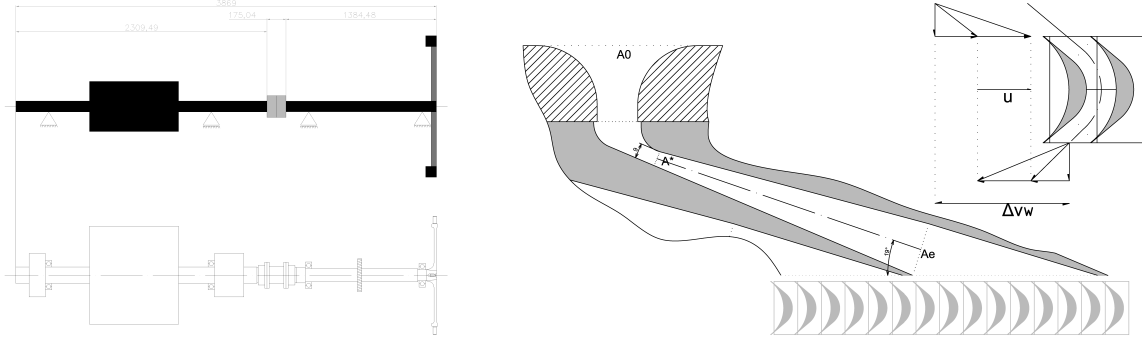
Figure 3: drive train with turbine and alternator arrangement

Figure 4: turbine nozzle and blade geometry as wrapped section

Stodola's Law of Cones [4] and Cooke's approach [2] to fit the imperfections of the real unit. The coefficient k_t represents the number of nozzles multiplied by the critical cross section of each nozzle and a correction factor, the discharge coefficient of a nozzle flow. The latter accounts for non-ideal effects in the De Laval Nozzle caused by imperfections in shape, such as edges.

$$\dot{m}_{turb} = k_t \times \sqrt{p_{in} \times \rho_{in}} \times \sqrt{1 - \left(\frac{p_{out}}{p_{in}}\right)^{\frac{\kappa+1}{\kappa}}} \tag{1}$$

The above reciprocal pressure ratio is usually denoted as r_s. As a first approximation the exponent of r_s was found to be 2 [1]. As the poly-tropic exponent of Octamethyltrisiloxane (MDM) for the states observed here ranges from 1.0 to 1.1 Cooke's value can be a reasonable simplification. Additionally to Cooke's version, two variations of the model have been introduced and tested. In variation one the parameter κ is set to an average

value for MDM. For variation two the value κ is calculate for each time step.

It is assumed that the isentropic efficiency is a function of the pressure ratio and the rotational speed and therefore of the resulting outflow angle from the nozzle to the blade. As this unit has an almost constant speed, under normal operation, the isentropic efficiency can be described only as a function of the pressure ratio β.

$$\eta_s = a \times atan\left(b \times \beta^2 + \frac{c}{\beta}\right) + d \times \beta + f \tag{2}$$

Fluid model: the property calculations for the fluid Octamethyltrisiloxane (MDM) in the system are based on the formulation of Nannan and Colonna implemented by Lemmon et al. [3] in the REFPROP library of NIST. For this work REFPROP has been used via the Fluidprop interface of MATLAB and modelica.

Mechanical model: the drive train at hands, including the turbine, a turbine shaft, a coupling and an alternator has a significant moment of inertia. In terms of the oscillating turbine speed, those tensors must be respected. The speed is kept within a range of $50\,\mathrm{Hz}$ $\pm 0.5\,\mathrm{Hz}$. This dispersion complies to ± 30 RPM. As a simplification the rotating masses are assumed to have homogeneous densities. This approach has a good fit for shafts and couplings. In the case of the rotating alternator masses the distribution in the rotor (e.g. copper coils, iron rotor) is surely not constant, but is the best possible guess. The inertia moment of this combination is governed by the turbine and alternator rotor. In order to validate those values a shut down procedure can be used. Measured data of a emergency stop provide the turbine speed from 3000 RPM to a full stop. We assume that the friction power loss of this arrangement at $50\,\mathrm{Hz}$ equals 2% of the nominal power, namely 20 kW. Furthermore, if we define that all relevant friction and inertia is concentrated in on node, the equation for the entire power train (see Fig. 1) can be written as:

$$\left(\tau_{shaft} + \tau_{fric}(\omega) + \sum J \times \dot{\omega}\right) \times \omega = \dot{m}_{tur} \times \Delta h_s \times \eta_s(\beta) \tag{3}$$

The friction torque consists of various friction components with constant, linear and quadratic characteristics. In this case a quadratic correlation to the frequency was chosen as a simplification. Under the above assumptions, we consequently obtain a friction torque of $63.7\,\mathrm{Nm}$, at full speed. Taking a look at an average shut-down procedure for this plant, as it has been measured plenty times during the observation period, an average of 480 seconds from 3000 RPM until total stand still can be found. The resulting angular acceleration is $-0.65\,\mathrm{rad/s^2}$. Neglecting fluid friction at the surface of the rotating parts and other effects (e.g. magnetic), the resulting tensor turns out to be $97.9\,\mathrm{kg\,m^2}$.
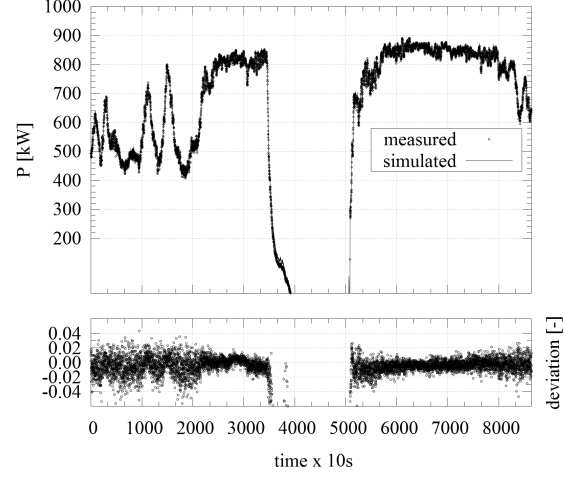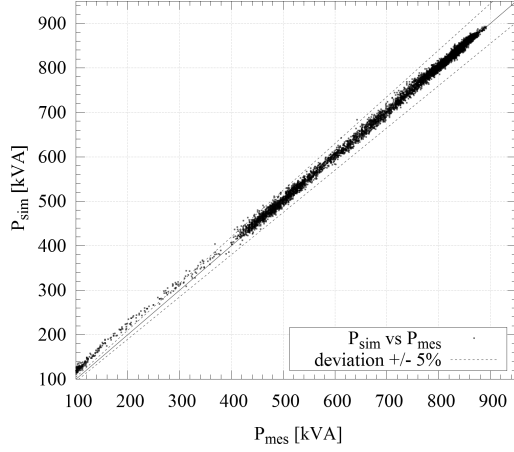
Alternator model: the alternator model is a four parameter fit based on design data of the manufacturer and validated through measured data:

$$\eta_{el} = a \times ln(x) + \frac{b}{x^2} + \frac{c}{x} + d \tag{4}$$

The variable x is the load in respect to the rated power of the unit.

RESULTS

As depicted in Fig. 5 and Fig. 6, the turbine model predicts the expected electric power

Figure 5: power output simulated versus measured (2012-01-02)

Figure 6: power output and deviation versus time (2012-01-02)

model	date	measured $\bar{P}_{el}$	W	simulated $\bar{P}_{el}$	W	deviation
-	[YYYY-MM-DD]	[kW]	[6xWh]	[kW]	[6xWh]	[-]
	2012-01-01	760.28	1094807.64	756.62	1089529.92	-0.4821%
$\kappa = \bar{\kappa}$	2012-01-02	592.47	853152.37	589.21	848467.89	-0.5491%
	2012-01-03	338.38	487265.80	337.09	485403.04	-0.3823%
	2012-01-01	760.28	1094807.64	756.62	1089526.21	-0.4824%
$\kappa(p,T)$	2012-01-02	592.47	853152.37	589.17	848402.75	-0.5567%
	2012-01-03	338.38	487265.80	337.06	485365.61	-0.3900%

Table 1: comparison of model and measured values for three data sets (1 day / 10 second steps)

well within a maximum deviation of $\pm 5\%$ across the whole load range. Larger errors can be found during load changes with an increment of more than $\pm 10\%$ per hour. The model shows the largest deviation in the case of a shut-down. Over a daily range the $\kappa(p,T)$-model has a tendency to under-predict the yield by 0.47%. The constant κ-model is slightly worse (0.48%). For an arrangement like the one at hand, the turbine can be described with six parameters. Five parameters for the isentropic efficiency characteristic and one for Stodola's k_t. Furthermore, a function for the friction torque is necessary. For a satisfying description of the alternator, a logarithmic fit with four parameters is sufficient. The mechanical inertia of the arrangement can be well estimated if data of shut-down procedures are available. In this model, the behaviour is represented by one tensor for the entire drive train. If there are no measured data available, the efficiency characteristic can be estimated by the geometry of the nozzle and turbine arrangement. More reliable results can be expected if the isentropic characteristic is fitted with real data.

LITERATURE

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