Internal model control for shank movement around the knee joint

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Summary. This paper derives an elementary mathematical model of the shank movement around the knee joint in the 2D sagittal plane. The resulting model is nonlinear which will be linearized in order to design an Internal Model Control (IMC) strategy. The control purpose is to regulate the angle of the shank by manipulating the torque applied to the knee joint. The performance of the IMC strategy is compared with that of a standard PID controller. The effect of the filter parameter a of the IMC strategy on the control performance is investigated. The results show that the control performance is enhanced by implementing IMC and the response of the controller is faster when $|a|$ is smaller.

Introduction

The knee joint is the biggest and most complicated joint in the entire human body and therefore, most exposed to injuries. Two possible treatments are discussed: Total Knee Replacement (TKR) and active orthoses which empower the human movement [1]. An important device in the evaluation of TKR is a dynamic knee rig [2]. Both the knee rig and the active orthoses have a major necessity for robust closed-loop control in order to ensure repeatability of the mimicked natural movement. Current control strategies include visual control or control of the quadriceps force for the dynamical knee rig [3]. Active orthoses are currently controlled by PID controllers [4] but also through measurements of EMG signals [5].
This paper describes an elementary model of the movement of the lower part of the leg, i.e. the shank, around the knee joint. The shank movement around the knee joint is described by a nonlinear equation of motion. To control the angle of the shank during the movement, the authors choose to design an Internal Model Control (IMC) strategy.
This paper is structured as follows: the next section gives the derivation of the mathematical model of the movement of the shank around the knee joint. Afterwards, the IMC strategy is explained and simulations and results are given. A summary of this paper is given in a conclusion section.

Mathematical model

A 2D mathematical model in the sagittal plane is derived to obtain a transfer function of the system describing the shank movement around the knee joint. Figure 1 shows the biomechanical model of the shank movement when the upper leg is fixed in horizontal position. Both segments are connected by a revolute joint. The angle θ has a range of motion of 90° i.e. the shank can go from a vertical position to a horizontal position.
The equation of motion can be obtained by taking the sum of all the torques around the knee joint. There is a torque caused by gravity, a torque caused by the inertia of the shank, a torque due to viscous damping, a torque as a result of the joint stiffness and the applied torque $T(t)$. Summing all these torques and taking into account their direction we get $I_T\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt} + K\theta(t) + m_T g r_T sin(\theta(t)) = T(t)$ with I_T the inertia of the shank around the knee joint, B the viscous damping coefficient of the knee joint and K the knee joint stiffness. The value for the shank inertia can be found in [6] as 0.4419 kgm^2. The values for the viscous damping coefficient and the joint stiffness were determined in [7] ($K = 42.2102$ Nm/rad and $B = 6.7453$ Nms/rad).
After linearization around the equilibrium point $\theta^* = 0°$ the equation of motion is $I_T\ddot{\theta} + B\dot{\theta} + K\theta(t) + m_T g r_T\theta(t) = T(t)$ with $\theta(t)$ and $T(t)$ representing deviation values. Taking the Laplace transformation, where all initial conditions are set to zero, results in the transfer function of the damped system (with the model parameters inserted) $TF(s) = \dfrac{1}{0.44s^2 + 6.75s + 49.07}$ or in discrete time $TF(q^{-1}) = \dfrac{0.00023(q^{-1} + 0.92q^{-2})}{1 - 1.77q^{-1} + 0.79q^{-2}}$ with sampling time $T_s = 0.015$ s.

Design of IMC controller

The IMC strategy is a model based control technique that has the potential to achieve good closed loop performance, while taking into account the model structure of the process [8]. In a discrete-time formulation, the process input is used at each

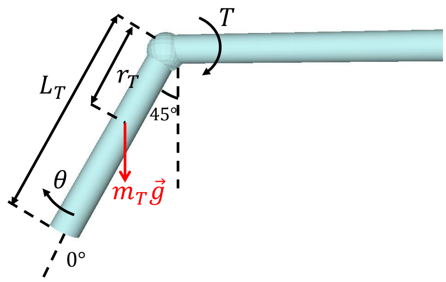

Figure 1: Biomechanical model consisting of the upper part of the leg and the shank. Both segments are connected by a revolute joint. T is the torque applied at the knee joint and the input of the system while the angle θ is the output. The value $\theta = 0$ corresponds with a position at 45° from the vertical line. L_T is the length of the shank (0.435 m). r_T is the distance between center of mass of the shank and the knee joint axis (0.188 m). m_T is the mass of the shank (3.72 kg). g which is the gravitational constant (9.81 N/kg). Values for these model parameters can be found in literature [6].

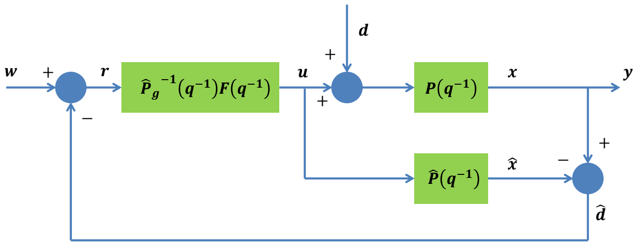

Figure 2: IMC scheme where $P(q^{-1})$ is the process, $\hat{P}(q^{-1})$ is the linear process model, $\hat{P}_q^{-1}(q^{-1})$ the inverse of the invertable part of $\hat{P}(q^{-1})$ and $F(q^{-1})$ the IMC filter. d is a disturbance signal, w is the reference, y is the process output and u is the control effort.

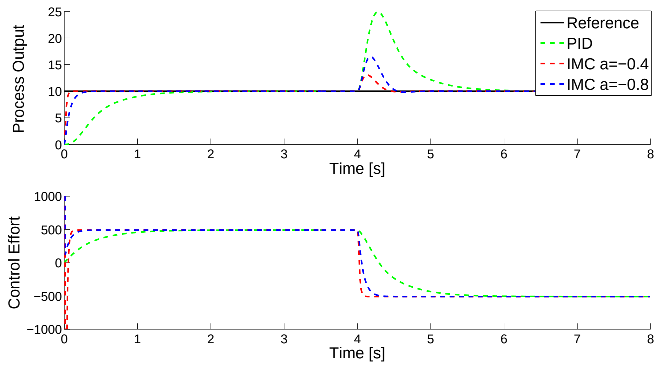

Figure 3: Results of the simulations. Notice that the IMC strategy performs better than the classical PID controller, resulting in a faster response. When looking at the effect of the a parameter of the IMC, we can clearly see that $|a|$ is smaller, the response of the system is faster. The filter parameter it is related to the closed-loop speed: if $|a|$ is closer to 1, the settling time will be bigger.

sampling instant to calculate an inverse function which compensates for the process dynamics while ensuring a desired closed loop performance trajectory. To calculate the inverse of the process, the process must be split in an invertible (good) part $\hat{P}_g(q^{-1})$ and a noninvertible (bad) part $\hat{P}_b(q^{-1})$. The closed loop control scheme is depicted in figure 2.

A 'basic' IMC filter is designed to make the controller transfer function fully proper: $F(q^{-1}) = \dfrac{(1+a)^n}{(1+aq^{-1})^n}$ with steady state gain $F(1) = 1$ and a the filter parameter defined as $a = -e^{T_s/\lambda}$. The (negative) values of this design parameter are in the range $0 << |a| < 1$. In simulation we will use $a = -0.4$ and $a = -0.8$.

Results and Simulations

To compare the control performance, a classic PID controller for the system is tuned with FRtool [9] with following specifications: Robustness > 0.9, Settling time < 2 s, % overshoot < 1 %. The resulting controller parameters are: $K_p = 12$, $T_i = 0.1$ and $T_d = 0.025$ for the standard PID form: $PID(s) = K_p * \left(1 + \frac{1}{T_i s} + T_d s\right)$.

A step reference with an amplitude of 10 at $t = 0$ is applied to the system in figure 2. At $t = 4$ a disturbance with amplitude 1000 is applied to check the disturbance rejection of the designed controllers. The resulting process output and control effort is shown in figure 3.

Conclusions

In this paper an IMC strategy is applied to an elementary model of the shank movement around the knee joint. The control purpose is to regulate the angle of the shank by manipulating the torque applied to the knee joint. We can conclude that the performance of the closed-loop control is increased by applying the IMC strategy instead of the classical PID controller. We conclude that the IMC design parameter a has an effect on the speed of the response. When $|a|$ is smaller, the response of the system is faster. As a general conclusion, we can state that IMC is a good strategy to control the movement of the shank around the knee joint.

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