Generalized Laguerre-based ROM Methods Applied To FDTD-like Problems

G. Lippens * L. Knockaert * D. De Zutter *

Abstract — **In this contribution a survey of Laguerre-based Reduced Order Modeling methods applied to FDTD systems is presented. The combination of robust Laguerre-based reduction methods and a Laplace transformed state space description of the discretized Maxwell's equations results in accurate broadband reduced models.**

1 INTRODUCTION

The discretization of Maxwell's equations on a Yee grid combined with reduced order modeling is a recent approach to solving interconnect problems. In this survey we will restrict ourselves to a space discretization using traditional Yee cells. In contrast with the FDTD method, the time variable remains undiscretized. This allows a representation of the electromagnetic system as a state space model. In this overview, attention will be focussed on the advantages offered by the application of a specific subset of reduced order modeling techniques. A generalization will be proposed for bandlimited systems. The Laguerre based techniques for model order reduction guarantee a more or less constant accuracy over the simulation bandwidth. The Laguerre-SVD (LSVD) method presented in [1] and [2] results in a reduced model valid from DC up to a specified frequency. In [3] this approach is extended to bandlimited systems.

The combination of the Yee cell based discretization and the reduced order modeling methods under scrutiny allows to circumvent the small time step problems of classical FDTD due to the Courant limit. In this context, the advantages of the Laguerre-based methods when large-size broadband systems are to be reduced, become clear. An interconnect type example is included in order to illustrate the feasibility of the presented methods.

Specific attention is devoted to the comparison of the simulation results with results obtained using the Padé Via Lanczos (PVL) [4] and the Multipoint Padé reduced order modeling algorithm [5].

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2 STATE SPACE FORMULATION OF YEE CELL EQUATIONS

2.1 THE 3D SISO APPROACH

In order to obtain a state space description, the spatial derivatives of the three dimensional Maxwell's equations for a charge-free region are discretised. The E- and H-fields are staggered according to the standard Yee-type arrangement of fields. The time derivative remains untouched. The equations for the time derivatives of e.g. the x-components of the electric and magnetic fields for a source free region are :

$$\begin{aligned}\mu_0\mu_{r;i,j,k}\dot{H}^x_{i,j,k} &= \Delta^{-1}E^y_{i,j,k+1/2} - \Delta^{-1}E^y_{i,j,k-1/2} + \\ &\quad \Delta^{-1}E^z_{i,j-1/2,k} - \Delta^{-1}E^z_{i,j+1/2,k} \\ \varepsilon_{r,i,j,k}\varepsilon_0\dot{E}^x_{i,j,k} &= \Delta^{-1}H^z_{i,j+1/2,k} - \Delta^{-1}H^z_{i,j-1/2,k} \\ &\quad +\Delta^{-1}H^y_{i,j,k-1/2} - \Delta^{-1}H^y_{i,j,k+1/2} \\ &\quad -\sigma_{i,j,k}E^x_{i,j,k} \qquad (1)\end{aligned}$$

where Δ is the space step. The equations for the y- and z-directions are cyclic permutations of (1). We now write the field equations (1), together with the current excitation and the recording of the voltage, in a descriptor state space format :

$$\begin{aligned}\boldsymbol{C}\dot{x} &= -\boldsymbol{G}x + \boldsymbol{B}u_I \\ y_V &= \boldsymbol{L}^T x \qquad (2)\end{aligned}$$

In the SISO approach vector x contains all the E and H field variables, the scalar u_I is the imposed current and the scalar y_V is the recorded voltage. The C-matrix contains the permittivities of the media and the G-matrix stores the conductivities and the topological connectivity information. The $C-$ and $G-$matrices are large but sparse. We refer to [6] for more detailed information. The system can be rewritten in the Laplace domain as :

$$\begin{aligned}s\boldsymbol{C}x &= -\boldsymbol{G}x + \boldsymbol{B}u_I \\ y_V &= \boldsymbol{L}^T x \qquad (3)\end{aligned}$$

For simplicity we took the same notation in the Laplace domain as in the time domain. The rational transfer function $H(s) = \boldsymbol{L}^T(s\boldsymbol{C} + \boldsymbol{G})^{-1}\boldsymbol{B}$ is a

scalar impedance function of s with $y_V(s)/u_I(s) = H(s)$. Since the McMillan degree (more simply the number of poles) of the state space description (3) is too large, we need to perform a model order reduction. The reduced system can be written as :

$$\begin{aligned} s\boldsymbol{C}_r z &= -\boldsymbol{G}_R z + \boldsymbol{B}_R u_I \\ y_V &= \boldsymbol{L}_R^T z \end{aligned} \tag{4}$$

with $\boldsymbol{C}_R$, $\boldsymbol{G}_R$, $\boldsymbol{L}_R$ and $\boldsymbol{B}_R$ the reduced counterparts of $\boldsymbol{C}$, $\boldsymbol{G}$, $\boldsymbol{L}$ and $\boldsymbol{B}$ respectively and with z the internal variables vector of the reduced system, with $q = \dim(z) \ll \dim(x)$ and with $H_R(s) = \boldsymbol{L}_R^T(s\boldsymbol{C}_R + \boldsymbol{G}_R)^{-1}\boldsymbol{B}_R$ the transfer function of the reduced system. We need to project the system (3) onto (4) with a rectangular projection matrix $\boldsymbol{V}$. The projection is determined by :

$$\begin{aligned} \boldsymbol{C}_R &= \boldsymbol{V}^T\boldsymbol{C}\boldsymbol{V} \\ \boldsymbol{G}_R &= \boldsymbol{V}^T\boldsymbol{G}\boldsymbol{V} \\ \boldsymbol{B}_R &= \boldsymbol{V}^T\boldsymbol{B} \\ \boldsymbol{L}_R &= \boldsymbol{V}^T\boldsymbol{L} \end{aligned} \tag{5}$$

2.2 THE 2D SUBCELL APPROACH

In the context of subcell modeling, the combination of reduced order modeling with Laguerre techniques has been applied to 2D problems [7] [8]. The method uses reduced order modeling in order to generate subcell equations for a portion of space only. Large 2D structures with a repetitive geometry e.g. photonic crystals [9] were also succesfully simulated with this technique. However, the technique appears to be too cumbersome for 3D problems, due to the high number of inputs/outputs in comparison with the systems' dimension. Nevertheless, the basic state space formulation is completely parallel to the approach in the previous section.

3 LAGUERRE-SVD REDUCED ORDER MODELING

In order to determine $\boldsymbol{V}$, the transfer function $H(s)$ is expanded in a Laguerre basis:

$$\phi_n(s) = \sqrt{2\gamma}\frac{(s-\gamma)^n}{(s+\gamma)^{n+1}} \quad n = 0, 1, \ldots \tag{6}$$

where $\gamma > 0$. Note that $\phi_n(s)$ is the product of the low pass filter $\frac{2\gamma}{s+\gamma}$ and the all pass filter $\left(\frac{s-\gamma}{s+\gamma}\right)^n$. As a consequence, advantegeous frequency properties emerge. We can rewrite [3] the transfer function as :

$$\begin{aligned} H(s) &= \boldsymbol{L}^T(s + \boldsymbol{C}^{-1}\boldsymbol{G})^{-1}\boldsymbol{C}^{-1}\boldsymbol{B} \\ &= \boldsymbol{L}^T \sum_{n=0}^{\infty} \phi_n(\boldsymbol{C}^{-1}\boldsymbol{G})\boldsymbol{C}^{-1}\boldsymbol{B}\,\phi_n(s) \end{aligned} \tag{7}$$

This last expression is a consequence of the reproducing kernel identity [3] :

$$\frac{1}{s+u} = \sum_{n=0}^{\infty} \phi_n(s)\phi_n(u) \quad \Re(s), \Re(u) \geq 0 \tag{8}$$

Defining the column vectors T_n as :

$$\begin{aligned} T_n &= \phi_n(\boldsymbol{C}^{-1}\boldsymbol{G})\boldsymbol{C}^{-1}\boldsymbol{B} \\ &= \frac{1}{2\pi}\int_{-\infty}^{\infty} (i\omega\boldsymbol{C} + \boldsymbol{G})^{-1}\boldsymbol{B}\,\overline{\phi_n(i\omega)}\,d\omega \\ &\qquad n = 0, \ldots, q-1 \end{aligned} \tag{9}$$

we consequently construct the matrix

$$\boldsymbol{K}_q = [T_0, T_1, \ldots, T_{q-1}] \tag{10}$$

which is a Krylov matrix, due to the simple polynomial form of the Laguerre basis functions (6). As a result, the T_n can be written as $T_n = \boldsymbol{A}T_{n-1}$. The T_n can thus be calculated recursively, and it is not needed to calculate the integrals (9) explicitly. More details on the straightforward algorithm can be found in [2]. The last step is then to orthogonalize $\boldsymbol{K}_q$, which provides the projection matrix $\boldsymbol{V}$. This matrix contains all the neccesary information to reduce the system. In the generic method [2], the SVD algorithm is used for its numerical accuracy. Depending on the particular problem, other orthogonalization methods can also be used.

4 BANDLIMITED LAGUERRE REDUCED ORDER MODELING (BLSVD)

The technique from the previous section can be generalized to bandlimited reduced order modeling. The crux is to find a new set of basis functions, orthonormal over the compact support $\mathcal{B} = [-\beta, -\alpha] \cup [\alpha, \beta]$, with $\beta > \alpha > 0$. A frequency transformation can be defined that maps the Laguerre basis (6) onto the desired new basis, thus preserving the advantageous frequency properties associated with the Laguerre basis functions. As a result, we obtain the following set of orthonormal expansion functions :

$$\psi_n(s) = \tau(s)\,\phi_n(\eta(s)) \quad n = 0, 1, \ldots \tag{11}$$

where

$$\eta(s) = \frac{\beta^2}{s}\frac{s^2+\alpha^2}{s^2+\beta^2}$$
$$\tau(s) = \beta\frac{s^2+s\sqrt{\beta^2+2\alpha\beta-3\alpha^2}+\alpha\beta}{s(s^2+\beta^2)} \quad (12)$$

Completely analogous to the previous section, we are then able to construct a matrix $\tilde{K}_q$ built from the column vectors :

$$\tilde{T}_n = \frac{1}{2\pi}\int_{\mathcal{B}}(i\omega\boldsymbol{C}+\boldsymbol{G})^{-1}\boldsymbol{B}\,\overline{\psi_n(i\omega)}\,d\omega \quad n=0,\ldots,q-1 \quad (13)$$

The resulting matrix

$$\tilde{\boldsymbol{K}}_q = \left[\tilde{T}_0, \tilde{T}_1, \ldots, \tilde{T}_{q-1}\right] \quad (14)$$

is not a Krylov matrix anymore and has to be calculated by evaluating the integrals (13) by means of numerical integration. The number of quadrature points is m. It is then sufficient to orthogonalize $\tilde{\boldsymbol{K}}_q$ to obtain the projection matrix $\boldsymbol{V}$.

5 EXAMPLE

As an illustration of the Laguerre-based methods, we consider a parallel-plate transmission line examples, one of which is depicted in Fig. 1. The line is terminated symmetrically by two lumped resistances of $240\pi\Omega$ each. In this way, the DC input impedance of the line should be the free space wave impedance. The elementary space step is $1mm$. The transmission line length is $L2 = 30mm$.

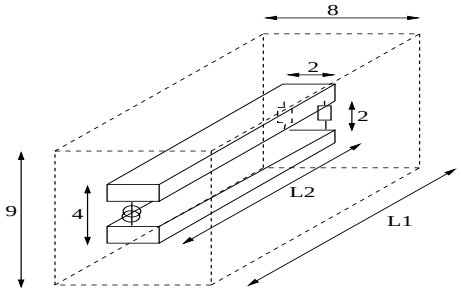

Figure 1: Parallel plate transmission line example.

The simulation domain has a length of $L1 = 35mm$, with cross-sectional dimensions of $8mm$ by $9mm$. The transversal dimensions of the conductors are $2mm$ by $1mm$. The dimension of the unreduced system is 14295. The PEC transmission line is embedded in free space. The real part of the input impedance (simulated with LSVD) is depicted in Fig. 2 as a function of frequency up to 10 GHz, together with the result for a reduction of the state space description with the PVL method. The reduced systems have dimensions of $q = 22$ for the LSVD reduction and $q = 25$ for the PVL reduction. PVL matches $2q = 50$ moments whereas LSVD only needs $q = 22$ moments. The correspondence between both results is very good over the entire range. For longer transmission lines, the difference in number of required moments between LSVD and PVL becomes more pronounced. The technique is able to reduce other relatively simple interconnect type problems of size over a million internal variables on a 2GByte machine correctly, the reduced system having a dimension of 240.

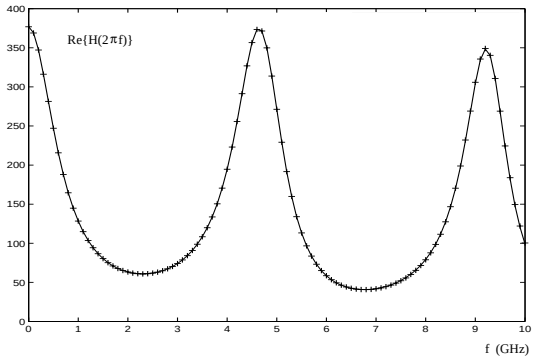

Figure 2: The transfer function of the example of Fig. 1 (L2 = 30 mm and L1 = 35 mm) simulated with LSVD (q = 22, full line) and with PVL (q = 25, crosses (+)).

As a next illustration, we compare the interconnect example, reduced with BLSVD, against the Multipoint Padé algorithm. In Fig. 3 the mean of the relative L_1 error norm of the real parts of the transfer functions with respect to their 'converged' transfer function at $q = 40$ is shown. The number of integration points is $m = 30$. The same frequency points were chosen for both reduction methods. The Multipoint Padé method thus uses complex matching points. The curves shown correspond with only one Krylov vector per matching point. It is apparent that the Multipoint Padé reductions are excellent in the lower part of the spectrum. The bandlimited Laguerre algorithm is slower in providing a low error in the lower part of the spectrum, but guarantees a more or less equal error over the complete simulation bandwidth. The Multipoint Padé method performs not so well in the higher parts of the spectrum, even for a relatively

high reduction order.

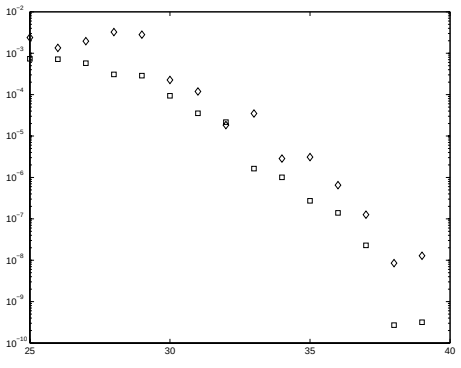

Figure 3: Mean error of the curves of Fig. 2 as a function of reduction q. Bandlimited Laguerre (squares) and Multipoint Padé (diamonds).

6 CONCLUSIONS

An overview of Laguerre-based methods in combination with Yee-cell type discretizations has been presented. The robustness of the methods with respect to the ability of obtaining a broadband reduced model was stressed and illustrated with a numerical example.

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