Future Network and MobileSummit 2010 Conference Proceedings
Paul Cunningham and Miriam Cunningham (Eds)
IIMC International Information Management Corporation, 2010
ISBN: 978-1-905824-16-8

A Novel Quantize-and-Forward Cooperative System: Channel Parameter Estimation Techniques

Iancu AVRAM[1], Nico AERTS[2], Marc MOENECLAEY[3]
[1,2,3]*Ghent University (TELIN), St.-Pietersnieuwstraat 41, Gent, 9000, Belgium*
Tel: +32 9 264 {8900[1,2],3413[3]}, Email:{iancu.avram[1],nico.aerts[2],mm[3]}@telin.ugent.be

Abstract: The Quantize and Forward cooperative communication protocol improves the reliability of data transmission by allowing a relay to forward to the destination a quantized version of the signal received from the source. In prior studies of the Quantize and Forward protocol, all channel parameters are assumed to be perfectly known at the destination, while in reality these need to be estimated. This paper proposes a novel Quantize and Forward protocol in which the relay compensates for the rotation on the source-relay channel using a crude channel estimate, before quantizing the phase of the received M-PSK data symbols. Therefore, as far as the source-relay channel is concerned, only an SNR estimate is needed at the destination. Further, the destination applies the EM algorithm to improve the estimates of the source-destination and relay-destination channels. The resulting performance is shown to be close to that of a system with known channel parameters.

Keywords: Cooperative communication, Quantize and Forward, EM algorithm

1. Introduction

The reliability of a classic point-to-point communication system is determined by the fading probability of the link between both terminals. In a cooperative communications system, the presence of a relay creates an additional independent channel between source and destination. The spatial diversity introduced by the relay improves the reliability of the system, which is now determined by the probability that both channels are simultaneously in fading [1]. In an Amplify and Forward (AF) system, the relay amplifies the signal received from the source and forwards it to the destination [2]. However, when using half-duplex terminals that cannot transmit and receive data at the same time, the relay needs to store the received information in order to forward it later on. The AF protocol assumes this data can be stored with an infinite precision. In a more realistic system this data is quantized before storage, yielding the Quantize and Forward (QF) protocol.

Several quantization schemes have been discussed in literature [3] [4]. In [3] the relay quantizes the phase of the received M-PSK modulated signal without knowing the source-relay channel. The destination is assumed to know all the channel coefficients when decoding the received symbols. It is shown that uniform quantization of the phase with $\log_2 M + 1$ bits is sufficient to closely approach the performance of a pure AF system. In a realistic system however, the different channel parameters need to be estimated. In this contribution we propose a novel QF protocol, where the relay first makes a crude estimate of the source-relay channel based on pilot symbols received from the source. This estimate is used to compensate for the channel rotation of this channel before quantizing the received signal. As will be shown, the proposed protocol requires only $\log_2 M$ bits for the quantization of each symbol to achieve a performance similar to that of a pure AF system. At the destination an initial estimate of the source-destination and relay-destination channel is obtained from the received pilot symbols. These initial estimates are then refined using the EM algorithm [5]. In an attempt to reduce the computational complexity of the EM algorithm, an approximation is discussed that yields only a minor loss in performance.

2. System Model

At the source, blocks of K information bits are encoded into blocks of N coded bits which are then mapped on K_d M-PSK symbols. In a first timeslot, the source transmits K_p pilot symbols along with the K_d coded data symbols, which are received by both the relay and the destination. In a second timeslot, the relay sends to the destination K_p pilot symbols followed by a quanitzed version of the noisy K_d coded symbols received from the source, along with the information on the estimated instantaneous signal-to-noise ratio (SNR) on the source-relay channel.

2.1 Communication channels

The communication channels involved are modelled as independent flat Rayleigh fading channels with additive white Gaussian noise. The source-destination, source-relay and relay-destination channel coefficients are denoted h_0, h_1 and h_2, respectively. Considering the channel model, the output of the different channels can be written as

$$\begin{aligned} \mathbf{r}_0 &= h_0\mathbf{c}_s + \mathbf{n}_0 \\ \mathbf{r}_1 &= h_1\mathbf{c}_s + \mathbf{n}_1 \\ \mathbf{r}_2 &= h_2\mathbf{c}_r + \mathbf{n}_2, \end{aligned} \tag{1}$$

with $\mathbf{c}_s$ the symbols sent by the source and $\mathbf{c}_r$ the symbols sent by the relay. All vectors are denoted as row vectors. The channel coefficients h_i are constant during a timeslot and have a zero mean circular symmetric complex gaussian (ZMCSCG) distribution with variance $N_{h_i} = 1/{d_i}^4$ and d_i the distance between the two terminals involved ($i = 1, 2, 3$). The elements of the vector $\mathbf{n}_i$ are also ZMCSCG distributed with variance N_i ($i = 1, 2, 3$). The energy of the symbols sent by the source and the relay equals E_s. Taking into account the transmission of the pilot symbols, the energy required by the source to send one information bit, denoted E_b, can be expressed in terms of E_s:

$$E_b = \frac{(K_d + K_p)}{K_d}\frac{N}{K\log_2 M}E_s. \tag{2}$$

2.2 Structure of the relay terminal

We propose a relay that compensates for the channel rotation caused by the source-relay channel h_1, before quantizing the received signal. This compensation makes use of an estimate $\hat{h_1}$ of this channel, based on pilot symbols transmitted by the source. The i-th symbol $c_{r,i}$ is a quantized version of the i-th element $r_{1,i}$ of $\mathbf{r}_1$:

$$c_{r,i} = e^{jq_i}, \tag{3}$$

where q_i is defined by the relationship

$$q_i = \frac{2\pi k_i}{2^Q} \qquad \text{if } \frac{\pi}{2^Q}(2k_i - 1) < \arg(r_{1,i}\hat{h_1}^*) < \frac{\pi}{2^Q}(2k_i + 1) \tag{4}$$

with $k \in \{0, 1, ..., 2^Q - 1\}$ and Q the number of quantization bits. When using this quantization scheme, the destination will only be required to know the instantaneous SNR on the source-relay channel, given by $\gamma = |h_1|^2/N_1$, and not the exact value of h_1, as will be proven in the next subsection. This instantaneous SNR is estimated by the relay, quantized, encoded, mapped to M-PSK symbols and forwarded to the destination.

2.3 Signal combining at the destination

For decoding purposes, the likelihoods of the received symbols must be determined by the destination. Because the source-destination and relay-destination channels are orthogonal, the likelihood of the i-th received source symbol $c_{s,i}$ equals

$$p(r_{0,i}, r_{2,i}|c_{s,i}, h_0, h_1, h_2) = p(r_{0,i}|c_{s,i}, h_0)p(r_{2,i}|c_{s,i}, h_1, h_2), \tag{5}$$

The first factor from (5) can be written as

$$p(r_{0,i}|c_{s,i}, h_0) = \frac{1}{\pi N_0} e^{-\frac{|r_{0,i} - h_0 c_{s,i}|^2}{N_0}}. \tag{6}$$

The second factor from (5) can be expressed as the marginal of $p(r_{2,i}, k_i, \hat{h_1}|c_{s,i}, h_1, h_2)$. This yields

$$p(r_{2,i}|c_{s,i}, h_1, h_2) = \sum_{k=0}^{2^Q-1} p(r_{2,i}|k_i = k, h_2) \int P(k_i = k|c_{s,i}, \hat{h_1}, h_1)p(\hat{h_1}|h_1)\mathrm{d}\hat{h_1}. \tag{7}$$

The evaluation of $p(r_{2,i}|k_i = k, h_2)$ proceeds similarly to (6). The first factor in the integrand from (7) can be calculated using the phase density function [3]

$$f_\Theta(\theta) = \frac{1}{2\pi}\left[e^{-\gamma} + \sqrt{\pi\gamma}\cos(\theta)e^{-\gamma\sin^2(\theta)}\mathrm{erfc}(-\sqrt{\gamma}\cos(\theta))\right].$$

This function describes the distribution of the received phase when a symbol with amplitude 1 and phase 0 is sent over an AWGN channel. The variable γ is the SNR ratio at the receiving terminal (the relay in this case). Using this function, one obtains

$$P(k_i = k|c_{s,i}, \hat{h_1}, h_1) = \int_{\phi_k^l}^{\phi_k^u} f_\Theta(\theta - \arg(c_{s,i}h_1\hat{h_1}^*))\mathrm{d}\theta, \tag{8}$$

where the integration in (8) is over the quantization interval (4) for $k_i = k$. When using a ML estimate for $\hat{h_1}$ based on K_p pilot symbols, the second factor in the integrand from (7) equals

$$p(\hat{h_1}|h_1) = \frac{1}{\pi N_1/K_p} e^{-\frac{|\hat{h_1} - h_1|^2}{N_1/K_p}}. \tag{9}$$

Using (8) and (9), the integral in (7) can be evaluated numerically, for a given h_1 and $c_{s,i}$. The resulting likelihood (5) of $c_{s,i}$ contains the channel parameters h_0, h_1 and h_2. As these parameters at not known at the destination, the likelihood (5) will be computed at the destination with the true channel parameters replaced by estimates.

The channel gains h_0 and h_2 are estimated at the destination, while an estimate of h_1, computed by the relay, could be sent from the relay to the destination. However, in order to avoid the numerical integration in (7), the destination will use the simplifying assumption that the relay makes a perfect estimate of h_1, so that

$$p(\hat{h}_1|h_1) = \delta(\hat{h}_1 - h_1). \tag{10}$$

In this case, (7) reduces to

$$p(r_{2,i}|c_{s,i}, h_1, h_2) = \sum_{k=0}^{2^Q-1} p(r_{2,i}|k_i = k, h_2)P(k_i = k|c_{s,i}), \tag{11}$$

where

$$P(k_i = k|c_{s,i}) = \int_{\phi_k^l}^{\phi_k^u} f_\Theta(\theta - \arg(c_{s,i}))\mathrm{d}\theta. \tag{12}$$

As a result, as far as the source-relay channel is concerned, only the value $\gamma = |h_1|^2/N_1$ now needs to be known by the destination; an estimate of γ is sent from the relay to the destination. Although the approximation (10) does not hold for small values of h_1, it does not significantly affect the error performance as the likelihood of the i-th received symbol is calculated using only the source-destination path when the value of h_1 approaches zero.

3. Estimation

When the channel coefficients are unknown at the receiver, they need to be estimated. The first step in the estimation process is the calculation of an initial estimate using known pilot symbols sent by the source and the relay. Thereafter, the estimates of the source-destination and relay-destination channels will be improved using the EM algorithm [5] at the destination. The EM algorithm alternates between an estimation step and a maximization step in order to iteratively improve the channel estimate. Introducing $\mathbf{r}_d = (\mathbf{r}_0, \mathbf{r}_2)$, $\mathbf{c}_d = (\mathbf{c}_s, \mathbf{c}_r)$, and $\mathbf{h}_d = (h_0, h_2)$, the estimation step during iteration k involves calculating the function

$$Q(\mathbf{h}_d, \hat{\mathbf{h}}_d^{(k-1)}) = \mathbf{E}_{\mathbf{c}_d}\left[\ln p(\mathbf{r}_d|\mathbf{c}_d, \mathbf{h}_d) \,\middle|\, \mathbf{r}_d, \hat{\mathbf{h}}_d^{(k-1)}\right]. \tag{13}$$

The maximization step involves determining a value for h_0 and h_2 that maximizes the Q function from (13), so the new estimates calculated at iteration k are equal to

$$\hat{\mathbf{h}}_d^{(k)} = \arg\max_{\mathbf{h}_d} Q(\mathbf{h}_d, \hat{\mathbf{h}}_d^{k-1}),$$

where $\hat{\mathbf{h}}_d^{(0)}$ contains the estimate of (h_0, h_2) obtained from the pilot symbols only. Using factorization (5) for $p(\mathbf{r}_d|\mathbf{c}_d, \mathbf{h}_d)$, one obtains

$$\begin{aligned}\hat{h}_0^{(k)} &= \frac{\mathbf{r}_0\mathbf{u}_s^H}{(K_p + K_d)E_s}\\ \hat{h}_2^{(k)} &= \frac{\mathbf{r}_2\mathbf{u}_r^H}{(K_p + K_d)E_s},\end{aligned} \tag{14}$$

with $\mathbf{u}_s$ and $\mathbf{u}_r$ denoting the a posteriori expectations (conditioned on $\mathbf{r}_d$ and $\hat{\mathbf{h}}_d^{(k-1)}$) of the symbol vectors $\mathbf{c}_s$ and $\mathbf{c}_r$, respectively.

The components of $\mathbf{u}_s$ and $\mathbf{u}_r$ that correspond to the pilot symbols are equal to these pilot symbols. Using in (14) only the pilot symbols allow calculating an initial estimate of the source-destination and relay-destination channel, respectively. The computation of the components of $\mathbf{u}_s$ and $\mathbf{u}_r$ that correspond to the data symbols is outlined below. The i-th elements of the vectors $\mathbf{u}_s$ and $\mathbf{u}_r$ are equal to:

$$u_{s,i} = \sum_{c_{s,i},c_{r,i}} c_{s,i} p(c_{s,i}, c_{r,i}|\mathbf{r}_d, \hat{\mathbf{h}}_d^{(k-1)}) = \sum_{c_{s,i}} c_{s,i} p(c_{s,i}|\mathbf{r}_d, \hat{\mathbf{h}}_d^{(k-1)}) \tag{15}$$

$$u_{r,i} = \sum_{c_{s,i},c_{r,i}} c_{r,i} p(c_{r,i}|c_{s,i}, \mathbf{r}_d, \hat{\mathbf{h}}_d^{(k-1)}) p(c_{s,i}|\mathbf{r}_d, \hat{\mathbf{h}}_d^{(k-1)}). \tag{16}$$

The summations in (15) and (16) run over all values that $c_{s,i}$ and/or $c_{r,i}$ can adopt. Further development of the conditional distribution of $c_{r,i}$ in (16) yields

$$p(c_{r,i}|c_{s,i}, \mathbf{r}_d, \hat{\mathbf{h}}_d^{(k-1)}) = \frac{p(c_{r,i}, r_{d,i}|c_{s,i}, \hat{\mathbf{h}}_d^{(k-1)})}{p(r_{d,i}|c_{s,i}, \hat{\mathbf{h}}_d^{(k-1)})} = \frac{p(r_{2,i}|c_{r,i}, \hat{h}_2^{(k-1)}) p(c_{r,i}|c_{s,i})}{\sum_{\tilde{c_{r,i}}} p(r_{2,i}|\tilde{c_{r,i}}, \hat{h}_2^{(k-1)}) p(\tilde{c_{r,i}}|c_{s,i})}.$$

The distribution $p(c_{r,i}|c_{s,i})$ follows from (12). When evaluating (12), the destination makes use of the estimate $\hat{\gamma}$, forwarded by the relay. The marginal a posteriori probabilities of the data symbols $c_{s,i}$ can be calculated by the decoder at the destination [6]; therefore, this EM approach is referred to as code-aided.

3.1 Assumption of uncoded transmission

To lower the computational complexity, the calculation of the marginal a posteriori symbol expectations (15) and (16) can be carried out under the (false) assumption that the M-PSK symbols transmitted by the source are uncoded: the symbols contained in $\mathbf{c}_s$ are considered statistically independent and uniformly distributed over the M-PSK constellation. This approximation involves the following substitution in (15), (16) :

$$p(c_{s,i}, c_{r,i}|\mathbf{r}_d, \hat{\mathbf{h}}_d^{(k-1)}) = C p(\mathbf{r}_{0,i}|c_{s,i}, \hat{h}_0^{(k-1)}) p(\mathbf{r}_{2,i}|c_{r,i}, \hat{h}_2^{(k-1)}) p(c_{r,i}|c_{s,i}), \tag{17}$$

where C is a normalization constant. When using this approximation no decoding steps are required within the EM algorithm. After the EM algorithm has completed, the resulting estimates are forwarded to the decoder. This approach significantly reduces computational complexity while still achieving an acceptable performance as will be shown in the next section.

4. Simulations

We consider a source that encodes frames of 1024 information bits by means of a $(1, 13/15)_8$ RSCC turbo code [7], and maps the encoded bits to M-PSK symbols. The relay is located halfway between source and destination. By means of computer simulations, the Frame Error Rate (FER) performance of the proposed system with the different estimation strategies is determined as function of the E_b/N_0 ratio, with E_b given by (2). All noise variances are assumed equal ($N_0 = N_1 = N_2$) and known to both destination and relay.

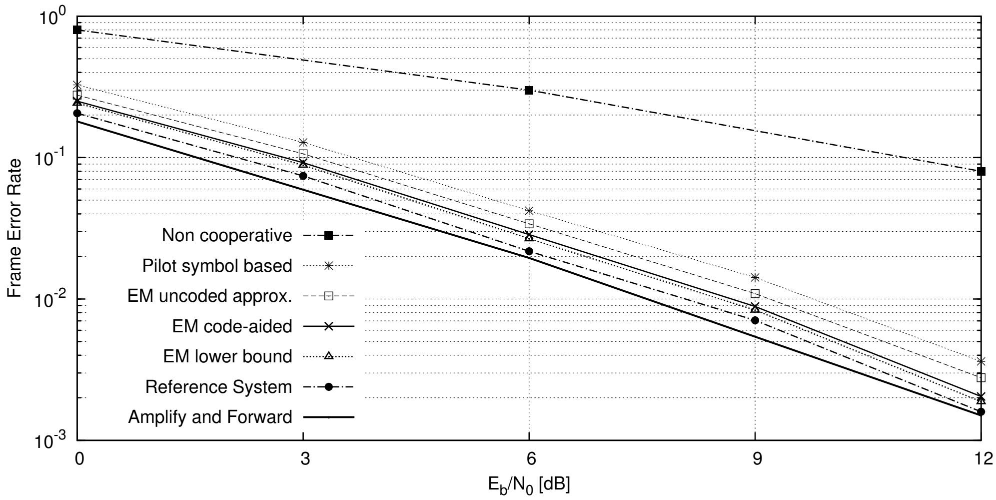

Figure 1: Frame Error Rate of the different proposed estimation techniques using 8-PSK mapping.

To be able to calculate an initial estimate for the channel coefficients, K_p pilot symbols are sent by both source and relay. To maintain a nearly fixed $(K_d + K_p)/K_d$ ratio in (2), 9, 5 and 3 pilot symbols are sent when using BPSK, QPSK and 8PSK mapping, respectively. The relay converts the estimated value $\hat{\gamma}$ of the instantaneous SNR to dB and uniformly quantizes it between $\gamma_{min,db}$ and $\gamma_{max,db}$ using 5 bits. We have selected the values of $\gamma_{min,db}$ and $\gamma_{max,db}$ such that they minimize, at $E_b/N_0 = 6$ dB, the FER of a QF system with known channel parameters, but with the value of γ unknown to the destination. For all values of E_b/N_0 in (0 dB, 12 dB), we used the $\gamma_{min,db}$ and $\gamma_{max,db}$ that are optimum at $E_b/N_0 = 6$ dB. The quantized bits are encoded with a simple $(1, 3)_8$ convolutional code, mapped on M-PSK symbols and sent to the destination. The EM iterations and turbo decoding iterations are merged as explained in [8]. For each frame 12 EM-code iterations are used. When using the approximation of uncoded symbols discussed in section 3.1, the EM algorithm is allowed 5 iterations, after which the turbo code is decoded using 12 iterations.

The FER performance resulting from the considered estimation technique is compared to an EM lowerbound. This EM lowerbound on the FER corresponds to the best performance the EM algorithm can achieve and is calculated by assuming the data-symbols sent by the source and relay are known at the destination when calculating the estimates of h_0 and h_2. As compared to the reference system with known channel parameters and no pilot symbols transmitted, this EM lowerbound has a worse FER performance due to channel estimation errors (especially the estimation of the source-relay channel coefficient, where only pilot symbols are used) and the smaller E_s from (2) because of the pilot symbols (assuming a constant total transmit energy per frame).

Fig. 1 illustrates the FER performance of the different estimation techniques by expressing the FER versus E_b/N_0 in the case of 8-PSK mapping and 3 bit quantization. The figure also shows the FER performance of a pure Amplify and Forward system and a non-cooperative system, both with known parameters. Fig. 1 shows that quantization with $\log_2 M$ bits is sufficient to closely approach the performance of a pure AF system. We have verified (results not shown here) this is also valid for BPSK and QPSK

Table 1: E_b/N_0 (dB) and degradation (dB) w.r.t. reference system @ FER = 0.01.

	BPSK	QPSK	8-PSK
Reference system	5.94 (0)	6.06 (0)	8.10 (0)
EM lower bound	6.04 (0.10)	6.29 (0.23)	8.52 (0.42)
EM code-aided	6.04 (0.10)	6.32 (0.26)	8.67 (0.57)
EM uncoded approx.	6.05 (0.11)	6.42 (0.36)	9.18 (1.08)
Pilot based only	6.52 (0.58)	6.91 (0.85)	9.78 (1.68)

mapping. The effect of the different estimation schemes on the error performance for BPSK, QPSK and 8-PSK mapping is summarized in Table 1 for FER $= 0.01$. The results indicate that the effect of channel estimation errors on the FER becomes more severe as the number of bits per symbol increases. The simulation results also show the assumption of uncoded symbols works very well for BPSK, but the performance deteriorates as the number of bits per symbol increases.

The effect of the constellation size on the FER performance degradation can be explained by investigating the Mean Square Error (MSE) values resulting from the different estimations, shown in Fig. 2 for h_0. The MSE values for the different estimates of h_2 are similar to those for h_0 and are not shown here. The deterioration in FER performance for higher constellations when using the assumption of uncoded symbols is also reflected in the increasing MSE of the estimate of h_0. The difference between the likelihoods of the different symbols in (17) will become less pronounced when there are more constellation points, making it harder to determine which symbol has been sent and thus making an accurate estimation difficult. The MSE of the code-aided approach is closer to the EM lowerbound compared to the uncoded approximation for the same constellation, but also rises with increasing number of bits per symbol due to a higher symbol error rate (QPSK) and more decoding errors (8-PSK) than in the case of BPSK. From (15) and (16), one notices the a posteriori expectation of the symbol vectors sent by both source and relay is conditioned on the observation of both communication channels (direct link and relaypath). This cooperative nature accounts for the very accurate estimate of the source-destination and relay-destination channel.

5. Conclusions

In this paper a novel Quantize and Forward protocol has been introduced, which involves the relay making a crude estimate of the source-relay channel using only the received pilot symbols. Doing so, it is shown that quantization with only $\log_2 M$ bits is sufficient to approach the performance of an Amplify and Forward system. At the destination, the EM algorithm allows improving the pilot based estimates of the source-destination and relay-destination channels. The EM algorithm results in a very good FER performance, but it also increases the computational complexity. This complexity can be reduced by using an approximation that assumes the received signal consists of uncoded M-PSK symbols and does not require the a-posteriori symbol probabilities provided by the channel decoder. This way, no decoding step is required within the EM algorithm. This approximation performs very well with BPSK mapping, but deteriorates with increasing number of bits per symbol. When using high density constellations like 8-PSK, the code-aided EM algorithm should be used to achieve a Frame Error Rate that

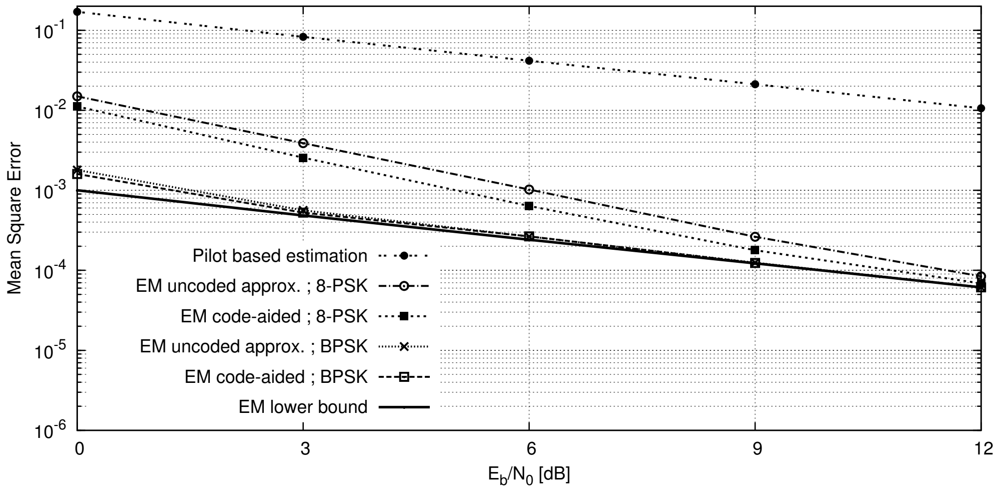

Figure 2: Mean Square Error values for the estimate of h_0.

is very close to that of a system with known channel coefficients.

Acknowledgement

The authors wish to acknowledge the activity of the Network of Excellence in Wireless COMmunications NEWCOM++ of the European Commission (contract n. 216715) that motivated this work.

References

[1] A. Sendonaris, E. Erkip and B. Aazhang, "User cooperation diversity - part I : System description," *IEEE Trans. on Commun.*, pp. 1927–1938, Nov 2003.

[2] J. N. Laneman, *Cooperative Diversity in Wireless networks: Algorithms and Architectures*. PhD thesis, Massachusetts Institute of Technology, 2002.

[3] M. Souryal and H. You, "Quantize-and-forward relaying with M-ary phase shift keying," in *WCNC Proceedings*, 2008.

[4] B. Djeumou, S. Lasaulce, and A. G. Klein, "Practical quantize-and-forward schemes for the frequency division relay channel," *EURASIP Journal on Wireless Communications and Networking*, p. 11, 2007.

[5] A. Dempster, N. Laird and D.B. Rubin, "Maximum like-lihood from incomplete data via the EM algorithm," *J. Royal Statistical Soc.*, vol. 39, pp. 1–38, 1977.

[6] H. Wymeersch, *Iterative Receiver Design*. Cambridge University Press, 2007.

[7] S. Lin and D. Costello, *Error Control Coding*. Pearson Education Inc., second edition ed., 2004.

[8] N. Noels, C. Herzet, A. Dejonghe, V. Lottici, H. Steendam, M. Moeneclaey, M Luise and L. Vandendorpe, "Turbo-synchronization: an EM algorithm approach," *Proc IEEE ICC*, May 2003.

