Piecewise Smoothed Value Picking Regularization Applied to 2D TM and TE Inverse Scattering

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***Abstract*—The Stepwise Relaxed Value Picking (SRVP) regularization technique, proposed earlier for the iterative reconstruction of piecewise (quasi-)homogeneous objects, is a non-spatial technique, whereby the reconstruction unknowns are clustered around a limited number of—*a-priori* unknown—reference values. Artifacts have been observed in some 2D and 3D complex permittivity reconstructions. This paper therefore combines the non-spatial SRVP technique with a spatial smoothing technique, whereby the reference values provided by the former—in each iteration—are employed by the latter to define separate smoothing regions. This way edges are preserved, since the spatial smoothing constraints in the cost function are active within but not accross the region boundaries. This combined technique, denoted as Stepwise Relaxed Piecewise Smoothed Value Picking (SRPSVP) regularization, is formulated for the 2.5D microwave inverse scattering problem and is illustrated with reconstructions from the Institut Fresnel 2D scattering database.**

***Index Terms*—Regularization, piecewise smoothing, reconstruction, optimization, inverse scattering, microwave imaging, complex permittivity.**

I. Introduction

Regularization by imposing spatial smoothing constraints on the entire reconstructed profile as in [1]–[3] is less suitable for piecewise homogeneous objects. Imaging piecewise homogeneous objects is of interest in various applications in non-destructive testing [4] and subsurface sensing [5]. Several spatial smoothing techniques have been proposed to enhance edges in nonlinear inverse scattering algorithms, e.g. L1-norm total variation (TV) regularization [6], edge-preserving regularization [7] with weighted L2-norm TV [8] and with various potential functions [9], [10]. Other methods are dedicated specifically to piecewise homogeneous profiles, e.g. the level-set algorithm for binary or *n*-ary objects [11], [12]. Stepwise Relaxed Value Picking (SRVP) regularization [13] was proposed for piecewise (quasi-)homogeneous objects and has yielded promising results for the inversion of the three-dimensional (3D) [14] and 2D [15] microwave scattering databases from Institut Fresnel and for 2.5D millimeter-wave imaging of concealed objects [16].

Value Picking (VP) regularization is a non-spatial technique—it does not operate on the spatial neighborhood of the reconstruction variable—which gradually clusters the reconstruction unknowns around a limited number of *a-priori* unknown reference values—the VP values—which in turn are adjusted during the iterative reconstruction process by means of a well-chosen regularizing function. The regularization thus encourages each reconstruction variable to converge towards one of these VP values. The basic idea of enforcing piecewise homogeneity by introducing reference values in the cost function has been explored for binary objects in [17], [18] and for one extra permittivity value in [19], but the choice function in those previous works differs from the one used in the VP regularization technique. The VP choice function is defined for any number of permittivities and has well-documented properties [13]; in particular it is "less than quadratic", hence it easily can be incorporated in the Gauss-Newton algorithm through a sequence of quadratic approximations. The Stepwise Relaxed (SR) approach refers to applying a severe regularization in the beginning of the iterations, by using only one VP value—the complex permittivity of the background—and gradually relaxing the regularization by adding new VP values. This considerably improves the convergence of the algorithm. Some reconstructions with SRVP regularization [13]–[15] have shown isolated (groups of) cells that are attracted to a wrong VP-value. Note that those reconstructions were quite challenging, since only single frequency data were used. When the information content of the data is low with respect to the number of degrees of freedom [20], additional regularization is recommended.

This paper thus proposes to combine the non-spatial SRVP technique with a piecewise spatial smoothing technique. This combined technique, further denoted as Stepwise Relaxed Piecewise Smoothing Value Picking (SRPSVP) regularization, is more explicit in enforcing homogeneity within the image parts that are recognized to be so: it additionally imposes spatial smoothing within but not across a group of neighbor cells that are attracted to the same VP value. Note in this context also the approach in the Bayesian estimation framework presented in [21] for a finite number of dielectric and conductive materials, which applies a Gauss-Markov field for the distribution of the contrast with a hidden Potts-Markov field for the class of materials. Furthermore the presence of one-cell-artifacts can be strongly reduced if smoothing accross such cells is allowed. Ofcourse, this should be avoided in applications where the object actually contains small inclusions with the size of one cell.

The proposed method is discussed in Section III and illustrated in Section IV with permittivity reconstructions from the Institut Fresnel 2D database [22], which contains multiple-frequency scattered field data for piecewise homogeneous objects from TM- and TE-polarized 2D incident fields—only single frequency data are used here. It is advantageous to employ the 2.5D forward scattering solver [23], since both

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polarization cases then can be computed with the same solver. Consequently, the inverse scattering problem is formulated in Section II for this more general 2.5D configuration, thus with 2D material properties and 3D fields. An expression for the 2.5D field derivatives is derived in the Appendix.

II. The Electromagnetic Inverse Scattering Problem

Consider an inhomogeneous, possibly lossy, dielectric cylinder with an arbitrary cross-sectional shape and with the axis along the z-direction in a 3D cartesian coordinate system $\boldsymbol{\rho} = \mathbf{r} + z\mathbf{u}_z$, where $\mathbf{r} = x\mathbf{u}_x + y\mathbf{u}_y$. A 2D rectangular investigation domain $\mathcal{D}$ is defined as the area in the cross-sectional x, y-plane where the (unknown) complex permittivity $\epsilon(\mathbf{r}) = \epsilon_0\epsilon_r(\mathbf{r}) + j\sigma(\mathbf{r})/\omega$ can differ from the free space permittivity ϵ_0. Here $\epsilon_r(\mathbf{r})$ is the relative dielectric permittivity, $\sigma(\mathbf{r})$ the conductivity and ω the angular frequency. To numerically solve the inverse problem, the unknown $\epsilon(\mathbf{r})$ is parameterized over $\mathcal{D}$ by approximating it as a piecewise constant function on a uniform grid with N^x and N^y identical square cells in, respectively, the x- and y-directions :

$$\epsilon(\mathbf{r}) \approx \sum_{m=0}^{N^x-1} \sum_{l=0}^{N^y-1} \epsilon_0 \; \epsilon_{m,l} \; \Phi_{m,l}(\mathbf{r}), \quad \mathbf{r} \in \mathcal{D} \tag{1}$$

with $\Phi_{m,l}$ the unit expansion function and $\epsilon_{m,l}$ the unknown coefficients. The latter are gathered in the $N = N^x N^y$-dimensional relative complex permittivity vector $\boldsymbol{\epsilon}$, also $\{\epsilon_\nu\}$, $\nu = 1 \ldots N$.
The input data of the inverse problem are a set of scattered fields $\mathbf{E}_i^{\mathrm{s}}(\mathbf{r}_r, z_r) = \mathbf{E}_i(\mathbf{r}_r, z_r) - \mathbf{E}_i^{\mathrm{i}}(\mathbf{r}_r, z_r)$, obtained by successively illuminating the scatterer with known incident fields $\mathbf{E}_i^{\mathrm{i}}(\mathbf{r}, z)$, $i = 1 \ldots N^i$, and by measuring the corresponding total fields $\mathbf{E}_i(\mathbf{r}, z)$ in a set of receiver points $(\mathbf{r}_r, z_r)$, $r = 1 \ldots N^r$. A time dependency $\exp(-j\omega t)$ of the fields is assumed. The measured scattered fields are collected in the N^d-dimensional vector $\mathbf{e}^{meas}$, with $N^d = 3N^iN^r$, the factor 3 stands for the x, y, z components. The excitations are realized by (3D) angular diversity (N^t transmitter positions or propagation directions) and polarization diversity (N^p polarizations), hence $N^i = N^tN^p$, where for $N^p = 1$, TM- or TE- and for $N^p = 2$, TM- and TE-polarizations are applied.
The relation between the scattered field and the permittivity is governed by a non-linear integral equation [16]. The inverse problem thus is solved iteratively by minimizing a cost function $\mathcal{F}$, which consists of a data fit and regularization terms:

$$\mathcal{F}(\boldsymbol{\epsilon}, \mathbf{c}) = \mathcal{F}^{\mathcal{LS}}(\boldsymbol{\epsilon}) + \gamma\mathcal{F}^{\mathcal{P}}(\boldsymbol{\epsilon}, \mathbf{c}) + \zeta\mathcal{F}^{\mathcal{PS}}(\boldsymbol{\epsilon}) \tag{2}$$

with $\mathcal{F}^{\mathcal{P}}$ the VP regularizing function, $\mathbf{c}$ a $P-$dimensional vector ($P \ll N$) with the complex VP values c_p, $\mathcal{F}^{\mathcal{PS}}$ the Piecewise Smoothing (PS) regularizing function and γ and ζ positive regularization parameters. The least squares data fit,

$$\mathcal{F}^{\mathcal{LS}}(\boldsymbol{\epsilon}) = \frac{\|\mathbf{e}^{scat}(\boldsymbol{\epsilon}) - \mathbf{e}^{meas}\|^2}{\|\mathbf{e}^{meas}\|^2} \tag{3}$$

is a measure for the difference between the experimentally obtained scattered field and the corresponding simulated scattered field $\mathbf{e}^{scat}(\boldsymbol{\epsilon})$—also an N^d-dimensional vector—for the current value of $\boldsymbol{\epsilon}$, which is computed with a 2.5D volume integral equation solver [23].

III. Regularization and Optimization

A. The VP regularization term

The function $\mathcal{F}^{\mathcal{P}}$ is given by [13]

$$\mathcal{F}^{\mathcal{P}}(\boldsymbol{\epsilon}, \mathbf{c}) = \frac{1}{N} \sum_{\nu=1}^{N} f^P(|\epsilon_\nu - c_1|^2, \ldots, |\epsilon_\nu - c_P|^2) \tag{4}$$

where f^P is the P-dimensional choice function, defined as

$$f^P(u_1, \ldots, u_P) = F^P(u_1, \ldots, u_P; 0) \tag{5}$$

with $F^P(u_1, \ldots, u_P; x)$ defined through the recursion formula

$$F^P(u_1, \ldots, u_P; x) = (u_P + x)\,\frac{F^{P-1}(u_1, \ldots, u_{P-1}; x)}{F^{P-1}(u_1, \ldots, u_{P-1}; u_P + x)} \tag{6}$$

with $F^1(u_1; x) = u_1 + x$.
The function (4) can be reformulated as a weighted sum of penalty functions $|\epsilon_\nu - c_p|^2$ [13]:

$$\mathcal{F}^{\mathcal{P}}(\boldsymbol{\epsilon}, \mathbf{c}) = \frac{1}{N} \sum_{\nu=1}^{N} \sum_{p=1}^{P} b_{p,\nu}^P(\boldsymbol{\epsilon}, \mathbf{c})|\epsilon_\nu - c_p|^2. \tag{7}$$

The behavior of $\mathcal{F}^{\mathcal{P}}$ is as follows: (i) when the permittivity of a cell is close to one of the VP values, the choice function tries to enforce equality with this VP value (i.e. the corresponding weight b^P of that term in (7) is close to 1), (ii) when there is no clear preference of a permittivity cell for a particular VP value, no choice is made (b^P being somewhere intermediate between 0 and 1) and (iii) VP values that are clearly far away from the considered permittivity cell are neglected (their b^P are almost zero). Each VP value is initialized randomly (but different from already present VP values) within some predefined upper and lower bounds—the values of these bounds are not critical—and is updated in each iteration, see Section III-C, except for c_P, which is kept fixed to the background permittivity.

B. The PS regularization term

PS regularization penalizes permittivity fluctuations within but not across image parts that are considered—at a given iteration—to be homogeneous, further denoted as smoothing regions. Smoothing regions are derived from a mapping of the grid cells to VP-groups. A VP-group is the collection of cells that clusters around one VP value and is determined as follows: at a given iteration, the weights $b_{p,\nu}^P(\boldsymbol{\epsilon}, \mathbf{c})$ in (7) for cell ν indicate how close the permittivity of this cell is to each VP value c_p; the two largest weights for cell ν are compared and if their difference is larger than a threshold value (e.g. 0.2), then cell ν is assigned to the VP-group that corresponds with the largest weight; otherwise, it is assigned to the indefinite group. VP-groups (and also the indefinite group) can consist of several spatially disconnected image parts. A smoothing region is an as large as possible spatially connected group of cells belonging to one VP-group (or to the indefinite group). Whereas with multiplicative smoothing (MS) as in [3]—the cost function then is $\mathcal{F} = \mathcal{F}^{\mathcal{LS}}\left(1 + \alpha\mathcal{F}^{\mathcal{R}}\right)$, with α a positive regularization parameter—the regularizing function $\mathcal{F}^{\mathcal{R}}$ penalizes permittivity variations over all cell boundaries

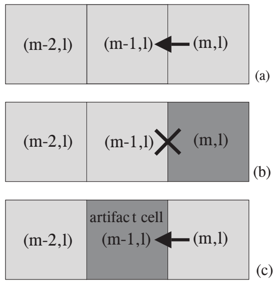


Fig. 1. Illustration of defining $\mathbf{S}^1_{m,l}$. Cells of the same color belong to the same VP-group. Allowed (arrow) and prohibited (cross) smoothing.

in the grid, $\mathcal{F}^{\mathcal{PS}}$ only does so over cell boundaries within each smoothing region. This is achieved by two matrices, $\mathbf{S}^1$ and $\mathbf{S}^2$ for the x- and y-directions, respectively,

$$\begin{aligned} \mathcal{F}^{\mathcal{PS}}(\boldsymbol{\epsilon}) &= \frac{1}{\mathcal{N}^{\mathcal{R}}} \sum_{m=0}^{N^x} \sum_{l=0}^{N^y-1} \mathbf{S}^1_{m,l} \, |\epsilon_{m,l} - \epsilon_{m-1,l}|^2 \\ & + \frac{1}{\mathcal{N}^{\mathcal{R}}} \sum_{m=0}^{N^x-1} \sum_{l=0}^{N^y} \mathbf{S}^2_{m,l} \, |\epsilon_{m,l} - \epsilon_{m,l-1}|^2 \end{aligned} \qquad (8)$$

where $\mathcal{N}^{\mathcal{R}}$ is a normalization constant which accounts for the dimensions of the object and the size of the cells. If cell $(m-1,l)$ belongs to the same VP-group as cell (m,l), smoothing in the x-direction is allowed and $\mathbf{S}^1_{m,l} = 1$, otherwise $\mathbf{S}^1_{m,l} = 0$. This is also illustrated in Fig. 1(a) and (b), where cells having the same color are assigned to the same VP-group. Similarly, if cells $(m, l-1)$ and (m,l) belong to the same VP-group (or to the indefinite group), $\mathbf{S}^2_{m,l} = 1$, otherwise $\mathbf{S}^2_{m,l} = 0$. Note that the MS regularizing function $\mathcal{F}^{\mathcal{R}}$ is as (8) with all entries in $\mathbf{S}^1$ and $\mathbf{S}^2$ equal to 1. To smooth out one-cell-artifacts nested in a quasi-homogeneous image part, the second neighbor is taken into account. For example, if the neighbor cell $(m-1,l)$ does not belong to the same VP-group as cell (m,l) but cell $(m-2,l)$ does, it is assumed that cell $(m-1,l)$ was attracted to a wrong VP value, hence it is also allowed to smooth towards this cell and $\mathbf{S}^1_{m,l} = 1$, see Fig. 1(c).

C. The optimization

Each iteration k in the optimization of (2) is a three-step procedure. Firstly, the permittivity profile is updated from iteration k to $k+1$ as:

$$\boldsymbol{\epsilon}_{k+1} = \boldsymbol{\epsilon}_k + \beta_k \mathbf{s}_k \qquad (9)$$

where $\mathbf{s}_k$ is a Gauss-Newton descent direction and β_k is the step size which approximately minimizes the cost function $\mathcal{F}$ along this direction, computed with a line search [24]. During this step, the VP values $\mathbf{c}$ in $\mathcal{F}^{\mathcal{P}}$ and the smoothing matrices $\mathbf{S}^1, \mathbf{S}^2$ in $\mathcal{F}^{\mathcal{PS}}$ are kept fixed. The Gauss-Newton direction $\mathbf{s}_k$ for $\mathcal{F}$ is the solution of

$$\left(\mathbf{J}^H_k \mathbf{J}_k + \lambda^2 \boldsymbol{\Sigma}_k\right) \mathbf{s}_k = -\left(\mathbf{J}^H_k \left[\mathbf{e}^{scat}_k - \mathbf{e}^{meas}\right] + \lambda^2 \boldsymbol{\Omega}^*_k\right) \qquad (10)$$

where λ^2 denotes $||\mathbf{e}^{meas}||^2$, $(.)^H$ and $(.)^*$ stand for conjugate transpose and complex conjugate, respectively and

$$\begin{aligned} \boldsymbol{\Omega}_k &= \gamma \, \boldsymbol{\Omega}^{\mathcal{P}}_k + \zeta \, \boldsymbol{\Omega}^{\mathcal{PS}}_k \\ \boldsymbol{\Sigma}_k &= \gamma \, \boldsymbol{\Sigma}^{\mathcal{P}}_k + \zeta \, \boldsymbol{\Sigma}^{\mathcal{PS}}_k. \end{aligned} \qquad (11)$$

Equation (10) follows from the equation for the Newton correction in complex notation [25] for the cost function (2), by neglecting in the Hessian matrix the second order derivatives of the scattered field with respect to the permittivity unknowns, in a manner similar as in [13]. In the following, the subscript k is mostly omitted. Elements of the Jacobian matrix $\mathbf{J}$ are $\mathbf{J}_{n\nu} = \partial \left(\mathbf{e}^{scat}\right)_n / \partial \epsilon_\nu$, an expression is derived in the Appendix. $\boldsymbol{\Omega}^{\mathcal{P}}$ and $\boldsymbol{\Omega}^{\mathcal{PS}}$ contain the first order derivatives of the regularizing functions:

$$\Omega^{\mathcal{P}*}_\nu = \frac{\partial Q^{\mathcal{P}}(\boldsymbol{\epsilon}, \mathbf{c}; \boldsymbol{\epsilon}_k, \mathbf{c}_k)}{\partial \epsilon^*_\nu} = \frac{1}{N} \sum_{p=1}^{P} b^P_{p,\nu}(\boldsymbol{\epsilon}_k, \mathbf{c}_k)(\epsilon_\nu - c_p) \qquad (12)$$

$$\begin{aligned} \Omega^{\mathcal{PS}*}_\nu = \frac{\partial \mathcal{F}^{\mathcal{PS}}}{\partial \epsilon^*_{m,l}} &= \frac{1}{\mathcal{N}^{\mathcal{R}}} \Big[\mathbf{S}^1_{m,l}\left(\epsilon_{m,l} - \epsilon_{m-1,l}\right) + \mathbf{S}^2_{m,l}\left(\epsilon_{m,l} - \epsilon_{m,l-1}\right) \\ &+ \mathbf{S}^1_{m+1,l}\left(\epsilon_{m,l} - \epsilon_{m+1,l}\right) + \mathbf{S}^2_{m,l+1}\left(\epsilon_{m,l} - \epsilon_{m,l+1}\right)\Big] \end{aligned} \qquad (13)$$

where $Q^{\mathcal{P}}$ is an approximation to $\mathcal{F}^{\mathcal{P}}$, obtained by considering the weights at their current values $b^P_{p,\nu}(\boldsymbol{\epsilon}_k, \mathbf{c}_k)$ as constants. $\boldsymbol{\Sigma}^{\mathcal{P}}$ and $\boldsymbol{\Sigma}^{\mathcal{PS}}$ contain the second order derivatives:

$$\Sigma^{\mathcal{P}}_{\nu,\mu} = \frac{\partial^2 Q^{\mathcal{P}}(\boldsymbol{\epsilon}, \mathbf{c}; \boldsymbol{\epsilon}_k, \mathbf{c}_k)}{\partial \epsilon_\mu \partial \epsilon^*_\nu} = \delta_{\nu,\mu} \frac{1}{N} \sum_{p=1}^{P} b^P_{p,\nu}(\boldsymbol{\epsilon}_k, \mathbf{c}_k) \qquad (14)$$

with $\delta_{i,j}$ the Kronecker symbol; for the diagonal elements:

$$\Sigma^{\mathcal{PS}}_{\nu,\nu} = \frac{\partial^2 \mathcal{F}^{\mathcal{PS}}}{\partial \epsilon_{m,l} \partial \epsilon^*_{m,l}} = \frac{1}{\mathcal{N}^{\mathcal{R}}} \left[\mathbf{S}^1_{m,l} + \mathbf{S}^2_{m,l} + \mathbf{S}^1_{m+1,l} + \mathbf{S}^2_{m,l+1}\right] \qquad (15)$$

and for the non-diagonal elements (which are zero except if ν denotes a neighbor of μ):

$$\begin{aligned} \Sigma^{\mathcal{PS}}_{\nu,\mu} = \frac{\partial^2 \mathcal{F}^{\mathcal{PS}}}{\partial \epsilon_{p,q} \partial \epsilon^*_{m,l}} &= -\frac{1}{\mathcal{N}^{\mathcal{R}}} \Big[\mathbf{S}^1_{m,l} \delta_{p,m-1} \delta_{q,l} + \mathbf{S}^2_{m,l} \delta_{p,m} \delta_{q,l-1} \\ &+ \mathbf{S}^1_{m+1,l} \delta_{p,m+1} \delta_{q,l} + \mathbf{S}^2_{m,l+1} \delta_{p,m} \delta_{q,l+1}\Big]. \end{aligned} \qquad (16)$$

Note that with MS regularization (used as a benchmark in Section IV) the employed modified Gauss-Newton direction satisfies an equation as (10) with $\boldsymbol{\Omega}_k = \boldsymbol{\Omega}^{\mathcal{R}}_k$, $\boldsymbol{\Sigma}_k = \boldsymbol{\Sigma}^{\mathcal{R}}_k$ and $\lambda^2 = \alpha ||\mathbf{e}^{meas}||^2 \mathcal{F}^{\mathcal{LS}}_k / (1 + \alpha \mathcal{F}^{\mathcal{R}}_k)$ [3].

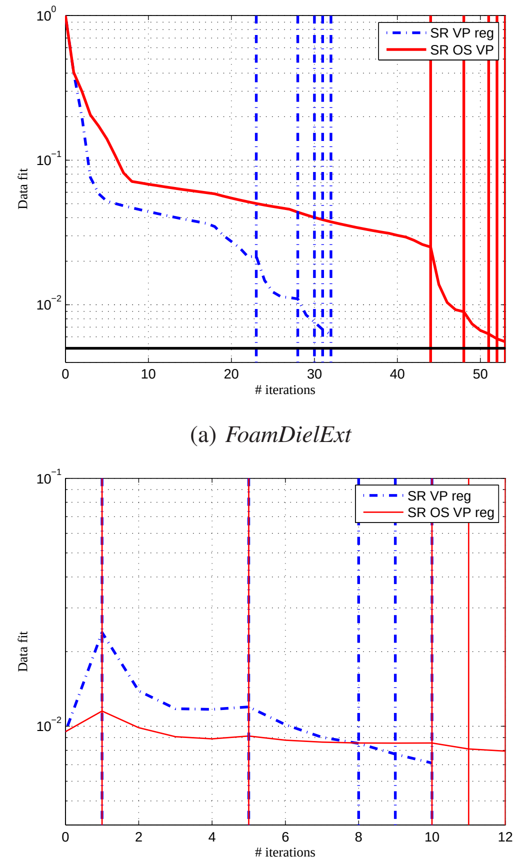
Secondly, the VP values $\{c_p\}$ are updated, while the permittivity and smoothing matrices are kept fixed. Since they are subject to upper and lower bounds on their real and imaginary parts, a constrained optimization is performed by an active set method [24], which acts on $\mathcal{F}^{\mathcal{P}}$ only. Similarly as with SRVP regularization, VP values are also updated whenever a new VP value is introduced.

Thirdly, the VP-groups/indefinite group and next the smoothing matrices $\mathbf{S}^1$ and $\mathbf{S}^2$ are updated, see Section III-B. The iterations are stopped when the data fit reaches the noise level, $\mathcal{F}^{\mathcal{LS}} \approx T^N$. The noise level $T^N = \mathcal{F}^{\mathcal{LS}}(\epsilon^0)$ is defined as the data fit for ϵ^0, which is the discretized permittivity profile that yields the closest approximation to the true profile.

With *MS regularization* it was observed [3] that the data fit is able to reach T^N with choices of the regularization parameter α in a wide range of values and that $\mathcal{F}^{\mathcal{LS}}$ is not much further minimized once this happens; a rather large α in this range yields an appropriately smoothed reconstruction, see the discrepancy principle [26].

With *SRVP regularization* the iterations start with only one VP value (strong regularization) and proceed until a local minimum of the corresponding cost function $\mathcal{F}$ is reached (i.e. its gradient is small) or until $\mathcal{F}^{\mathcal{LS}}$ increases again. For a sufficiently large regularization parameter γ, this first step terminates with $\mathcal{F}^{\mathcal{LS}} > T^N$. The regularization then is relaxed by adding an extra VP value and the optimization proceeds as before. New VP values are added this way until $\mathcal{F}^{\mathcal{LS}}$ reaches (an estimate of) T^N. Ideally, $P = P_0$ at this stage, with P_0 the number of different permittivity values in the exact profile, but when γ is chosen too large, the algorithm typically stops with $P > P_0$; some of these VP values tend to merge such that still a satisfactory reconstruction is obtained. When γ is too small, $\mathcal{F}^{\mathcal{LS}}$ easily reaches T^N, even with too few VP values, but the reconstruction then is of poorer quality.

In this paper, the choice of γ results from numerical experimentation, but *a priori* knowledge of the exact profile—apart from its piecewise-homogeneous character with $P_0 \ll N$—is not assumed: if the final reconstruction shows insufficient clustering of the permittivity unknowns around the VP values, a larger γ is tried; if the clustering is sufficient, a smaller γ can be tried to see if a comparable clustering can be achieved with fewer VP values and/or with fewer iterations. We did not perform an in-depth study on the choice of the parameters γ and ζ in case of *SRPSVP regularization*. In the examples of Section IV, γ is chosen as with the SRVP regularization case and ζ as with the MS regularization.



(a) *FoamDielExt*

(b) *FoamTwinDiel*

Fig. 2. Data fit $\mathcal{F}^{\mathcal{LS}}$ as a function of the iteration number: SRVP (dash-dots), SRPSVP (solid). Vertical lines indicate when a VP value was added.

IV. Reconstructions From Experimental Data

A. Objects, measurement set-up and general settings

Single frequency scattering data at 4 GHz ($\lambda_0 = 74.9$ mm) are considered for two objects from the Institut Fresnel 2D database [22]: the *FoamDielExt* object, which consists of a plastic cylinder with radius $r_a = 15.5$ mm $\approx 0.2\lambda_0$ and relative permittivity $\epsilon_{r,a} = 3 \pm 0.3$ that is placed against a foam cylinder with $r_b = 40$ mm $\approx 0.5\lambda_0$ and $\epsilon_{r,b} = 1.45 \pm 0.15$ (Fig. 3(a)) and the *FoamTwinDiel* object, which is as *FoamDielExt* plus an extra plastic cylinder off-centered inside the foam cylinder, with their centers 5mm apart (Fig. 3(e)).

The illumination - receiver configuration, with 360 (1° spaced) possible antenna positions on a circle with radius 1.67 m in the xy-plane, is detailed in [27]. Here, we use a subset of 8 (45° spaced) transmitter positions for *FoamDielExt* and 18 (20° spaced) positions for *FoamTwinDiel*, and 241 receiving antenna positions on an arc (from 60° to 300°) facing the source (when at 0°). We invert both TM and TE data simultaneously, whence separate inversions are presented in [28], the only contribution in [22] that exploits both polarizations for the considered objects. This means that we include both polarizations in the field vectors $\mathbf{e}^{meas}$ and $\mathbf{e}^{scat}$ in (3), e.g. by filling each vector first with the TE-data followed by the TM-data. Since the fields in this paper are considered purely 2D [27], all TM-fields are parallel to the z-axis, in particular $\mathbf{E}^{\mathrm{s,TM}} = E_z^{\mathrm{s,TM}}\mathbf{u}_z$, $E_x^{\mathrm{s,TM}} = 0, E_y^{\mathrm{s,TM}} = 0$, and all TE-fields are parallel to the xy-plane, in particular $\mathbf{E}^{\mathrm{s,TE}} = E_x^{\mathrm{s,TE}}\mathbf{u}_x + E_y^{\mathrm{s,TE}}\mathbf{u}_y$, $E_z^{\mathrm{s,TE}} = 0$ (the TE scattered field furthermore is tangential to the measurement circle). For this configuration, the 2.5D solver [23] solves both polarization cases at once, if the incident field is chosen as the sum of the TE and TM incident fields at a given source position, i.e. $\mathbf{E}^{\mathrm{i}} = E_x^{\mathrm{i,TE}}\mathbf{u}_x + E_y^{\mathrm{i,TE}}\mathbf{u}_y + E_z^{\mathrm{i,TM}}\mathbf{u}_z$ yields $\mathbf{E}^{\mathrm{s}} = E_x^{\mathrm{s,TE}}\mathbf{u}_x + E_y^{\mathrm{s,TE}}\mathbf{u}_y + E_z^{\mathrm{s,TM}}\mathbf{u}_z$. The dimension of the data vector $\mathbf{e}^{\mathrm{meas}}$ then is $N^d = 3N^tN^r = 5784$ complex numbers for *FoamDielExt* and $N^d = 13014$ for *FoamTwinDiel*. A simple calibration is applied to match phase and energy between measured and simulated fields [27]: all measured field values are multiplied by a complex factor, which is the ratio of the simulated and measured incident fields at the receiver location opposite to the source. The incident fields are treated as plane waves.

In each experiment, the foam cylinder was positioned in the center of the antenna circle (within the positioning uncertainty), which is also the center of the reconstruction grid. This grid is a 150 mm $\times$ 150 mm square, that is discretized in 30×30 square cells with edge 5 mm (roughly 15 cells per λ_0), yielding a total of 900 permittivity unknowns. This relatively small cell size should facilitate the reconstruction of the curved object contours. For the forward problem solution, each cell is subdivided further in $2 \times 2 = 4$ forward problem cells, the tolerance for the BICGSTAB routine is set to 10^{-3}

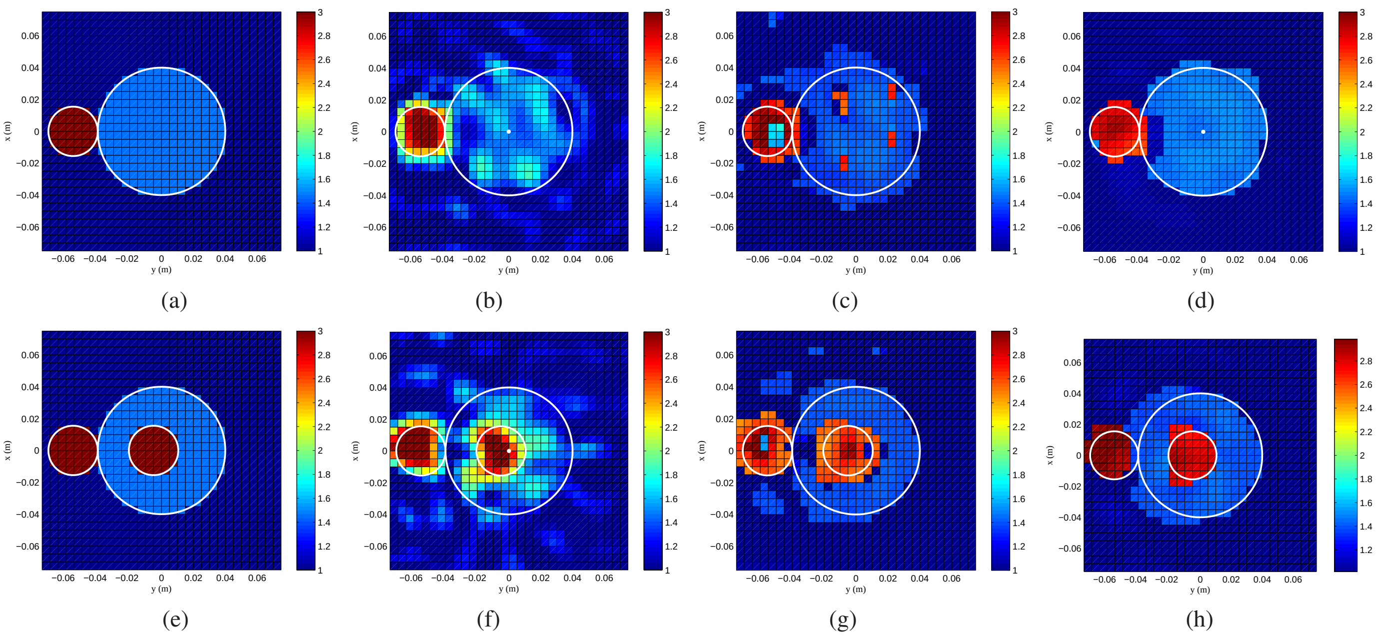


Fig. 3. Real part of the permittivity for *FoamDielExt* (a)-(d): exact (a), reconstructions with MS (b), SRVP (c), SRPSVP (d) and for *FoamTwinDiel* (e)-(g): exact (e), reconstructions with MS (f), SRVP (g), SRPSVP (h). The white lines indicate the actual object contours.

[23] and a marching-on-in-source-position approach [29] using three previous solutions is applied. Since the fields are 2D, the 2.5D computations only need to be done for one Fourier component $k_z = 0$.

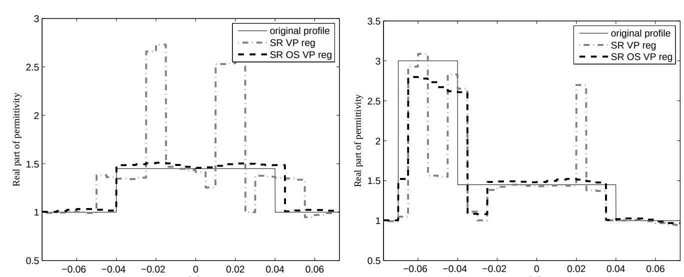
During the reconstructions, constraints are imposed on the VP values but not on the permittivity profile. They are $1.1 < \Re(c_p) < 5$ and $-0.001 < \Im(c_p) < 0.001, p = 1 \ldots P-1$, knowing that the permittivities under test do not have a significant imaginary part. Consequently, most figures in this paper only show the real part of the permittivity (see [15] for the imaginary parts). The iterations are stopped when the data fit reaches $\mathcal{F}^{\mathcal{LS}} = 5\ 10^{-3}$ or as soon as a sixth extra VP value is to be introduced. All computations are performed on a machine with two AMD Opteron 270 Quad core processors occupying all 8 CPU cores (each core solves a set of forward problems). In the following, reconstructions with MS, SRVP and SRPSVP regularization are discussed.

B. Reconstructions of FoamDielExt

For all the reconstructions of *FoamDielExt* a free space grid is chosen as the initial permittivity estimate. First a reconstruction with *MS regularization* is performed with a regularization parameter $\alpha = 2\ 10^{-3}$. Fig. 3(b) shows the result after 16 iterations, when the data fit stagnates around $\mathcal{F}^{\mathcal{LS}} = 1.4\ 10^{-3}$. The plastic cylinder is clearly visible and its permittivity is well estimated (approximately 3), but the foam cylinder is rather blurry without a clear shape or permittivity and artifacts are present in the background. Due to the globally imposed smoothness, permittivities are not well clustered.

Next a reconstruction with *SRVP regularization* is performed with $\gamma = 3$ and $\zeta = 0$ in (2). It was observed that with $\gamma = 1$ the data fit decreased too fast, leaving insufficient influence for the regularization, while with $\gamma = 5$ convergence was too slow. The reconstruction is obtained after 32 iterations (3h 40min), see Fig. 2(a). The algorithm adds more VP values than there are materials (in the last iterations a new VP value is introduced in each step), but most of these end up merging or approaching one another. The final VP values are $c_1 = 2.99$, $c_2 = 1.39$, $c_3 = c_4 = 2.70$ and $c_5 = 2.8$. They all lie within the specified uncertainties on the object properties ($\epsilon_{r,a} = 3 \pm 0.3$ and $\epsilon_{r,b} = 1.45 \pm 0.15$). None of the weights (7) corresponding with c_1, c_3, c_4 and c_5 are dominant, but the cells of the plastic cylinder are slightly more attracted to $c_3 = 2.70$, which may explain why the cylinder dimensions are somewhat overestimated, see Fig. 3(c). Some artifacts are clearly visible in the three permittivity regions: cells in the plastic cylinder pick the VP value corresponding to the foam cylinder and vice versa; a similar exchange of VP values is observed for the background and foam. These artifacts also appear in the dash-dot curves in Fig. 4, which show the permittivity along lines parallel to the x- and y- axes of the grid at $y = -5$ mm and $x = -5$ mm respectively.

Let us therefore apply the proposed *SRPSVP regularization*, with $\gamma = 3$ and $\zeta = 2\ 10^{-3}$ ($\mathcal{N}^{\mathcal{R}} = 1$). Compared to the SRVP



(a) Along x at $y = -5$ mm (b) Along y at $x = -5$ mm

Fig. 4. Real part of the reconstructed permittivity along $x-$ and $y-$directions for *FoamDielExt*. Solid: exact profiles from Fig. 3(a), Dash-dots: SRVP profiles from Fig. 3(c), Dashes: SRPSVP profiles from Fig. 3(d).

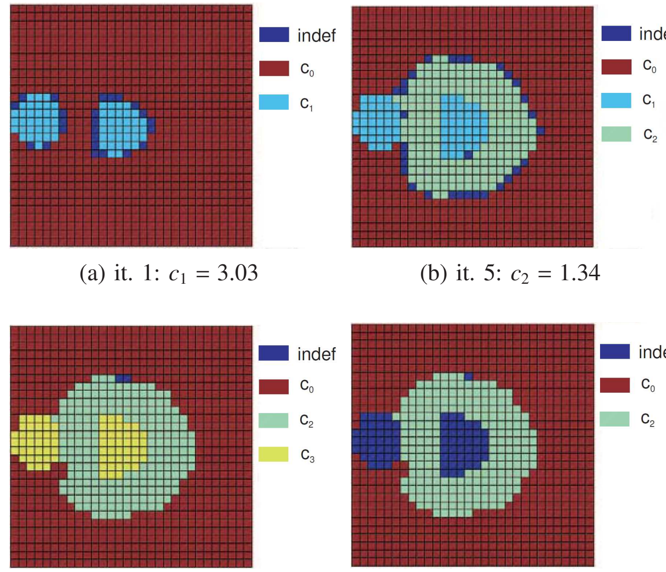
cost function, the weight of the regularizing terms relative to the data fit term has increased, resulting in a slower decrease of the data fit, see Fig. 2(a). Consequently, VP values are added later in the optimization process. The final VP values (after 53 iterations) are $c_1 = 3.00$, $c_2 = 1.48$ and $c_3 = c_4 = c_5 = 2.75$. They fit even better within the uncertainties on the object properties than those obtained with SRVP. None of the weights corresponding with c_1, c_3, c_4, c_5 are dominant, hence all cells within the plastic cylinder end up in the indefinite group and the smoothing is performed over this complete cylinder. Also the cells in almost the exact foam cylinder contour belong to the same VP-group. The reconstructed permittivity profile in Fig. 3(d) shows that most artifacts of Fig. 3(c) have disappeared. Dimensions and positions of both cylinders are correctly reconstructed and the shapes are better than with SRVP regularization alone. The dashed curves in Fig. 4 show that the permittivity of the foam cylinder is accurately reconstructed while that of the plastic cylinder is somewhat underestimated.

C. *Reconstructions of FoamTwinDiel*

A reconstruction with *MS regularization* is performed, with $\alpha = 2\ 10^{-3}$ and starting from a free space permittivity grid. Figure 3(f) shows the result after 22 iterations, when $\mathcal{F}^{\mathcal{LS}} = 2.4\ 10^{-3}$. The presence of the two plastic cylinders is clearly visible, although their shape is harder to determine. As with *FoamDielExt* the foam cylinder is less perceptible and there are fluctuations in the background.

Next a reconstruction with *SRVP regularization* is performed with $\gamma = 3$ and $\zeta = 0$ in (2). Since the number of transmitters now is about twice that for *FoamDielExt*, resulting in a longer computation time, the initial permittivity is chosen as the available profile at iteration 5 of the MS reconstruction, see Fig. 5 (with $\mathcal{F}^{\mathcal{LS}} = 9.5\ 10^{-3}$ after 2h 30min). Note that the final result of Fig. 3(f) cannot be used since the corresponding data fit is on the noise level and leaves no room for further optimization. The reconstruction is obtained after 10 iterations (3h 35min), see Fig. 2(b). The VP values then are given by $c_1 = 3.00$, $c_2 = 1.39$ and $c_3 = c_4 = c_5 = 2.59$. The values c_1 and c_2 lie well within the uncertainties on the object properties, while $c_3 = c_4 = c_5$ are slightly too low. However, the clustering of the permittivities (Fig. 3(g)) compared to the MS reconstruction (Fig. 3(f)) is apparent. As with *FoamDielExt* artifacts are visible in all permittivity regions: cells in the plastic cylinders pick VP values corresponding to the foam cylinder and background and vice versa. Figure 7 shows cross-sections along the x- and y- axis and through the center of the reconstruction grid.

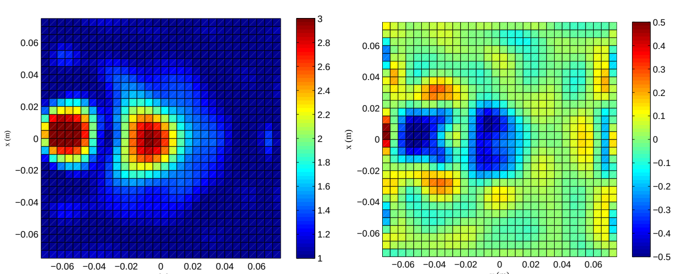
Let us again apply SRPSVP with $\gamma = 3$ and $\zeta = 2\ 10^{-3}$. The final VP values after 12 iterations are $c_1 = 3.50$, $c_2 = 1.40$ and $c_3 = c_4 = c_5 = 2.88$. Fig. 6 shows the VP-groups mapping at some iterations when a new VP value is added. It follows that no cells finally are attracted to $c_1 = 3.50$ (Fig. 6(d)). However, the introduction of c_1 has not been useless, as appears from Fig. 6(a), when $c_1 = 3.03$ and the two plastic cylinders start to appear at the correct locations. After introducing $c_2 = 1.34$ (Fig. 6(b)), the foam cylinder also appears at the correct position. In Fig. 6(d), all cells at the location of the plastic cylinders pick $c_3 = c_4 = c_5 = 2.88$ and belong to the indefinite group, while those located in the foam cylinder are attracted to $c_2 = 1.40$. We can conclude that also for *FoamTwinDiel* the smoothing is performed in the appropriate regions. The reconstructed permittivity in Fig. 3(h) shows that the artifacts



(a) it. 1: $c_1 = 3.03$ (b) it. 5: $c_2 = 1.34$

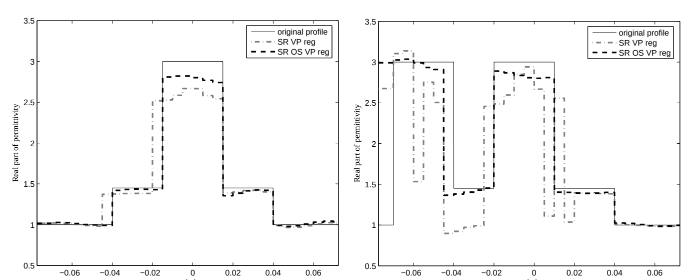
(c) it. 10: $c_3 = 2.92$ (d) it. 11: $c_4 = c_3 = 2.88$

Fig. 6. Mapping of the permittivity cells into VP-groups at different iterations (it.) during the reconstruction of *FoamDielTwin*. Each VP-group is represented by its VP value, c_0 stands for the fixed background VP value and *indef* for the indefinite group.



(a) Real part (b) Imaginary part

Fig. 5. Initial estimate for the reconstructions of *FoamTwinDiel* with SRVP and SRPSVP.



(a) along x at $y = 0$ mm (b) along y at $x = 0$ mm

Fig. 7. Real part of the reconstructed permittivity along $x-$ and $y-$directions for *FoamTwinDiel*. Solid: exact profiles from Fig. 3(e), Dash-dots: SRVP profiles from Fig. 3(g), Dashes: SRPSVP profiles from Fig. 3(h).

of Fig. 3(g) have disappeared. The reconstruction quality is comparable to that of Fig. 3(d): shape, dimensions and positions of all cylinders are correctly reconstructed, except for a slight translation (approximately two cells) of the foam cylinder to the left. The dashed curves in Fig. 7 show that the permittivities of the foam and exterior plastic cylinders are almost exactly reconstructed while the permittivity of the inner plastic cylinder is somewhat underestimated.

Concluding, the combination of piecewise smoothing with SRVP regularization can significantly improve the results compared to applying SRVP regularization alone. The reconstructions obtained here from single frequency data visibly are of superior quality than those presented in [22] for the same objects and also using single frequency data, e.g. with a multi-resolution inversion technique [30]. They even are comparable to and in many cases better than the reconstructions from multi-frequency or frequency-hopping data in [22], see e.g. with *n*-ary level-sets [11], with an adaptive multiscale approach [31], with a frequency-weighted data fit cost function [32], with an extended Born inversion with Tikonov regularization [33], with multiplicative weighted TV regularized contrast source inversion (CSI) [28], with a diagonal tensor approximation CSI [34], and with compound Markov modelling [21]. Compared to the weighted L2-norm TV regularization applied to CSI in [28] or applied to a hybrid technique in [35], both using multi-frequency data, the SRPSVP regularization yields sharper edges, especially for the foam cylinders in [28] and for the plastic cylinder in [35]. Also in the 3D case, edges are clear when using SRVP regularization [14] while some smoothing is observed with weighted L2-norm TV regularization [36]. A possible explanation is that with SRPSVP regularization, the smoothing strength is set equal to zero accross cell boundaries that are assumed, at a given iteration, to coincide with edges, while with L2-norm TV regularization, the spatially dependent smoothing weights then are small but still non-zero. Therefore, SRPSVP regularization may present advantages compared to TV when dealing with piecewise constant objects.

V. Conclusion

In this contribution we have presented a regularization strategy for piecewise (quasi-)homogeneous objects, that combines a non-spatial value picking regularization method with a piecewise spatial smoothing regularization. In each iteration of the optimization scheme, the VP values provided by the former serve to determine the separate smoothing regions in the image for the latter. The new method is validated by reconstructions of real world objects from experimental data. In particular reconstructions from single-frequency TM- and TE-polarized scattering data from the Institut Fresnel 2D database were presented for two piecewise homogeneous objects, *FoamDielExt* and *FoamTwinDiel*. These reconstructions are quite accurate and show a significant improvement compared to those obtained with the non-spatial technique only or with a global spatial smoothing approach. They also compare well to reconstructions of the same objects with various techniques by other authors. It can be concluded that SRPSVP is indeed a valuable approach to deal with artifacts that may appear when SRVP only is applied to piecewise (quasi-)homogeneous objects.

Appendix
Scattered Field Derivatives

A closed form expression is derived for the scattered field derivatives $\partial \mathbf{E}^{s}(\mathbf{r},z)/\partial\epsilon_\nu$. In the 2.5D formulation [23] Fourier field expansions are employed, hence the derivative is written as

$$\frac{\partial \mathbf{E}^{s}(\mathbf{r},z)}{\partial\epsilon_\nu} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\partial \widehat{\mathbf{E}}^{s}(\mathbf{r},k_z)}{\partial\epsilon_\nu}e^{jk_z z}\mathrm{d}k_z \tag{17}$$

where $\widehat{.}$ stands for the Fourier transform,

$$\widehat{g}(\mathbf{r},k_z) = \int_{-\infty}^{\infty} g(\mathbf{r},z)e^{-jk_z z}\mathrm{d}z. \tag{18}$$

An operator $\mathcal{G}^{\mathcal{V}}$ acting on a vector function $\mathbf{p}$ with support $\mathcal{V}$ is defined as

$$[\mathcal{G}^{\mathcal{V}}(\mathbf{p})](\mathbf{r}) = j\omega\mu_0\left(\mathbf{I}+\frac{1}{k_0^2}\widehat{\nabla}\widehat{\nabla}\right)\cdot\int_{\mathcal{V}}\widehat{G}(\mathbf{r},\mathbf{r}';k_z)\mathbf{p}(\mathbf{r}')\mathrm{d}\mathbf{r}' \tag{19}$$

where $\mathbf{I}$ is the 3×3 identity dyadic, $\widehat{\nabla} = (\frac{\partial}{\partial x},\frac{\partial}{\partial y}, jk_z)$ and

$$\widehat{G}(\mathbf{r},\mathbf{r}';k_z) = \frac{j}{4}H_0^{(1)}\left(\sqrt{k_0^2-k_z^2}\,|\mathbf{r}-\mathbf{r}'|\right) \tag{20}$$

which corresponds to the 2D scalar Green's function of homogeneous space with relative permittivity $\epsilon_r = 1 - k_z^2/k_0^2$. The CSIE [23] for the total field $\widehat{\mathbf{E}}(\mathbf{r},k_z)$ then is formulated as

$$\widehat{\mathbf{E}}(\mathbf{r},k_z) = [\mathcal{G}^{\mathcal{S}}(\widehat{\mathbf{J}}^{\mathrm{i}})](\mathbf{r}) + [\mathcal{G}^{\mathcal{D}}(-j\omega[\epsilon-\epsilon_0]\widehat{\mathbf{E}})](\mathbf{r}) \tag{21}$$

where $\mathbf{J}^{\mathrm{i}}$ and $-j\omega[\epsilon-\epsilon_0]\widehat{\mathbf{E}}$ are the applied and induced current densities, respectively. Since the incident field does not depend on the permittivity, derivation of (21) and using (1) yields

$$\begin{aligned}\frac{\partial \widehat{\mathbf{E}}^{s}(\mathbf{r},k_z)}{\partial\epsilon_\nu} &= [\mathcal{G}^{\mathcal{D}}(-j\omega\epsilon_0\Phi_\nu\widehat{\mathbf{E}})](\mathbf{r}) \\ &+ [\mathcal{G}^{\mathcal{D}}(-j\omega[\epsilon-\epsilon_0]\frac{\partial \widehat{\mathbf{E}}^{s}(\mathbf{r},k_z)}{\partial\epsilon_\nu})](\mathbf{r}).\end{aligned} \tag{22}$$

Comparing (22) with (21), it follows that $\partial\widehat{\mathbf{E}}^{s}(\mathbf{r},k_z)/\partial\epsilon_\nu$ satisfies an equation as (21) corresponding to an applied current density $-j\omega\epsilon_0\Phi_\nu\widehat{\mathbf{E}}$ in cell ν. Consequently, an expression is readily obtained from the total field solution of (21) when it is expressed as a function of $\widehat{\mathbf{J}}^{\mathrm{i}}$, by replacing $\widehat{\mathbf{J}}^{\mathrm{i}}$ with $-j\omega\epsilon_0\Phi_\nu\widehat{\mathbf{E}}$. Therefore the 3×3 dyadic Green's function of inhomogeneous space $\widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r},\mathbf{r}';k_z)$ is constructed by applying a 2D elementary dipole current density

$$\widehat{\mathbf{J}}_{\delta,p}(\mathbf{r}-\mathbf{r}') = \frac{1}{j\omega\mu_0}\delta(\mathbf{r}-\mathbf{r}')\mathbf{u}_p \tag{23}$$

along a unit vector $\mathbf{u}_p$ in a point $\mathbf{r}'$ in presence of the scatterer for the three orthogonal directions $p = x,y,z$. The resulting total fields $\widehat{\mathbf{E}}_p^{\mathrm{dipole}}$ yield the columns of the inhomogeneous dyadic Green's function:

$$\begin{aligned}\widehat{\mathbf{E}}_p^{\mathrm{dipole}}(\mathbf{r},k_z) &= j\omega\mu_0\int_{\mathcal{D}}\widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r},\mathbf{r}'';k_z)\cdot\widehat{\mathbf{J}}_{\delta,p}(\mathbf{r}''-\mathbf{r}')\mathrm{d}\mathbf{r}'' \\ &= \widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r},\mathbf{r}';k_z)\cdot\mathbf{u}_p.\end{aligned} \tag{24}$$

The total field resulting from the current density $\widehat{\mathbf{J}}^{\mathrm{i}}(\mathbf{r}, k_z)$ in presence of the scatterer then is expressed as

$$\widehat{\mathbf{E}}(\mathbf{r}, k_z) = j\omega\mu_0 \int_{\mathcal{D}} \widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r}, \mathbf{r}'; k_z) \cdot \widehat{\mathbf{J}}^{\mathrm{i}}(\mathbf{r}', k_z) \mathrm{d}\mathbf{r}'. \quad (25)$$

It follows that

$$\frac{\partial \widehat{\mathbf{E}}^{\mathrm{s}}(\mathbf{r}, k_z)}{\partial \epsilon_\nu} = k_0^2 \int_{\mathcal{D}} \Phi_\nu(\mathbf{r}') \widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r}, \mathbf{r}'; k_z) \cdot \widehat{\mathbf{E}}(\mathbf{r}', k_z) \mathrm{d}\mathbf{r}'. \quad (26)$$

Now the elements $\mathbf{E}^{\mathrm{s}}_{t,p}(\mathbf{r}_r, z_r) \cdot \mathbf{u}_{r,p'}$ of the scattered field vector $\mathbf{e}^{\mathrm{scat}}$ are considered. These are the x, y, z scattered field components in receiver points $(\mathbf{r}_r, z_r)$ resulting from illuminations $E^i_{t,p}\mathbf{u}_{t,p}$. It follows that

$$\frac{\partial \widehat{\mathbf{E}}^{\mathrm{s}}_{t,p}}{\partial \epsilon_\nu}(\mathbf{r}_r, k_z) \cdot \mathbf{u}_{r,p'} = k_0^2 \int_{\mathcal{D}} \Phi_\nu(\mathbf{r}')\ \mathbf{u}_{r,p'} \cdot \widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r}_r, \mathbf{r}'; k_z) \cdot \widehat{\mathbf{E}}_{t,p}(\mathbf{r}', k_z) \mathrm{d}\mathbf{r}'. \quad (27)$$

Due to reciprocity $\widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r}_r, \mathbf{r}'; k_z) = \widehat{\mathbf{G}}^{\mathrm{T}}_{\mathrm{inh}}(\mathbf{r}', \mathbf{r}_r; k_z)$ and from (24) it follows that

$$\begin{aligned} \mathbf{u}_{r,p'} \cdot \widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r}_r, \mathbf{r}'; k_z) &= \widehat{\mathbf{G}}_{\mathrm{inh}}(\mathbf{r}', \mathbf{r}_r; k_z) \cdot \mathbf{u}_{r,p'} \\ &= \widehat{\mathbf{E}}^{\mathrm{dipole}}_{r,p'}(\mathbf{r}', k_z). \end{aligned} \quad (28)$$

Here, $\widehat{\mathbf{E}}^{\mathrm{dipole}}_{r,p'}(\mathbf{r}', k_z)$ is the total field generated by a 2D dipole in the point $\mathbf{r}_r$ oriented along $\mathbf{u}_{r,p'}$ in presence of the scatterer. Introducing (28) into (27) finally yields

$$\frac{\partial \widehat{\mathbf{E}}^{\mathrm{s}}_{t,p}}{\partial \epsilon_\nu}(\mathbf{r}_r, k_z) \cdot \mathbf{u}_{r,p'} = k_0^2 \int_{\mathcal{D}} \Phi_\nu(\mathbf{r}')\ \widehat{\mathbf{E}}_{t,p}(\mathbf{r}', k_z) \cdot \widehat{\mathbf{E}}^{\mathrm{dipole}}_{r,p'}(\mathbf{r}', k_z) \mathrm{d}\mathbf{r}'. \quad (29)$$

To compute (29), two types of forward problems must be solved for each spectral component k_z of the incident field: (i) a *regular* forward problem to compute $\widehat{\mathbf{E}}_{t,p}(\mathbf{r}', k_z)$ on $\mathcal{D}$ for each incidence (t, p), or a total of $\#k_z N^t N^p_t$ forward problems; these have been solved already to determine the data fit term (3); (ii) a *dipole* forward problem to compute $\widehat{\mathbf{E}}^{\mathrm{dipole}}_{r,p'}(\mathbf{r}', k_z)$ on $\mathcal{D}$ for dipole excitations in each receiver position $\mathbf{r}_r$, or a total of $\#k_z N^r N^p_r$ forward problems.

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