# Projection Based quasi-Helmholtz Decompositions: Loop/Star-like Schemes Without the Search for Global Loops

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Electric Field Integral Equations (EFIEs), while widely used, are known to be no panacea. The EFIE operator is composed of vector and scalar potential contributions that scale directly and inversely proportional to the frequency, respectively. These operators' divergent properties is the source of the ill-conditioning of the discretized EFIE for low-frequencies. This so-called low-frequency breakdown phenomenon in the past has been dealt with by using loop-star and loop-tree quasi-Helmholtz decompositions, which regularize the EFIE by separating its solution into properly scaled solenoidal and non-solenoidal components. Their historical importance notwithstanding, classical loop-star and loop-tree decompositions today constitute outdated technology when used for curing EFIE low-frequency breakdown.

More recently, several studies have identified the benefits of using quasi-Helmholtz decompositions for the low-frequency discretization of the second kind operators in MFIEs and Calderon-preconditioned EFIEs. Unfortunately, when loop-star/tree decompositions are used, these approaches are fraught with two significant problems. (i) The loop-star and loop-tree Gram matrices are ill-conditioned, resulting in ill-conditioning of the discretized second-kind operator after decomposition. (ii) When the scatterer contains handles, i.e. when its genus is greater than zero, the quasi-Helmholtz decomposition contains a basis subset comprised of global loops, i.e. basis functions that are discretized versions of the harmonic functions on the surface. Finding this basis subset often is nontrivial: standard algorithms use tree searches which can be cumbersome, especially if the geometry under consideration is complicated and if no a priori information on connectivity is available.

This presentation will describe a new way to obtain Helmholtz decomposed equations. The proposed scheme is based on the use of suitable and efficiently computable projectors. The proposed decompositions leave unaltered the condition number of the undecomposed equations. Thus they give rise to well-conditioned Helmholtz decomposed second kind equations. Moreover, the new decompositions do not require one to search for global loops; these are accounted for implicitly. Numerical examples will show the relevance and effectiveness of the new schemes when applied to several widely used integral equations.