Piping in loose sands – the importance of geometrical fixity of grains

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Abstract: Piping is one of the possible failure mechanism for dams and levees with a sandy foundation. Water flowing through the foundation causes the onset of grain transport, due to which shallow pipes are formed at the interface of the sandy layer and an impermeable blanket layer. In the past, the mechanism has been investigated predominantly in densely packed sands, in which the process was observed to start at the downstream side (backward erosion). Recently performed experiments in loose sand (van Beek et al. 2009) showed a different failure mechanism (forward erosion). In this article additional experiments of piping in loose sands are described for investigating the relevance of the forward process for practice. In these experiments the type of process was found to be dependent on the presence of shear resistance between sand box cover and top sand grains, that causes grains to be fixed geometrically. Without this shear resistance the process was found to be forward, whereas with this shear resistance the process was found to be backward oriented. The change in degree of fixity and relative density as a result of loading is investigated with electrical density measurements. The experiments show that the forward process is not relevant for levees in practice, in which the cohesive blanket layer causes the sand grains to be fixed properly.

Keywords: piping, relative density, forward erosion, backward erosion, electrical density.

1 INTRODUCTION

In dams and levees with a sandy foundation, the onset of grain movement, as a result of water transport, poses a threat to the stability of the levee. Various forms of internal erosion can take place dependent on the local soil conditions. In areas with uniform sands, covered by a cohesive relative impermeable blanket layer, the grain movement results in the formation of shallow pipes in the top of the sand bed. These pipes can exist due to the presence of the cover layer that serves as a roof to the pipes.

This process has been investigated experimentally by various authors (a.o. De Wit 1981, Hanses 1985, Miesel 1978). They observed that the process starts at the downstream side and progressed towards the upstream side. In the field it has often been observed that the sand deposition stabilizes in time. This has led to the development of a model for prediction of the critical head, the model of Sellmeijer, in which the equilibrium of forces on grains in the pipe is the criterium for the onset of erosion (Sellmeijer 1988). Using this model, it can be determined whether a pipe can progress once it has formed. Calculations with this model show that in the Netherlands piping is a dominant failure mechanism for primary levees (Floris study 2005).

Most experimental work to study the process of piping is performed on dense sand samples. In reality however, the sandy foundations of levees are often loose. In an extensive research program in which the model of Sellmeijer was validated, experiments have been performed on both dense and loose sand samples. In the small-scale experiments on loose sands (relative density <50%) it was found that a different process occurred than in the dense sand samples (van Beek et al. 2009). The piping process started at the upstream side and appeared to develop towards the downstream side by local densification of the sand sample (Figure 1).

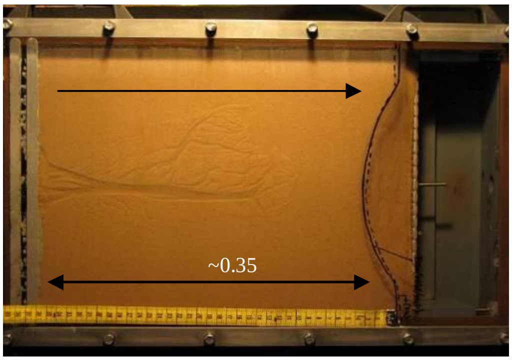

Figure 1. Forward erosion as observed in small-scale experiments. The arrow indicates the flow direction (van Beek et al. 2009).

Similar processes have been observed by Chevalier et al. (2009) and Johnson et al. (2008), who have studied injection of water into granular suspensions in a Hele-Shaw cell (consisting of parallel flat plates).

Doubts were raised on the relevance of this phenomenon for real dams and levees, as in reality the sand is usually loaded, which was not the case for the sand sample in the laboratory set up. Additional experiments have been performed, to assess the influence of the connection between sand bed and cover (causing geometrical fixity of grains) on the piping process. With electrical density measurements, pore and total stress sensors the loading process has been monitored, to assess whether the loading process causes densification of the top layer and to assess the connection between the sand and cover.

2 EXPERIMENTAL WORK

The experimental work consists of several small-scale tests in which a fine sand sample has been subjected to vertical loading and to an increasing hydraulic gradient.

2.1 Setup

The setup consists of a small PVC box with Perspex cover, with inner dimensions of 0.5x0.3x0.1 m. The box is filled with sand, which is retained by two filters, such that the obtained seepage length is around 0.30-0.35 m. A constant head is applied to the sand, with a range of 0-1m. A transparent Perspex cover permits to observe the piping process (Figure 2).

The sand has been prepared using the “dry sieving method”, in which dry sand is scattered into water while the box is placed in vertical direction (Poel and Schenkeveld 1998). In this way a loose sample is created. As the used sand type, Baskarp sand (B15), is uniform, this method ensures a homogeneous and well-saturated sample. The properties of the sand type used are shown in Table 1.

Between the cover and the box a compressible strip is placed (Figure 2). The compressible strip allows for loading of the sand sample after preparation, by compressing the cover closer to the box. Using two total stress sensors and two pore pressure sensors, which are placed in the Perspex cover (both at a distance of 125 and 250 mm from the upstream filter, see Figure 3), the vertical effective stresses in the sand can be determined. Hereto, the measured total stresses are corrected for the pore pressures that have been determined at the same distance from the upstream filter (125 or 250 mm).

At 11 locations in the cover and 8 locations in the side of the box, rows of copper needles have been glued in for the measurement of electrical resistance, both in the cover (Figure 3) and at the side of the box. The needles have been inserted in such a way that only the rounded top of the needle (with length of 1 mm) protrudes from the inside of the cover. Each row has four needles with intermediate distance of 5 mm. An alternating current of 10 Hz is applied to the outer needles of each row and the electrical resistance is measured between the inner needles. A linear relationship exists between porosity and the ratio of specific electrical resistance of the water and the specific electrical resistance of the soil, when configuration and other soil and water properties remain the same. This relation is established by calibration.

The experiments were performed by increasing the hydraulic head with 1 cm per 5 minutes, until erosion of sand grains is observed. The short time interval is feasible as no gradual changes are expected in the sample due to ground water flow. In case of grain transport, the increase of hydraulic head is delayed until the pipe is stable for at least two minutes. In the experiments the hydraulic head at breakthrough is measured, which is the head at which the pipe has grown from the downstream side towards the upstream side, so that a connection is present between the upstream and downstream side.

Table 1. Soil parameters Baskarp sand.

Parameter	Value	Dimension
Density grains	2.647	kg/m3
d10/d50/d90	90/130/200	µm
Min./max porosity	34/46.9	%

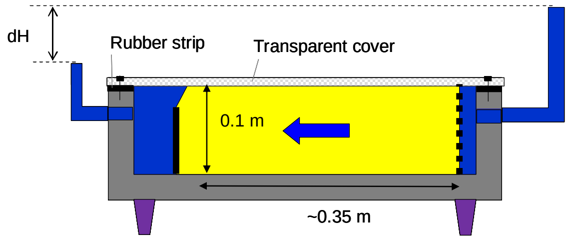

Figure 2. Cross-section of set up

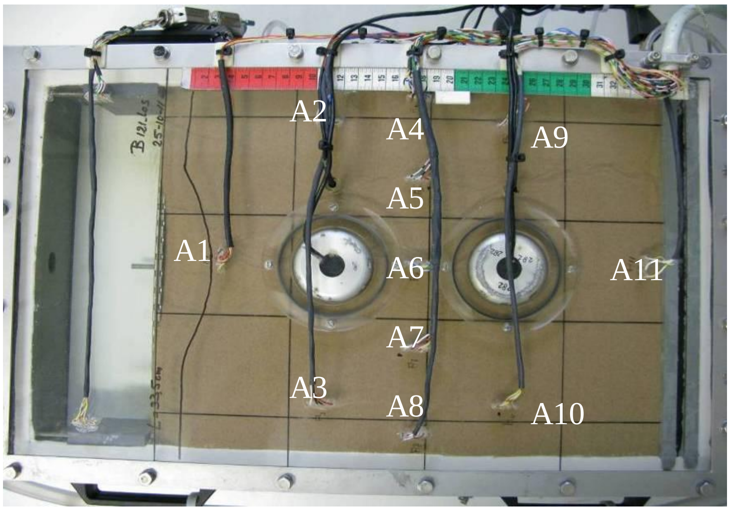

Figure 3. Eleven rows of four needles for measurement of electrical resistance

2.2 Test program

In the experiments on loose sands that have been performed for the validation of the model of Sellmeijer, the sand samples have not been loaded. The lack of loading does not influence the backward piping process in dense sands (Schmertmann 2000, De Wit 1984, van Beek et al. 2011), but may influence the occurrence of forward piping in loose sands.

Therefore the influence of vertical loading on the piping process has been investigated in additional tests. Several types of tests have been performed. In the first test series, the loose sand samples were loaded by compression of the strip between cover and sand box, to investigate the erosion process under loaded conditions. To apply the load, the cover needed to be compressed to the sand box over a distance of a few millimeters. In these tests it was therefore uncertain whether the relative density of the top layer of the sand sample had been altered as a result of the loading of the sand bed. Therefore extra series of tests have been performed with electrical resistance measurements, to determine the change porosity in the top of the sample and in the depth of the sample. After loading the sample, the sample was unloaded again before applying hydraulic gradient, to distinguish between the influence of the alteration of the density of the sand sample and the loading itself. A third series of tests has been done to observe whether a pipe that has formed upstream can continue to develop in a loaded sand sample.

2.3 Test results

2.3.1 Influence of a vertical load on initiation of piping

In the first test series, the sand sample is loaded, to investigate the influence of vertical load on the initiation of the piping process. After preparation of the sand sample, compression of the cover to the sand box was performed to establish a load on the sand sample. The degree of loading was restricted, as loading appeared to cause deformation and densification of the sand sample and the bulk relative density had to remain below 50%. The distance between cover and box served as a measure for the compression of the sand sample. This distance was decreased a few millimeters to generate effective stresses in the top of the sand sample. It is noted that at the downstream slope the effective stresses are absent, as a result of the open outflow area, whereas at the upstream side the stresses are at a maximum value. The effective stresses as derived from the stress sensors are given in Table 2, for three different tests.

The initial bulk relative density was determined afterwards by measuring the amount of sand in the box. After loading, the number of millimeters of compression was used to correct the bulk relative density. In all tests the bulk relative density remained below 50%. In all tests backward erosion was observed, at a critical head of 8 cm.

Table 2. Test characteristics

Parameter	B101	B103	B104
Initial bulk relative density [%] (porosity [-])	31 (0.429)	9 (0.457)	9 (0.457)
Bulk relative density after loading [%](porosity [-])	47 (0.408)	24 (0.438)	24 (0.438)
Effective stress location 1 [kPa]	5.9	1.2	1.0
Effective stress location 2 [kPa]	12.7	3.7	4.2
Critical head [m]	0.08	0.08	0.08

The effective stresses in the area around the pipe are expected to be very low and absent at the location of the pipe. The pipe development can therefore be monitored with the stress sensors; the derived effective stresses drop as soon as the pipe passes the specific stress sensor (Figure 4). In Figure 4 it can be seen that the pipe first passes the downstream located stress sensors (grey line) and passes the upstream located stress sensors (black line) a little later.

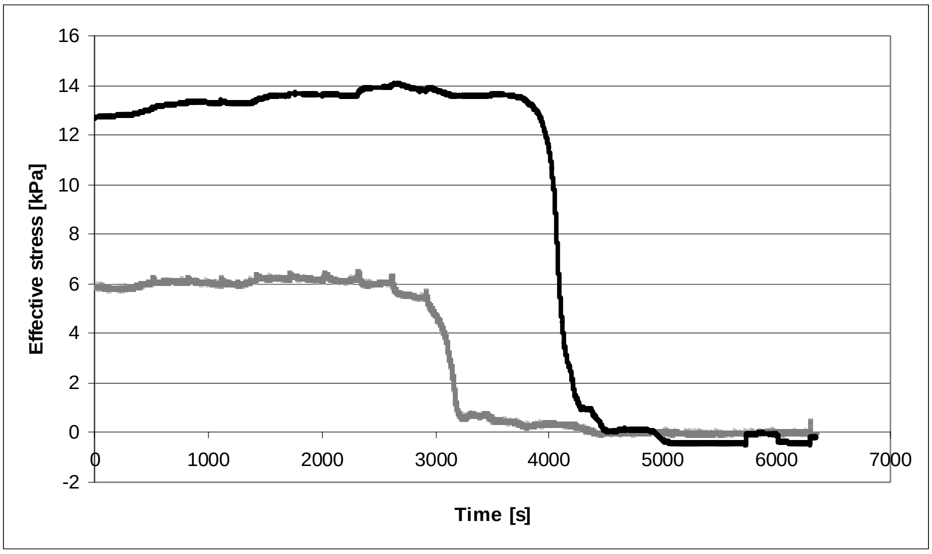

Figure 4. Effective stresses derived from stress sensors in test B101. The effective stress drops to 0 kPa when the pipe passes the stress sensors.

2.3.2 *Alteration of the sand sample due to loading and its influence on the piping process*

As was found in the tests with vertical loading described in the previous paragraph, several millimeters of compression of the cover to the sand box were necessary to generate sufficient pressure on the sand bed. This might have caused a change in the relative density of the sand sample. If this change is predominantly taking place in the upper centimeters of the sand bed, the upper layer may not be classified as loose anymore, thereby affecting the outcome of the experiment.

In the second series of tests, the influence of the loading process on the relative density has been investigated using the electric density method. It is known that the porosity is related to the specific electrical resistance of the sand by the following equation (Vlasblom 1977)

$$n = a + b(\rho_w / \rho_g) \qquad (1)$$

in which a and b are constants that are related to the type of soil, which cannot be determined analytically and ρ_w and ρ_g are the specific electrical resistances of the pore water and the soil respectively. The specific electrical resistance of the soil can be obtained by measuring the electrical resistance (R) of the soil, which can be translated to the specific electric resistance by means of calibration. For a specific soil type, temperature and water type, the parameters a and b can be determined for each row of needles. A column experiment, with three measurement locations in the sand sample, shows that indeed the relation between $\rho_w/\ \rho_g$ and n is approximately linear in the area of interest.

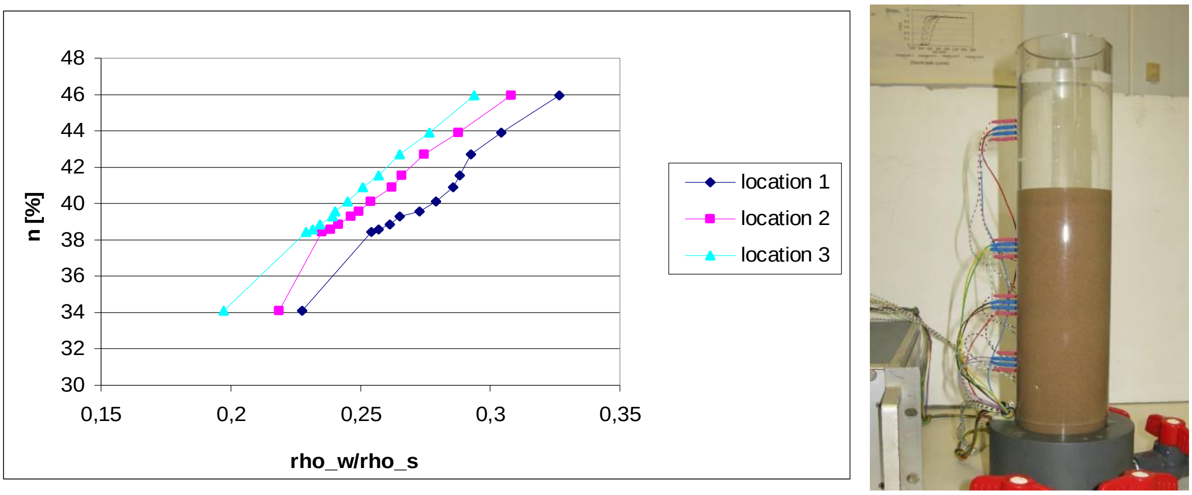

Figure 5. Results of column experiment with electric resistance measurements

Two piping tests have been performed. In both tests, the electrical resistance measurements in the cover showed that the initial porosity (before loading the sample) was exceptionally high (around 0,70). The values of the electrical density measurements at the side of the box matched quite well with the bulk porosity as determined from the weight of the sand in the box (Table 3), although the porosity obtained with the electric density method is slightly higher than the porosity obtained based on the weight of the sample. This is most likely caused by a small difference in temperature between the calibration test and the piping tests. Based on visual observations some space was present between cover and the sand sample before loading.

When the cover was compressed towards the sand box, the porosities in the top of the sample as calculated from the electric resistance measurements gradually decreased to values that matched better with the initial bulk porosity, whereas the values at the side of the sample remained constant. It was observed that the connection between cover and sand bed improved due to this compression. At some point the values for porosity as measured in the cover remained more or less constant, which corresponded to the point where stress is built up in the sand sample (Figure 6 and 7).

After loading, the load has been decreased by gently releasing the cover, in a way that the connection between sand and cover is retained. This was confirmed by the measurement of electric resistance, which did not change during the process of unloading. The hydraulic head is gradually applied until pipe-formation occurred. In both tests backward erosion occurred.

Table 3. Test characteristics

Parameter	B121	B122
Initial bulk relative density, based on weight [%] (porosity [-])	13 (0.452)	12 (0.454)
Average initial porosity cover, based on electrical resistance measurement [-]	0.741	0.705
Average initial porosity side of box, based on elec. resistance measurement [-]	0.501	0.484
Bulk relative density after loading, based on mm compression [%] (porosity [-])	30 (0.430)	33 (0.426)
Average porosity after loading, based on electrical density, in cover [-]	0.472	0.476
Average porosity after loading, based on electrical resistance, side of the box [-]	0.488	0.482
Effective stress location 1 at start of piping test [kPa]	0	0
Effective stress location 2 at start of piping test[kPa]	1.5	1.5
Critical head [m]	0.09	0.08

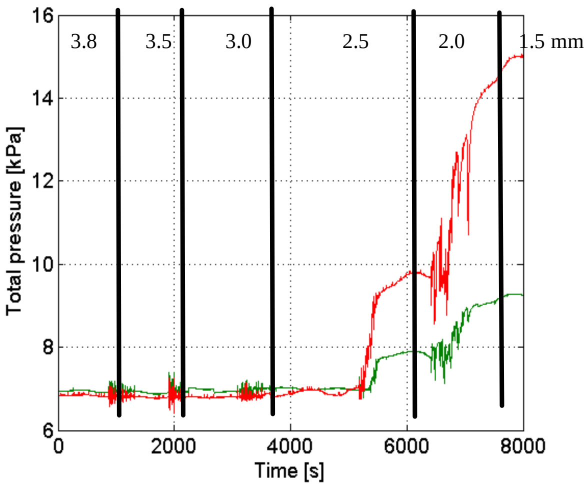

Figure 6. Total pressure development in time as a result of compression of strip between cover and sand box (the degree of compression is indicated by the number of mm between cover and sand box).

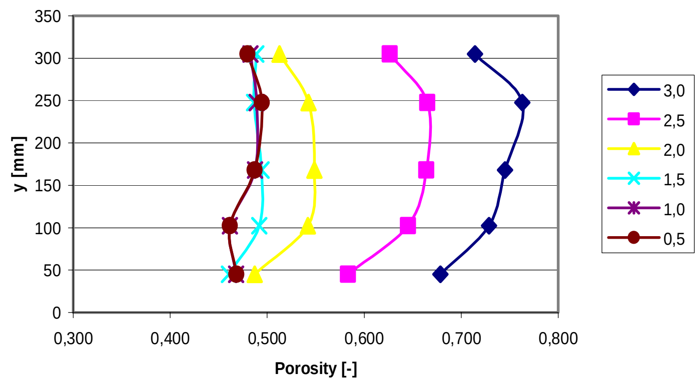

Figure 7. Porosity derived from electric resistance measurements in the cover of the set up in test B122 (location A4-A8), for different steps of compression of the strip between cover and sand box (legend indicates the distance in mm between cover and sand box)

It is concluded that the connection between cover and sand sample was not optimal at the start of the loading phase, resulting in exceptionally high porosity values. Compression of the cover to the sand box resulted in a better connection between sand sample and cover, so that effective stresses could be built up in the sand sample. The static pressure build-up does not cause a large decrease of porosity, indicating that the top layer is still well below a relative density of 50%. The fact that backward erosion occurred after releasing some of the load, indicates that forward erosion is not only prohibited by the influence of load, but can neither take place when grains are geometrically fixed by the cover.

2.3.3 Influence of a load on the progression of forward piping

Although the forward piping process cannot initiate when the sand grains are fixed by shear resistance, the process might be able to continue once it has started, as the stresses around an existing pipe are very low. Hereto an experiment has been performed. In this experiment, the loading phase is not fully completed. The strip between cover and sand box was compressed until, based on visual observations, the sand was in good connection with the cover. The electrical resistance measurements showed that at this point the porosity was still quite high, indicating limited space between cover and sand bed (Table 4). Consequently, a hydraulic head was applied to the sand sample, causing a forward pipe of several centimeters to develop, after which the hydraulic head was brought back to zero. In order to monitor the progression of the pipe in a sand bed subjected to vertical loading, the strip between the cover and the sand box was compressed until effective stresses build up in the sand sample. During this loading process the larger pipes remained intact, whereas smaller pipes disappeared (Figure 8).

The consequent gradual re-application of the hydraulic head did not result in continuation of the forward process, but in backward erosion, starting at the downstream side of the set up. The hydraulic head to create backward erosion (13 cm) exceeded the critical head for the creation of forward erosion in the first stage of the experiment (8 cm).

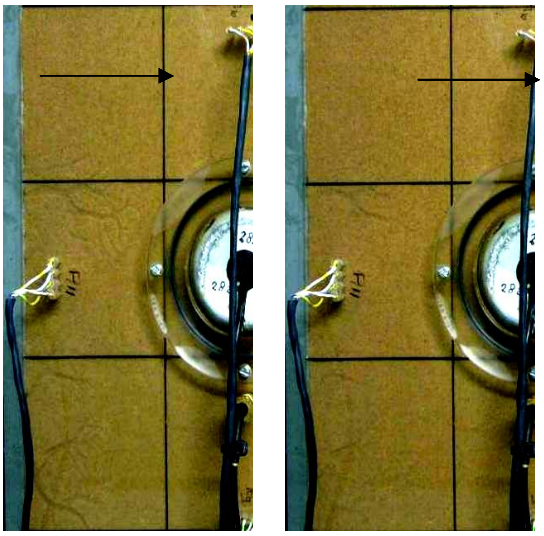

Figure 8. Pipes at the upstream side of the set up, before (left) and after loading (right)

Table 4. Test characteristics test B123, first phase

Parameter	B123a
Initial bulk relative density, based on weight [%] (porosity [-])	12 (0.454)
Average initial porosity cover, based on electrical resistance meas. [-]	0.677
Average initial porosity, side of the box, based on electrical resistance meas. [-]	0.479
Bulk relative density after compression, based on mm compression [%] (porosity [-])	18 (0.446)
Average porosity after compression, based on electrical resistance meas., in cover [-]	0.577
Average porosity after compression, based on electrical res. meas., side of the box [-]	0.483
Critical head [m] (forward erosion)	0.08

Table 5. Test characteristics test B123, second phase

Parameter	B123b
Bulk relative density after loading, based on mm compression [%] (porosity [-])	30 (0.430)
Average porosity after loading, based on electrical resistance, in cover [%]	*
Average porosity after loading, based on electrical resistance, side of the box [%]	*
Effective stress location 1 [kPa]	1.0
Effective stress location 2 [kPa]	5.8
Critical head [m] (continuation)	0.13

* Unreliable data due to change in temperature as a result of water flow in the previous phase

The first phase of the experiment shows that whereas visually the sand bed is in well connection to the cover, the electric resistance measurements show that the porosity at this stage is still rather high, indicating a poor connection between cover and sand bed. The loading process in the second phase of the experiment causes a proper fixity of grains, preventing the forward process to continue.

3 PREDICTION OF BACKWARD EROSION IN LOOSE SAND

In the experiments in which the grains were properly fixed as a result of the good connection between cover and sand bed, backward erosion took place. A model to predict the progression of backward erosion is Sellmeijer's model (1988). This model relates the sand and aquifer characteristics to the critical head, which is the head above which the pipe can develop from downstream to upstream side, assuming the presence of a pipe. Earlier small-scale experiments have been used to validate the model for the influence of sand characteristics and scale (Sellmeijer et al. 2011). The model has accordingly been adjusted for the influence of (among others) relative density. In this validation study only experiments with relatively densities higher than 50% were included, as experiments with relative densities below 50% resulted in forward erosion. Now it has been observed that once grains are properly geometrically fixed, backward erosion will also occur for loose sand samples, it is investigated whether the model also functions for samples with relative density below 50%.

Figure 9 shows the critical heads obtained in different small-scale experiments with Baskarp sand. A clear trend is observed between relative density and critical head. It is noted that a change in relative density causes a change in permeability as well. In this graph it can be observed that forward erosion often occurs at lower critical head than backward erosion. Also it is noted that the bulk permeability of samples in which forward erosion took place is also a little higher (Figure 10), perhaps due to the presence of very little space between cover and sand sample.

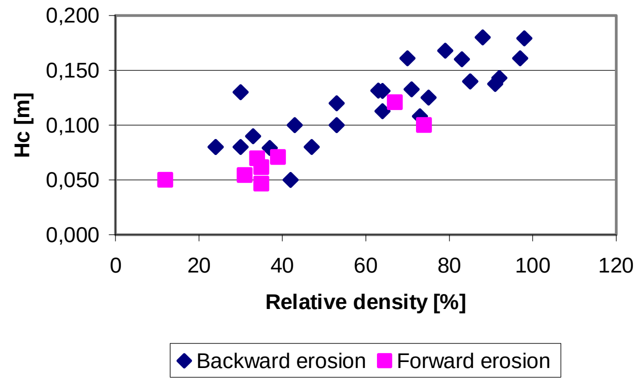

Figure 9. Experimentally obtained critical heads versus relative density of the sand sample.

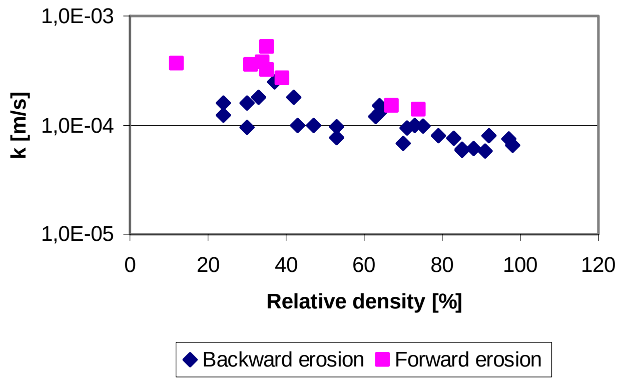

Figure 10. Relation between permeability and relative density.

For all small-scale experiments, the critical head is calculated using Sellmeijer's model. The adapted rule as described in Sellmeijer et al. (2011) is derived for a standard dike geometry and is given in equation 2

$$\frac{H_c}{L} = \frac{1}{c} = F_R \, F_S \, F_G$$
$$F_R = \eta \, \frac{\gamma'_p}{\gamma_w} \tan \vartheta \left(\frac{RD}{RD_m}\right)^{0.35} \left(\frac{U}{U_m}\right)^{0.13} \left(\frac{KAS}{KAS_m}\right)^{-0.02} \tag{2}$$
$$F_S = \frac{d_{70}}{\sqrt[3]{\kappa L}} \left(\frac{d_{70m}}{d_{70}}\right)^{0.6}$$
$$F_G = 0.91 \left(\frac{D}{L}\right)^{\frac{0.28}{\left(\frac{D}{L}\right)^{2.8} - 1} + 0.04}$$

The small scale experiments have a different geometry than the standard dike geometry, for which the geometry factor (F_G) needs an adjustment. Using calculations in MSeep, which is a numerical ground water flow program in which Sellmeijer's model has been implemented (Sellmeijer 2006), the geometry factor for this specific situation was found to be:

$$F_G = 0.87 \left(\frac{D}{L}\right)^{\frac{0.28}{\left(\frac{D}{L}\right)^{2.8} - 1} + 0.04} \tag{3}$$

Using equation 2 and 3 the critical heads according to the model of Sellmeijer are calculated for all experiments on Baskarp sand in which backward erosion took place. To check whether the adjustment on relative density $(RD/RD_m)^{0.35}$ is correct, the critical head is also determined with equation 2 and 3 in which $(RD/RD_m)^{0.35}$ is set equal to one, thereby removing the influence of relative density, other than its influence on permeability.

The calculated critical gradients are compared to the experimentally obtained critical gradients (Figure 11). It can be observed that especially for low critical gradients, which correspond to the experiments on loose sand, the model including the influence of relative density outperforms the model in which the influence of relative density is excluded. However, compared to the scatter in the results, the influence is small.

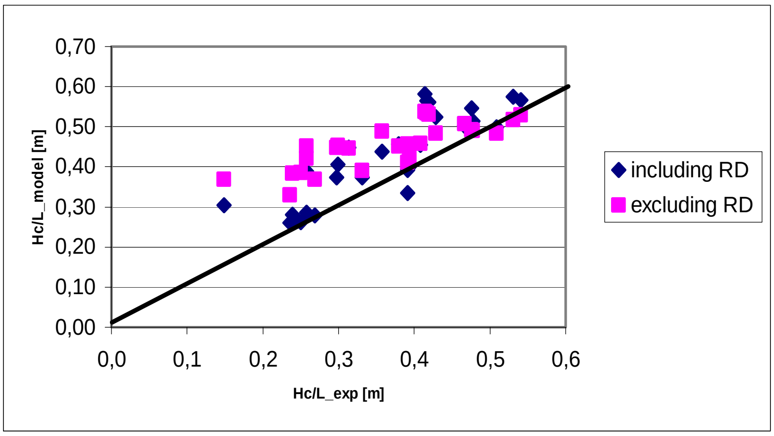

Figure 11. Experimentally obtained critical gradients vs. calculated critical gradients

This finding is confirmed by the difference between the experimentally obtained values and the calculated values (defined as $H_c / L_{\exp} - H_c / L_{calc}$). The standard deviation and the average of the difference for the model including relative density are lower than those of the model excluding relative density (Table 6).

Table 6. Average and standard deviation of the difference between Hc/L_exp and Hc/L_calc.

Parameter	Average [-]	Standard deviation [-]
Adjusted Sellmeijer model including relative density factor	-0.062	0.057
Adjusted Sellmeijer model excluding relative density factor	-0.087	0.066

4 DISCUSSION AND CONCLUSIONS

Experimental work has been performed to investigate the process of piping in loose and uniform sand samples. The influence of loading and fixity of the grains on the type of process has been investigated.

The electrical density method has been applied to assess the porosity of the sand sample. This method appears to be very suitable for measuring of local porosities in laboratory experiments, in which conditions can be controlled relatively well. Using this method, porosity changes due to loading of a loose sand sample could be measured, from which it has been concluded that before loading the connection between sand sample and cover is not fully established. The occurrence of forward erosion in loose sand beds, as observed in piping experiments described in van Beek et al. (2009), therefore appears to be related to the lack shear stresses between top sand grains and cover.

Although several measures had been taken in the experiments described in van Beek et al. (2009) to improve the connection between sand bed and cover, it is likely that some space existed between cover and sand bed. The measures included silicon treatment of the inside of the cover and compression of the cover (effective stresses in the sand bed have not been measured in these earlier experiments). Also color injections and black lines on the inside of the cover had been used to improve visual judgment of the connection between sand and cover. It is therefore concluded that the earlier investigation of phenomena in loose sand beds must be executed with caution and that it is essential to measure the effective stresses in the soil to be sure that and there is contact and some effective stress between the cover and the sand grains.

Apparently even with very limited space (<1 mm) between cover and sand bed, the forward mechanism may occur. In the case of a soft or natural transition between sand and cover, such as is the case for most levees, the grains are geometrically fixed by a shear resistance between cover and sand grains, and the forward mechanism is unlikely to occur. However, in the case of a rigid transition, such as is not uncommon for artificial water-retaining structures, the grains may not be geometrically fixed and forward piping may cause instability problems.

The existing prediction model (model of Sellmeijer), such as described in Sellmeijer et al. (2011), which has been extended with a factor for relative density, appears to be well suited for prediction of backward erosion in both loose and dense sands.

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