Pilot Based Time Delay Estimation for KSP-OFDM Systems

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***Abstract*—We propose a time delay estimator for known symbol padding (KSP) orthogonal frequency division multiplexing (OFDM) in a multipath fading environment. The estimator makes use of pilot symbols in the guard interval and known pilot carriers. The performance of the estimator is illustrated by means of simulation results for the mean squared error (MSE) and the bit error rate (BER). There is a degradation in performance compared with a receiver with perfect synchronization, especially for high E_s/N_0, but the KSP-OFDM system with the proposed estimator outperforms a cyclic prefix OFDM system with the time delay estimator from [1].**

I. INTRODUCTION

The number of wired and wireless services has increased a lot during the last years. This increase has made it necessary to find a technique that combines high data rates with a high reliability. Orthogonal frequency division multiplexing (OFDM) is a strong candidate as it is a flexible technique that can support high data rates, and is able to combat frequency selective channels [2]. These advantageous properties have made OFDM a hot research topic and the OFDM technique has already been applied in various standards like digital audio broadcasting (DAB) [3], digital video broadcasting (DVB) [4], in modems for digital subscriber lines (xDSL) [5], in wireless local area networks (WLAN) [6], ...

An OFDM system can be efficiently implemented by the usage of fast Fourier transforms (FFT), which is a great advantage. Before the transmission, an inverse FFT (IFFT) is applied to the information to be transmitted, in order to convert the data that are modulated in the frequency domain on the different carriers into a time domain signal. Further, a guard interval is inserted to avoid inter block interference (IBI) between successively transmitted OFDM blocks. In the literature, there exist different types of guard intervals. The two most popular guard interval techniques are the cyclic prefix (CP) and the zero padding (ZP) techniques [7]. In the cyclic prefix technique, the guard interval is transmitted before each OFDM block and consists of the last samples of the OFDM block. In ZP-OFDM, the guard interval is filled with zeros, i.e. during the guard interval no signal is transmitted. In this paper however, we will consider a third guard interval technique, i.e. the known symbol padding (KSP) technique [8]. In this technique, the guard interval is filled with known samples or pilots.

Synchronization of the OFDM receiver with the OFDM transmitter requires to find the starting point of the OFDM symbol: time offsets can cause inter carrier interference (ICI) and IBI [9], [10]. For CP-OFDM, several time delay estimation algorithms have been proposed in the literature. The authors of [1] derive the maximum likelihood (ML) estimator for a time delay in the presence of additive white Gaussian noise (AWGN). The redundancy of the cyclic prefix and pilot symbols on the carriers are exploited. The blind estimator of [11] is a special case of the previous estimator and only makes use of the correlation of the cyclic prefix and the last samples of the transmitted OFDM block. A time delay estimator that makes use of a specially designed training symbol is proposed in [12] for the AWGN channel. However, as it does not employ all available information, the estimator is suboptimal. In [13], the ML time delay estimator is derived in the case of dispersive channels under the assumption of perfect channel knowledge. The estimator uses the cyclic prefix only. However, as it is in practice very difficult to obtain a channel estimate without knowledge about the time delay, the performance of this estimator can be seen as a lower bound on the performance of an estimator which does not assume any knowledge about the channel.

To our knowledge, no research has been done about time delay estimation algorithms for KSP-OFDM. This motivated us to derive an ML-based timing delay estimator for KSP-OFDM in dispersive channels. The performance of the proposed estimator is compared with the estimator for CP-OFDM from [1] in terms of the mean squared error (MSE) of the time delay estimate, and in terms of the bit error rate (BER).

II. SYSTEM MODEL

Consider a KSP-OFDM system with N carriers and a guard interval of length ν. M is defined as the total number of transmitted pilot symbols of which ν are transmitted during the guard interval and $M-\nu$ on the carriers. On the different carriers we transmit a block of symbols $\mathbf{a} = (a(0), \ldots, a(N-1))^T$ consisting of $M-\nu$ pilot symbols denoted as $\mathbf{b}_c = (b_c(0), \ldots, b_c(M-\nu-1))^T$ and $N+\nu-M$ data symbols denoted as $\mathbf{a}_d = (a_d(0), \ldots, a_d(N+\nu-M-1))^T$. The guard interval consists of ν pilot symbols denoted as $\mathbf{b}_g = (b_g(0), \ldots, b_g(\nu-1))^T$. We define E_s as the transmitted energy per symbol: $E_s = \mathrm{E}\left[|a(n)|^2\right] = \mathrm{E}\left[|b_g(k)|^2\right]$. The transmitted symbol $\mathbf{a}$ is modulated on the different carriers using the N-point IFFT. The guard interval is inserted after the N

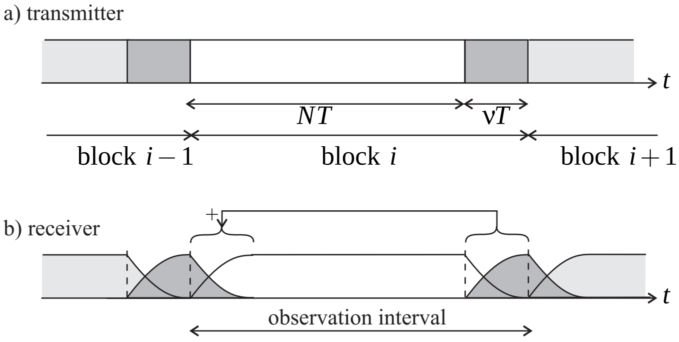

Figure 1. Time-domain signal of a KSP-OFDM block a) transmitted signal b) received signal and observation interval

IFFT outputs. The samples of the transmitted time domain signal $\mathbf{s} = (s(0),\ \ldots,\ s(N+\nu-1))^T$ are given by

$$\mathbf{s} = \sqrt{\frac{N}{N+\nu}} \begin{pmatrix} \mathbf{F}^H\mathbf{a} \\ \mathbf{b}_g \end{pmatrix} \tag{1}$$

where $\mathbf{F}$ denotes the $N \times N$ FFT matrix with elements $(\mathbf{F})_{k,l} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{kl}{N}};\ k,l = 0,\ \ldots,\ N-1$. Figure 1 shows the time domain signal. We define the vectors $\mathbf{s}_p$ and $\mathbf{s}_d$ as

$$\begin{aligned} \mathbf{s}_p &= \sqrt{\frac{N}{N+\nu}} \mathbf{F}_p \mathbf{b}_c \\ \mathbf{s}_d &= \sqrt{\frac{N}{N+\nu}} \mathbf{F}_d \mathbf{a}_d \end{aligned}$$

where $\mathbf{F}_p$ consists of the $M-\nu$ columns of $\mathbf{F}^H$ which correspond to the pilot carriers and $\mathbf{F}_d$ is given by the $N+\nu-M$ columns of $\mathbf{F}^H$ that correspond to the data carriers. So $\mathbf{s}_p$ can be seen as the N-point IFFT of the pilot carriers only, while $\mathbf{s}_d$ is the N-point IFFT of the data carriers only. We define $\mathbf{b}$ as the total transmitted pilot signal, so $\mathbf{b}$ collects the contribution from the pilot carriers and the pilot symbols in the guard interval

$$\mathbf{b} = \begin{pmatrix} \mathbf{s}_p \\ \sqrt{\frac{N}{N+\nu}} \mathbf{b}_g \end{pmatrix}. \tag{2}$$

The samples $\mathbf{s}$ are transmitted over a frequency selective channel with an impulse response of length L denoted as $\mathbf{h} = (h(0),\ \ldots,\ h(L-1))^T$. In order to avoid inter block interference, the length of the guard interval ν is chosen so that the guard interval exceeds the duration of the channel impulse response: $\nu \geq L-1$. The received signal $r(k)$ can be written as

$$r(k) = \sum_{l=0}^{L-1} h(l)\, s(k-k_0-l) + w(k) \tag{3}$$

where k_0 is the integer time delay and $w(k)$ is additive white Gaussian noise with variance N_0 and zero mean. Note that $s(k) = 0$ for $k \geq N+\nu$ and $k < 0$. Expression (3) can be written in the following matrix form

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} \tag{4}$$

where $\mathbf{r} = (r(k_0),\ \ldots,\ r(k_0+N+\nu+L-2))^T$, $\mathbf{w} = (w(k_0),\ \ldots,\ w(k_0+N+\nu+L-2))^T$, $\mathbf{s}$ is defined in (1) and $\mathbf{H}$ is the $(N+\nu+L-1) \times (N+\nu)$ Toeplitz channel matrix whose entries are defined as $(\mathbf{H})_{i:i+L-1,i} = \mathbf{h};\ i = 0,\ \ldots,\ N+\nu-1$. The contribution of the useful signal in (4) can be written as the sum of the contribution of the data symbols and the pilot symbols:

$$\mathbf{H}\mathbf{s} = \mathbf{H}_d\mathbf{s}_d + \mathbf{B}\mathbf{h} \tag{5}$$

where $\mathbf{H}_d$ consists of the first N columns of $\mathbf{H}$ and $\mathbf{B}$ is the $(N+\nu+L-1) \times L$ Toeplitz matrix with entries $(\mathbf{B})_{i:i+N+\nu-1,i} = \mathbf{b};\ i = 0,\ \ldots,\ L-1$.

For data detection, the contribution from the pilot symbols of the guard interval is first subtracted from the received signal, and the last ν samples of the observation interval are added to the first ν samples of the OFDM symbol (see figure 1b). The resulting block of N samples is then applied to the FFT.

III. Time Delay Estimation

In this section we derive an ML-based estimator for the time delay k_0, starting from the joint log likelihood function of k_0 and $\mathbf{h}$. We derive the algorithm under the assumption that only one OFDM block is transmitted, so before and after the OFDM block only noise is received. In the simulation results section, we will evaluate the proposed algorithm in a continuous transmission mode, where continuously OFDM symbols are transmitted.

The joint ML estimate of k_0 and $\mathbf{h}$ can be obtained by maximizing the log likelihood function of k_0 and $\mathbf{h}$, i.e. $\Lambda(k_0,\ \mathbf{h})$, given by (see appendix)

$$\begin{aligned} \Lambda(k_0,\ \mathbf{h}) = &-\frac{1}{2}\log\det(\mathbf{R}) \\ &-\frac{1}{N_0}\left(\sum_{k=-\infty}^{k_0-1} |r(k)|^2 + \sum_{k=k_0+N+\nu+L-1}^{+\infty} |r(k)|^2\right) \\ &-(\mathbf{r}-\mathbf{B}\mathbf{h})^H \mathbf{R}^{-1} (\mathbf{r}-\mathbf{B}\mathbf{h}) \end{aligned} \tag{6}$$

where $\det(\mathbf{R})$ is the determinant of the matrix $\mathbf{R}$, and $\mathbf{R}$ is defined as

$$\mathbf{R} = N_0\mathbf{I} + \frac{NE_s}{N+\nu}\mathbf{H}_d\mathbf{F}_d\mathbf{F}_d^H\mathbf{H}_d^H. \tag{7}$$

The maximization of (6) with respect to $\mathbf{h}$ is difficult because of the presence of $\frac{NE_s}{N+\nu}\mathbf{H}_d\mathbf{F}_d\mathbf{F}_d^H\mathbf{H}_d^H$ in the matrix $\mathbf{R}$. To solve this problem we simplify the log likelihood function (6) by neglecting $\frac{NE_s}{N+\nu}\mathbf{H}_d\mathbf{F}_d\mathbf{F}_d^H\mathbf{H}_d^H$ in $\mathbf{R}$, which means that we neglect the contribution from the unknown data symbols

$$\begin{aligned} \Lambda(k_0,\ \mathbf{h}) = &-\frac{1}{2}\log\det(N_0\mathbf{I}) \\ &-\frac{1}{N_0}\left(\sum_{k=-\infty}^{+\infty} |r(k)|^2 - \mathbf{r}^H\mathbf{B}\mathbf{h} - \mathbf{h}^H\mathbf{B}^H\mathbf{r} + \mathbf{h}^H\mathbf{B}^H\mathbf{B}\mathbf{h}\right). \end{aligned} \tag{8}$$

The first two terms in (8) do not depend on k_0 and $\mathbf{h}$, and can therefore be neglected. The estimate of $\mathbf{h}$ given k_0 is obtained by deriving (8) with respect to $\mathbf{h}$ and results in

$$\hat{\mathbf{h}}(k_0) = \left(\mathbf{B}^H\mathbf{B}\right)^{-1}\mathbf{B}^H\mathbf{r} \tag{9}$$

When we substitute this estimate of $\mathbf{h}$ in (8) we obtain the function $\Gamma(k_0)$ which only depends on k_0:

$$\Gamma(k_0) = \frac{1}{N_0}\mathbf{r}^H\mathbf{B}\left(\mathbf{B}^H\mathbf{B}\right)^{-1}\mathbf{B}^H\mathbf{r}. \qquad (10)$$

The estimate of k_0 is then given by

$$\hat{k}_0 = \arg\max_{k_0}\{\Gamma(k_0)\}. \qquad (11)$$

Although we derive the joint estimate of $\mathbf{h}$ and k_0 in this algorithm, only the estimate for k_0 is used. Indeed, the estimate for $\mathbf{h}$ will perform badly at high SNR, as the contributions from the data symbols in (6) and (7) have been neglected, resulting in an error floor in the MSE of $\mathbf{h}$ and the BER (see [14] and [15]). The derivation of the estimate of $\mathbf{h}$ is only needed to remove its contribution from (8) in order to obtain a simple expression for the estimate of k_0. For channel estimation, better estimators are available in the literature, e.g. [16], [17], having better performance at high SNR than the estimator (9).

If we take a closer look at (10), we see that the function $\Gamma(k_0)$ computes the correlation between the received signal and the pilot vector $\mathbf{b}$ at L successive time instants as can be seen from the matrix product $\mathbf{B}^H\mathbf{r}$. The estimator (11) tries to find the $\hat{k}_0$ that maximizes a function of the L successive correlations between the received signal and the pilot vector.

IV. Simulation Results

In this section the performance of the time delay estimator is evaluated by means of simulations. We compare the performance of the estimator with the ML time delay estimation algorithm for CP-OFDM from [1]. We consider $N = 1024$ sub carriers and a guard interval of length $\nu = 100$ for KSP-OFDM and CP-OFDM respectively. To make a fair comparison between CP-OFDM and KSP-OFDM, we assume that the number of pilot symbols transmitted on the carriers in the CP-OFDM signal is equal to $M - \nu$. The transmitted symbols consist of randomly generated QPSK symbols. Although we derived the estimator for k_0 under the assumption that only one OFDM block is transmitted, we simulate a continuous transmission of OFDM symbols. As we want to focus on the impact of time delay estimation errors, it is assumed for the simulation of the BER that possible phase rotations of the symbol constellation caused by time delay estimation errors, are perfectly compensated and that the channel is perfectly estimated after the time delay estimation. For KSP-OFDM, these assumptions mean that the interference caused by pilot symbols from the guard interval can be perfectly removed.

The performance of both estimators in a dispersive channel is shown in figures 2-7. We consider a frequency selective Rayleigh fading channel consisting of $L = 50$ channel taps. Figure 2 shows the results for the MSE on the time delay estimate. The KSP-OFDM estimator outperforms the estimator for CP-OFDM as could be expected: our estimator takes the dispersive nature of the channel into account while the estimator from [1] was designed for an AWGN channel. When we add extra pilot carriers, we see that the MSE of the

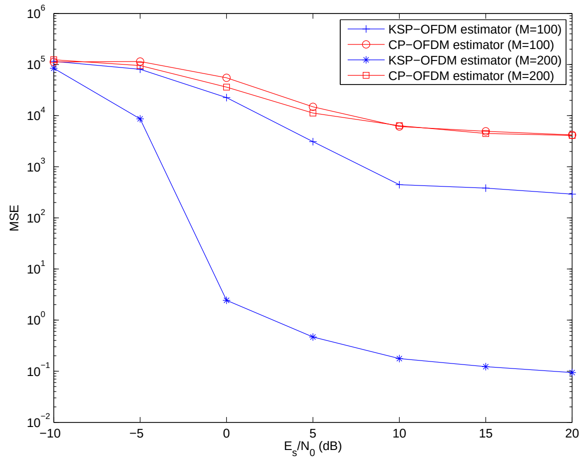

Figure 2. MSE results for a frequency selective channel, $L = 50$, $N = 1024$, $\nu = 100$

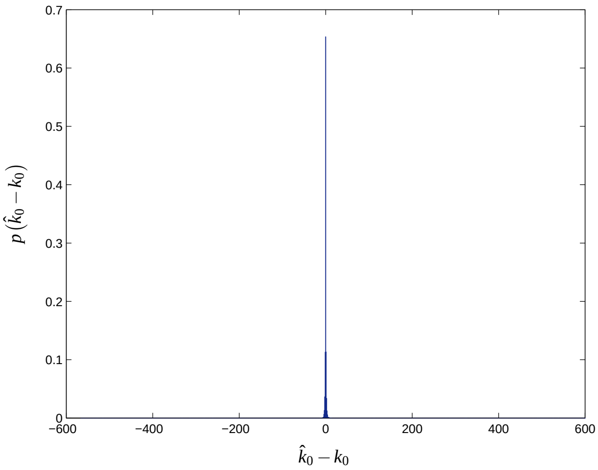

Figure 3. Histogram of the time delay estimation error for the KSP-OFDM, $E_s/N_0 = 20$ dB, $M = 100$

KSP-OFDM estimator decreases while the MSE of the CP-OFDM estimator does not change much. This indicates that the performance of the estimator [1] is not robust to a dispersive channel.

Figures 3 and 4 show a histogram of the estimation error $\hat{k}_0 - k_0$ for the KSP-OFDM estimator and the CP-OFDM estimator respectively for $E_s/N_0 = 20$ dB. For these figures, we only put pilot symbols in the guard interval ($M = \nu$). In almost 70% of all simulated cases, the KSP-OFDM estimator finds the real k_0 and in more than 90% of all simulated cases, we find that $\left|\hat{k}_0 - k_0\right| \leq 2$ samples. The performance of the CP-OFDM estimator is much worse: the true k_0 is almost never found and less than 1% of all cases results in $\left|\hat{k}_0 - k_0\right| \leq 2$ samples.

Figures 5 and 6 show a histogram of the estimation error $\hat{k}_0 - k_0$ for the KSP-OFDM estimator and the CP-OFDM estimator respectively for $E_s/N_0 = 20$ dB and 100 pilot carriers ($M = 200$). The KSP-OFDM estimator finds the real k_0 in

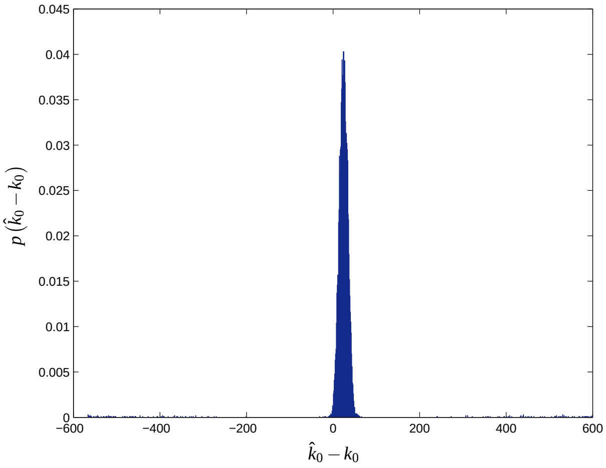

Figure 4. Histogram of the time delay estimation error for the CP-OFDM estimator, $E_s/N_0 = 20$ dB, no pilot carriers

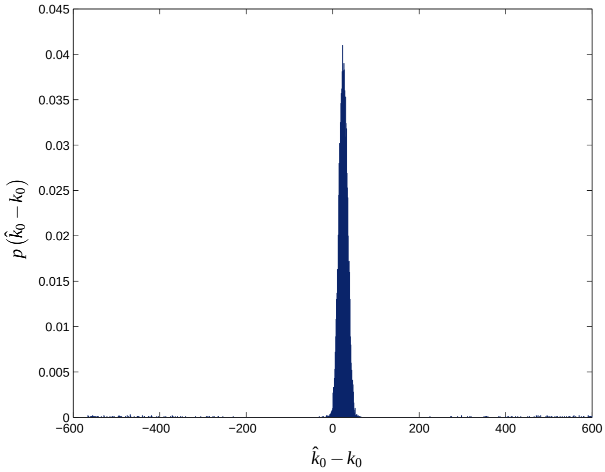

Figure 6. Histogram of the time delay estimation error for the CP-OFDM estimator, $E_s/N_0 = 20$ dB, 100 pilot carriers

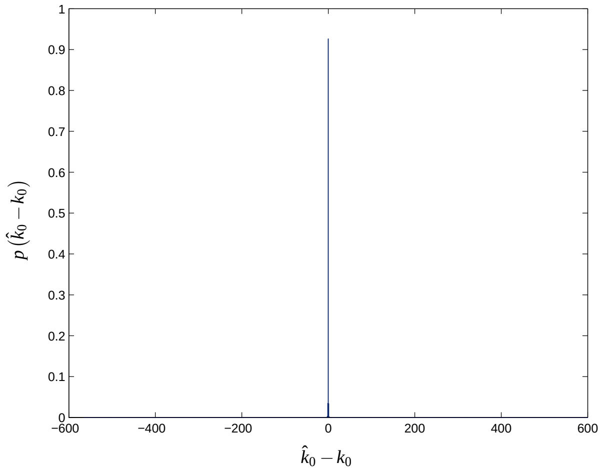

Figure 5. Histogram of the time delay estimation error for the KSP-OFDM, $E_s/N_0 = 20$ dB, $M = 200$

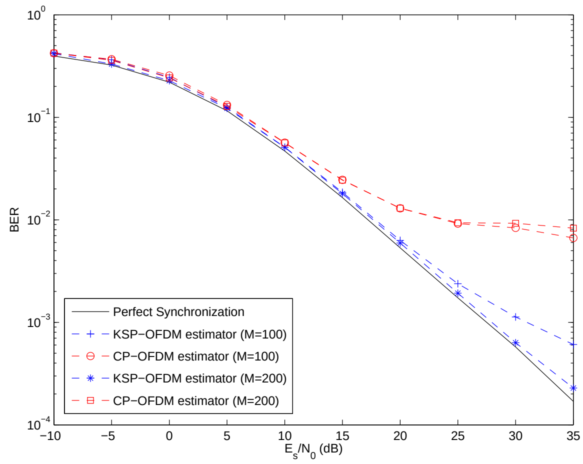

Figure 7. BER results for a frequency selective channel, $L = 50$, $N = 1024$, $\nu = 100$

more than 90% of all simulated cases and in more than 99% of all simulated cases, $\left|\hat{k}_0 - k_0\right|$ is smaller than or equal to 2 samples. Hence, a large improvement is obtained by using 100 pilot carriers in the proposed estimator. The performance of the CP-OFDM estimator does not benefit from the pilot carriers: the performance is similar to the case without pilot carriers.

The BER results for a dispersive channel are shown in figure 7. We see that the KSP-OFDM estimator exhibits a lower BER than the CP-OFDM system with the time delay estimator from [1]. Both estimators exhibit an error floor for higher E_s/N_0, but the CP-OFDM has a significantly higher error floor. This error floor of the proposed estimator is caused by the assumptions made in the derivation of this estimator, i.e. that only one OFDM symbol is transmitted whereas in the simulations continuous transmission is considered, and by the neglection of the data symbols in the channel estimate used to estimate the timing offset. As can be seen on the figure, the performance of the KSP-OFDM estimatoris further improved by adding extra pilot symbols on some carriers.

V. Conclusion

We have derived a time delay estimator for KSP-OFDM in multipath fading environments. The estimator is based on the correlation between the received signal and the pilot symbols in the guard interval and the correlation between the received signal and the time domain contribution from the pilot carriers. We compared the proposed time delay estimator with the ML time delay estimator for a CP-OFDM system [1]. The KSP-OFDM system with our estimator outperforms the considered CP-OFDM system. The KSP-OFDM system with our time delay estimator results in a lower BER than the considered CP-OFDM system. We have seen that adding extra pilot carriers improves the performance of our estimator while the estimator for the CP-OFDM system does not benefit from the extra pilot symbols.

APPENDIX

For the derivation of the time delay estimation algorithm we assume that only one OFDM block is transmitted. This means that before and after the OFDM block, only white Gaussian noise is received. We define $\mathbf{r}_\infty$ as the vector of all observations:

$$\mathbf{r}_\infty = \{r(k) \mid k = -\infty, \ldots, +\infty\}$$

The contribution of the unknown data symbols in the received OFDM block $\mathbf{r}$ can be modelled as an extra noise term. Each element of the vector $\mathbf{s}_d$ consists of a large weighted sum of i.i.d random variables $\mathbf{a}_d$ (if the number of data carriers is sufficiently large) and can therefore be modelled as a Gaussian random variable. Assuming the joint probability density function of $\mathbf{s}_d$ is Gaussian, it has zero mean and covariance matrix equal to $\frac{N}{N+\nu} E_s \mathbf{F}_d \mathbf{F}_d^H$. The distribution of the observation $\mathbf{r}_\infty$ given k_0, the channel impulse response $\mathbf{h}$ and the pilot vectors $\mathbf{b}_c$ and $\mathbf{b}_g$ is given by

$$\begin{aligned} p(\mathbf{r}_\infty | k_0, \mathbf{h}, \mathbf{b}_c, \mathbf{b}_g) = \\ C(\det(\mathbf{R}))^{-\frac{1}{2}} \exp\Bigg\{ -\frac{1}{N_0} \Bigg(\sum_{k=-\infty}^{k_0-1} |r(k)|^2 \\ + \sum_{k=k_0+N+\nu+L-1}^{+\infty} |r(k)|^2 \Bigg) \\ -\frac{1}{N_0} (\mathbf{r}-\mathbf{Bh})^H \mathbf{R}^{-1} (\mathbf{r}-\mathbf{Bh}) \Bigg\} \end{aligned} \quad (12)$$

where C is some constant, $\det(\mathbf{R})$ is the determinant of the matrix $\mathbf{R}$, and the covariance matrix $\mathbf{R}$ is defined as

$$\mathbf{R} = N_0 \mathbf{I} + \frac{N E_s}{N+\nu} \mathbf{H}_d \mathbf{F}_d \mathbf{F}_d^H \mathbf{H}_d^H .$$

The log likelihood function of k_0 and $\mathbf{h}$, neglecting irrelevant terms, is then given by

$$\begin{aligned} \Lambda(k_0, \mathbf{h}) = -\frac{1}{2} \log\det(\mathbf{R}) \\ -\frac{1}{N_0} \left(\sum_{k=-\infty}^{k_0-1} |r(k)|^2 + \sum_{k=k_0+N+\nu+L-1}^{+\infty} |r(k)|^2 \right) \\ -\frac{1}{N_0} (\mathbf{r}-\mathbf{Bh})^H \mathbf{R}^{-1} (\mathbf{r}-\mathbf{Bh}) \end{aligned} \quad (13)$$

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