The Föllmer-Schweizer decomposition: Comparison and description

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Abstract

This paper proposes two main contributions concerning the Föllmer-Schweizer decomposition (called hereafter FS-decomposition). First we completely elaborate the relationship between this decomposition and the Galtchouk-Kunita-Watanabe decomposition under the minimal martingale measure. The difference between these two decompositions is highlighted on a very practical example, and the martingale tools that enhance this difference are illustrated in the semi-martingale framework as well. The second main contribution focuses on the description of the FS-decomposition using the predictable characteristics.

Keywords: [class=AMS] Minimal martingale measure, Galtchouk-Kunita-Watanabe decomposition, Föllmer-Schweizer decomposition, incomplete markets, local risk-minimization, predictable characteristics.

1. Introduction

The quadratic criterion of local risk-minimization is among the earliest concepts of hedging in incomplete markets. It is an extension -to the semimartingale framework- of the risk-minimization concept discussed in [19]. This local risk-minimization concept was introduced in [27] and [28], and is based essentially on the minimal martingale measure introduced in [18]. In later works, the author realized that the local risk-minimization concept boils down to a decomposition which is called the Föllmer-Schweizer decomposition (FS-decomposition hereafter), which was discussed in many papers at different levels of generality and for different purposes, see for example [2], [8], [9], [15], [29], [30], [31], and [32] and the references therein. When the price of the discounted risky asset is a martingale, this decomposition coincides with the Galtchouk-Kunita-Watanabe decomposition (GKW-decomposition hereafter).

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There has been an upsurge interest in the FS-decomposition (or equivalently the local riskminimization concept) since it was introduced. In fact, this technique has been used for hedging risks in different types of incomplete markets, such as the (life or non-life) insurance markets see for example [25] and [33] and the references therein, and defaultable markets see for example [5], [6], and [7] and the reference therein. In most of these works, the authors formulate their local riskminimization results based on the key fact that the FS-decomposition and the GKW-decomposition under the minimal martingale measure coincide. This fact remains true when the discounted price process of the risky assets is continuous, while it breaks down in the case that involves jumps. Our paper fills this gap by elaborating clearly the relationship between the two decompositions, and highlights the difference on a simple market model with jumps.

This paper is organized in the following manner: The next section addresses the mathematical model, notations, and recall the existing results (as well as add new results) that we will use frequently throughout the paper. Then the third section presents our first main contribution that deals with comparing the FS-decomposition and the GKW-decomposition. The description of the FS-decomposition in terms of the predictable characteristics of the discounted stock price process is illustrated in Section 4.

2. Preliminaries

In this section we will introduce the setting we work in, for all unexplained notations we refer the reader to [22].

The market is represented by a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0 \le t \le T}, P)$. Here the filtration is supposed to be right-continuous, complete and \mathcal{F}_0 is trivial. On this space, we consider a *d*dimensional semimartingale $S = (S_t)_{0 \le t \le T}$ that represents the discounted price processes of *d* risky assets. We assume that the nondecreasing process $\left(\sup_{0 \le s \le t} |S_s|\right)_{0 \le t \le T}$ is locally square integrable, and the Doob-Meyer decomposition of *S* is given by

$$S = S_0 + M + B, (2.1)$$

where M is a locally square integrable local martingale, and B is a predictable process with finite variation. No-arbitrage assumptions on the market model lead to the existence of a predictable process $\tilde{\lambda}$ satisfying

$$dB_t = -d\langle M \rangle_t \widetilde{\lambda}_t, \qquad \int_0^T \widetilde{\lambda}_u^* d\langle M \rangle_u \widetilde{\lambda}_u < +\infty, \quad P\text{-a.s.}$$
(2.2)

This property is called Structure Condition (SC), was introduced in [29] and discussed in many papers see for instance [2] and [9] and the references therein, while for arbitrage theory we refer the reader to [16] and the references therein.

Now we recall the definition of the predictable characteristics of the semimartingale S (see Section II.2 of [22]). The random measure μ associated to its jumps is defined by

$$\mu(dt, dx) = \sum I_{\{\Delta S_s \neq 0\}} \delta_{(s, \Delta S_s)}(dt, dx),$$

with δ_a the Dirac measure at point a. The continuous local martingale part of S is denoted by S^c (it coincides with that of M, that is $M^c = S^c$). This leads to the following decomposition, called "the canonical representation" (see Theorem 2.34, Section II.2 of [22]):

$$S = S_0 + S^c + x \star (\mu - \nu) + B, \qquad (2.3)$$

where the random measure ν is the compensator of the random measure μ , and C is the matrix with entries $C^{ij} := \langle S^{c,i}, S^{c,j} \rangle$. The triple (B, C, ν) is called **predictable characteristics** of S. Furthermore, we can find a version of the characteristics triple satisfying

$$B = b \cdot A, \quad C = c \cdot A, \quad \nu(\omega, \ dt, \ dx) = dA_t(\omega)F_t(\omega, \ dx). \tag{2.4}$$

Here A is an increasing and predictable process which is continuous if and only if S is quasi-left continuous, b and c are predictable processes, $F_t(\omega, dx)$ is a predictable kernel, $b_t(\omega)$ is a vector in \mathbb{R}^d and $c_t(\omega)$ is a symmetric $d \times d$ -matrix, $\forall (\omega, t) \in \Omega \times [0, T]$. In the sequel we will often drop ω and t and write, for instance, F(dx) as a shorthand for $F_t(\omega, dx)$. These characteristics, B, C, and ν , satisfy

$$F_t(\omega, \{0\}) = 0, \quad \int (|x|^2 \wedge 1) F_t(\omega, dx) \le 1, \quad \Delta B_t = \int x \nu(\{t\}, dx), \\ c = 0 \quad \text{on} \quad \{\Delta A \neq 0\}.$$

Set

$$\nu_t(dx) := \nu(\{t\}, dx), \quad a_t := \nu_t(\mathbb{R}^d) = \Delta A_t F_t(\mathbb{R}^d) \le 1$$

The set of all probability measures that are absolutely continuous with respect to (respectively equivalent to) P is denoted by \mathbb{P}_a (respectively \mathbb{P}_e). The set of martingales under the probability Q is denoted by $\mathcal{M}(Q)$. $\mathcal{M}^{e}(S)$ is the set of probabilities $Q \sim P$ such that S is a Q-local martingale. If C is a class of processes, we denote by C_0 the set of processes X with $X_0 = 0$ and by C_{loc} the set of processes X such that there exists a sequence of stopping times (τ_n) increasing stationarily to T and the stopped process X^{τ_n} belongs to \mathcal{C} . We put $\mathcal{C}_{0,loc} = \mathcal{C}_0 \cap \mathcal{C}_{loc}$.

As usual, \mathcal{A}^+ denotes the set of increasing, right-continuous, adapted and integrable processes. On the set $\Omega \times [0, T]$, we define two σ -fields \mathcal{O} and \mathcal{P} generated by the adapted and càdlàg processes and the adapted and continuous processes respectively. On the set $\Omega \times [0, T] \times \mathbb{R}^d$, we consider the σ -field $\widetilde{\mathcal{P}} = \mathcal{P} \otimes \mathcal{B}(\mathbb{R}^d)$ (resp. $\widetilde{\mathcal{O}} = \mathcal{O} \otimes \mathcal{B}(\mathbb{R}^d)$), where $\mathcal{B}(\mathbb{R}^d)$ is the Borel σ -field for \mathbb{R}^d .

For any process g, $\widetilde{\mathcal{O}}$ -measurable (hereafter denoted by $g \in \widetilde{\mathcal{O}}$), we define $M^P_{\mu}(g \mid \widetilde{\mathcal{P}})$ the unique $\widetilde{\mathcal{P}}$ -measurable process, when it exists, such that for any bounded $W \in \mathcal{P}$,

$$M^P_{\mu}(Wg) := E\Big(\int_0^T \int W(s,x)g(s,x)\mu(ds,dx)\Big) = M^P_{\mu}\Big(WM^P_{\mu}(g \mid \widetilde{\mathcal{P}})\Big).$$

For the following representation theorem which is a key tool for our analysis, we refer to [21] (Theorem 3.75, page 103) and to [22] (Lemma 4.24, page 185).

Theorem 2.1. Let $N \in \mathcal{M}_{0,loc}$. Then there exist a predictable and S^c-integrable process ϕ , N' \in $\mathcal{M}_{0,loc}$ with [N', S] = 0 and functionals $f \in \widetilde{\mathcal{P}}$ and $g \in \widetilde{\mathcal{O}}$ such that

$$\int_{0}^{T} \int_{I\!\!R^{d} \setminus \{0\}} |f| \wedge |f|^{2} \nu(dt, dx) < +\infty, \quad \left(\sum_{s=0}^{t} g(s, \Delta S_{s})^{2} I_{\{\Delta S_{s} \neq 0\}}\right)^{1/2} \in \mathcal{A}_{loc}^{+}$$

$$M^{P}_{\mu}(g \mid \mathcal{P}) = 0,$$

$$N = \phi \cdot S^{c} + W \star (\mu - \nu) + g \star \mu + N', \quad W = f + \frac{\hat{f}}{1 - a} I_{\{a < 1\}}$$
(2.5)

where $\hat{f} = \int f(x)\nu(\{t\}, dx)$ and f has a version such that $\{a = 1\} \subset \{\hat{f} = 0\}$. Moreover

$$\Delta N_t = \left(f_t(\Delta S_t) + g_t(\Delta S_t) \right) I_{\{\Delta S_t \neq 0\}} - \frac{\widehat{f_t}}{1 - a_t} I_{\{\Delta S_t = 0\}} + \Delta N'_t.$$

$$(2.6)$$

The following lemma sounds new to us and is dealing with the uniqueness of the decomposition of Theorem 2.1.

Lemma 2.2. The decomposition in (2.5) is unique (up to indistinguishability) in the following sense: If there exists a quadruplet (ϕ, f, g, N') as in Theorem 2.1 satisfying

$$0 = \phi \cdot S^{c} + W \star (\mu - \nu) + g \star \mu + N', \qquad (2.7)$$

then

$$c\phi = 0$$
 $dP \otimes dA - a.e., \quad f(x) = g(x) = 0 \quad \mu - a.e., \quad N' = 0$

Proof: If N'^c denotes the continuous local martingale part of N', then from (2.7) we deduce that

$$\phi \cdot S^c + {N'}^c \equiv 0.$$

Due to [S, N'] = 0, the orthogonality of $\phi \cdot S^c$ and N'^c follows. This combined with the above equation implies that

$$\phi \cdot S^c = 0, \qquad N'^c = 0.$$
 (2.8)

Thus, the first equation above is equivalent to $c\phi = 0$ $dP \otimes dA - a.e.$

On $\{\Delta S \neq 0\}$, $\Delta N' = 0$ and hence (2.6) leads to $f(\Delta S) + g(\Delta S) = 0$, which is equivalent to

$$f(x) + g(x) = 0 \qquad \mu - a.e.$$

By taking conditional expectation under M^P_{μ} , and using $M^P_{\mu}\left(g|\widetilde{\mathcal{P}}\right) = 0$ we conclude that

$$f = g = 0$$
 $M^P_\mu - a.e.$ (2.9)

This implies that $\hat{f} = 0$, hence due to (2.6) again we get $\Delta N' = 0$. This together with the second equation in (2.8) leads to N' = 0. This completes the proof of the lemma.

Now we define a set of strategies that we will consider throughout the paper:

$$\Theta := \left\{ \theta \in L(S) \mid \|\theta \cdot S\|_{\mathcal{H}^2(P)}^2 := E\left(\int_0^T \theta_u^* d\langle M \rangle_u \theta_u + \left[\int_0^T |\theta_u^* dB_u|\right]^2\right) < +\infty \right\}.$$

Definitions 2.3.

1) Two local martingales K and L are said to be orthogonal under a probability measure if the process [L, K] is a local martingale under that probability measure.

2) A contingent claim is any \mathcal{F}_T -measurable and P-square integrable random variable. 3) Let H be a contingent claim. Then H is said to have the Föllmer-Schweizer decomposition if there exist a constant, H_0 , a S-integrable process $\xi^H \in \Theta$, and a square integrable martingale L^H

$$H = H_0 + (\xi^H \cdot S)_T + L_T^H.$$
(2.10)

Throughout the paper, the triplet (H_0, ξ^H, L^H) will be called the FS-decomposition components.

3. The FS-decomposition versus the GKW-decomposition

such that $[L^H, M]$ is a local martingale, and

This section addresses the relationship between the Galtchouk-Kunita-Watanabe decomposition under the minimal martingale measure and the Föllmer-Schweizer decomposition. The minimal martingale measure \tilde{Q} is the martingale measure such that any *P*-local martingale which is orthogonal to *M*, as defined in (2.1), under *P* remains a local martingale under \tilde{Q} . We start by stating the assumption under which we elaborate our results and which guarantees the existence of the FS-decomposition. See Section 4 for a further discussion about the existence of this decomposition. Throughout the rest of the paper, \tilde{N} denotes $\tilde{\lambda} \cdot M$ with $\tilde{\lambda}$ given by (2.2).

Assumptions 3.1. We assume that $\mathcal{E}(\tilde{N}) > 0$, and there exists a constant C > 0 such that for any stopping time σ ,

$$E\left[\left(\mathcal{E}_T\left(\widetilde{N}-\widetilde{N}^{\sigma}\right)\right)^2 \mid \mathcal{F}_{\sigma}\right] \le C, \qquad P-a.s.$$
(3.11)

Remark: Thanks to Theorem 5.5 in [8], the Föllmer-Schweizer decomposition of any contingent claim exists under Assumptions 3.1.

From [30] and the references therein, we know that $\mathcal{E}(\tilde{\lambda} \cdot M)$ is the density of the signed minimal martingale measure for S. It is clear that the assumption (3.11) implies that $\mathcal{E}(\tilde{N})$ is a true martingale, and the minimal martingale measure that we denote throughout this section by

$$\widetilde{Q} := \mathcal{E}_T(\widetilde{N}) \cdot P, \tag{3.12}$$

really exists. When S is a continuous process, it is generally known that the two decompositions coincide see e.g. [32]. However this fact is no longer true in the general framework due to the presence of jumps in S. The correct relationship between the two decompositions will be completely determined in the following theorem.

Theorem 3.2. Suppose that Assumptions 3.1 are satisfied. Let H be a contingent claim whose FS-decomposition components are denoted by (H_0, ξ^H, L^H) . Suppose that the \tilde{Q} -martingale, $\tilde{V}_t = E^{\tilde{Q}}(H|\mathcal{F}_t)$, admits the Galtchouk-Kunita-Watanabe decomposition which is given by

$$\widetilde{V} = \widetilde{V}_0 + \widetilde{\xi} \cdot S + \widetilde{L}, \qquad (3.13)$$

where $\tilde{\xi} \in L(S)$ (i.e. S-integrable), $\tilde{\xi} \cdot S$ and \tilde{L} are \tilde{Q} -local martingales, and \tilde{L} is \tilde{Q} -orthogonal to S. Then the following holds:

(i) If $(\tilde{\beta}, \tilde{f}, \tilde{g}, \tilde{L}')$ denotes the quadruplet associated with \tilde{L} under \tilde{Q} through Theorem 2.1, then

$$\widetilde{\Phi} := \Sigma^{inv} \int x \widetilde{f}(x) \left[\widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda} \right] F(dx), \qquad (3.14)$$

is a well defined predictable process, S-integrable, and satisfies

$$\xi^{H} = \widetilde{\xi} - \widetilde{\Phi}, \qquad \qquad L^{H} = \widetilde{L} + \widetilde{\Phi} \cdot S. \qquad (3.15)$$

Here, Σ^{inv} denotes the Moore-Penrose pseudoinverse of the matrix Σ given by

$$\Sigma := c + \int x x^* F(dx) := \frac{d\langle S \rangle}{dA}.$$
(3.16)

(ii) If there exists a sequence of stopping times $(T_n)_n$ increasing stationarily to T such that $\tilde{\xi}I_{[0,T_n]} \in \Theta$, then the process $\langle \tilde{N}, [\tilde{L}, S] \rangle$ exists, and is absolutely continuous with respect to $\langle S \rangle$ of which the Radon-Nikodym derivative is a version of $\tilde{\Phi}$. Furthermore $\tilde{\Phi}I_{[0,T_n]} \in \Theta$.

Before proving this theorem, we would like to discuss briefly the existence of the GKWdecomposition for two local martingales, and provide some conditions that guarantee the integrability assumption on $\tilde{\xi}$ in Theorem 3.2 assertion (ii).

Remarks:

1) Let X and Y be two local martingales (for simplicity we suppose that both processes are realvalued) such that Y is locally square integrable. Then X admits the GKW-decomposition with respect to Y if the process $\langle X, Y \rangle$ exists, $\lambda_t := \frac{d\langle X, Y \rangle_t}{d\langle Y, Y \rangle_t}$ is Y-integrable, and $\lambda \cdot Y$ is a local martingale. These three conditions are fulfilled when X and Y are both locally square integrable local martingales, or when Y is a continuous process. For more details about the GKW-decomposition and related subject, we refer the reader to [3] and the references therein.

2) Suppose that there exists a sequence of stopping times T_n increasing stationarily to T, and a sequence of positive numbers, δ_n , such that $1 \ge \delta_n > 0$ and

$$\delta_n \le 1 + \Delta \widetilde{N}^{T_n} \le \delta_n^{-1}. \tag{3.17}$$

Then, under Assumptions 3.1, for any contingent claim H the process $\tilde{V}_t = E^{\tilde{Q}}(H|\mathcal{F}_t)$ admits the GKW-decomposition under \tilde{Q} described in (3.13), and there exists a sequence of stopping times (σ_n) increasing stationarily to T such that $\sigma_n \leq T_n$ and $\tilde{\xi}I_{[0,\sigma_n]} \in \Theta$. In other words, the assumption in assertion (ii) of Theorem 3.2 is fulfilled. To prove this fact, we proceed into two steps: In the first step we will prove that \tilde{V} is a \tilde{Q} -locally square integrable martingale, while the second step will deal with $\tilde{\xi}I_{[0,\sigma_n]} \in \Theta$. Indeed, due to Assumptions 3.1 and Theorem 4.9 in [8] we deduce that $E[\tilde{V}, \tilde{V}]_T < +\infty$ in one hand. On the other hand, due to the RHS inequality in (3.17), we get

$$E\left[(1+\Delta\widetilde{N})\cdot[\widetilde{V},\widetilde{V}]_{T_n}\right] \leq \delta_n^{-1}E[\widetilde{V},\widetilde{V}]_{T_n} < +\infty.$$

This proves that the compensator of $(1 + \Delta \widetilde{N}) \cdot [\widetilde{V}, \widetilde{V}]^{T_n}$ exists and is integrable, which coincides with the \widetilde{Q} -compensator of $[\widetilde{V}, \widetilde{V}]^{T_n}$. This proves that \widetilde{V}^{T_n} is a \widetilde{Q} -square integrable martingale. This ends the first step. Then it is obvious that the GKW-decomposition under \widetilde{Q} for \widetilde{V} described in (3.13) exists (see Remark 1 above). Furthermore, since \tilde{V}^{T_n} is a \tilde{Q} -square integrable (the first step), the process $\tilde{\xi} \cdot S^{T_n}$ is a \tilde{Q} -square integrable martingale. Notice that the \tilde{Q} -compensator of $[\tilde{\xi} \cdot S, \tilde{\xi} \cdot S]$ coincides with the *P*-compensator of $(1 + \Delta \tilde{N}) \cdot [\tilde{\xi} \cdot S, \tilde{\xi} \cdot S]$. Hence there exists a sequence of stopping times (τ_n) increasing stationarily to *T*, such that the latter process stopped at τ_n is *P*-integrable. Then, due to $\delta_n \leq 1 + \Delta \tilde{N}^{T_n}$, we get

$$E[\widetilde{\xi} \cdot S, \widetilde{\xi} \cdot S]_{\tau_n \wedge T_n} \leq \delta_n^{-1} E\left[(1 + \Delta \widetilde{N}) \cdot [\widetilde{\xi} \cdot S, \widetilde{\xi} \cdot S]_{\tau_n \wedge T_n} \right] < +\infty.$$

By combining this with Theorem 4.9 in [8], we conclude that $\tilde{\xi}I_{[0,\tau_n\wedge T_n]} \in \Theta$, and the proof of the claim is achieved.

3) If we consider the condition of $\delta_n \leq 1 + \Delta \tilde{N}^{T_n}$ instead of (3.17), then the results in the above remark (Remark 2) are still valid for contingent claims that are \tilde{Q} -square integrable. In fact, for a contingent claim H that is \tilde{Q} -square integrable, the process \tilde{V} is a \tilde{Q} -square integrable martingale. Hence, the remaining part of the proof follows exactly the second step in the proof of Remark 2. 4) The integrability of $\tilde{\xi}$ described in Theorem 3.2 assertion (i) is enough to achieve our goal and to prove the main idea of the theorem which lies in describing the difference between the two decompositions. This remark was noticed by an anonymous referee, who also suggested that $\tilde{\xi}$ may belong to Θ . We are doubtful about this latter fact, due to the fact that the process $[\tilde{L}, M]$ may not be a P-local martingale (\tilde{L} is given by (3.13)), nor even a special semimartingale under P.

Proof of Theorem 3.2. (i) A key tool in this proof is Theorem 2.1 applied under the probability measure \tilde{Q} . To this end, we start with describing the representation of S under this measure. First, the compensator of the random measure μ under \tilde{Q} will be denoted by $\nu^{\tilde{Q}}$ and is given by

$$\nu^{\widetilde{Q}}(dt, dx) = \left(1 + \widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}\right) \nu(dt, dx).$$
(3.18)

Then, the process S takes the following canonical decomposition under \widetilde{Q} ,

$$S = S_0 + S^{c,\widetilde{Q}} + x \star (\mu - \nu^{\widetilde{Q}}), \qquad S^{c,\widetilde{Q}} := S^c - c\widetilde{\lambda} \cdot A.$$
(3.19)

We remark that the *P*-local martingale L^H is also a \tilde{Q} -local martingale by definition of the minimal martingale measure. Applying Theorem 2.1 to the \tilde{Q} -local martingale L^H , provides

$$L^{H} = \theta^{H} \cdot S^{c,\widetilde{Q}} + W^{H} \star (\mu - \nu^{\widetilde{Q}}) + g^{H} \star \mu + \overline{L}^{H}, \qquad [\overline{L}^{H}, S] = 0, \qquad M^{\widetilde{Q}}_{\mu}(g^{H}|\widetilde{\mathcal{P}}) = 0, \quad (3.20)$$

where $W^H(x) = f^H(x) + \left(1 - \nu^{\widetilde{Q}}(\{t\}, \mathbb{R}^d)\right)^{-1} \int f^H(x) \nu^{\widetilde{Q}}(\{t\}, dx)$. Analogously, we find for the \widetilde{Q} -local martingale \widetilde{L}

$$\widetilde{L} = \widetilde{\theta} \cdot S^{c,\widetilde{Q}} + \widetilde{W} \star (\mu - \nu^{\widetilde{Q}}) + \widetilde{g} \star \mu + \widetilde{L}', \qquad [\widetilde{L}', S] = 0, \qquad M^{\widetilde{Q}}_{\mu}(\widetilde{g}|\widetilde{\mathcal{P}}) = 0, \tag{3.21}$$

with $\widetilde{W}(x) = \widetilde{f}(x) + \left(1 - \nu^{\widetilde{Q}}(\{t\}, \mathbb{R}^d)\right)^{-1} \int \widetilde{f}(x)\nu^{\widetilde{Q}}(\{t\}, dx).$

Due to the integrability conditions, we deduce that $\xi^H \cdot S$ and L^H are martingales under \widetilde{Q} , and

$$H_0 + \xi^H \cdot S + L^H = \widetilde{V}_0 + \widetilde{\xi} \cdot S + \widetilde{L}.$$
(3.22)

Notice that using (3.19), we get $\xi^H \cdot S = \xi^H \cdot S^{c,\tilde{Q}} + x^*\xi^H \star (\mu - \nu^{\tilde{Q}})$ and $\tilde{\xi} \cdot S = \tilde{\xi} \cdot S^{c,\tilde{Q}} + x^*\tilde{\xi} \star (\mu - \nu^{\tilde{Q}})$. By plugging these two equations together with (3.20) and (3.21) into (3.22), we conclude that the two processes $H_0 + (\xi^H + \theta^H) \cdot S^{c,\tilde{Q}} + (x^*\xi^H + W^H) \star (\mu - \nu^{\tilde{Q}}) + g^H \star \mu + \overline{L}^H$ and $\tilde{V}_0 + (\tilde{\xi} + \tilde{\theta}) \cdot S^{c,\tilde{Q}} + (x^*\tilde{\xi} + \widetilde{W}) \star (\mu - \nu^{\tilde{Q}}) + \tilde{g} \star \mu + \tilde{L}'$ are identical. Therefore, due to the uniqueness of Jacod's decomposition (see Lemma 2.2), we derive $H_0 = \tilde{V}_0, g^H(x) = \tilde{g}(x), \overline{L}^H = \tilde{L}'$, and

$$c\widetilde{\xi} + c\widetilde{\theta} = c\xi^H + c\theta^H \qquad P \otimes dA\text{-a.e.}, \quad x^*\widetilde{\xi} + \widetilde{f}(x) = x^*\xi^H + f^H(x), \quad F(dx) \otimes dA\text{-a.e.}$$
(3.23)

Since L^H is a *P*-local martingale orthogonal to M ($\langle L^H, M \rangle = \langle L^H, S \rangle = 0$) and \widetilde{L} is a \widetilde{Q} -local martingale orthogonal to S ($\langle \widetilde{L}, S \rangle^{\widetilde{Q}} = 0$), we deduce that dA-a.e.,

$$c\theta^{H} + \int x f^{H}(x) F(dx) = 0, \quad c\tilde{\theta} + \int x \tilde{f}(x) \left[1 + \tilde{\lambda}^{*} x + \tilde{\lambda}^{*} \Delta \langle M \rangle \tilde{\lambda} \right] F(dx) = 0.$$
(3.24)

The second equation in (3.23) leads to $\int xx^* \tilde{\xi}F(dx) + \int x\tilde{f}F(dx) = \int xx^* \xi^H F(dx) + \int xf^H(x)F(dx)$. By adding this to the first equation of (3.23), taking into account the first equation in (3.24), and using $\Sigma_t := c_t + \int xx^* F_t(dx)$, we obtain

$$\Sigma \xi^{H} = \Sigma \widetilde{\xi} - \int x \widetilde{f}(x) \left[\widetilde{\lambda}^{*} x + \widetilde{\lambda}^{*} \Delta \langle M \rangle \widetilde{\lambda} \right] F(dx).$$

Therefore, we conclude that the process $\widetilde{\Phi}$ defined in (3.14) is a well defined predictable process, S-integrable (since ξ^H and $\widetilde{\xi}$ are S-integrable), and satisfies the first equation in (3.15) (this follows from the above equation). The second equation in (3.15) result from inserting this first equation of (3.15) in (3.22). This ends the proof of assertion (i).

(ii) Since $\widetilde{\xi}I_{[0,T_n]} \in \Theta$, and $\sup_{t \leq \cdot} |\widetilde{V}_t|^2 \in \mathcal{A}^+(P)$ (due to Theorem 4.9 in [9]), we deduce that the process $\sup_{s \leq \cdot} |\widetilde{L}_s|^2 \in \mathcal{A}^+_{loc}(P)$. Thus, the process $[\widetilde{L}, S]$ has a *P*-locally integrable variation, and hence the process $[\widetilde{N}, [\widetilde{L}, S]]$ has a *P*-locally integrable variation (since $[\widetilde{L}, S]$ is a \widetilde{Q} -local martingale which is equivalent to $[\widetilde{L}, S] + [\widetilde{N}, [\widetilde{L}, S]]$ is a *P*-local martingale). Therefore its compensator, $\langle \widetilde{N}, [\widetilde{L}, S] \rangle$, exists. Furthermore, we calculate

$$\langle \widetilde{N}, [\widetilde{L}, S] \rangle = \left\{ \int x \widetilde{f}(x) \Big[\widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda} \Big] F(dx) \right\} \cdot A.$$

Thus, $\widetilde{\Phi}$ is a version of the Radon-Nikodym derivative, $\widetilde{\Psi}$, of $\langle \widetilde{N}, [\widetilde{L}, S] \rangle$ with respect to $\langle S \rangle$ (here by version, we mean $\Sigma \widetilde{\Psi} = \Sigma \widetilde{\Phi}$ or equivalently $\widetilde{\Phi} - \widetilde{\Psi} \in \text{kernel}(\Sigma)$). This completes the proof of the theorem.

Remarks:

(i) The process $\widetilde{\Phi}$ can also be explained as the Radon-Nikodym derivative of $\Sigma^{inv} d\langle [\widetilde{L}, S], \widetilde{N} \rangle$ with respect to dA, that is

$$\widetilde{\Phi} = \Sigma^{inv} \frac{d\langle [\widetilde{L}, S], \widetilde{N} \rangle}{dA},$$

- (ii) Through Theorem 3.2, we can easily claim that the two decompositions FS-decomposition and GKW-decomposition - are equivalent when S is a continuous process. Indeed, in this case, both processes $[\widetilde{L}, S]$ and $\widetilde{\Phi}$ vanish, implying that $\widetilde{L} = L^H$ and $\widetilde{\xi} = \xi^H$.
- (iii) This theorem also allows us to decide whether the two decompositions coincide or differ for any \mathcal{F}_T -measurable random variable and market model through the following statement: The two decompositions coincide if and only if

$$E\left[\int_0^T I_{\left\{(\omega,t) : \Sigma_t^{inv}(\omega)\Lambda_t(\omega)\notin \operatorname{kernel}(\Sigma_t(\omega))\right\}} dA_t\right] = 0, \quad \Lambda := \int x\widetilde{f}(x) \left[\widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}\right] F(dx).$$

3.1. A practical counterexample

Consider the following one-dimensional discounted process

$$S_t := S_0 \mathcal{E}_t(X), \qquad X_t := \sigma W_t + \gamma \widetilde{p}_t + \mu t, \qquad 0 \le t \le T, \qquad (3.25)$$

where $(p_t)_{t\geq 0}$ is the standard Poisson process with intensity 1, $\tilde{p}_t = p_t - t$ is the compensated Poisson process, W_t is the standard Brownian motion, $S_0 > 0$, $\sigma > 0$, and γ and μ are real numbers such that

$$\gamma > -1, \qquad 0 \neq \mu\gamma < \sigma^2 + \gamma^2. \tag{3.26}$$

The process S represents the discounted stock price process that constitutes the market model. Then, the processes M, B, and A (defined in (2.1) and (2.4) respectively) for this model are given by

$$dM_t = S_{t-} \left(\sigma dW_t + \gamma d\widetilde{p}_t \right), \quad dB_t = \mu S_{t-} dt, \qquad A_t = t.$$

Hence, we deduce that

$$\widetilde{\lambda}_t = \frac{1}{S_{t-}} \frac{-\mu}{\sigma^2 + \gamma^2}, \qquad \widetilde{N}_t = \sigma_1 W_t + \gamma_1 \widetilde{p}_t, \qquad \sigma_1 := \frac{-\mu\sigma}{\sigma^2 + \gamma^2}, \qquad \gamma_1 := \frac{-\mu\gamma}{\sigma^2 + \gamma^2}.$$

Thus if (3.26) holds, then $\mathcal{E}_t(\widetilde{N})$ is a square integrable and positive martingale, and the minimal martingale measure exists and is given by $Q := \mathcal{E}(N)_T \cdot P$.

Now consider the European put option with strike price K whose payoff is given by $H = (K - S_T)^+$. In the following we will calculate the processes $\tilde{V}, \tilde{\xi}, \tilde{\Phi}$ and \tilde{L} . Due to the independent increments of X, we deduce that $V_t = f(t, S_t)$, where

$$f(t,x) = E^{\tilde{Q}} \left[\left(K - x \frac{S_T}{S_t} \right)^+ \right].$$
(3.27)

Consider the - in the variable y - strictly increasing distribution function

$$F(s,y) := \widetilde{Q} \left(\sigma W_s + \log(1+\gamma)\widetilde{p}_s + \overline{\mu}s \le y \right), \quad \overline{\mu} := \mu - \frac{1}{2}\sigma^2 + \log(1+\gamma) - \gamma, \quad y \in \mathbb{R}, \ s \in [0,T].$$
(3.28)

Thanks to the stationary property of X and the notation in (3.28), the function f(t, x) in (3.27) takes the following form

$$f(t,x) = KF\left(T - t, \log(K/x)\right) - x \int_{-\infty}^{\log(\frac{K}{x})} e^y F_y(T - t, y) dy, \qquad x > 0, \ t \in [0,T].$$
(3.29)

As a result, $f(t,x) \in C^{1,2}((0,T) \times (0,+\infty))$, and by applying Itô's formula to $f(t,S_t)$ we derive

$$\widetilde{V}_{t} = \widetilde{V}_{0} + \int_{0}^{t} f_{t}(u, S_{u-}) du + \int_{0}^{t} f_{x}(u, S_{u-}) dS_{u} + \frac{1}{2} \int_{0}^{t} f_{xx}(u, S_{u-}) S_{u}^{2} \sigma^{2} du + \sum_{0 < u \leq t} \left[f(u, S_{u}) - f(u, S_{u-}) - f_{x}(u, S_{u}) \Delta S_{u} \right].$$

Remark that $\sum_{0 < u \le t} [f(u, S_u) - f(u, S_{u-}) - f_x(u, S_u) \Delta S_u] = \Gamma \cdot p$, where

$$\Gamma_u := f(u, S_{u-}(1+\gamma)) - f(u, S_{u-}) - f_x(u, S_u)\gamma S_{u-}.$$
(3.30)

Thus, since \widetilde{V} is a \widetilde{Q} -martingale, we deduce that the function f(t, x) satisfies a PDE equation (a fact that can be verified directly since the function f(t, x) is explicitly calculated in (3.29)), and

$$\widetilde{V}_t = \widetilde{V}_0 + \int_0^t f_x(u, S_{u-}) dS_u + \left(\Gamma \cdot \widetilde{p}^{\widetilde{Q}}\right)_t, \quad p_t^{\widetilde{Q}} := p_t - (1 + \gamma_1)t.$$
(3.31)

Here $\tilde{p}^{\tilde{Q}}$ is the compensated Poisson process under \tilde{Q} . Now we will focus on calculating $\tilde{\xi}$ as follows

$$d[\widetilde{V}, S]_t = \sigma^2 S_{t-}^2 f_x(t, S_{t-}) dt + \left(S_{t-}^2 \gamma^2 f_x(t, S_{t-}) + \Gamma_t S_{t-} \gamma\right) dp_t.$$

Recall that the compensator of p_t under \tilde{Q} coincides with $(1 + \gamma_1)t$, thus using the fact that $\tilde{\xi}_t = \frac{d\langle \tilde{V}, S \rangle_t^{\tilde{Q}}}{d\langle S \rangle_t^{\tilde{Q}}}$, we derive the components of the GKW-decomposition under \tilde{Q} for \tilde{V} as follows

$$\tilde{\xi}_{t} = f_{x}(t, S_{t-}) + \frac{\Gamma_{t}\gamma(1+\gamma_{1})}{\sigma^{2}S_{t-} + S_{t-}\gamma^{2}(1+\gamma_{1})}, \quad \tilde{L} = \Gamma \cdot \tilde{p}^{\tilde{Q}} - \frac{\gamma\Gamma(1+\gamma_{1})}{S_{-}[\sigma^{2}+\gamma^{2}(1+\gamma_{1})]} \cdot S.$$
(3.32)

This allows us to state the following.

Corollary 3.3. Consider the model described by (3.25)-(3.26). Then the following assertions hold.

(i) The GKW-decomposition of \widetilde{V} under \widetilde{Q} is given by

$$\widetilde{V} = \widetilde{V}_0 + \widetilde{\xi} \cdot S + \widetilde{L},$$

where $\tilde{\xi}$ and \tilde{L} are given by (3.32).

(ii) The FS-decomposition of H and the GKW-decomposition under \widetilde{Q} for \widetilde{V} differ.

Proof. The first assertion is already proved, while the second assertion will follow after proving that the process $\tilde{\Phi}$ defined in Theorem 3.2 for this model never vanishes. The calculation of this process requires the calculation of $[[\tilde{L}, S], \tilde{N}]$, and $[\tilde{L}, S]$. Due to (3.32), these processes are given by

$$d[\widetilde{L},S]_t = \frac{S_{t-\gamma}\Gamma_t\sigma^2}{\sigma^2 + \gamma^2(1+\gamma_1)}dp_t - \frac{\gamma\Gamma_tS_{t-\sigma}^2(1+\gamma_1)}{\sigma^2 + \gamma^2(1+\gamma_1)}dt, \qquad [[\widetilde{L},S],\widetilde{N}] = \frac{\gamma_1S_{-\gamma}\Gamma\sigma^2}{\sigma^2 + \gamma^2(1+\gamma_1)} \cdot p.$$

As a result, we derive

$$\widetilde{\Phi}_t = \frac{-\mu\gamma\sigma^2}{(\gamma^2 + \sigma^2)^2(\sigma^2 + \gamma^2(1 + \gamma_1))} \frac{\gamma\Gamma_t}{S_{t-}}.$$
(3.33)
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Then, by putting $s_1(t) := \log(\frac{K}{S_{t-}})$ and $s_2(t) := \log(\frac{K}{S_{t-}(1+\gamma)})$, and using

$$f_x(t,x) = -\int_{-\infty}^{\log(K/x)} e^y F_y(T-t,y) dy,$$

we obtain

$$\begin{split} \Gamma_t &= f(t, S_{t-}(1+\gamma)) - f(t, S_{t-}) - f_x(t, S_{t-})S_{t-}\gamma \\ &= KF(T-t, s_2(t)) - KF(T-t, s_1(t)) - S_{t-}(1+\gamma) \int_{-\infty}^{s_2(t)} e^y F_y(T-t, y) dy + \\ &+ S_{t-} \int_{-\infty}^{s_1(t)} e^y F_y(T-t, y) dy + S_{t-}\gamma \int_{-\infty}^{s_1(t)} e^y F_y(T-t, y) dy \\ &= \int_{s_1(t)}^{s_2(t)} \Big[K - S_{t-}(1+\gamma) e^y \Big] F_y(T-t, y) dy. \end{split}$$

This proves that Γ_t is a positive process if $\gamma \neq 0$. By (3.33) the process $\tilde{\Phi}$ then also has a constant sign and never vanishes if (3.26) holds. Therefore, $\tilde{\xi}$ and ξ^H (see the FS-decomposition of H in (2.10)) never coincide and hence the FS-decomposition and the GKW-decomposition under \tilde{Q} differ for this model.

Through this practical example, we proved that Riesner's results in [25] (which are based on the fact that the FS-decomposition and the GKW-decomposition under the minimal martingale measure coincide) as well as Cont-Tankov's result in [14], Section 10.4, are wrong (this fact was noticed in [33] without any proof).

In the following subsection, we will detail the difference between the two decompositions.

3.2. Martingales under \widetilde{Q} versus P-martingales

The main difference between the FS-decomposition and the GKW-decomposition consists of two facts: The first one deals with the inheritance of the *P*-orthogonality to M from the \tilde{Q} orthogonality to S for a \tilde{Q} -local martingale (see the definition below for the orthogonality of semimartingales). The second fact is concerned with the characterization of \tilde{Q} -local martingales that are *P*-local martingales. Both facts are intimately related to each other, while the first fact can be incorporated into the second fact through the definition below. Thus, due to the use of predictable characteristics of S and Theorem 2.1, we will identify the \tilde{Q} -local martingales parts that preserve the local martingale property under P.

Definitions 3.4. Let Q be a probability measure, and K and L be two Q-semimartingales.

1) K is said to be orthogonal to L under Q if the process [L, K] is a Q-local martingale.

2) K is said to be Q-locally integrable if the nondecreasing process, $\sup_{s \leq \cdot} |K_s|$, is Q-locally integrable

(i.e. it belongs to $\mathcal{A}^+_{loc}(Q)$, or in other words K is a special semimartingale under Q).

It is obvious that, through this extension of the definition of the orthogonality to semimartingales, the preservation of the orthogonality when switching from \tilde{Q} to P reduces to the preservation of the martingale property under the same change of measures, and this enhances our focus on characterizing the preservation of the martingale property only (Proposition 3.6). However, due to the specificity of the measure \tilde{Q} , in the following we will show that also the preservation of the orthogonality implies the preservation of the martingale property. **Proposition 3.5.** Let L be a \widetilde{Q} -local martingale. Then, L is P-locally integrable and is P-orthogonal to M if and only if L is a P-local martingale that is orthogonal to M.

Proof. L is P-locally integrable if and only if there exist a P-local martingale, \overline{L} , and a predictable process with finite variation, \overline{B} , such that

$$L = \overline{L} + \overline{B}.$$

Then, due to $\langle L, M \rangle = \langle \overline{L}, M \rangle$, we deduce that L is P-orthogonal to M if and only if \overline{L} is P-orthogonal to M in one hand. On the other hand, L is a \widetilde{Q} -local martingale if and only if

$$\overline{B} = -\langle \overline{L}, \widetilde{N} \rangle = -\langle \overline{L}, M \rangle \cdot \widetilde{\lambda}.$$

Thus we deduce that if L is a \tilde{Q} -local martingale and is P-orthogonal to M, then $\overline{B} = 0$. This ends the proof of the lemma.

In the following we will elaborate our main results in this subsection.

Proposition 3.6. The following assertions hold:

(i) Let X and Y be two \tilde{Q} -local martingales such that [S, X] = 0, and there exists an \tilde{O} -measurable functional, g, such that $Y = g \star \mu$ (with $M^{\tilde{Q}}_{\mu}(g|\tilde{\mathcal{P}}) = 0$). Then X (respectively Y) is a P-local martingale if and only if X (respectively Y) is P-locally integrable.

(ii) Let Z be a Q-local martingale whose decomposition through Theorem 2.1 is given by

$$Z = Z_0 + \beta \cdot S^{c,\tilde{Q}} + W \star (\mu - \nu^{\tilde{Q}}) + g \star \mu + Z', \qquad W_t(x) := f_t(x) + \frac{\int f_t(y)\nu^Q(\{t\}, dy)}{1 - \nu^{\tilde{Q}}(\{t\}, I\!\!R^d)}.$$

Then Z is a P-local martingale if and only if (a) The processes $(|f| \wedge |f|^2) \star \mu$, $g \star \mu$ and Z' are P-locally integrable, and (b) For $P(d\omega)dA_t(\omega)$ -almost all (t,ω) , we have

$$\widetilde{\lambda}_t^* c_t \beta_t + \int \left[\widetilde{\lambda}_t^* x + \widetilde{\lambda}_t^* \Delta \langle M \rangle_t \widetilde{\lambda}_t \right] W_t(x) F_t(dx) = 0.$$
(3.34)

Proof. (i) Suppose that X and Y are P-locally integrable. Notice that for any \tilde{Q} -local martingale, L, we have the following: L is a P-local martingale if and only if $[L, \tilde{X}]$ is a \tilde{Q} -local martingale, where

$$\widetilde{X} := -\widetilde{N} + \frac{1}{1 + \Delta \widetilde{N}} \cdot [\widetilde{N}, \widetilde{N}]$$
(3.35)

describes the change of measure from \tilde{Q} to P and follows from $1/\mathcal{E}(\tilde{N}) = \mathcal{E}(\tilde{X})$. For any semimartingale X, we calculate

$$[X, \widetilde{X}] = -\left(1 + \Delta \widetilde{N}\right)^{-1} \cdot [X, \widetilde{N}]$$

= $-\widetilde{\lambda} \left(1 + \Delta \widetilde{N}\right)^{-1} \cdot [X, S] - \sum \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda} \left[1 + \widetilde{\lambda}^* \Delta S + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}\right]^{-1} \Delta X.$ (3.36)

Now suppose that X satisfies [X, S] = 0, then in particular, we have $\Delta S = 0$ on the set $\{\Delta X \neq 0\}$, and (3.36) becomes

$$[X,\widetilde{X}] = -\sum \frac{\widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}}{1 + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}} \Delta X = -\frac{\widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}}{1 + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}} \cdot X.$$
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The last equality is due to the fact that the process $\frac{\tilde{\lambda}^* \Delta \langle M \rangle \tilde{\lambda}}{1 + \tilde{\lambda}^* \Delta \langle M \rangle \tilde{\lambda}}$ is thin and is bounded. Hence the process $[X, \tilde{X}]$ is a \tilde{Q} -local martingale when X is a \tilde{Q} -local martingale with [X, S] = 0. Now suppose that $X = g \star \mu$ with $M^{\tilde{Q}}_{\mu} \left(g \mid \tilde{\mathcal{P}}\right) = 0$. Then we get

$$\begin{split} [X,\widetilde{X}] &= -\sum g(\Delta S) \frac{\widetilde{\lambda}^* \Delta S + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}}{1 + \widetilde{\lambda}^* \Delta S + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}} I_{\{\Delta S \neq 0\}} = G \star \mu_{2} \\ G_t(x) &:= -g_t(x) \frac{\widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}}{1 + \widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}}. \end{split}$$

Then, it is obvious that

$$M^{\widetilde{Q}}_{\mu}\left(G \mid \widetilde{\mathcal{P}}\right)(t,x) = \frac{\widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}}{1 + \widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}} M^{\widetilde{Q}}_{\mu}\left(g \mid \widetilde{\mathcal{P}}\right)(t,x) = 0.$$

Thus, $X = g \star \mu$ is a *P*-local martingale.

(ii) The proof of this assertion will be outlined in two steps. The first step (parts 1), 2), and 3) below) will show that Z is P-locally integrable if and only if the assertion (ii)-(a) holds, while the second step (part 4)) will prove that under the P-local integrability of Z, the P-compensator of Z is zero if and only if the assertion (ii)-(b) is satisfied.

1) We start the first step by noticing that $|f| \wedge |f|^2 \star \mu$ is a process with finite variation, since its \tilde{Q} -compensator exists $(|f| \wedge |f|^2 \star \nu_T^{\tilde{Q}} < +\infty, P-a.s.)$. Therefore, $|f| \wedge |f|^2 \star \mu$ is P-locally integrable if and only if $|f|I_{\{|f|>1\}} \star \mu$ is P-locally integrable, since the process $|f|^2 I_{\{|f|\leq 1\}} \star \mu$ is a locally bounded process. We also recall a result that constitutes a crucial tool in proving the first step, which is Theorem 25 (Chapter VII) in [17]. Thanks to this theorem, a semimartingale K is P-locally integrable if and only if the nondecreasing process $\sup_{s\leq \cdot} |\Delta K_s|$ is P-locally integrable (i.e.

it belongs to $\mathcal{A}^+_{loc}(P)$). This is also equivalent to the fact that both processes $\sup_{s \leq \cdot} \left[|\Delta K_s| I_{\{\Delta S_s \neq 0\}} \right]$ and $\sup_{s \leq \cdot} \left[|\Delta K_s| I_{\{\Delta S_s = 0\}} \right]$ are *P*-locally integrable.

2) Due to (2.6) and the fact that [Z', S] = 0, the process

$$\sup_{s \le t} \left[|\Delta Z_s| I_{\{\Delta S_s \ne 0\}} \right] = \sup_{s \le t} \left[|W_s(\Delta S_s) - \widehat{W}_s^{\widetilde{Q}} + g_s(\Delta S_s)| I_{\{\Delta S_s \ne 0\}} \right] = \sup_{s \le t} |f_s(\Delta S_s) + g_s(\Delta S_s)| I_{\{\Delta S_s \ne 0\}}$$

is *P*-locally integrable if and only if the two processes $\sup_{s \leq t} |f_s(\Delta S_s) + g_s(\Delta S_s)| I_{\{|f_s(\Delta S_s)| > 1, \Delta S_s \neq 0\}}$ and $\sup_{s \leq t} |f_s(\Delta S_s) + g_s(\Delta S_s)| I_{\{|f_s(\Delta S_s)| \leq 1, \Delta S_s \neq 0\}}$ are *P*-locally integrable.

and $\sup_{s \leq t} |J_s(\Delta c_s)| + g_s(-c_{s_{j+1}}|J_s(\Delta c_{s_{j+2}}, -c_{s_{j+2}})|$ It is obvious that $\sup_{s \leq t} |f_s(\Delta S_s) + g_s(\Delta S_s)| I_{\{|f_s(\Delta S_s)| \leq 1, \Delta S_s \neq 0\}}$ is *P*-integrable if and only if the process, $\sup_{s \leq t} |g_s(\Delta S_s)| I_{\{|f_s(\Delta S_s)| \leq 1, \Delta S_s \neq 0\}}$, is *P*-locally integrable or equivalently $gI_{\{|f| \leq 1\}} \star \mu$ is *P*-locally integrable since the latter process exists as semimartingale. Again, since the two processes $fI_{\{|f|>1\}} \star \mu$ and $gI_{\{|f|>1,\}} \star \mu$ exist as semimartingales, then we deduce that $\sup_{s \leq t} |f_s(\Delta S_s) + g_s(\Delta S_s)|I_{\{|f_s(\Delta S_s)|>1, \Delta S_s \neq 0\}}$ is *P*-locally integrable if and only if $(f + g)I_{\{|f|>1\}} \star \mu$ is *P*-locally integrable. It is easy to verify that the P-compensator of $K := (f + g)I_{\{|f| > 1\}} \star \mu$ coincides with the \tilde{Q} -compensator of

$$\left(1+\Delta\widetilde{N}\right)^{-1}\cdot K = \frac{f+g}{1+\widetilde{\lambda}^*x+\widetilde{\lambda}^*\Delta\langle M\rangle\widetilde{\lambda}}I_{\{|f|>1\}}\star\mu,$$

which is given by

$$M^{\widetilde{Q}}_{\mu}\left(\frac{f+g}{1+\widetilde{\lambda}^{*}x+\widetilde{\lambda}^{*}\Delta\langle M\rangle\widetilde{\lambda}}\mid\widetilde{\mathcal{P}}\right)I_{\{|f|>1\}}\star\nu^{\widetilde{Q}}=\frac{f}{1+\widetilde{\lambda}^{*}x+\widetilde{\lambda}^{*}\Delta\langle M\rangle\widetilde{\lambda}}I_{\{|f|>1\}}\star\nu^{\widetilde{Q}}=fI_{\{|f|>1\}}\star\nu.$$

As a result, this proves that $(f+g)I_{\{|f|>1\}}\star\mu$ is *P*-locally integrable if and only if both $|f|I_{\{|f|>1\}}\star\mu$ and $gI_{\{|f|>1\}}\star\mu$ are *P*-locally integrable. By combining all these conclusions we deduce that $|f|I_{\{|f|>1\}}\star\mu$ and $g\star\mu$ are *P*-locally integrable.

3) Now consider the following process

$$\sup_{s \le t} \left[|\Delta Z_s| I_{\{\Delta S_s = 0\}} \right] = \sup_{s \le t} \left[|-\widehat{W}_s^{\widetilde{Q}} + \Delta Z_s'| I_{\{\Delta S_s = 0\}} \right].$$
(3.37)

Thanks to [17] (Chapter VIII, Section 11), the process $\sup_{s \leq t} |\widehat{W}_s^{\widetilde{Q}}|$ is locally bounded, and as a result the *P*-local integrability of $\sup_{s \leq t} \left[|\widehat{W}_s^{\widetilde{Q}}| I_{\{\Delta S_s = 0\}} \right]$ follows. This implies that the process in (3.37) is *P*-locally integrable if and only if $\sup_{s \leq t} |\Delta Z'_s|$ is *P*-locally integrable, or equivalently Z' is *P*-locally integrable. Thus, by combining parts 1), 2), and 3), we conclude that the first step of our proof for assertion (ii) is achieved.

4) Thanks to assertion (i) and the first step, we deduce that -under assertion (ii)(a)- Z is a P-local martingale if and only if

$$Z^{(1)} := \beta \cdot S^{c,\widetilde{Q}} + W \star (\mu - \nu^{\widetilde{Q}}),$$

has a null *P*-compensator. As a consequence the process $Z^{(1)}$ is *P*-locally integrable or equivalently the process $W \star (\nu - \nu^{\tilde{Q}})$ makes sense. Hence, since β is *S*^c-integrable (in the semimartingale sense), we obtain

$$W \star (\mu - \nu^{\widetilde{Q}}) = W \star (\mu - \nu) + W \star (\nu - \nu^{\widetilde{Q}}), \qquad \beta \cdot S^{c,\widetilde{Q}} = \beta \cdot S^{c} - \widetilde{\lambda}^{*} c\beta \cdot A.$$

Then, these equations imply that $Z^{(1)}$ is a *P*-local martingale if and only if

$$0 = W \star (\nu - \nu^{\widetilde{Q}}) - \widetilde{\lambda}^* c\beta \cdot A = -\left[\widetilde{\lambda}^* x + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}\right] W \star \nu - \widetilde{\lambda}^* c\beta \cdot A.$$

Therefore, (3.34) follows. This ends the proof of the proposition.

Remarks: 1) A particular case of the first assertion in Proposition 3.6 is the case when X is a continuous \tilde{Q} -local martingale that is orthogonal to S under \tilde{Q} , then X is a P-local martingale orthogonal to M.

2) As a consequence of the assertion (ii) of the proposition, we can immediately characterize the

 \widetilde{Q} -local martingales that are orthogonal to S under \widetilde{Q} and remain P-local martingales by combining equation (3.34) and the equation related to the orthogonality with S.

3) We conclude this section by illustrating the results of this subsection on those of Theorem 3.2 as follows. It can be shown that $\widetilde{\Phi}$ is a null process if and only if \widetilde{L} (the martingale component in the GKW-decomposition of \widetilde{V}^H under \widetilde{Q}) is *P*-orthogonal to *M*. Indeed, notice that $[\widetilde{L}, S]$ is a \widetilde{Q} -local martingale if and only if $[\widetilde{L}, M]$ is a \widetilde{Q} -local martingale if and only if

$$0 = \langle \widetilde{L}, M \rangle + \langle \widetilde{N}, [\widetilde{L}, M] \rangle.$$
(3.38)

Now we calculate

$$\langle S \rangle \cdot \widetilde{\Phi} = \langle \widetilde{N}, [\widetilde{L}, S] \rangle = \langle \widetilde{N}, [\widetilde{L}, M] \rangle - \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda} \cdot \langle M, \widetilde{L} \rangle.$$

Therefore by inserting this equation into (3.38), we obtain

$$0 = \left(1 + \widetilde{\lambda}^* \Delta \langle M \rangle \widetilde{\lambda}\right) \cdot \langle \widetilde{L}, M \rangle + \langle S \rangle \cdot \widetilde{\Phi}.$$

Thus, $\widetilde{\Phi}$ is a null process if and only if $\langle \widetilde{L}, M \rangle \equiv 0$. This ends the proof of the claim.

4. The FS-decomposition via the predictable characteristics

This section proposes a description of the FS-decomposition – under some integrability conditions that guarantee the existence of this decomposition – in terms of the predictable characteristics of S. The following assumptions hold throughout the whole section.

Assumptions 4.1. We assume that there exists a constant C > 0 such that (3.11) holds.

Remarks:

(1) It is obvious that Assumptions 4.1 is weaker than Assumptions 3.1. That is in this section, the minimal martingale measure may not exist as a measure, and/or its density may vanish. This is an interesting generalization, especially when one is working with models that involve jumps such as Lévy market models. In our view, the integrability condition of (3.11) is less restrictive than the positivity of $\mathcal{E}(\tilde{N})$, since in many models considered in the literature the authors (see for instance [6], and [7]) assume that $\int_0^T \tilde{\lambda}_s^* d\langle M \rangle_s \tilde{\lambda}_s$ is bounded. Thanks to Proposition 3.7 in [8], this condition implies (3.11).

(2) Due to Doob's inequality, we deduce that for any n, $T_n \mathcal{E}(\widetilde{N}) := \mathcal{E}\left(\widetilde{N} - \widetilde{N}^{T_n}\right)$ is a true martingale, where

$$T_0 = 0, \quad T_{n+1} := \inf\{t > T_n : \Delta N_t = -1\} \land T, \quad n \ge 0.$$

In [8], we refer to this property by saying that $\mathcal{E}(\tilde{N})$ is *regular*. Therefore, Assumptions 4.1 guarantees for us the existence of the FS-decomposition for any square integrable \mathcal{F}_T -measurable H (see Theorem 5.5 in [8] for details).

(3) Throughout this section, for any square integrable \mathcal{F}_T -measurable H we denote

$$\widetilde{V}_t^H := \left[{}^{T_n} \mathcal{E}(\widetilde{N})_t \right]^{-1} E\left({}^{T_n} \mathcal{E}(\widetilde{N})_T H \mid \mathcal{F}_t \right), \qquad T_n \le t < T_{n+1}.$$
(4.39)

Proposition 4.2. The following assertions hold.

(1) The process

$$\widetilde{K}_t := \widetilde{V}_t^H - \widetilde{V}_0^H + \langle \widetilde{V}^H, \widetilde{N} \rangle_t, \qquad (4.40)$$

is a P-local martingale.

(2) If (H_0, ξ^H, L^H) are the FS-decomposition components of H, then

$$\widetilde{V}_t^H = H_0 + \left(\xi^H \cdot S\right)_t + L_t^H.$$
(4.41)

Proof:

(1) This assertion follows from a combination of Proposition 3.12-(iii), and Corollary 3.16 of [8]. (2) Since $S + [S, \tilde{N}]$ and $[L^H, \tilde{N}]$ are *P*-local martingales, it is easy to check that for any $n \ge 0$, the processes ${}^{T_n} \mathcal{E}(\tilde{N}) \left[(\xi^H \cdot S) - (\xi^H \cdot S)^{T_n} \right]$ and ${}^{T_n} \mathcal{E}(\tilde{N}) \left[L^H - L^H_{T_n \wedge} \right]$ are *P*-local martingales. Furthermore, these processes are uniformly integrable due to (3.11) and the integrability of $\xi^H \cdot S$ and L^H . Then, for $t \ge T_n$ we derive

$$E\begin{bmatrix}T_{n}\mathcal{E}_{T}(\widetilde{N})\Big((\xi^{H}\cdot S)_{T}-(\xi^{H}\cdot S)_{T_{n}}\Big)|\mathcal{F}_{t}\end{bmatrix} = T_{n}\mathcal{E}_{t}(\widetilde{N})\Big((\xi^{H}\cdot S)_{t}-(\xi^{H}\cdot S)_{T_{n}}\Big),\\E\begin{bmatrix}T_{n}\mathcal{E}_{T}(\widetilde{N})\Big(L_{T}^{H}-L_{T_{n}}^{H}\Big)|\mathcal{F}_{t}\end{bmatrix} = T_{n}\mathcal{E}_{t}(\widetilde{N})\Big(L_{t}^{H}-L_{T_{n}}^{H}\Big).$$

As a result, due to $T_n \mathcal{E}_t(\widetilde{N}) \neq 0$ on $\{T_n \leq t < T_{n+1}\}$, we deduce that

$$\widetilde{V}_t^H = H_0 + (\xi^H \cdot S)_t + L_t^H.$$

This ends the proof of the second assertion.

Now we will state the main result in this section.

Theorem 4.3. Consider a square integrable \mathcal{F}_T -measurable random variable H, and denote by (H_0, ξ^H, L^H) its FS-decomposition components. Then the following holds

$$\xi^{H} = \Sigma^{inv} \left\{ c \widetilde{\phi} + \int x \widetilde{f}(x) F(dx) \right\}, \qquad L^{H} = \widetilde{V}^{H} - \xi^{H} \cdot S.$$
(4.42)

Here $(\tilde{\phi}, \tilde{f}, \tilde{g}, \tilde{K}')$ is the quadruplet associated to \tilde{K} through Theorem 2.1, and Σ is a random symmetric matrix defined in (3.16).

Proof. By applying Jacod's Theorem (Theorem 2.1) to the *P*-local martingale \widetilde{K} , we obtain

$$\widetilde{K} = \widetilde{\phi} \cdot S^c + \widetilde{W} \star (\mu - \nu) + \widetilde{g} \star \mu + \widetilde{K}',$$

$$\widetilde{W}_t(x) := \widetilde{f}_t(x) + \frac{1}{1 - a_t} \int \widetilde{f}_t(y) \nu(\{t\}, dy).$$
(4.43)

Another application of Theorem 2.1 now to L^H leads to

$$L^{H} = \phi^{\perp} \cdot S^{c} + W^{\perp} \star (\mu - \nu) + g^{\perp} \star \mu + \overline{L^{H}},$$

$$W_{t}^{\perp}(x) := f_{t}^{\perp}(x) + \frac{1}{1 - a_{t}} \int f_{t}^{\perp}(y)\nu(\{t\}, dy).$$
(4.44)

Since $\langle \widetilde{V}^H, \widetilde{N} \rangle = \widetilde{\lambda} \cdot \langle \widetilde{V}^H, M \rangle$, then thanks to (4.40) we calculate

$$\langle \widetilde{V}^H, M \rangle = \langle \widetilde{K}, M \rangle = \langle \widetilde{K}, S \rangle = \left\{ c \widetilde{\phi} + \int x \widetilde{f}(x) F(dx) \right\} \cdot A,$$
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and by plugging this resulting quantity into (4.40) while taking into account (4.43), we get on one hand

$$\widetilde{V}^{H} = \widetilde{V}_{0}^{H} + \widetilde{\phi} \cdot S^{c} + \widetilde{W} \star (\mu - \nu) + \widetilde{g} \star \mu + \widetilde{K}' - \left(\widetilde{\lambda}^{*} c \widetilde{\phi} + \int \widetilde{\lambda}^{*} x \widetilde{f}(x) F(dx)\right) \cdot A$$

On the other hand, by using (4.41), (4.44), and $\langle M \rangle + \sum \left(\Delta \langle M \rangle \widetilde{\lambda} \right) \left(\Delta \langle M \rangle \widetilde{\lambda} \right)^* = \langle S \rangle$, we obtain

$$\widetilde{V}^{H} = H_{0} + \left(\xi^{H} + \phi^{\perp}\right) \cdot S^{c} + \left(W^{\perp} + x^{*}\xi^{H}\right) \star (\mu - \nu) + g^{\perp} \star \mu + \overline{L^{H}} - \frac{\widetilde{\lambda}^{*}c\xi^{H} + \int \widetilde{\lambda}^{*}xx^{*}\xi^{H}F(dx)}{1 + \widetilde{\lambda}^{*}\Delta\langle M\rangle\widetilde{\lambda}} \cdot A.$$

Therefore, due to the uniqueness of Jacod's decomposition (Lemma 2.2) and that of the Doob-Meyer decomposition, we conclude that

$$c\widetilde{\phi} = c\xi^H + c\phi^{\perp}, \qquad \widetilde{f}(x) = x^*\xi^H + f^{\perp}(x) \qquad \widetilde{g}(x) = g^{\perp}(x), \qquad \overline{L^H} = \widetilde{K}'.$$
 (4.45)

Thus by transforming the first two equations above, we derive

$$c\widetilde{\phi} + \int x\widetilde{f}(x)F(dx) = c\xi^{H} + c\phi^{\perp} + \int xx^{*}\xi^{H}F(dx) + \int xf^{\perp}(x)F(dx)$$

= $\Sigma\xi^{H} + c\phi^{\perp} + \int xf^{\perp}(x)F(dx).$ (4.46)

Since L^H satisfies

$$\langle L^H, M \rangle = \langle L^H, S \rangle = \left(c \phi^{\perp} + \int x f^{\perp}(x) F(dx) \right) \cdot A = 0,$$

then the equation (4.46) reduces to

$$\Sigma \xi^H = c \widetilde{\phi} + \int x \widetilde{f}(x) F(dx),$$

and the first equation in (4.42) follows immediately. This ends the proof of the theorem.

Remark When the FS-decomposition exists, it is clear that the ingredient ξ^H can be obtained as the Radon-Nikodym derivative of $d\langle \tilde{V}^H, M \rangle$ with respect to $d\langle M \rangle$. However, our description of this ingredient using the predictable characteristics has other impacts:

- (1) Through the use of the predictable characteristics, the variation of the FS-decomposition with additional jumps and/or uncertainty will be easy to handle. Furthermore, this illustration using the predictable characteristics is helpful in avoiding pitfalls and misleading generalizations of results such as those of [25] and [14] (Section 10.4). Many practical market models are described using the predictable characteristics such as Barndorff-Nielsen-Shephard models, see [4] and [26] and the reference therein about these models and related subjects. Hence, we think that our description of the FS-decomposition will be useful for those models.
- (2) Recently, the more explicitly characterized optimal martingale measures in the literature are expressed in terms of the predictable characteristics, see [10], [11], and [12] for semimartingale framework, and [4], [20], [23], and [24] for models driven by Lévy processes. Thus, we believe that our current description of the FS-decomposition is suitable for those contexts.

(3) Finally, as it will be illustrated in the following example, our description generalizes the approach of [13] and [33] to the semimartingale context where the predictable martingale representation may be violated in one hand. On the other hand the predictable characteristics are the extension of Lévy characteristics for models driven by semimartingales.

A Practical Example: Consider a market model for which $A_t = t$ and Assumption 4.1 holds. Let H be a \mathcal{F}_T -measurable random variable such that the process \tilde{V}^H satisfies

$$\widetilde{V}_t^H = E^Q(H|\mathcal{F}_t) = f(t, S_t), \qquad (4.47)$$

where f(t, x) is a $C^{1,2}((0, T) \times \mathbb{R}^d)$ -function. This case generalizes the examples that are frequently used in the literature, such as those treated in [13] and [33]. By applying Itô's formula, we find

$$\widetilde{V}_{t}^{H} = \widetilde{V}_{0}^{H} + \int_{0}^{t} f_{x}(s, S_{s-}) dS_{s} + \int_{0}^{t} [f_{t}(s, S_{s}) + \frac{1}{2} c_{s} f_{xx}(s, S_{s-})] ds + \sum_{0 < s \le t} [f(s, S_{s}) - f(s, S_{s-}) - f_{x}(s, S_{s-}) \Delta S_{s}].$$

Since \widetilde{V}^H is a special semimartingale, then by compensating the last term in the RHS of the above equation and simplifying the resulting equation, we obtain

$$\widetilde{V}^H = \widetilde{V}_0^H + f_x(\cdot, S_-) \cdot S^c + \left[f(\cdot, S_- + x) - f(\cdot, S_-) \right] \star (\mu - \nu) + \widetilde{B},$$

where \widetilde{B} is a predictable process with finite variation. Therefore, this leads to the description of the process \widetilde{K} defined in (4.40), and hence to the FS-decomposition of H as follows.

Corollary 4.4. The following assertions hold:

(1) The process \widetilde{K} is given by

$$\widetilde{K} = f_x(\cdot, S_-) \cdot S^c + \left[f(\cdot, S_- + x) - f(\cdot, S_-) \right] \star (\mu - \nu).$$
(4.48)

(2) The FS-decomposition of H is given by

$$\xi^{H} = \Sigma^{inv} \left[cf_{x}(\cdot, S_{-}) + \int_{\mathbb{R}^{d}} x[f(\cdot, S_{-} + x) - f(\cdot, S_{-})]F(dx) \right], \quad L^{H} = \widetilde{V}^{H} - \widetilde{V}_{0}^{H} - \xi^{H} \cdot S.$$

Proof. The proof of the first assertion is obvious from the previous calculation, while the second assertion is an immediate application of Theorem 2.1 and the fact that the quadruplet of \widetilde{L} through Theorem 4.3 is $(f_x(\cdot, S_-), [f(\cdot, S_- + x) - f(\cdot, S_-)], 0, 0)$.

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