Thermal Design of Transformers and Inductors in Power Electronics

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An overview of convection and radiation heat transfer in magnetic components is proposed. Firstly an introduction in 'rule of the thumb' is given. Secondly an improved modeling is proposed. It comprises natural convection heat transfer including also the effects of the orientation of the component and the influence of the ambient temperature. The proposed modeling is verified by comparison with experimental data obtained for an experimental box shape. The carried out accurate measurements for four different kinds of surfaces of the experimental model allow a fine-tuning of the improved expression for convection heat transfer coefficient. Thirdly an approach for forced convection is given. The derived results can be used in thermal design of magnetic components as well for other electronic equipment.

***Index Terms*— Electromagnetic devices, inductors, Thermal analysis, transformers**

I. INTRODUCTION

The thermal design of inductors and transformers is important for achieving high efficiency, reliability and low volume of the equipment. The classical approach [1] is based on thermal resistance networks. In an *isotherm surface model* (all open surfaces of the component have the same temperature), eqn (1) gives the total heat transfer rate q is the sum of radiation q_r and convection transfer rate q_c and possible heat conduction q_g:

$$q = q_r + q_c + q_g$$
$$= \varepsilon\,\sigma\,A_r\,(T_s^4 - T_a^4) + h_c\,A_c\,(T_s - T_a) + k\,\frac{A_g}{l_g}(T_s - T_h) \qquad (1)$$

where
q_r : radiation heat transfer rate [W];
q_c : convection heat transfer rate [W];
q_g : conduction heat transfer rate [W];
ε is the *emissivity* of the radiating surface;
σ is the Stefan-Boltzmann constant, $\sigma = 5.67\times 10^{-8}$ [W/m^{2}K^4]
k: conduction coefficient [W/(m^2 K)]
S_r is the radiating area, i.e. the component open surface, [m^2].
T_s is the surface temperature of the component, T_a the ambient temperature;
h_c is the *convection heat transfer coefficient* of the material, [W/m^2 K];
S_c is the convection surface of the component, [m^2],
S_g , l_g are conduction path and length: by the coil former to the printed circuit board or to a heat sink
T_h : temperature of the heat sink

$A_c = A_r$ if the component is convex. and . $A_c > A_r$ for concave surfaces.

Fig 1. gives a view how relevant surfaces for convection and radiation can be derived.

The derivation seems logic, but even calculating such surfaces need a lot of details to apply the principles of heat transfer.

The conduction path may be to a heat sink or printed circuit board. This way is also used in many designs. This article is an a revised overview, it contains elements of [2] chapter 6 thermal aspects and [3] for the convection modelling.

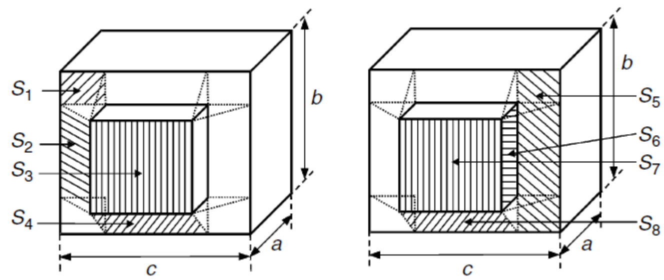

Fig. 1. The equivalent surfaces of an EE core transformer:
a) the 'envelope' surface for radiation, .
$S_r = 2a\,b + 2a\,c + 2\,(4S_1 + 2S_2 + S_3 + 2S_4)$
b) the equivalent surface for convection, .
$S_c = 2a\,b + 2a\,c + 2\,(2S_5 + 2S_6 + S_7 + 2S_8)$

II. RULE OF THE THUMB MODELS

Engineers like often to have an order of magnitude, even when knowing that more accurate models exist.

This was done by 'reverse engineering' of older manufacturer data [4] for 50Hz transformers. It was derived for 40°C ambient and 75°C temperature rise, They seem to be a bit on the safe side. It appeared that the stack height was not a very important parameter, except when it becomes high compared to other dimensions.

So the following "rule of the thumb" was tried out:

The allowable heat dissipation is about 0.2W/cm^2 considering the multiplication of the two largest dimensions :

$$P_{th} = a\,b \times 2000\,\frac{W}{m^2} \qquad (2)$$

It is possibly between 0 and 40% oversizing, but it sets an order of magnitude. We know that the accuracy is not perfect, but at least it can be fast calculated even by heart.

If we look at a number of transformers one gets fig (2), out of data of [4] which supports eqn. (2).

For an EI60 scrapples core, a shell type transformer, the two largest dimensions are 50 and 60mm, and results in 7.5W.

In UI transformers, one of the largest dimensions is determined by the copper. An UI30 core (cut EI60 in two) will result in: (3+2x0.5)x5x0.25 = 5W, as the transformer is a core type and two coils are coming out.

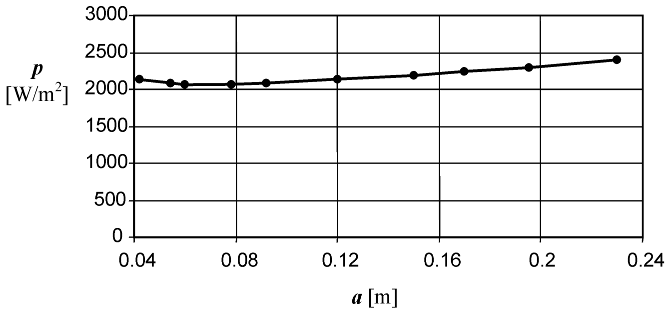

Fig 2. Observed total losses of 50Hz transformers, depending on the largest dimensions

Normally, one would think that small 50Hz transformers cool better in convection than larger ones. The reason why it is not the case in practice is that the copper losses in small transformers are much higher than the iron losses. This results in a much higher copper temperature. and so the average heat loss removal is reduced. If the transformer is impregnated, the thermal resistance between copper and iron gets better and the internal temperature drop in copper reduces, allowing 20-30% more heat removed from the copper.

In ferrite transformers, the ratio of copper to iron loss may vary a lot, so the analysis is less easy.

Another way is using information on thermal resistances given by manufacturers as [5] "bell transformers"

III. More precise analysis

A. Usual models

The coefficient h_c in the expression (1) for convection heat transfer is quite critical. Most authors [6,7] give present the following simplified expression for it as a function of the height L of the component:

$$h_c = C\left(\frac{\Delta T}{L}\right)^{1/4} \tag{3}$$

where C=1.32...1.42

ΔT is the temperature rise, $\Delta T = T_s - T_a$ [K];

L is the height of the component, [m], in classical theory L is a characteristic dimension equal to the height of an infinite vertical surface.

For magnetic components the values of h_c are in the range h_c = 6...10 W/(m^2 K) for a temperature rise of ΔT=50K. [6, 7]

The expression (3) is only valid under specific conditions, which limit its validity for magnetic component design:

- Convection heat transfer is a quite complex process and the expression (3) derived for infinite surfaces is not completely applicable for magnetic components.
- The conductivity, viscosity and density of air are assumed to be constant in the temperature range, where the expression is used, which is only an approximation.
- Expression (3) is not valid for natural convection in an enclosed space or in close proximity of other heated surfaces. Usually the 'ambient temperature' is adapted to some inside average temperature in the enclosure.

As a result of the above mentioned limitations of expression (3), the convection heat transfer is estimated with some inaccuracy of 20…30%. This fact results in poor prediction of the component temperature rise. Magnetic components have similar shapes, but they are never 'infinite or thin plates'. So, the heat transfer coefficient h_c could be well defined, but still different from the classical thermal approach for horizontal and vertical plates.

This part presents a study about natural convection and convection heat transfer coefficient h_c. The results are relevant in the design of magnetic components for power electronics and other equipment.

B. Experimental set-up

To find a more precise expression for the coefficient h_c, equilibrium temperatures of an experimental model were measured under different conditions and different type of its surfaces. A box or brick type shape was used with dimensions 42/42/15 mm, which are the outer dimensions of an EE42 core.

The experimental model was made from copper, 1mm thickness.

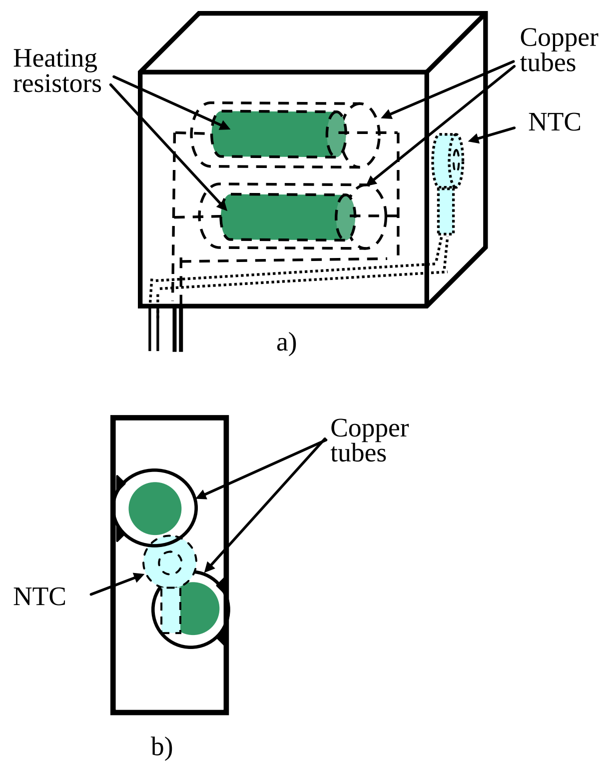

Fig.3 A sketch of the experimental model. a) transparent view
b) side cross section

The temperature was measured by NTC thermistors, type JR203R5, with the nominal resistance 20kΩ at 20°C. The NTC's were mounted on the inner surface of the model (see Fig.1). Care was taken to realise good thermal contacts and to avoid cooling the sensors by the wires.

The heating of the model was realised by two heating resistors (2 x 5W), put with silicone paste in copper tubes with an outer diameter of 10mm, 22mm long, 0.5mm thickness, which were soldered inside the model. The model is close to an isothermal surface model because the thermal conductivity of copper is quite high.

To avoid the influence of the air current in the room, the models were put in a box open at the top with dimensions 220mm by 220mm on 300mm high. As a supporting element, a thin iron wire was used and the model was kept in a chosen distance (10mm…100mm) above the bottom of the box.

To convert the measured resistance values R_{meas} in temperature (T_{meas}) the following model of the thermistor characteristic was used:

$$T_{meas} = \frac{B}{\ln \dfrac{R_{meas}}{20000} + \dfrac{B}{T_b + 25}} - T_b \tag{4}$$

The values of the parameters T_b and B were adapted to fit the data provided by the NTC manufacturer and the results of the calibration of the NTC's. Thus, the found values for the used NTC's are: B = 5102, T_b = 316. The resistivity of the NTC's was checked by finding their resistance for 0°C and 100°C. The deviations of the temperature values calculated by model given by equation (4) and the nominal values given in the data of NTC's are less than 0.4K.

The experiments were carried out in ambient temperature in the range of 25…27°C. The results were corrected (normalised) to 25°C, taking in account the difference in radiation heat transfer for different ambient temperatures. All the measurements were repeated several times and the differences between the different measurements are below 0.3K, which proves a sufficient repeatability of the experiment. Without box open at the top, one obtains a cloud at low power levels, the natural convection is influenced by small air movements in the room.

C. *Thermal measurements*

The aims of the experiments carried out with the box type model were to collect enough data in order to derive a more precise expression for the convection heat transfer coefficient h_c. The measurements were done with four different surfaces of the model:

- New but unpolished copper, which is the original surface of the model;
- Enamelled copper, which is the real open surface of windings;
- Black painted surface, which has the emissivity close to the emissivity of transformer iron and ferrites;
- Bright aluminium covered model. The model was covered by a thin aluminium foil.

To find the influence of the horizontal and vertical surfaces area on the coefficient h_c, the measurements were done for both horizontal and vertical orientation of the model.

The measured results for ambient temperature T_a = 25°C are shown in Fig.3. The figure represents the dependence of the temperature rise ΔT on the dissipated power P_{diss}.

The bold line represents the dependence $\Delta T = f(P_{diss})$ in horizontal orientation of the model and the light line in vertical orientation of the model. To find out the convection heat rate we need the values of the emissivity ε of the different surfaces.

From Fig.3 we can find the differences between the emissivity of the surfaces because the convection heat transfer is the same for the same temperature rise (assuming the same

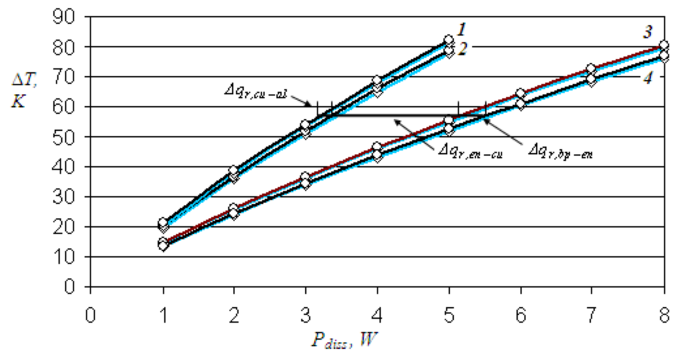

Fig.4 Temperature rise ΔT as function of the dissipated power P_{diss} for different surfaces, box 42x42x15mm, bold curves: horizontal orientation of the model; light curves: vertical orientation of the model.
(note that the light curves are just below the bold curves)
1: bright aluminium; 2: unpolished copper; 3: enamelled copper 4: black painted copper.

ambient temperature). The results are corrected (normalized) to 25 °C ambient temperature, during the measurements the ambient temperature was in the range 25…27 °C.

The difference $\Delta\varepsilon$ between the emissivity of any two surfaces is:

$$\Delta\varepsilon = \varepsilon_1 - \varepsilon_2 = \frac{\Delta q_r}{A\sigma(T_s^4 - T_a^4)} \tag{5}$$

where

T_s : measured temperature of the radiating surface;
T_a : ambient temperature;
$\Delta q_r = q_{r1} - q_{r2}$ is the difference in the radiation heat rate of the compared surfaces; in the case Δq_r is the difference in the dissipated power for one and the same ΔT (see Fig.4).

The found differences between the emissivity of the investigated surfaces are:

$\Delta\varepsilon_{bp-en}$ = 0.115 (between black and enamelled surfaces),
$\Delta\varepsilon_{en-cu}$ = 0.0.67 (between enamelled and unpolished surfaces),
$\Delta\varepsilon_{cu-al}$ = 0.0.07 (between unpolished and aluminium surfaces).
We chose a value of ε_{bp} = 0.925 for black painted surface as a reference value. Then, the emissivity of the other surfaces are:

Enamelled copper:

$$\varepsilon_{en} = \varepsilon_{bp} - \Delta\varepsilon_{bp-en} = 0.925 - 0.115 = 0.81$$

Unpolished copper:

$$\varepsilon_{cu} = \varepsilon_{en} - \Delta\varepsilon_{en-cu} = 0.81 - 0.67 = 0.14$$

Bright aluminium:

$$\varepsilon_{al} = \varepsilon_{cu} - \Delta\varepsilon_{cu-al} = 0.14 - 0.07 = 0.07$$

D. Presentation of the convection heat transfer coefficient

A curve fitting was done, based on the widely used presentation of the convection coefficient h_c mentioned in eqn. (4), where C= 1.32...1.42 , and L is the height of the component [m]. The results are quite bad matching the experimental data and theoretical model, both with respect to the temperature rise and dimensions of the component. The reason for the bad matching is that the convection process is a quite complex phenomenon. Properties of air such as heat conductivity k, kinematic viscosity ν and specific weight (density) ρ, which influence the convection process, change a lot in the considered temperature range 250...400K.

TABLE I
PROPERTIES OF AIR: HEAT CONDUCTIVITY K, VISCOSITY ν AND DENSITY ρ IN THE TEMPERATURE RANGE 250...400 K [2,6]

Temperature, [K]	250	300	350	400
Conductivity k, [W/m K]	0.02227	0.02624	0.03003	0.03365
Kinematic viscosity ν, [10^{-6} m^2/s]	11.31	15.69	20.76	25.29
Density ρ, [kg/m^3]	1.4128	1.1774	0.9980	0.8826
Prandtl number Pr,[.]	0.722	0.708	0.697	0.689

Thus, the heat transfer parameters: Nusselt number Nu, Grashof number Gr and Rayleigh number Ra, which are used in classical convection heat transfer theory, are quite influenced by the temperature and as a result, the simplified proportionality $h_c \propto (\Delta T / L)^{0.25}$ is not observed in the real experiment. The definitions for Prandtl number Pr, Grashof number Gr and Rayleigh number Ra are:

$$Pr = \frac{\nu}{\alpha} \tag{6}$$

$$Gr = \frac{g\left(\frac{2}{T_s + T_a}\right)(T_s - T_a)L^3}{\nu^2} \tag{7}$$

$$Ra = Gr\,Pr \tag{8}$$

where ν is kinematic viscosity, [m^2/s];
α is an accommodation coefficient, [s/m^2];
g is the gravity, =9.81m/s^2.

The convection coefficient h_c is defined by the Nusselt number Nu as follows:

$$h_c = Nu\frac{k}{L} \tag{9}$$

where k is the thermal conductivity.

One precise presentation of the Nusselt number, applicable over wide range of the Rayleigh number has been provided by Churchill and Chu [10]:

$$Nu = 0.68 + \frac{0.670\,Ra^{1/4}}{\left(1 + (0.492/Pr)^{9/16}\right)^{4/9}}$$

for $Ra < 10^9$ (10)

Substituting the different equations in (9), results in the following expression of h_c ; for the temperature dependency of ν and α, the average between ambient and surface temperature is used:

$$h_c = \left[0.68 + \frac{0.670\left\{g\left(\frac{2}{T_s + T_a}\right)(T_s - T_a)L^3\,\mathrm{Pr}/\nu^2\right\}^{1/4}}{\left\{1 + (0.492/\mathrm{Pr})^{9/16}\right\}^{4/9}}\right]\frac{k}{L} \tag{11}$$

From eqn. (11) it is clear that:

- the exponent, giving the final dependence of h_c on temperature rise, is lower than 0.25 as T_s , ν, Pr and k are quite temperature dependant.
- the exponent giving the dependence of h_c on the height L is higher than 0.25 because of the additional term 0.68 in (9) and (10).

Those conclusions imply the need of more precise values of the exponents. Considering that facts *our investigation aims were the following*:

1. To obtain more precise values of the exponents in a simplified expression of h_c:

$$h_c = C\frac{(\Delta T)^{\alpha_T}}{L^{\alpha_L}} \tag{12}$$

The exponents α_T , α_L and the coefficient C are to be found (note that α_T and α_L are not equal like in (2)).

2. To extend the expression (12) and to derive the dependence of h_c on the pressure p, on the ambient temperature T_a and on the orientation (horizontal or vertical) of the component, i.e. to define a complete presentation of h_c in the way:

$$h_c = C\left(\frac{p}{p_{ref}}\right)^{\alpha_p}\left(\frac{T_a}{T_{a,ref}}\right)^{\alpha_{Ta}}\frac{(\Delta T)^{\alpha_T}}{L^{\alpha_L}} \tag{13}$$

where the exponents α_p and α_{Ta} and the coefficient C (depending on the orientation) are to be found.

First, using MATCAD and the table I data, we derive the following analytical expressions:

$$k = f_1(T),\ \mu = f_2(T),\ \rho = f_3(T),\ Pr = f_4(T)$$,

which match the corresponding table data very well and the difference is below 0.1% (μ is dynamic viscosity, $\nu = \mu / \rho$). Secondly, those expressions are substituted in (11) and we obtain the complete classical expression for h_c:

$$h_c = F(\Delta T, T_a, L, p) \tag{14}$$

Finally, the precise values of the exponents α_T , α_L, α_p and α_{Ta} giving the dependence of h_c on the corresponding quantity and the coefficient C were found. Each exponent was found individually by comparing the results obtained by (13) and the results of an expression consisting of an adaptation coefficient and the corresponding quantity $\Delta T, T_a, L, p$ and the wanted exponent.

For the final modelling of the convection coefficient h_c obtained after the above-proposed approach we propose:

$$h_c = C\left(\frac{p}{p_{ref}}\right)^{0.477}\left(\frac{T_a}{T_{a,ref}}\right)^{-0.218}\frac{(\Delta T)^{0.225}}{L^{0.285}} \tag{15}$$

where C is: $C_h = 1.53$ for horizontal orientation and $C_v = 1.58$ for vertical orientation of the component;

L is the total distance passed by the air cooling the component (see Fig.5);

ΔT is the temperature rise, $\Delta T = T_s - T_a$, [K];

p_{ref} is the reference pressure at the sea altitude;

$T_{a,ref}$ is the reference ambient temperature, $T_{a,ref} = 25°C$.

A. Proposed dependence of h_c on temperature rise

The value found by MATCAD for the exponent α_T is $\alpha_T = 0.225$. The deviations between the values of h_c calculated by (15) and the expression $h_c = A_1(\Delta T)^{\alpha_T}$ are below 0.5% in the range of 10...90K for the temperature rise ΔT (A_1 is an adaptation coefficient). The same exponent $\alpha_T = 0.225$ matches well the results for convection of vertical and horizontal plates in the considered temperature range 250...400 K in the software, included to the classical book of Holman [6].

B. *Proposed dependence of h_c on the size of the component*

The observed dependence for combined vertical and horizontal surfaces, which is the case of magnetic components, includes two new aspects:

1. A more precise exponent for L in the considered range $L = 10...400\ mm$ is $\alpha_L = 0.285$ with deviations 4% at the end of the range (for comparison, the exponent 0.25 results in deviations above 22% in the considered range).

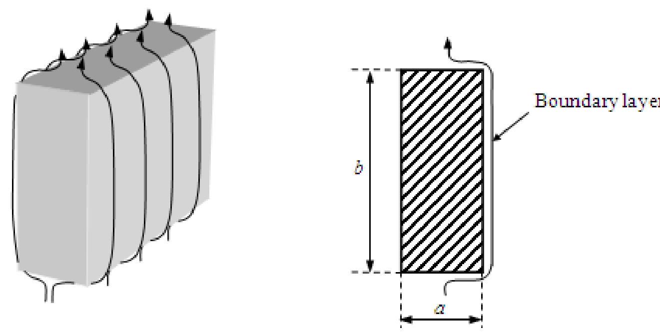

Fig.5 Fig.4 Parameter L as the total distance passed by the air cooling the component: $L = a + b$.

2. The parameter L is the total distance passed by the air cooling the component (see Fig.5). In general L could be described as "half of the length of the shortest path around a vertical mid section of the object". Notice that L is not the height of the component. In the box-shape model, for example the model with EE42 dimensions, the parameter $L = a + b$=57mm .

C. *Proposed dependence of h_c on the orientation of component*

The difference in convection for horizontal and vertical orientation of a component is proved by the experiments to be very low. This difference can be presented by different values of the coefficient C for both orientations. The experimentally obtained values are: $C_h = 1.53; C_v = 1.58$, respectively for horizontal and vertical orientation of the model.

D. *Proposed dependence of h_c on pressure*

The influence of the pressure p on the coefficient h_c was found to be given by the exponent $\alpha_p = 0.477$. The deviations are below 0.2% for the range of $50...200\%\ p_{ref}$. A similar dependence $h_c \propto \sqrt{p}$ can also be found in [3].

E. *Proposed dependence of h_c on the ambient temperature*

The value found by MATCAD for the exponent α_{Ta} is $\alpha_{Ta} = -0.218$. The deviations between the values of h_c calculated by (14) and the expression $h_c = A_2(T_a / T_{a,ref})^{\alpha_{Ta}}$ are below 0.04% in the range of $0 \sim 120\ °C$ for the ambient temperature rise T_a (A_2 is an adaptation coefficient).

F. *Dependence of h_c on the shape of the component*

Till now we considered only box shape components. For more complex component shapes an equivalent surface can be used to find the convection heat transfer. This surface is closed to the envelope surface, which is quite lower than the total component open surface. For example, for an EE42 transformer (including coil ends) the total open surface is $7.872\times10^{-3}[m^2]$, the 'envelope' surface is $6.895\times10^{-3}[m^2]$ and the box surface, corresponding to the ferrite's dimensions is $6.048\times10^{-3}[m^2]$.

The derived expression (14) can be used also in more complex thermal models including inner thermal resistances and different copper and iron temperatures, representing the complexity in the construction details of the component.

E. Comparison of the experimental results and the convection fit formulae

The experimental results where compared with the analytical curves obtained by the final fit formulae (14). The experimental and theoretical curves were matched for enameled and black painted surfaces of the model. The matching is quite good as it can be seen in Fig.5 and Fig.6 showing the dependence $\Delta T = f(P_{diss})$ for ambient temperature T_a = 25°C and proves the validity of the proposed expression for h_c as well as the found values of the emissivity of enameled copper and black paint surface.

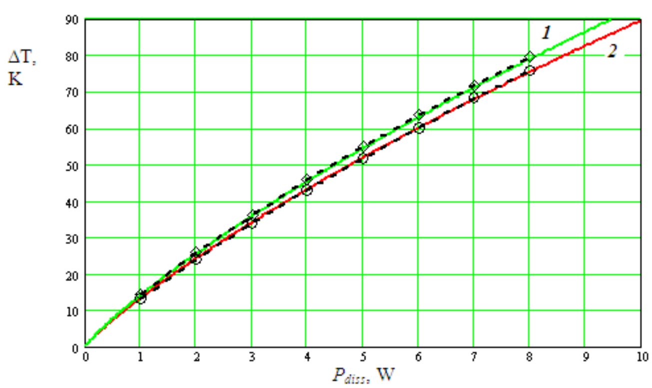

Fig.5 Temperature rise ΔT as function of the dissipated power P_{diss} for enameled (***1***) and black painted (***2***) surfaces, box 42x42x15mm, horizontal orientation, T_a=25°C.
solid curves: model results; dash lines: experimental results.

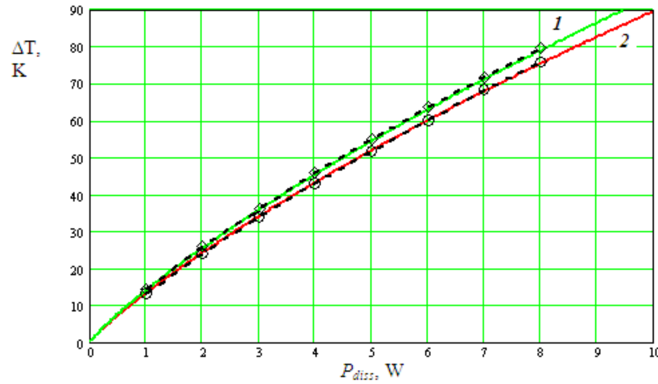

Fig.6 Temperature rise ΔT as function of the dissipated power Pdiss for enameled (1) and black painted (2) surfaces, box 42x42x15mm, vertical orientation, Ta=25°C.
solid curves: model results; dash lines: experimental results.

IV. Forced convection equation

Most of forced convection equations predict no heat transfer if no forced air speed is present. An natural convection is in fact a forced convection where the own heat creates circulation. We suppose that the air flow helps the natural convection. So we made the effort to match a natural convection with a forced convection in a single equation, the natural convection part is a bit simplified. To simplify the calculations coming from the equation (15) we propose the following expression for forced convection in air at atmospheric pressure:

$$h_c = \left(3.33 + 4.8u_\infty^{0.8}\right)L^{-0.288} \qquad (16)$$

where L is the total distance of the boundary layer of the component (see Fig. 4).

The expression (16) is consistent with the classical reference [11] up to u_∞=12m/s as well with [3] from a few m/s.. The advantage of eqn. (16) is that it combines both natural and forced convection processes. The offsets of the corresponding curves when the velocity of the approaching flow u_∞ is zero, correspond to the values of the natural convection coefficient h_c given by the equation (16) in the previous section. Fig. 7 presents the convection coefficient h_c for different values of the parameter L for a temperature difference of 30 °C in accordance with the equation (16). Fig 7 gives a fast result for the forced convection coefficient h_c, including the scale effect of the component size.

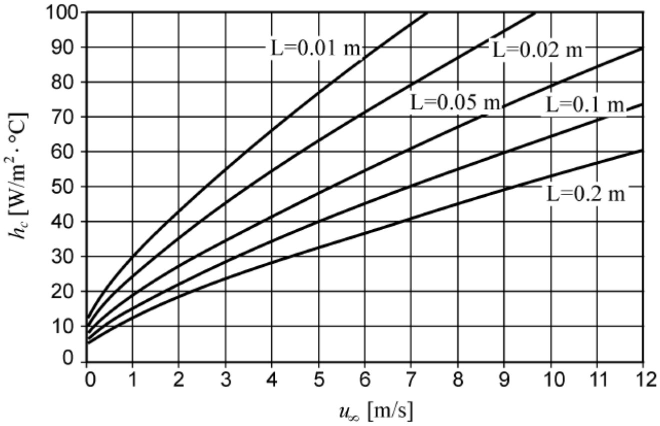

Fig.7. Temperature rise as function of the characteristic dimension of the component and the air speed at sufficient distance and 30K rise.

In forced cooling there are a lot of details to be considered to find the accurate heat transfer as the position and orientation of the component, the near by components. Thus, the accuracy of the equation (16), which is about 15%, which is quite acceptable for most of the designs in Power Electronics.

Concerning forced convection, some warnings should be given:

- The forced convection reduces the surface-to-ambient thermal resistance, but does not change the internal hot spot-to-ambient thermal resistance.
- An intensive forced cooling results in a high temperature gradient within the component. In extreme cases the thermal stresses caused by such a cooling can break the ferrites or reduce the lifetime of the isolation.

V. Conclusion

An overview of thermal modelling of magnetic components was proposed, ranging from fast approximations to more accurate ones. An investigation has been done on the dependency of the convection heat transfer coefficient h_c on the temperature rise ΔT, the dimensions and orientation of the magnetic component. The advantage of the model is that it uses the simple classical representation of h_c, but with more precise values of the exponents of the parameters ΔT and characteristic dimension L. The influence of the orientation of the component, the ambient temperature and pressure are also more precisely defined. The experimental results show a good matching with the model results and prove its validity in the considered temperature range. The presented convection model improves the heat convection modelling of magnetic components.

The proposed isotherm surface model can also be used as an element in more complex, multiple thermal resistance models of magnetic components as well for other electronic equipment.

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