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Maximal Combinations of Fairness Definitions

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Abstract

The so-called 'Impossibility Theorem' for fairness definitions is one of the more striking research results with both theoretical and practical consequences, as it states that satisfying certain combinations of fairness definitions is impossible. To date, this negative result has not yet been complemented with a positive one: a characterization of which combinations of fairness notions *are* possible. This work aims to fill this gap by identifying maximal sets of commonly used fairness definitions for binary classification that can be simultaneously satisfied. The fairness definitions used are demographic parity, equal opportunity, predictive equality, predictive parity, false omission rate parity, overall accuracy equality and treatment equality. We conclude that in total 12 maximal sets of these fairness definitions are possible, among which are seven combinations of two definitions, and five combinations of three definitions. Our findings also shed light on the practical relevance and utility of each of these 12 *maximal fairness* definitions in various scenarios, regarding the accuracy of the classifier and ratios of false positives and false negatives, considering the base rates.

1. Introduction

The field of fairness in AI started gaining more attention after the ProPublica article about the recidivism risk assessment system COMPAS (Angwin et al., 2016). Due to this article, questions arose on which characteristics an AI system needs to satisfy for it to be fair in a practical, intuitive, or legal sense. One of the main criticisms from the authors of the ProPublica article was the difference between the false positive and false negative rates for black people compared to white people. The creators of the COMPAS-tool defended their system by arguing that it was properly calibrated, meaning that defendants with the same scores had similar rearrest rates for both demographic groups, and that this made the system fair. One could argue that ideally these properties would both be satisfied. However, the works of Chouldechova (2016), Barocas et al. (2019), and Kleinberg et al. (2016) all concluded that these properties cannot be satisfied simultaneously. This result is colloquially referred to as the "Impossibility Theorem", stating it is mathematically *impossible* to achieve calibration, equal false positive and equal false negative rates simultaneously, except in a practically irrelevant, degenerate case. (See Sec. 2 for further details.)

The usage of fairness definitions has often been limited to enforcing one at a time. This severely limits the complexity of the fairness requirements imposed on a system. Creating a contrasting "Possibility Theorem" might open up the possibility of simultaneously imposing multiple compatible fairness definitions. These combinations could allow to enforce multiple perspectives on fairness simultaneously (Rahman et al., 2024a; Park et al., 2022; Luo et al., 2024). Additionally, it would also indicate which perspectives are incompatible.

Contributions To the best of our knowledge, this negative result has never been complemented with a positive one, namely the characterisation of maximal sets of fairness notions that *can* be simultaneously satisfied. In this paper, we investigate which and how many of the more frequently used fairness definitions can be combined, leading to a set of maximal notions of fairness on which no additional of the commonly used fairness notions can be imposed without making the problem infeasible. We do this for the simplest but most commonly studied case of binary classification.

Out of a range of seven commonly used fairness definitions (Demographic Parity (Dwork et al., 2012), Equal Opportunity (Hardt et al., 2016), Predictive Equality (Hardt et al., 2016), Predictive Parity (Chouldechova, 2016), False Omission Rate Parity, Overall accuracy Equality (Berk et al., 2021) and Treatment Equality (Berk et al., 2021)), we identify a total of 12 maximal combinations, including seven maximal combinations of two and five maximal combinations of three fairness definitions. Figure 1 shows the possible combinations of fairness definitions. In addition, we investigate the constraints these combinations impose on the accuracies of both groups and on the confusion matrices. We find that imposing a combination of two or more fairness definitions imposes a strict behaviour on the relation of errors made between two sensitive groups. This behaviour is influenced by the base rates of both groups, which means that for some datasets a combination of fairness definitions would heavily constrain the performance for one sensitive group.

2. Related Work

The field of fairness in AI knows two large, distinct research topics. The first one is based on the explainability or interpretability of AI systems. This research aims to create AI models that are explainable or to create explanations for existing models in order to investigate if the models exhibit unfairness, which is identifiable for people. This approach is related to the legal concept of procedural fairness (Australian Law Reform Commission, 2016), meaning that the decision process itself should be deemed fair.

The second research focus, adopted in the current paper, concerns the formulation and study of mathematical definitions of fairness. The aim is to have the model adhere to a certain fairness notion, defined in terms of certain mathematical properties on the outcome of the model. This method if often referred to as outcome fairness and relates to the legal concept of distributive justice (Lamont & Favor, 2017), meaning that the distribution of the outcome decision should be deemed fair.

Both approaches are not mutually exclusive, but work with a different ideology on how fairness should be achieved. It is possible to have a fair procedure, thus procedural fairness, that results in an unfair distribution and vice versa. In this section, we briefly elaborate on both these research lines.

2.1 Procedural Fairness Approaches: Explainability and Interpretability

The simplest approach in explainability is to only use non-discriminatory features. For example, Grgic-Hlaca et al. (2018) let human participants decide which features are acceptable to base a prediction on given a specific context. A special case of this fair feature selection is fairness through unawareness, wherein no sensitive attributes are included in the features.

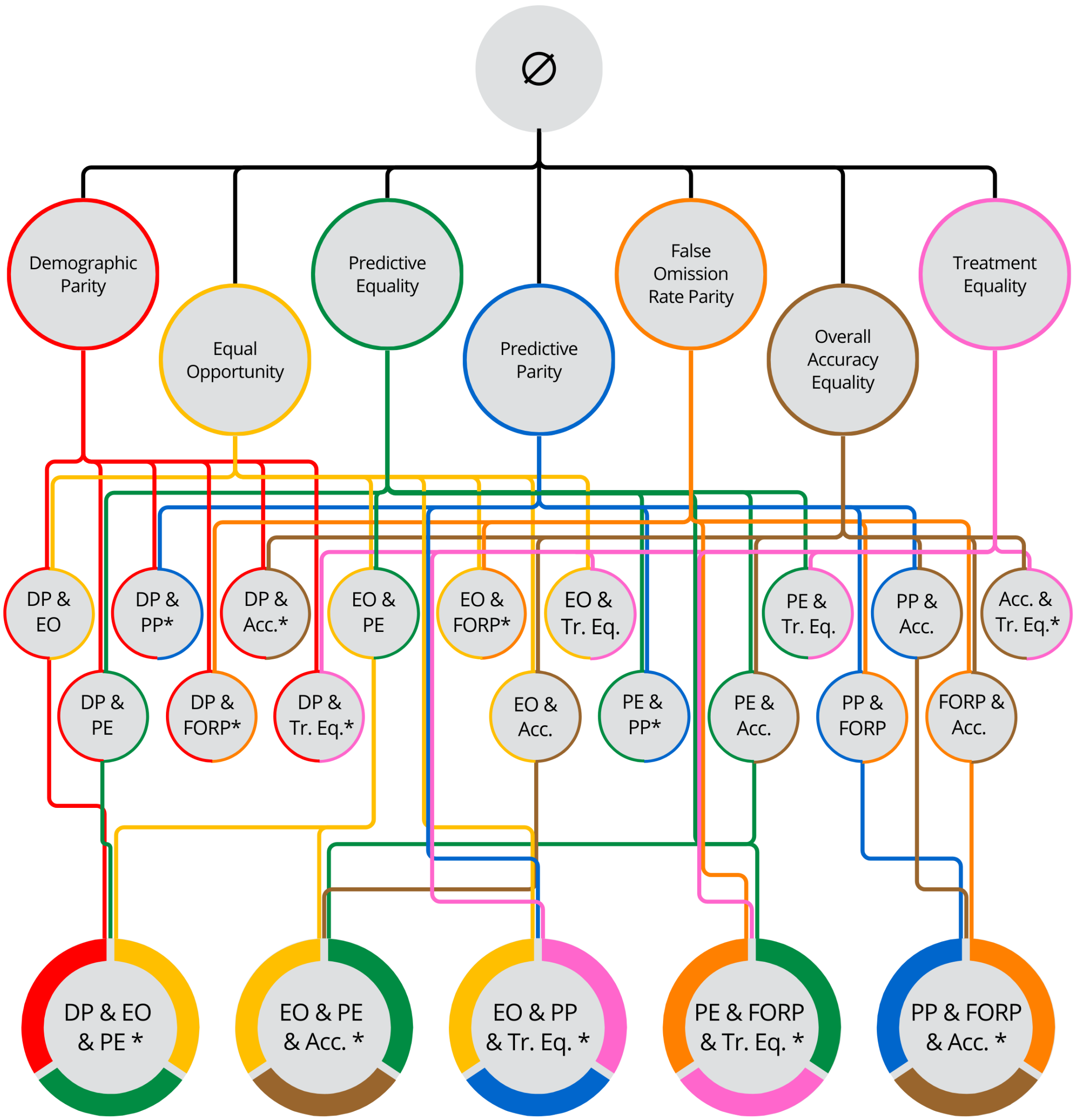


Figure 1: The possible sets of fairness definitions are represented by nodes, according to Definition 1. The nodes with an asterisk are maximal sets which cannot be extended with an extra fairness definition, whilst satisfying Definition 1.

A second explainability approach to AI fairness focuses on the output behaviour of a model, and aims to assess the fairness of outputs per individual. This can be achieved by interchanging the values of the sensitive attributes, and checking if doing so changes the prediction (Agarwal et al., 2018; Galhotra et al., 2017). Counterfactuals extend this principle by analysing the model's behaviour through the difference between the real-world data and a counterfactual one. This counterfactual data is created by changing the sensitive attribute and using a causal model to subsequently adjust the other features (Kusner et al., 2017). This approach is favoured to account for the correlation between the sensitive attribute and the other features. Other methods besides causal models can also be used to adjust for these correlations (Black et al., 2020). The work of Grari et al. (2023) extends the capabilities of counterfactuals to also function with continuous sensitive variables.

Another method using counterfactuals is investigating the difference in effort certain groups would have to make in order to change their predictions (Sharma et al., 2020). The work of Goethals et al. (2023) has a similar goal: they investigate which attributes have an imbalanced influence on the prediction for a group with a certain sensitive attribute.

2.2 Outcome Fairness: Fairness Definitions on the Output Distribution

The research around fairness definitions can be categorised in two main groups. The first group consists of methods enforcing a certain fairness definition on a learned model, often called fairness methods. The other consist of more conceptual research on these definitions. In general this group is about either the introduction of new fairness definitions, surveys on fairness definitions, the interpretation of previously proposed fairness notions, or about so-called impossibility theorems showing the incompatibility of certain combinations of fairness definitions. Here we survey the work most directly related to the present paper.

2.2.1 Surveys on Fairness Definitions

Verma & Rubin (2018) discuss different fairness definitions for classification and focus on creating a human-understandable interpretation of these definitions and applying them to a specific use case. An elaborate discussion about the design of a predictive model concerning pitfalls and biases that occur in the process can be found in Mitchell et al. (2021). It also covers different fairness definitions and relates the choice between these definitions back to the discussion of design choices. They touch upon the impossibilities of combining certain fairness definitions when working with scores instead of binary decisions.

The survey of Mehrabi et al. (2021) contains an extensive list of possible biases and fairness definitions. The fairness definitions are grouped by type, group, subgroup or individual. It also discusses methods to satisfy them. A similar recent survey was done by Caton & Haas (2024). Pessach & Shmueli (2022) published a similar survey. They conclude with several emerging research sub-fields of algorithmic fairness. Ruf & Detyniecki (2021) strive to create a type of *Fairness Compass* which should help practitioners decide what fairness definition is best suited to a given problem. Throughout the work, different fairness definitions are clearly explained through the use of confusion matrices and with a theoretical use case. Congruently, a similar tool was developed with the Aequitas Toolkit (Saleiro et al., 2019). However, their *Fairness Tree* is constructed based on the use of the system rather than the properties in the data, on which the *Fairness Compass* is designed.

2.2.2 Prior Work on Combining Fairness Definitions

The majority of research on combining fairness definitions focusses on the impossibility of certain combinations. Chouldechova (2016) discuss how adherence to a certain fairness criterion can lead to considerable disparate impact between groups. They state it is impossible to achieve equality between groups for false positive rate (FPR), positive predictive value (PPV) and false negative rate (FNR) when the base rates differ between groups.

Kleinberg et al. (2016) discuss combining three specific fairness definitions in the context of risk scoring. The work focusses on scoring rather than binary classification. They conclude that combining those three definitions is only possible under unique circumstances, namely if the groups have equal base rates or if the model is capable of perfect prediction.

Berk et al. (2021) focus on the trade-off that would occur when using multiple incompatible fairness definitions such as discussed in the work by Chouldechova (2016). They also provide different techniques for achieving this goal.

Related to the two original *Impossibility Theorems*, Beigang (2023) created a new impossibility theorem, which concerns the combination of counterfactual fairness with equalized odds and predictive parity. They also provide some relaxations to the fairness definitions in order for them to be compatible. Bell et al. (2023) investigated the impossibility theorems in practice. In this work, experiments are used to investigate the effects of enforcing equal FPRs, FNRs and PPVs on the accuracy of the system. A similar result is found in the work by Hsu et al. (2022). They use integer programming to solve this multi-objective problem. In their experiment they also reaffirm the existing impossibility theorems.

The work of Rosenblatt & Witter (2023) shows that counterfactual fairness and demographic parity are basically equivalent. Pleiss et al. (2017) created a relaxed version of equalised odds and calibration for which the combination is feasible in more situations. However, they indicate themselves that the models which satisfy this combination of constraints have limited usefulness.

Another approach was taken by Rahman et al. (2024b). They designed a novel set of fairness definitions. The paper focusses on intra-marginal and intersectional concepts of fairness and express these concepts for both individual and group fairness.

2.2.3 Combining Fairness Definitions with Other Metrics

The work of Cummings et al. (2019) shows that given a system with non-trivial accuracy, it is impossible to combine pure differential fairness with strict equal opportunity. Trivial accuracy is the maximal accuracy a system could achieve when only predicting the same label. They also show that combining differential fairness with a relaxed version of equal opportunity is feasible. A subsequent work of Agarwal (2021) refutes this possibility of the combination with a relaxed notion of fairness. Furthermore, they prove that pure differential fairness is impossible to combine with either demographic parity, equal opportunity or equalised odds while achieving non-trivial accuracy.

The work of Pinzón et al. (2023) finds that given a probabilistic data source, equal opportunity is incompatible with non-trivial accuracy under certain conditions. This is complementary to the work of Hardt et al. (2016), which proves that equal opportunity and non-trivial accuracy are compatible for a deterministic data source.

3. Maximal Combinations of Fairness Definitions

Common fairness definitions can be expressed as functions of the variables in confusion matrices (such as Table 1) for the different protected demographic groups. More specifically, they are defined as the equality of a statistic, such as the false negative rate, computed on the confusion matrices for the different demographic groups. Often, these demographic groups are based on their sensitive attributes. We articulate these expressions for seven common fairness definitions in Sec. 3.1. This approach makes combining fairness definitions easy: simply combining the constraints they impose on the confusion matrices.

Our analysis considers two groups, referred to as groups a and b. However, later we show that our findings hold up for multiple groups, if these groups are disjoint. Note that confusion matrices commonly use counts. However, we normalize the confusion matrices such that the four variables in each confusion matrix sum to 1, as this simplifies the calculations.

Table 1: Symbolic representation of a confusion matrix.

		Predicted	
		Positive	Negative
True	Positive	TP	FN
	Negative	FP	TN

The values in the confusion matrix are constrained due to properties of the dataset itself:

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ 0 \leq TP_a \leq 1 - x, 0 \leq TP_b \leq 1 - y \\ 0 \leq TN_a \leq x, 0 \leq TN_b \leq y \\ 0 \leq FP_a \leq x, 0 \leq FP_b \leq y \\ 0 \leq FN_a \leq 1 - x, 0 \leq FN_b \leq 1 - y \end{cases} \tag{1}$$

The variables $1 - x$ and $1 - y$ in Equation (1) denote the base rates of group a and group b respectively. The base rate is the fraction of positive samples in the data per group. The inequalities will be left out of subsequent derivations to keep the notations concise.

Let us now define what we mean when we say that a certain combination of fairness definitions is ‘possible’. Before doing this, the following two observations are relevant.

First, if the base rates of the two groups are equal, i.e. if $x = y$, then it is always possible to satisfy any combination of fairness definitions, as the confusion matrices can then be identical. If the confusion matrices are identical, then any statistics computed on them will also be equal. However, base rates are a property of the dataset and hence situation dependent. Thus, any definition of what it means for a combination of fairness definitions to be ‘possible’, should be independent of the base rates.

Second, some combinations of fairness definitions may be too constraining that jointly enforcing them leads to certain variables in the confusion tables having a fixed (and possibly trivial) value. Although this means that the combination can strictly speaking be satisfied, it leaves so little freedom to the classifier that it becomes of limited practical use. Thus, we propose a definition where also such combinations are not deemed 'possible'.

The following definition determines the necessary properties for a combination of fairness definitions to be deemed 'possible'.

Definition 1 (Possibility of combining fairness definitions)**.** *A combination of fairness definitions is deemed* possible *if after combining the constraints, all elements in both confusion matrices can still take on values within a subset of* $[0,1]$ *with non-zero Lebesgue measure, and this for all pairs of base rates* $(1-x, 1-y)$ *from a subset of* $[0,1]^2$ *with non-zero 2-dimensional Lebesgue measure. Or informally speaking, the elements of the confusion matrices should be able to take on values within a non-trivial range, and this for all base rate pairs from within a non-trivial 2-dimensional range.*

Remark 1. *This definition is solely focussed on finding some configuration of the confusion matrix that will satisfy the fairness definitions. It would also be possible to include the performance of the resulting classifier as a requirement. This was done by Barocas et al. (2019). They additionally required that the prediction of the model must not be independent of the ground truth. In other words, it would require the classifier not to be random.*

Remark 2. *This paper only considers satisfying fairness definitions, meaning the equality must be satisfied. Other approaches to fairness aim to minimize the difference between two relevant quantities, instead of requiring an equality. In this line of research, several methods have been proposed to use this difference as a loss term in order to make a model more fair (Kamishima et al., 2011; Zemel et al., 2013; Padala & Gujar, 2020; Buyl et al., 2024) . The results in the present paper remain valid for this approach as an incompatibility will signify that minimizing to a difference of zero is impossible, thus signalling that the minimization task will be working on conflicting goals.*

3.1 Fairness Definitions Considered in the Present Paper

The present paper considers the following fairness definitions: demographic parity, equal opportunity, predictive equality, predictive parity, false omission rate parity, overall accuracy equality and treatment equality. Table 2 contains two equivalent definitions for each fairness definition: one expressed in terms of probabilities, and another in terms of a constraint on the confusion matrices. The table also notes the statistical property it requires to be equal between groups (if it has a standard name). In its last column, it states the 'orientation' of the constraint on the confusion matrices (horizontal, vertical, or diagonal).

Other frequently used fairness definitions exist, particularly in other settings than binary classification, but this paper focusses only on the binary prediction setting. A well known example is calibration as is used in Kleinberg et al. (2016) for risk scoring, its equivalent for binary classification, Predictive Parity, is used in this work. Note that each of these definitions only imposes one constraint on the confusion matrix, combining them increases the number of constraints.

Below we discuss each of the considered fairness definitions in greater detail. Figure 2 illustrates them more visually, highlighting the orientations on the confusion matrix, and thus showing their differences and similarities.

Table 2: The list of fairness definitions considered in this paper, defined in probabilistic terms (second column) and defined in terms of elements in the confusion matrices (third column). The fourth column names the statistic that is constrained to be equal across the confusion matrices. The fifth column mentions the orientation of the constraints on the confusion matrix as shown in Figure 2.

Probabilities	Confusion Matrix	Statistic	Orientation
Demographic Parity			
$P(\hat{Y} = y \mid A = 0) = P(\hat{Y} = y \mid A = 1)$	$\frac{FP_A+TP_A}{FP_A+TP_A+FN_A+TN_A} = \frac{FP_B+TP_B}{FP_B+TP_B+FN_B+TN_B}$	Positive Rate (PR)	Board (B_v)
Equal Opportunity			
$P(\hat{Y} = 1 \mid A = 0, Y = 1) = P(\hat{Y} = 1 \mid A = 1, Y = 1)$	$\frac{TP_A}{TP_A+FN_A} = \frac{TP_B}{TP_B+FN_B}$	True Positive Rate (TPR)	Horizontal (H)
Predictive Equality			
$P(\hat{Y} = 1 \mid A = 0, Y = 0) = P(\hat{Y} = 1 \mid A = 1, Y = 0)$	$\frac{FP_A}{FP_A+TN_A} = \frac{FP_B}{FP_B+TN_B}$	False Positive Rate (FPR)	Horizontal (H)
Predictive Parity			
$P(Y = 1 \mid A = 0, \hat{Y} = 1) = P(Y = 1 \mid A = 1, \hat{Y} = 1)$	$\frac{TP_A}{TP_A+FP_A} = \frac{TP_B}{TP_B+FP_B}$	Pos. Prediction Value (PPV)	Vertical (V)
False Omission Rate Parity			
$P(Y = 1 \mid A = 0, \hat{Y} = 0) = P(Y = 1 \mid A = 1, \hat{Y} = 0)$	$\frac{FN_A}{FN_A+TN_A} = \frac{FN_B}{FN_B+TN_B}$	False Omission Rate (FOR)	Vertical (V)
Overall accuracy Equality			
$P(\hat{Y} = Y \mid A = 0) = P(\hat{Y} = Y \mid A = 1)$	$\frac{TP_A+TN_A}{TP_A+FP_A+TN_A+FN_A} = \frac{TP_B+TN_B}{TP_B+FP_B+TN_B+FN_B}$	Accuracy (ACC)	Board (B_d)
Treatment Equality			
–	$\frac{FN_A}{FP_A} = \frac{FN_B}{FP_B}$	-	Diagonal (D)

Definition 2 (Demographic Parity - DP (Dwork et al., 2012))**.** *In order to satisfy demographic parity both groups proportionally must receive the positive prediction (TP + FP) equally regardless of the ground truth and thus also regardless of the base rates. This is unlike other fairness definitions, the result of which is that satisfying demographic parity can come at the cost of predictive performance. Conditional statistical parity (Corbett-Davies et al., 2017) is a similar fairness definition, which requires equal positive rates between sensitive groups when restricted to people that share a certain attribute value. Conditional statistical parity will not be considered in this paper.*

Definition 3 (Equal Opportunity - EOP (Hardt et al., 2016))**.** *Equal opportunity is both a definition of itself and also one of two conditions that is required for the popular fairness definition equalised odds (Hardt et al., 2016). Equal opportunity requires that people have the same probability of receiving a negative prediction given that they belong in the positive category, regardless of their sensitive attribute.*

Definition 4 (Predictive Equality - PE (Hardt et al., 2016))**.** *Predictive equality is the second condition required in equalised odds, alongside equal opportunity. It requires that the probability of receiving a positive prediction when the ground truth is negative is equal across demographic groups. If both predictive equality and equal opportunity are satisfied then equalised odds is satisfied.*

Definition 5 (Predictive Parity - PP (Chouldechova, 2016))**.** *Predictive parity is yet another kind of definition as it is conditional on the probability of the actual class and not of the predicted class. Expressed on the confusion matrix, it imposes a vertical ratio on it rather than a horizontal one like equal opportunity and predictive equality. In order to satisfy predictive parity the probability that a positive prediction is correct must be equal across sensitive groups.*

Definition 6 (False Omission Rate Parity - FORP)**.** *False omission rate parity is similar to predictive parity. It requires that the probability of a negative prediction to be correct is equal across sensitive groups. The combination of satisfying both predictive parity and false omission rate parity is also called conditional use accuracy equality (Verma & Rubin, 2018).*

Definition 7 (Overall accuracy Equality - OaE (Berk et al., 2021))**.** *Overall accuracy equality requires equal classifier accuracy between sensitive groups. Although straightforward and intuitive, this definition alone is often insufficient to guarantee intuitive fairness. Indeed, it implicitly assumes an equal importance of positive and negative predictions, which is often inaccurate in practice.*

Definition 8 (Treatment Equality - Tr. Eq. (Berk et al., 2021))**.** *Treatment equality is an atypical fairness definition as it cannot be expressed as a probability nor as a commonly used statistic. It is a constraint on the false positives and false negatives, requiring their ratio to be equal for sensitive groups.*

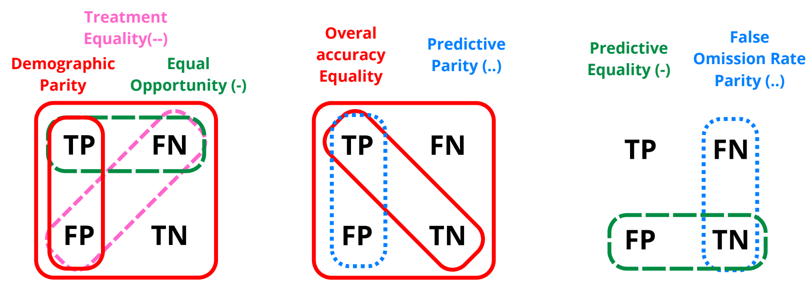


Figure 2: Orientations of the fairness definitions in the confusion matrix.

3.2 Combining Two Fairness Definitions

We begin with the *pairwise* combinations of the fairness definition discussed in Sec. 3.1.

Proposition 1. *All pairwise combinations of demographic parity, equal opportunity, predictive equality, predictive parity, false omission rate parity, overall accuracy equality and treatment equality are possible in accordance with Definition 1.*

Proof outline. To prove for a given pair of fairness definitions that they can be combined pairwise, we pair the constraints of the confusion matrices, as listed in Eq. (1), and the constraints from the respective fairness definitions to yield a combined system of equations. We solve this system of equations to express a subset of the variables in terms of the remaining free variables. The base rates must be such free variables, as required in Def. 1.

An example of calculating the constraints for combining two fairness definitions is shown in Eq. (2), where demographic parity is combined with equal opportunity:

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ \frac{TP_a}{TP_a+FN_a} = \frac{TP_b}{TP_b+FN_b} \\ \frac{TP_a+FP_a}{1} = \frac{TP_b+FP_b}{1} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ \frac{TP_a}{1-x} = \frac{TP_b}{1-y} \\ \frac{TP_a+FP_a}{1} = \frac{TP_b+FP_b}{1} \end{cases} \iff \begin{cases} FN_a = 1 - x - \frac{1-x}{1-y}TP_b \\ TN_a = x - \frac{x-y}{1-y}TP_b - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = \frac{1-x}{1-y}TP_b \\ FP_a = \frac{x-y}{1-y}TP_b + FP_b \end{cases} \tag{2}$$

By combining this result with the inequalities from Eq. (1), feasible ranges for each of the free variables and the base rates can be computed. For conciseness, a shortened version of the calculation of these ranges is shown in Eq. (3), while the full system of inequalities can be found in Appendix A.1.1:

$$\begin{cases} 0 \leq 1 - x - \frac{1-x}{1-y}TP_b \\ 1 - x - \frac{1-x}{1-y}TP_b \leq 1 - x \\ 0 \leq x - \frac{x-y}{1-y}TP_b - FP_b \\ x - \frac{x-y}{1-y}TP_b - FP_b \leq x \\ 0 \leq \frac{x-y}{1-y}TP_b + FP_b \\ \frac{x-y}{1-y}TP_b + FP_b \leq x \end{cases} \iff \begin{cases} \frac{1}{1-y}TP_b \leq 1 \\ \frac{1-x}{1-y}TP_b \geq 0 \\ FP_b \leq x + \frac{-x+y}{1-y}TP_b \\ \frac{-x+y}{1-y}TP_b \leq FP_b \\ \frac{-x+y}{1-y}TP_b \leq FP_b \\ FP_b \leq x + \frac{-x+y}{1-y}TP_b \end{cases} \tag{3}$$

DP & EOP (1) $TP_a = \frac{1-x}{1-y} TP_b$ $FP_a = \frac{x-y}{1-y} TP_b + FP_b$	**DP & PE** (1) $TP_a = \frac{y-x}{y} FP_b + TP_b$ $FP_a = \frac{x}{y} FP_b$	**DP & PP** (2) $TP_a = TP_b$ $FP_a = FP_b$	**DP & FORP** (2) $TN_a = TN_b$ $FN_a = FN_b$	**DP & OaE** (5) $FP_a = FP_b + \frac{x-y}{2}$ $TP_a = TP_b + \frac{y-x}{2}$
DP & Tr. Eq. (4) $FN_a = FN_b (\frac{x-y}{FP_b - FN_b} + 1)$ $FP_a = FP_b (\frac{x-y}{FP_b - FN_b} + 1)$	**EOP & PE** (3) $TP_a = \frac{1-x}{1-y} TP_b$ $FP_a = \frac{x}{y} FP_b$	**EOP & PP** (3) **EOP & Tr. Eq.** **PP & Tr. Eq.** $TP_a = \frac{1-x}{1-y} TP_b$ $FP_a = \frac{1-x}{1-y} FP_b$	**EOP & FORP** (3) $FN_a = \frac{1-x}{1-y} FN_b$ $TN_a = \frac{1-x}{1-y} TN_b$	**EOP & OaE** (7) $TP_a = \frac{1-x}{1-y} TP_b$ $TP_b = \frac{1-y}{y-x} (TN_b - TN_a)$
PE & PP (3) $FP_a = \frac{x}{y} FP_b$ $TP_a = \frac{x}{y} TP_b$	**PE & FORP** (3) **PE & Tr. Eq.** **FORP & Tr. Eq.** $TN_a = \frac{x}{y} TN_b$ $FN_a = \frac{x}{y} FN_b$	**PE & OaE** (7) $TN_a = \frac{x}{x-y} (TP_b - TP_a)$ $TN_b = \frac{y}{x-y} (TP_b - TP_a)$	**PP & FORP** (6) $TP_a = \frac{TP_b \cdot FP_a}{FP_b}$ $FN_a = \frac{FN_b \cdot TN_a}{TN_b}$	**PP & OaE** (4) $TP_a = FP_a (\frac{y-x}{FP_a - FP_b} + 1)$ $TP_b = FP_b (\frac{y-x}{FP_a - FP_b} + 1)$
	FORP & OaE (4) $TN_a = TN_b (\frac{x-y}{TN_b - FN_b} + 1)$ $FN_a = FN_b (\frac{y-x}{TN_b - FN_b} + 1)$	**OaE & Tr. Eq.** (2) $FP_a = FP_b$ $FN_a = FN_b$		

Figure 3: Resulting constraints when combining pairs of fairness definitions.

This system of inequalities shows a specific range for the free variables FP_b, TP_b, and both base rates $1-x$ and $1-y$ are unconstrained. The range for FP_b is $[max(0, \frac{-x+y}{1-y}), min(x + \frac{-x+y}{1-y}, y)]$ which simplifies to $[0, x + \frac{-x+y}{1-y}]$ if $x \geq y$, which means it has a non-zero Lebesgue measure, or $[\frac{-x+y}{1-y}, y]$ if $y > x$ then TP_b will have a non-zero Lebesgue measure if $x > y^2$. Thus the Lebesgue measure is non-zero for each free variable for a set of base rates with a non-zero 2-dimensional Lebesgue measure, meaning that the combination of demographic parity with equal opportunity is possible.

The proofs for the other pairwise combinations of fairness definitions follow a similar structure and can be found in Appendix A. The results are summarized in Figure 3. □

From Proposition 1 we can infer that none of the fairness definitions are in contradiction with each other. In other words, what is fair according to one of the fairness definitions considered, is not impossibly fair according to another.

Figure 3 shows that the combinations of fairness definitions result in a unique set of constraints. However, two exceptions exist: the three pairwise combinations of any two fairness definitions taken from the set (EOP, PP, Tr. Eq.) are identical to each other, and the same is true for all pairwise combinations of fairness definitions from the set (PE, FORP, Tr. Eq.).

The interpretability of these combinations differs greatly. The solution of the combination of predictive parity with false omission rate parity is too tedious to detail in Table 3, but can be found in the calculations of combining predictive parity, false omission rate parity and overall accuracy equality in Section B.25.

Table 3: The structure of the seven different constraint types and the associated combination of orientations.

Nr.	Orientation of the combined definitions	Constraint structure	
(1)	B_vH	$X_a = Factor_{BR} * X_b$	$Y_b = Factor_{BR} * X_b + Y_b$
(2)	$B_vV,\ B_dD$	$X_a = X_b$	$Y_a = Y_b$
(3)	HH, HV, HD, VD	$X_a = Factor_{BR} * X_a$	$Y_a = Factor_{BR} * Y_b$
(4)	$B_vD,\ VB_d$	$X_a = Factor_{BR,X_b,Y_b} * X_b$	$Y_a = Factor_{BR,X_b,Y_b} * Y_b$
(5)	B_vB_d	$X_a = X_b + Factor_{BR}$	$Y_a = Y_b + Factor_{BR}$
(6)	VV	$X_a = Factor_{BR,Y_a,Y_b} * Y_b + Factor_{BR,Y_a,Y_b} * Y_a$	$X_b = Factor_{BR,Y_a,Y_b} * Y_b + Factor_{BR,Y_a,Y_b} * Y_a$
(7)	HB_d	$X_a = Factor_{BR} * (Y_b - Y_a)$	$X_b = Factor_{BR} * (Y_b - Y_a)$

We point out that some structure can be found in the resulting constraints from the pairwise combinations. Seven types of constraint combinations can be discerned, as denoted in the top right corner of the combinations in Figure 3. Which of these seven types a pairwise combination belongs to, is dependent on the orientations of the fairness definitions as discussed in Section 3.1, Table 2 and Figure 2. A summary of the structures and the corresponding orientations of the fairness definitions can be found in Table 3.

3.2.1 The P%-Rule

We additionally investigate a relaxed fairness definition based on demographic parity, namely the p%-rule (Roth et al., 2021). Instead of requiring equality of the positive rates, as in demographic parity, the p%-rule requires that positive rate in any demographic group cannot be less than p% of the positive rate in any other demographic group. Formally:

$$\forall a, b : \frac{PR_{group\ a}}{PR_{group\ b}} \geq \frac{p}{100}. \tag{4}$$

The p%-rule is unique as it is based on a legal concept, the 80%-rule, from the Equal employment opportunity commission (1978). It requires that any selection rate for recruitment procedures is at least 4/5 of the group with the highest selection rate. If this is not satisfied then it is seen as adverse impact. In other words it requires that the positive rates between groups must satisfy the p%-rule, in this instance $p = 80$.

There are two special instances of the p%-rule, namely if $p = 0$ or $p = 100$. If $p = 0$ then no restrictions are placed on the system as a ratio is always larger than 0. The second possibility of $p = 100$ requires for the positive rates to be equal across all groups, which is equivalent to requiring demographic parity. Because each fairness definition can be combined with demographic parity and the p%-rule with $p < 100$ is a relaxation of demographic parity, all fairness definitions can be combined with the p%-rule.

3.2.2 Extending to Multiple Demographic Groups

Thus far only two demographic groups were taken into consideration. However, the analysis can easily be extended to multiple disjoint groups.

When combining any two fairness definitions, two degrees of freedom remain in the system of linear equations. It is then possible to parameterize these equations in such a way that those two free variables represent two of the four variables in the confusion matrix of just one demographic group. Let us refer to this demographic group as the 'first' one. The variables in the confusion matrix of the other demographic group are then expressed through those two free variables.

Then adding an additional demographic group can be done by simply constraining its confusion matrix in the same manner. This ensures that the resulting fairness notions are equal to those from the first demographic group. As the fairness notions in each additional demographic group are equal to those of the first group, they will be equal to each other.

In order for this to hold, it is necessary that the demographic groups are disjoint to prevent dependencies between groups. Disjoint groups can be achieved by creating a new group for each possible combination of sensitive attributes. Depending on the number of sensitive attributes, this could result in a large number of groups and lead to some very small groups. The work of Ghosh et al. (2021) argues that this intersectional fairness may be preferable for detecting every type of unfairness. We therefore argue that this property of extending to multiple sensitive groups is sufficient for most contexts.

3.3 Combining Three Fairness Definitions

We characterize the set of possible combinations of three fairness definitions.

Proposition 2. *When combining three fairness definitions out of demographic parity, equal opportunity, predictive equality, predictive parity, false omission rate parity, overall accuracy equality, and treatment equality only five out of 35 possible combinations are feasible (according to Definition 1).*

Proof outline. Section 3.2 shows that combinations of three fairness definitions are possible as certain pairwise combinations are equivalent. To examine other possible combinations of three fairness definitions, we combine the constraints of two pairwise combinations which share one fairness definition. Equation (5) shows an example of the calculations for combining demographic parity, equal opportunity, and predictive equality:

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{1-y} TP_b \\ FP_b = \frac{y}{1-y} TP_b \\ TP_b = TP_b \end{cases} \vee \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \\ 0 = 0 \end{cases} \tag{5}$$

The resulting constraints show that one variable, TP_b, is free, the base rates are unconstrained, and none of the variables are fixed to a particular value. Therefore, the combination of demographic parity, equal opportunity and predictive equality is possible.

Remark 3. *Note that the prediction of any classifier that satisfies demographic parity, equal opportunity, and predictive equality, is independent of the ground truth: the probability of any combination of a ground truth label value and predicted label value is equal to the marginal probabilities of these values. Indeed, independence can be expressed in terms of the variables in the confusion matrix:*

$$\begin{cases} TP_a = (TP_a + FP_a) \cdot (TP_a + FN_a), \\ TP_b = (TP_b + FP_b) \cdot (TP_b + FN_b), \\ TN_a = (TN_a + FN_a) \cdot (TN_a + FP_a), \\ TN_b = (TN_b + FN_b) \cdot (TN_b + FP_b). \end{cases} \tag{6}$$

It is easy to verify that given Eq. (5), this set of equations expressing independence is satisfied. While this is of course undesirable behaviour of a classifier, Definition 1 does not deem such combinations 'impossible', because it is possible to satisfy the equations in non-trivial value ranges – albeit only with an essentially random classifier.

It is worth reiterating that e.g. in Barocas et al. (2019), such combinations were defined as incompatible as well. We return to this point in Sec. 3.6.

However, not all combinations of three fairness definitions are possible. The combination of demographic parity, predictive parity, and false omission rate parity serves as an example. The calculations can be found in Eq. (7):

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ TN_a = TN_b \\ FN_a = FN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ x - FP_a = y - FP_a \\ -x - TP_a = -y - TP_a \end{cases} \iff \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \\ x = y \end{cases} \tag{7}$$

The final set of constraints requires $x = y$, which violates the condition of the base rates having a non-zero 2-dimensional Lebesgue measure. Therefore, these three fairness definitions can only be achieved simultaneously when the base rates are equal, which is entirely data-dependent. The combination of demographic parity, predictive parity and false omission rate parity is thus not possible.

Combining demographic parity, equal opportunity and predictive parity is also impossible, as shown in Eq. (8):

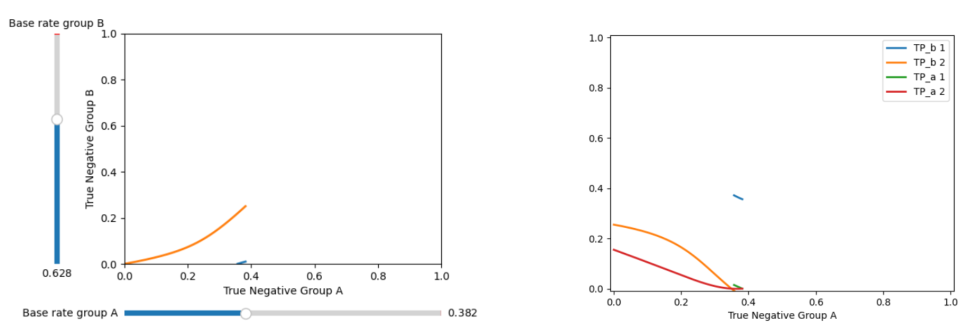
$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \iff \begin{cases} FN_a = 1 - x - TP_a \\ TN_a = x - FP_a \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ x = y \\ TP_a = TP_b \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \vee \begin{cases} FN_a = 1 - x \\ TN_a = x - FP_a \\ FN_b = 1 - y \\ TN_b = y - FP_b \\ TP_b = 0 \\ TP_a = 0 \\ TP_a = 0 \\ FP_a = FP_b \end{cases} \tag{8}$$

Eq. 8 shows that the solution either requires equal base rates or it requires for TP_b and TP_a to equal zero. The occurrence of the former was previously discussed in this section, how this leads to incompatibility. In that latter case, the feasible set of four out of eight variables in the confusion matrices will have a Lesbesgue measure of zero, which violates the requirements for being a possible combination from Definition 1.

The final kind of outcome, when combining three fairness definitions, arises when combining predictive parity, false omission rate parity and overall accuracy equality. In this case, the following second degree polynomial needs to be satisfied:

$$\begin{aligned}(-2x+1)TN_b^2 + (xy - y + x + 2yTN_a - x^2 + 2xTN_a - 2TN_a)TN_b \\ +(xy - x + TN_a - 2yTN_a + y - y^2)TN_a = 0.\end{aligned} \tag{9}$$

As this equation is not transparent, a numerical approach was used to solve it. The results are plotted in Figure 4. From Figure 4 we can see that a free variable, e.g. TN_a, exists. It is solvable for a range of base rates, although not for *all* base rates. However, this range of base rates consists of non-trivial real intervals and therefore satisfies Definition 1. The plots show that none of the variables have a Lebesgue measure of zero, such that the combination of predictive parity, false omission rate parity and overall accuracy equality is possible.



(a) Relation between TN_a and TN_b (b) Relation between TN_a, TP_a and TP_b

Figure 4: Relation between the free variable TN_a and the variables TN_b, TP_a, TP_b for (PP, FORP, OaE), showing multiple values are possible for TP_b and TP_a given TN_a.

Table 4: Summary of possible and impossible combinations of three fairness definitions.

Possible combinations	**Impossible combinations**		
(DP, EOP, PE)	(DP, PP, FORP)	(DP, EOP, PP)	(DP, FORP, Tr. Eq.)
(EOP, PE, OaE)	(DP, PP, OaE)	(DP, EOP, FORP)	(DP, OaE, Tr. Eq.)
(EOP, PP, Tr. Eq.)	(DP, FORP, OaE)	(DP, EOP, OaE)	(EOP, FORP, Tr. Eq.)
(PE, FORP, Tr. Eq.)	(DP, PP, Tr. Eq.)	(DP, PE, Tr. Eq.)	(EOP, PE, Tr. Eq.)
(PP, FORP, OaE)	(DP, PE, PP)	(EOP, PE, PP)	(EOP, PE, FORP)
	(EOP, OaE, PP)	(DP, PE, FORP)	(EOP, FORP, OaE)
	(DP, PE, FORP)	(EOP, OaE, Tr. Eq.)	(EOP, FORP, PP)
	(DP, PE, Tr. Eq.)	(PE, PP, FORP)	(FORP, OaE, Tr. Eq.)
	(PE, PP, OaE)	(PE, OaE, Tr. Eq.)	(PE, OaE, FORP)
	(PE, PP, Tr. Eq.)	(PP, OaE, Tr. Eq.)	(PP, FORP, Tr. Eq.)

The proofs for all other combinations of three fairness definitions can be found in Appendix B, and the results are summarized in Table 4.

□

3.3.1 P%-Rule

The p%-rule is investigated separately from the more strict fairness definitions. In order to determine the compatibility of the p%-rule with two other fairness definitions, we introduce the following two inequalities in the set of linear equations:

$$\begin{cases} \frac{TP_a+FP_a}{TP_b+FP_b} \geq \frac{p}{100}, \\ \\ \frac{TP_b+FP_b}{TP_a+FP_a} \geq \frac{p}{100}. \end{cases} \tag{10}$$

The combination of these equations constitute the p%-rule via the confusion matrix. The calculations for all combinations with two other fairness definitions can be found in Appendix C. The results can be divided into three categories: solvable for all p, an upper bound on p, or for $p = 1$ there is a specific value for certain variables. This property is noted for each combination in Table 5.

Table 5: The compatibility of the p%-rule with two fairness definitions.

$\forall \mathbf{p}$[1]	**Natural bound**[2]	**Set value for a variable**[3]		
(EOP, PE)	(EOP, Tr. Eq.)	(EOP, PP)	(PE, FORP)	(FORP, OaE)
	(PE, PP)	(EOP, FORP)	(PE, Tr. Eq.)	(FORP, Tr. Eq.)
	(PP, Tr. Eq.)	(EOP, OaE)	(PP, OaE)	(OaE, Tr. Eq.)

3.3.2 Extending to Multiple Sensitive Groups

Like for combining two fairness definitions, the calculations can be conveniently extended to multiple sensitive groups. The combinations (EOP, PP, Tr. Eq.) and (PE, FORP, Tr. Eq.) both contain two free variables; their extension to multiple sensitive groups follows the method discussed in Section 3.2.2. Other possible combinations have only one free variable, belonging to one sensitive group. The confusion matrices of other groups are defined by the base rates and the one free variables. This is repeated for every disjoint sensitive group.

3.4 Combining Four Fairness Definitions

There are only four free variables across the confusion matrices for two sensitive groups, due to the inherent constraints in a confusion matrix as noted in Eq. (1). Therefore, combinations of four *independent* fairness notions will not be possible. In other words: for a combination of four fairness definitions to be possible, at least one of the constraints should mathematically follow from the combination of other constraints in order for the values in the confusion matrix to have a non-zero Lebesgue measure.

Proposition 3. *Out of demographic parity, equal opportunity, predictive equality, predictive parity, false omission rate parity, overall accuracy equality and treatment equality no set of four fairness definitions is possible (according to Definition 1).*

Proof outline. If a set of four fairness definitions were possible, then all subsets of three out of those four definitions should be possible. Based on Table 4, we can exhaustively verify that this is not the case: no set of size four exists for which all its size three subsets are possible. Thus, no possible combination of four fairness definitions exist. □

3.4.1 P%-Rule

The compatibility of three fairness definitions with the p%-rule can be found by combining the previous constraints of the combination of the p%-rule with two fairness definitions. Simply put, the intersection of the possible values of p of all the combinations with two of the relevant fairness definitions will constitute the range of possible values for p when combined with three fairness definitions. It is not possible to satisfy three fairness definitions and the p%-rule for every possible value of p. This can be deduced from the incompatibility that no three fairness definitions can be extended with demographic parity, which can be seen as the strictest form of the p%-rule.

3.5 Integration of the Results

Combining the propositions above, we now state the main result of this paper.

Theorem 1. *Twelve sets form a maximal combination of fairness definitions that are possible according to Definition 1, out of demographic parity, equal opportunity, predictive equality, predictive parity, false omission rate parity, overall accuracy equality and treatment equality. We call these twelve combinations the **maximal fairness** notions. Seven of these sets are pairwise combinations and five are a combination of three fairness definitions. All subsets from these twelve sets are also possible.*

Proof outline. Figure 1 visualizes the sets of possible combinations out of the results in Propositions 1, 2, and 3. This tree of possible sets contains twelve leaf nodes, meaning that these twelve nodes are the maximal sets of possible fairness definitions. The nodes representing a maximal set of fairness definitions are marked with an asterisk(*). □

A relation is found between the possible combinations. This relation is shown in Table 6. It shows how often each fairness definition is possible to combine with two others, is impossible to combine as it requires a trivial 2-dimensional range for base rates, or requires a trivial range either for the variables or a trivial 2-dimensional range for the base rates.

Table 6: The prevalence of each fairness definition when combined with two others as the different characteristics of the outcome.

Definition	**Count possible combinations**	**Count requiring equal base rates**	**Count trivial range for variables or base rates**
Demographic Parity	1	3	11
Equal Opportunity	3	2	10
Predictive Equality	3	2	10
Predictive Parity	2	6	7
False Omission Rate Parity	2	6	7
Overall accuracy Equality	2	2	11
Treatment Equality	2	3	10

Table 6 reveals a symmetry between equal opportunity and predictive equality and also between predictive parity and false omission rate parity. This symmetry is due to their mathematical symmetry, as formalized by the following proposition.

Proposition 4. *Given any possible combination of fairness definitions, another possible combination can be obtained by swapping the semantics of the classes, i.e. by renaming the positive class as the negative and vice versa.*

Proof outline. When visualization fairness definitions on the confusion matrix, swapping the positive and negative classes is equivalent to a point inversion at the center of the confusion matrices. Figure 5 illustrates the point inversions of all possible combinations of three fairness definitions, revealing the symmetric relation between them.

A formal proof follows from the symmetry relations between the *individual* fairness definitions. Switching positives and negatives in the constraint of equal opportunity results in the constraint for predictive equality, and vice versa. The same relation holds between predictive parity and false omission rate parity. However, demographic parity becomes an equivalent constraint to the original definition. The definitions of overall accuracy equality and treatment equality remain unaffected by switching the positive and negative classes. □

Proposition 4 explains the symmetry in Table 6 of equal opportunity with predictive equality and of predictive parity with false omission rate parity. If for every possible combination their inversion is also a possible combination, then for every possible combination

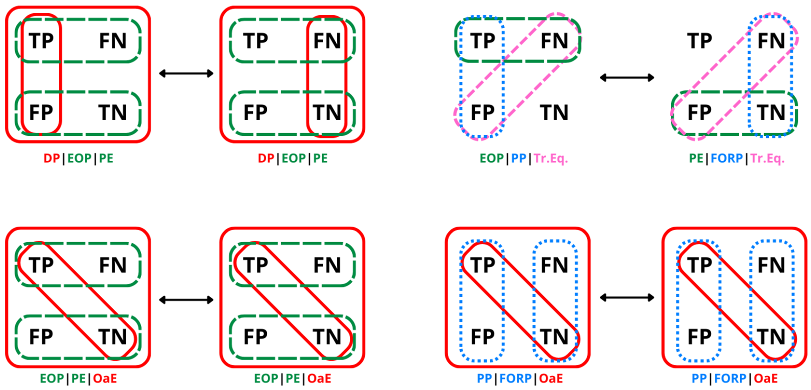


Figure 5: Point inversion of the possible combinations of three fairness definitions.

with predictive parity there will be a possible combination with false omission rate parity. The relation between equal opportunity and predictive equality is analogous.

Figure 5 shows that three, among the five maximal combinations of three fairness definitions, treat the positive and negative classes equally; they remain unaltered by swapping the role of the positive and negative class. These three combinations are (DP, EOP, FPP), (EOP, PP, OaE), and (PP, PE, OaE). The combinations (EOP, PP, Tr. Eq.) and (FPP, PE, Tr. Eq.) treat the classes differently, and represent each other's symmetrical analogue.

Finally, both Figure 1 and Table 6 indicate that possible combinations with demographic parity are most rare. Demographic parity does not relate to the base rates of the groups as it is independent from the ground truth. Therefore, it can only be combined with definitions that relate directly to the base rates such as equal opportunity and predictive parity.

3.6 Relation to Existing Impossibility Theorems

Our work is congruent with previous impossibility theorems. The work of Chouldechova (2016) states, it is not possible to have equal positive prediction values, false negative rates, and false positive rates between groups for different the base rates. We call these definitions predictive parity, predictive equality, and equalised odds. As can be seen in Figure 1, our work comes to the same conclusion as the work of Chouldechova.

Another impossibility theorem of Kleinberg et al. (2016) is centred on scoring instead of binary prediction. The incompatible fairness definitions for risk scoring can be mapped onto binary prediction. Calibration is seen as predictive parity, balance for the negative class as predictive equality, and balance for the positive class as equal opportunity. This matches what Chouldechova (2016) proved and our results.

Barocas et al. (2019) state that demographic parity is not compatible with equalised odds. Initially, this seems to contradict our results. However, Barocas et al. introduce the constraint on combining fairness definitions that predictions cannot be independent from the ground truth. As pointed out in Remark 3, this could be a desirable constraint for a definition of compatibility, but was not adopted in Def. 1. Def. 1 focusses on the possibility of finding a confusion matrix configuration satisfying multiple fairness definitions. Sec. 4 concerns the performance constrained seen when enforcing multiple fairness definitions.

4. Analysis of The Fairness Combinations

The following section discusses the performance and characteristics of the combinations of fairness definitions. We first discuss the maximal accuracies each group can achieve when enforcing multiple fairness definitions. In the last section we discuss the performance in more detail. We discuss the type of constraint that is posed on the confusion matrix in order to provide some insight on the resulting behaviour of the system.

4.1 Performance Bounds of Combinations of Fairness Definitions

Enforcing a set of fairness definitions imposes extra constraints on the classifier. This might affect the performance on the model. We discuss the effects each set of fairness definitions will have on the accuracy for each group. Below, for the sake of the argument, we take the positive class to be preferable over the negative class.

The calculations from Sections 3.2 and 3.3 can be reused to determine the minimal and maximal accuracy a predictor can achieve. This performance is dependent on the difference in base rates and for some combinations on the absolute values of the base rate. This relation between the accuracies of both groups is visualized for some combinations of fairness definitions in Figure 6.

Figure 6 shows that the behaviour of the accuracies vary greatly depending on the combinations of fairness definitions. Such as for Figure 6a, the combination of demographic parity with predictive parity, where for large differences between the base rates of groups A and B, the performance for group A is greatly limited. Contrarily, the combination of equal opportunity and predictive equality (Figure 6b) can achieve high performance for both groups regardless of the difference in base rates.

The formulas to calculate the minimal and maximal accuracy for each feasible combination can be found in Table 7. In calculations of the maximal accuracy, the variable ε_x either refers to the false positive or false negative rate within a specific group. For the minimal accuracy it signifies the true positive and true negative rate.

From Table 7, we can see that the combinations with demographic do not have accuracies within a the range $[0, 1]$. This range is dependant on the base rates. This means that a perfect predictor is impossible while enforcing a set of fairness definitions that contains demographic parity.

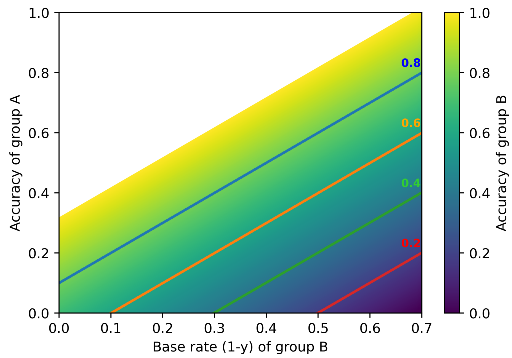
Example of Performance Influence with The COMPAS Dataset

We briefly show how these performance bounds have an effect on a real-world example. The COMPAS dataset (Angwin et al., 2016) is used for this example as this was used in the introduction. It has been argued that the COMPAS dataset is not appropriate to use in a benchmarking setting (Fabris et al., 2022). However, this example is fictional and the only information used in the subsequent calculations are the base rates.

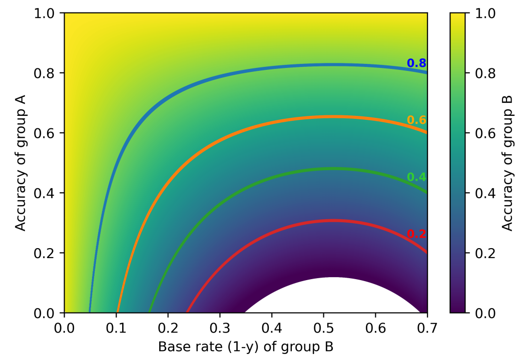
We limit ourselves to two sensitive groups, namely Caucasian and African American. The base rates are 0.43 and 0.54 respectively for the Caucasian and African American group. The COMPAS case is peculiar as an individual prefers the negative prediction. Common fairness use cases such as loan applications or job applications have a preferred positive prediction. Therefore, the advantaged group is the Caucasian as it has a lower base rate.

Table 7: The minimal and maximal accuracy of each possible combination of fairness definitions except for the combination of (PP, FORP) and (PP, FORP, OaE). ε_x denotes the false positive or false positive for a specific group in the maximal accuracy calculations and the true positive or true negative rates in the minimal accuracy calculations. The value of ε_x is thus constrained by the properties of the set of combinations. Specific ranges can be found in Appendix A and B.

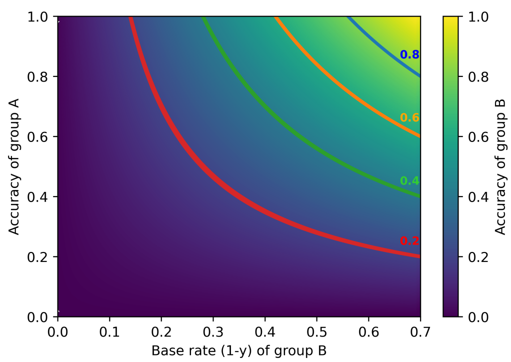
	Minimal accuracy		Maximal Accuracy	
	Advantaged	Disadvantaged	Advantaged	Disadvantaged
Pairwise Combinations				
(DP, EOP)	$x - y + \frac{1+y-2x}{1-y}\varepsilon_1 + \varepsilon_2$	$\varepsilon_1 + \varepsilon_2$	$1 + y - x - \frac{1+y-2x}{1-y}\varepsilon_1 - \varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$
(DP, PE)	$y - x + \frac{2x-y}{y}\varepsilon_2 + \varepsilon_1$	$\varepsilon_1 + \varepsilon_2$	$1 + x - y - \frac{y-2x}{y}\varepsilon_2 - \varepsilon_1$	$1 - \varepsilon_1 - \varepsilon_2$
(DP, PP)	$x - y + \varepsilon_1 + \varepsilon_2$	$\varepsilon_1 + \varepsilon_2$	$1+x-y-\varepsilon_1-\varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$
(DP, FORP)	$y - x + \varepsilon_1 + \varepsilon_2$	$\varepsilon_1 + \varepsilon_2$	$1-x+y-\varepsilon_1-\varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$
(DP, OaE)	0	0	$1 - \frac{y-x}{2}$	$1 - \frac{y-x}{2}$
(DP, Tr. Eq.)	$\varepsilon_1 + \varepsilon_2 + \frac{(y-x)(1-\varepsilon_1-\varepsilon_2)}{2*y-1-\varepsilon_2+\varepsilon_1}$	$\varepsilon_1 + \varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2 - \frac{(\varepsilon_1+\varepsilon_2)(x-y)}{\varepsilon_2-\varepsilon_1}$	$1 - \varepsilon_1 - \varepsilon_2$
(EOP, PE)	$\frac{1-x}{1-y}\varepsilon_1 + \frac{x}{y}\varepsilon_2$	$\varepsilon_1 + \varepsilon_2$	$1 - \frac{1-x}{1-y}\varepsilon_1 - \frac{x}{y}\varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$
(EOP, FORP)	$\frac{1-x}{1-y}(\varepsilon_1 + \varepsilon_2)$	$\varepsilon_1 + \varepsilon_2$	$\frac{1-x}{1-y}(1-\varepsilon_1-\varepsilon_2)$	$1 - \varepsilon_1 - \varepsilon_2$
(EOP, OaE)	$\frac{1-x}{y-x}\varepsilon_2 - \frac{1-y}{y-x}\varepsilon_1$	$\frac{1-x}{y-x}\varepsilon_2 - \frac{1-y}{y-x}\varepsilon_1$	$1 - \varepsilon_2$	$1 - \varepsilon_2$
(PE, PP)	$\frac{x}{y}(\varepsilon_1 + \varepsilon_2)$	$\varepsilon_1 + \varepsilon_2$	$\frac{x}{y}(1 - \varepsilon_1 - \varepsilon_2)$	$1 - \varepsilon_1 - \varepsilon_2$
(PE, OaE)	$\frac{x}{x-y}\varepsilon_2 - \frac{y}{x-y}\varepsilon_1$	$\frac{x}{x-y}\varepsilon_2 - \frac{y}{x-y}\varepsilon_1$	$1 - \varepsilon$	$1 - \varepsilon$
(PP, OaE)	$\varepsilon_1 + \varepsilon_2$	$\varepsilon_1 + \varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$
(FORP, OaE)	$\varepsilon_1 + \varepsilon_2$	$\varepsilon_1 + \varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$
(OaE, Tr. Eq.)	$\varepsilon_1 + \varepsilon_2$	$\varepsilon_1 + \varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$
Combination of three fairness definitions				
(DP, EOP, PE)	$x + \frac{1-2*x}{1-y}\varepsilon$	$y + \frac{1-2*y}{1-y}\varepsilon$	$1 - x - \frac{1-2*x}{1-y}\varepsilon$	$1 - y - \frac{1-2*y}{1-y}\varepsilon$
(EOP, PE, OaE)	$\frac{1}{y}\varepsilon$	$\frac{1}{y}\varepsilon$	$1 - \frac{\varepsilon}{y}$	$1 - \frac{\varepsilon}{y}$
(EOP, PP, Tr. Eq.)	$\frac{x-y}{1-y} + \frac{1-x}{1-y}(\varepsilon_1 + \varepsilon_2)$	$\varepsilon_1 + \varepsilon_2$	$1 - \frac{1-x}{1-y}\varepsilon_1 - \frac{1-x}{1-y}\varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$
(PE, FORP, Tr. Eq.)	$\frac{y-x}{y}+\frac{x}{y}(\varepsilon_1+\varepsilon_2)$	$\varepsilon_1 + \varepsilon_2$	$1 - \frac{x}{y}\varepsilon_1 - \frac{x}{y}\varepsilon_2$	$1 - \varepsilon_1 - \varepsilon_2$



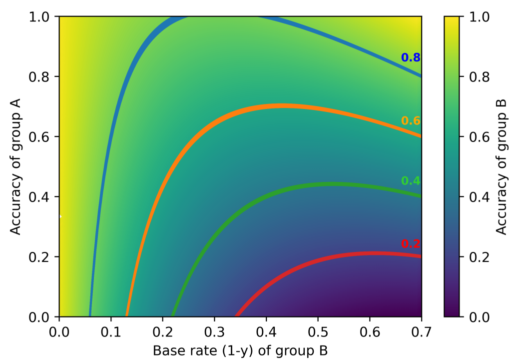
(a) (DP, PP)



(b) (EOP, PE)



(c) (EOP, FORP)



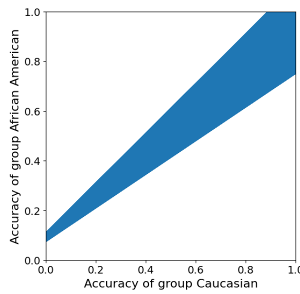
(d) (DP, OEP)

Figure 6: The maximal accuracies for groups A and B. We assume the base rate of group A $= 0.7$ and $FN_a = 2 * FP_a$ (Reducing the complexity to allow for visualizations). White regions in the graph denotes impossible results. Performance borders for group B are drawn for the values 0.2, 0.4, 0.6 and 0.8.

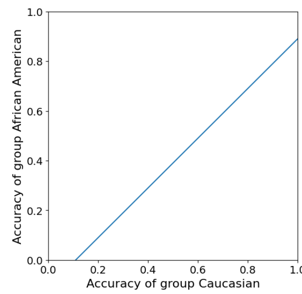
We posit that the most interesting fairness definitions for this use case are Predictive Equality and Predictive Parity as both these definitions aim to impose some type of parity related to the false positives. We focus on these false positive as these are the worst possible outcome for an individual. Figure 7 shows some examples of the performance characteristics when enforcing a combination of fairness definitions.

The combination of Predictive Equality and Predictive Parity could seem opportune in such a system. However, Figure 7d shows that the performance discrepancy between groups for these base rates is large, which can be considered undesirable. The performance of the fairness-unaware classifier often provides an upper bound (Liu & Vicente, 2022) for the performance of a fairness-aware classifier, providing a likely accuracy region of the fairness-aware classifier. For lower accuracy levels, a significant performance difference can be observed for the combinations *DP & PP* (Fig. 7b) and *PP & Tr.Eq.* (Fig. 7f). Therefore, these combinations can be seen as undesirable for lower accuracies.

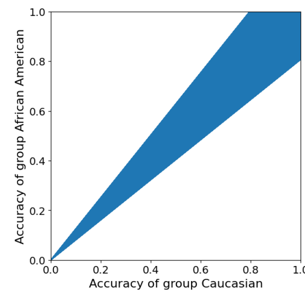
The other three combinations of fairness definitions shown in Figure 7 exhibit more preferable properties. Figure 7e shows that the combination *PE & Tr.Eq.* has a large breadth of possible combinations, meaning that a desirable solution could be found. The



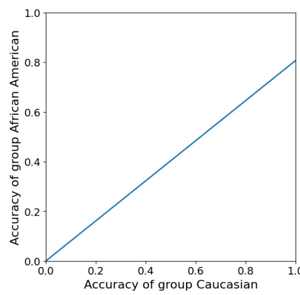
(a) Enforcing DP and PE



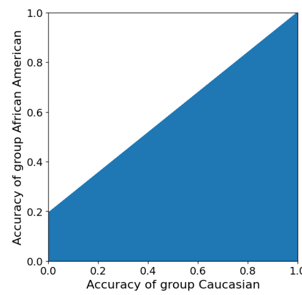
(b) Enforcing DP and PP



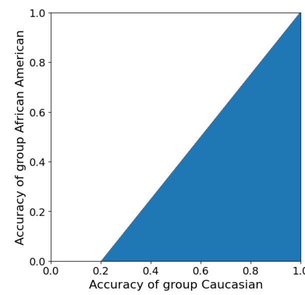
(c) Enforcing EOP and PE



(d) Enforcing PE and PP



(e) Enforcing PE and Tr. Eq.



(f) Enforcing PP and Tr. Eq.

Figure 7: Possible range of results when enforcing a set of fairness constraints for the COMPAS dataset for the groups of Caucasian and African American individuals.

combinations of *DP & PE* and *EOP & PE*, as shown in Figures 7a and 7c respectively, have a fairly wide range of solutions and require smaller differences in the accuracies of the groups compared to the other portrayed combinations of fairness definitions. From a performance perspective this is desirable as there is no risk of sizeable performance differences between sensitive groups when a model satisfies these combinations of fairness definitions.

Remark 4. *The desired combination of fairness definitions should not be determined based on performance. A choice of fairness definitions is an ethical choice that has to be made independently. However, negative influences on the performance of the model might eliminate certain combinations from consideration.*

4.2 Behaviour of Feasible Combinations of Fairness Definitions

The calculations in Sections 3.2 and 3.3 allow to group the sets of possible combinations based on the resulting constraints that are imposed on the confusion matrices. Five distinct behaviour types and their correspondent sets are noted in Table 8.

An example for each of the behaviour types identified in Table 8 is provided in Tables 9 and 9c in the form of the resulting confusion matrices when enforcing such a set of fairness definitions. We discuss these behaviour in the remainder of this section.

Lower bound on one variable A lower bound is imposed on one variable in the confusion matrix. This lower bound is dependent on the base rates of the groups. The lower bound is primarily dependent on the difference in base rates when combining two fairness definitions. An example of such behaviour in the confusion matrix can be found in Table 9a.

Table 8: The different feasible combinations of fairness definitions sorted on the behaviour they exhibit in the confusion matrices

Behaviour type	Combinations of fairness definitions
Lower bound on one variable	(DP, PP) (DP, FORP) (DP, OaE) (DP, EOP, PE)
Trade-off between errors	(DP, EOP) (DP, PE) (PP, OaE) (FORP, OaE) (EOP, OaE) (PE, OaE)
Factor on the accuracy	(EOP, FORP) (PE, PP)
Factor on the error	(EOP, PP, Tr.Eq.) (PE, FORP, Tr. Eq.) (EOP, PE, OaE) (EOP, PE)
No effect	(OaE, Tr. Eq.)

It shows that there will either be false negatives for the one group or for the other group as it is impossible to satisfy both $y - x - \varepsilon_1 = 0$ and $\varepsilon_1 = 0$ if the base rates differ.

The combination of demographic parity, equalised opportunity, and predictive equality also contains a lower bound, though on the false positives. Unlike the combinations with two fairness definitions, the relation between the base rates and the lower bound is more complicated as can be seen in Table 9b.

Trade-off between errors Two errors of the other group contribute to one of the variables in the confusion matrix. In this case, the value from one errors constrains the possible range of values for the other error. An example is shown in Table 9c for the combination of predictive parity and overall accuracy equality. The presence of $x - y$ in the equation means that the difference in base rates will influence the possible ranges of these errors.

Factor on the TP/TN True positives or true negatives are related through a multiplicative factor, derived from the base rates. An example of this relation on the true negatives is shown in Table 9d. Unless the base rate are equal, this causes that no perfect accuracy can be achieved for one of the groups. This can also be seen in Table 7, as the combinations *EOP & FORP* and *PE & PP* have that factor in their maximal accuracy.

Factor on the error This is closely related to the previous behaviour, however the factor is on the errors. Although similar, this characteristic does not impose the same limitation on the accuracies of the groups. If one group has perfect accuracy, then the other group will too, as all errors will be zero. An example of this can be found in Table 9e.

No effect Only the combination of overall accuracy equality and treatment equality have no special properties are present on the confusion matrix, as can be seen in Table 9f.

5. Conclusion

We investigated which and how many fairness definitions can be combined and imposed simultaneously for the simple setting of binary classification. We find that out of seven fairness definitions, namely demographic parity, equal opportunity, predictive equality, predictive parity, false omission rate parity, overall accuracy equality and treatment equality, in total twelve maximal combinations are possible. Five of these maximal combinations

(a) Lower bound on one variable: Demographic parity and predictive parity

Group A with base rate: $1-x$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-y-\varepsilon_1$	$y-x+\varepsilon_1$
	Neg.	ε_2	$x-\varepsilon_2$

Group B with base rate: $1-y$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-y-\varepsilon_1$	ε_1
	Neg.	ε_2	$y-\varepsilon_2$

(b) Lower bound on one variable: Demographic parity, equalised odds and predictive equality.

Group A with base rate: $1-x$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-x-\frac{1-x}{1-y}\varepsilon$	$\frac{1-x}{1-y}\varepsilon$
	Neg.	$x-\frac{x}{1-y}\varepsilon$	$\frac{x}{1-y}\varepsilon$

Group B with base rate: $1-y$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-y-\varepsilon$	ε
	Neg.	$y-\frac{y}{1-y}\varepsilon$	$\frac{y}{1-y}\varepsilon$

(c) Trade-off between errors: Predictive parity and overall accuracy equality

Group A with base rate: $1-x$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$\varepsilon_1\frac{y-x}{\varepsilon_1-\varepsilon_2}+\varepsilon_1$	$1-x-\varepsilon_1\frac{y-x}{\varepsilon_1-\varepsilon_2}-\varepsilon_1$
	Neg.	ε_1	$x-\varepsilon_1$

Group B with base rate: $1-y$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$\varepsilon_2\frac{y-x}{\varepsilon_1-\varepsilon_2}+\varepsilon_2$	$1-y-\varepsilon_2\frac{y-x}{\varepsilon_1-\varepsilon_2}-\varepsilon_2$
	Neg.	ε_2	$y-\varepsilon_2$

(d) Factor on the TP/TN: Equal opportunity and false omission rate parity.

Group A with base rate: $1-x$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-x-\frac{1-x}{1-y}\varepsilon_1$	$\frac{1-x}{1-y}\varepsilon_1$
	Neg.	$x-\frac{1-x}{1-y}(y-\varepsilon_2)$	$\frac{1-x}{1-y}(y-\varepsilon_2)$

Group B with base rate: $1-y$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-y-\varepsilon_1$	ε_1
	Neg.	ε_2	$y-\varepsilon_2$

(e) Factor on the error: Equal opportunity and predictive equality.

Group A with base rate: $1-x$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-x-\frac{1-x}{1-y}\varepsilon_1$	$\frac{1-x}{1-y}\varepsilon_1$
	Neg.	$\frac{x}{y}\varepsilon_2$	$x-\frac{x}{y}\varepsilon_2$

Group B with base rate: $1-y$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-y-\varepsilon_1$	ε_1
	Neg.	ε_2	$y-\varepsilon_2$

(f) No effect: Overall accuracy equality and treatment equality.

Group A with base rate: $1-x$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-x-\varepsilon_1$	ε_1
	Neg.	ε_2	$x-\varepsilon_2$

Group B with base rate: $1-y$

		Predicted: Pos.	Predicted: Neg.
True	Pos.	$1-y-\varepsilon_1$	ε_1
	Neg.	ε_2	$y-\varepsilon_2$

Table 9: Examples of the different behaviour types identified in Table 8 and their corresponding confusion matrices. $1-x$, $1-y$ are the base rates of groups A and B respectively. ε_x denotes an error of some type, and functions as a variable.

consist of three fairness definitions, while seven of them consist of just two fairness definitions. Each of these sets could be regarded as *maximal fairness* notions, which means that it is not possible to impose any further of the discussed fairness constraints without the problem becoming infeasible. Evidently, all subsets of these twelve maximal sets are also possible combinations of fairness definitions. An overview of the possible combinations, maximal as well as non-maximal, and their subset relations, is given in Figure 1. Our results confirm and extend related work, such as the theorem colloquially referred to as the fairness impossibility theorem.

We consider the five maximal sets of three fairness definitions as particularly interesting. Of these five, three treat the positive and negative classes symmetrically, in that they remain unaltered after swapping the class labels. The remaining two are each other's symmetrical analogue: one of these two sets of fairness definitions is equivalent to the other set after swapping the class labels. These properties are shown in Figure 5. They can provide initial guidance to determine which set of fairness notions is most appropriate in a given setting: if the costs of different types of misclassifications are very different, one of the sets of fairness notions that are not invariant with respect to a label swap might be more appropriate.

Furthermore, we investigated the properties of the possible combinations. This included the effects they have on the accuracy of the classifier for each of the groups. Those results showed that certain combinations of fairness definitions are impractical if the difference between the base rates are too large. Additionally, we studied the type of effect the possible combinations had on the confusion matrix and how they constrain the confusion matrices. Five behaviour types could be discerned.

An important next step building on these results would be to develop methods that are capable of imposing these (maximal) sets of fairness definitions. Subsequently, it would be interesting to investigate the impact of imposing (maximal) sets of fairness definitions on the learning capabilities (e.g. learning rate and convergence rate) of machine learning methods.

A separate line of further work would be towards including other fairness definitions, in addition to the seven definitions included in this work. Moreover, we consider extensions towards other problems besides binary classification as fruitful avenues for further research.

Finally, and perhaps most importantly, the meaning of the maximal fairness notions derived in the present paper needs to be better understood from an ethical and/or legal point of view. This is an important aspect of research in fairness in AI as this translation to real world concepts is a pre-requisite for wide-spread adoption.

Acknowledgments

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Appendix A. Combining two definitions

A.1 Equal Opportunity and Demographic Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ \frac{TP_a}{TP_a + FN_a} = \frac{TP_b}{TP_b + FN_b} \\ \frac{TP_a + FP_a}{TP_a + FP_a + FN_a + TN_a} = \frac{TP_b + FP_b}{TP_b + FP_b + FN_b + TN_b} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ \frac{TP_a}{1-x} = \frac{TP_b}{1-y} \\ TP_a + FP_a = TP_b + FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = TP_b + FP_b - \frac{1-x}{1-y} TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = (1 - \frac{1-x}{1-y}) TP_b + FP_b \end{cases} \iff \begin{cases} FN_a = 1 - x - \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{x-y}{1-y} TP_b - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \end{cases}$$

A.1.1 Applying Inequalities

$$\begin{cases} 0 \le 1 - x - \frac{1-x}{1-y} TP_b \\ 1 - x - \frac{1-x}{1-y} TP_b \le 1 - x \\ 0 \le x - \frac{x-y}{1-y} TP_b - FP_b \\ x - \frac{x-y}{1-y} TP_b - FP_b \le x \\ 0 \le 1 - y - TP_b \\ 1 - y - TP_b \le 1 - y \\ 0 \le y - FP_b \\ y - FP_b \le y \\ 0 \le \frac{1-x}{1-y} TP_b \\ \frac{1-x}{1-y} TP_b \le 1 - x \\ 0 \le \frac{x-y}{1-y} TP_b + FP_b \\ \frac{x-y}{1-y} TP_b + FP_b \le x \end{cases} \iff \begin{cases} \frac{1}{1-y} TP_b \le 1 \\ \frac{1-x}{1-y} TP_b \ge 0 \\ FP_b \le x + \frac{-x+y}{1-y} TP_b \\ \frac{-x+y}{1-y} TP_b \le FP_b \\ TP_b \le 1 - y \\ TP_b \ge 0 \\ FP_b \le y \\ FP_b \ge 0 \\ 0 \le \frac{1-x}{1-y} \\ \frac{1}{1-y} TP_b \le 1 \\ \frac{y-x}{1-y} \le \frac{FP_b}{TP_b} \\ FP_b + \frac{x-y}{1-y} TP_b \le x \end{cases}$$

$$\iff \begin{cases} \frac{y-x}{1-y} \le \frac{FP_b}{TP_b} \\ FP_b + \frac{x-y}{1-y} TP_b \le x \end{cases}$$

A.2 Demographic Parity and Predictive Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a + FP_a = TP_b + FP_b \\ FP_a = \frac{x}{y} FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ TP_a = TP_b + (1 - \frac{x}{y}) FP_b \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - \frac{y-x}{y} FP_b - TP_b \\ TN_a = x - \frac{x}{y} FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ FP_a = \frac{x}{y} FP_b \\ TP_a = \frac{y-x}{y} FP_b + TP_b \end{cases}$$

A.2.1 Added Constraints through Inequalities

$$\begin{cases} \frac{x-y}{y} \leq \frac{TP_b}{FP_b} \\ 1 - x \geq \frac{y-x}{y} FP_b + TP_b \end{cases}$$

A.3 Demographic Parity and Predictive Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a * FP_b = TP_b * FP_a \\ TP_a + FP_a = TP_b + FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b * FP_a}{FP_b} \\ \frac{TP_b * FP_a}{FP_b} + FP_a = TP_b + FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b * FP_a}{FP_b} \\ \frac{FP_b + TP_b}{FP_b} * FP_a = TP_b + FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b * FP_a}{FP_b} \\ FP_a = FP_b \vee FP_b + TP_b = 0 \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \vee \begin{cases} FN_a = 1 - x - FP_a \\ TN_a = x - FP_a \\ FN_b = 1 - y \\ TN_b = y \\ TP_a = FP_a \\ FP_b = TP_b = 0 \end{cases}$$

A.3.1 Added Constraints through Inequalities

$$\begin{cases} 1 - x \geq TP_b \\ x \geq FP_b \end{cases}$$

A.4 Demographic Parity and False Omission Rate Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a * TN_b = FN_b * TN_a \\ TP_a + FP_a = TP_b + FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * TN_a}{TN_b} \\ 1 - FN_a - TN_a = 1 - FN_b - TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * TN_a}{TN_b} \\ FN_a + TN_a = FN_b + TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * TN_a}{TN_b} \\ \frac{FN_b * TN_a}{TN_b} + TN_a = FN_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * TN_a}{TN_b} \\ \frac{FN_b + TN_b}{TN_b} * TN_a = FN_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = TN_b \vee FN_b = TN_b = 0 \\ FN_a = \frac{FN_b * TN_a}{TN_b} \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_a \\ FP_a = x - FN_a \\ TP_b = 1 - y \\ FP_b = y \\ FN_b = TN_b = 0 \\ FN_a = TN_a \end{cases} \vee \begin{cases} TP_a = 1 - x - FN_b \\ FP_a = x - TN_b \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ TN_a = TN_b \\ FN_a = FN_b \end{cases}$$

A.4.1 Added Constraints through Inequalities

$$\begin{cases} FN_b \leq 1 - x \\ TN_b \leq x \end{cases}$$

A.5 Demographic Parity and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a + FP_a = TP_b + FP_b \\ TP_a + TN_a = TP_b + TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a - TP_b = FP_b - FP_a \\ TP_a - TP_b = TN_b - TN_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_b - TN_a = FP_b - FP_a \\ TP_a - TP_b = FP_b - FP_a \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ y - FP_b - x + FP_a = FP_b - FP_a \\ TP_a - TP_b = FP_b - FP_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ y - x = 2FP_b - 2FP_a \\ TP_a - TP_b = FP_b - FP_a \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_b - FP_a = \frac{y-x}{2} \\ TP_a - TP_b = \frac{y-x}{2} \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - TP_b - \frac{y-x}{2} \\ TN_a = x - FP_b - \frac{x-y}{2} \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ FP_a = FP_b + \frac{x-y}{2} \\ TP_a = TP_b + \frac{y-x}{2} \end{cases}$$

A.5.1 Added Constraints through Inequalities

$$\begin{cases} TP_b \leq 1 - \frac{x+y}{2} \\ TP_b \geq \frac{x-y}{2} \\ FP_b \leq \frac{x+y}{2} \\ FP_b \geq \frac{y-x}{2} \end{cases}$$

A.6 Demographic Parity and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a + FP_a = TP_b + FP_b \\ \frac{FN_a}{FP_a} = \frac{FN_b}{FP_b} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{FN_a * FP_b}{FN_b} \\ 1 - x - FN_a + \frac{FN_a * FP_b}{FN_b} = 1 - y - FN_b + FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{FN_a * FP_b}{FN_b} \\ \frac{FP_b - FN_b}{FN_b} FN_a = x - y + FP_b - FN_b \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ TN_a = x - FP_b(\frac{x-y}{FP_b - FN_b} + 1) \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FN_a = FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ FP_a = FP_b(\frac{x-y}{FP_b - FN_b} + 1) \end{cases} \vee \begin{cases} TP_a = 1 - x - FN_a \\ TN_a = x - FN_a \\ TP_b = 1 - y - FN_b \\ TN_b = y - FN_b \\ FP_b = FN_b \\ x = y \\ FP_a = FN_a \end{cases}$$

A.6.1 Added Constraints through Inequalities

$$\begin{cases} \frac{x-y}{FP_b - FN_b} \geq -1 \\ 1 - x \geq FN_b \\ x \geq FP_b \end{cases}$$

A.7 Equalised Odds

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{FP_a}{FP_a + TN_a} = \frac{FP_b}{FP_b + TN_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{FP_a}{x} = \frac{FP_b}{y} \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{x}{y} FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \end{cases}$$

A.8 Equal Opportunity and Predictive Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{TP_a}{TP_a + FP_a} = \frac{TP_b}{TP_b + FP_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TP_a * FP_b = TP_b * FP_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{1-x}{1-y} * TP_b * FP_b = TP_b * FP_a \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{1-x}{1-y} FP_b = FP_a \vee TP_b = 0 \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y} FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases} \vee \begin{cases} FN_a = 1 - x \\ TN_a = x - FP_a \\ FN_b = 1 - y \\ TN_b = y - FP_b \\ TP_a = 0 \\ TP_b = 0 \end{cases}$$

A.8.1 Added Constraint through Inequalities

$$FP_b \leq x \frac{1-y}{1-x}$$

A.9 Equal Opportunity and False Omission Rate Parity

$$\begin{cases} TP_a + FN_a = 1-x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1-y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{FN_a}{FN_a+TN_a} = \frac{FN_b}{FN_b+TN_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1-x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1-y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FN_a * TN_b = FN_b * TN_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1-x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1-y \\ TN_b + FP_b = y \\ 1-x-FN_a = \frac{1-x}{1-y}(1-y-FN_b) \\ FN_a * TN_b = FN_b * TN_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1-x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1-y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ \frac{1-x}{1-y} FN_b * TN_b = FN_b * TN_a \end{cases} \iff \begin{cases} TP_a + FN_a = 1-x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1-y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ \frac{1-x}{1-y} TN_b = TN_a \lor FN_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a = 1-x-\frac{1-x}{1-y} FN_b \\ FP_a = x - \frac{1-x}{1-y} TN_b \\ TP_b = 1-y-FN_b \\ FP_b = y - TN_b \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{1-x}{1-y} TN_b \end{cases} \lor \begin{cases} TP_a = 1-x \\ FP_a = x - TN_a \\ TP_b = 1-y \\ FP_b = y - TN_b \\ FN_a = 0 \\ FN_b = 0 \end{cases}$$

A.9.1 Added Constraint through Inequalities

$$TN_b \leq \frac{1-y}{1-x} * x$$

A.10 Equal Opportunity and Overall accuracy equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TP_a + TN_a = TP_b + TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{1-x}{1-y} TP_b - TP_b = TN_b - TN_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{y-x}{1-y} TP_b = TN_b - TN_a \end{cases} \iff \begin{cases} FN_a = 1 - x - \frac{1-x}{y-x}(TN_b - TN_a) \\ FP_a = x - TN_a \\ FN_b = 1 - y - \frac{1-y}{y-x}(TN_b - TN_a) \\ FP_b = y - TN_b \\ TP_a = \frac{1-x}{y-x}(TN_b - TN_a) \\ TP_b = \frac{1-y}{y-x}(TN_b - TN_a) \end{cases}$$

A.10.1 Added Constraints through Inequalities

$$\begin{cases} y > x \\ TN_a \leq TN_b \\ y - x \geq TN_b - TN_a \end{cases} \vee \begin{cases} x > y \\ TN_b \leq TN_a \\ y - x \leq TN_b - TN_a \end{cases}$$

A.11 Equal Opportunity and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{FN_a}{FP_a} = \frac{FN_b}{FP_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ \frac{1-x}{1-y} \frac{FN_b}{FP_a} = \frac{FN_b}{FP_b} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = \frac{1-x}{1-y} FP_b \vee FN_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - \frac{1-x}{1-y} FN_b \\ TN_a = x - \frac{1-x}{1-y} FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases} \vee \begin{cases} TP_a = 1 - x \\ TN_a = x - FP_a \\ TP_b = 1 - y \\ TN_b = y - FP_b \\ FN_a = 0 \\ FN_b = 0 \end{cases}$$

A.11.1 Added Constraints through Inequalities

$$\frac{1-y}{1-x} x \geq FP_b$$

A.12 Predictive Equality and Predictive Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ TP_a * FP_b = TP_b * FP_a \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ TP_a * FP_b = TP_b * \frac{x}{y} FP_b \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - TP_a \\ TN_a = x - \frac{x}{y} FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ FP_a = \frac{x}{y} FP_b \\ TP_a = \frac{x}{y} TP_b \vee FP_b = 0 \end{cases}$$

A.12.1 Added Constraints through Inequalities

$$\frac{y}{x}(1-x) \geq TP_b$$

A.13 Predictive Equality and False Omission Rate Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ FN_a * TN_b = FN_b * TN_a \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ x - TN_a = \frac{x}{y}(y - TN_b) \\ FN_a * TN_b = FN_b * TN_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ FN_a * TN_b = FN_b * \frac{x}{y} * TN_b \end{cases} \iff \begin{cases} TP_a = 1 - x - FN_a \\ FP_a = x - \frac{x}{y} TN_b \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ TN_a = \frac{x}{y} TN_b \\ FN_a = \frac{x}{y} FN_b \vee TN_b = 0 \end{cases}$$

A.13.1 Added Constraints through Inequalities

$$\frac{y}{x}(1 - x) \geq FN_b$$

A.14 Predictive Equality and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a + TN_a = TP_b + TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a + \frac{x}{y} TN_b = TP_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ \frac{x-y}{y} TN_b = TP_b - TP_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_b = \frac{y}{x-y}(TP_b - TPa) \\ TN_a = \frac{x}{y} \frac{y}{x-y}(TP_b - TP_a) \end{cases} \iff \begin{cases} FN_a = 1 - x - TP_a \\ FP_a = x - \frac{x}{x-y}(TP_b - TP_a) \\ FN_b = 1 - y - TP_b \\ FP_b = y - \frac{y}{x-y}(TP_b - TPa) \\ TN_b = \frac{y}{x-y}(TP_b - TPa) \\ TN_a = \frac{x}{x-y}(TP_b - TP_a) \end{cases}$$

A.14.1 Added Constraints through Inequalities

$$\begin{cases} x > y \\ TP_a \leq TP_b \\ x - y \geq TP_b - TP_a \end{cases} \vee \begin{cases} x < y \\ TP_a \geq TP_b \\ x - y \leq TP_b - TP_a \end{cases}$$

A.15 Predictive Equality and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ \frac{FN_a}{FP_a} = \frac{FN_b}{FP_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ FN_a * FP_b = FN_b * \frac{x}{y} * FP_b \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_a \\ TN_a = x - \frac{x}{y} FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FP_a = \frac{x}{y} FP_b \\ FN_a = \frac{x}{y} * FN_b \vee FP_b = 0 \end{cases}$$

A.15.1 Added Constraints through Inequalities

$$\frac{y}{x}(1 - x) \geq FN_b$$

A.16 Predictive Parity and False Omission Rate Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b FP_a}{FP_b} \\ FN_a = \frac{FN_b TN_a}{TN_b} \end{cases}$$

A.17 Predictive Parity and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a * FP_b = TP_b * FP_a \\ TP_a + TN_a = TP_b + TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b * FP_a}{FP_b} \\ \frac{TP_b * FP_a}{FP_b} + x - FP_a = TP_b + y - FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b * FP_a}{FP_b} \\ \frac{FP_a - FP_b}{FP_b} TP_b = y - x + FP_a - FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b * FP_a}{FP_b} \\ TP_b = FP_b \frac{y - x}{FP_a - FP_b} + FP_b \vee (FP_a = FP_b \wedge x = y) \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - FP_a \frac{y - x}{FP_a - FP_b} - FP_a \\ TN_a = x - FP_a \\ FN_b = 1 - y - FP_b \frac{y - x}{FP_a - FP_b} - FP_b \\ TN_b = y - FP_b \\ TP_b = FP_b \frac{y - x}{FP_a - FP_b} + FP_b \\ TP_a = FP_a \frac{y - x}{FP_a - FP_b} + FP_a \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \wedge x = y \end{cases}$$

A.17.1 Added Constraints through Inequalities

$$\begin{cases} 1 - y \geq 1 - x \\ FP_a \geq FP_b \\ \frac{1 - y - FP_a}{1 - x - FP_a} \geq \frac{FP_b}{FP_a} \\ \frac{1 - y + FP_b}{1 - x + FP_b} \geq \frac{FP_b}{FP_a} \end{cases} \vee \begin{cases} 1 - y \leq 1 - x \\ FP_a \leq FP_b \\ \frac{1 - y - FP_a}{1 - x - FP_a} \leq \frac{FP_b}{FP_a} \\ \frac{1 - y + FP_b}{1 - x + FP_b} \leq \frac{FP_b}{FP_a} \end{cases}$$

A.18 Predictive Parity and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a * FP_b = TP_b * FP_a \\ FN_a * FP_b = FN_b * FP_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * FP_a}{FP_b} \\ (1 - x - \frac{FN_b * FP_a}{FP_b}) * FP_b = (1 - y - FN_b) * FP_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * FP_a}{FP_b} \\ FP_b - xFP_b - FN_b * FP_a = FP_a - yFP_a - FN_b * FP_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * FP_a}{FP_b} \\ FP_b - xFP_b = FP_a - yFP_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * FP_a}{FP_b} \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases} \iff \begin{cases} TP_a = 1 - x - \frac{1-x}{1-y} FN_b \\ TN_a = x - \frac{1-x}{1-y} FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases}$$

A.18.1 Added Constraints through Inequalities

$$\frac{1-y}{1-x} x \geq FP_b$$

A.19 False Omission Rate Parity and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a * TN_b = FN_b * TN_a \\ TP_a + TN_a = TP_b + TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * TN_a}{TN_b} \\ x + FN_a - TN_a = y + FN_b - TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * TN_a}{TN_b} \\ \frac{TN_b - FN_b}{TN_b} TN_a = x - y + TN_b - FN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = TN_b \frac{x-y}{TN_b - FN_b} + TN_b \vee (TN_b = FN_b \wedge x = y) \\ FN_a = \frac{FN_b * TN_a}{TN_b} \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_b \frac{x-y}{TN_b - FN_b} - FN_b \\ FP_a = x - TN_b \frac{x-y}{TN_b - FN_b} - TN_b \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ TN_a = TN_b \frac{x-y}{TN_b - FN_b} + TN_b \\ FN_a = FN_b \frac{x-y}{TN_b - FN_b} + FN_b \end{cases} \vee \begin{cases} TP_a = 1 - x - TN_a \\ FP_a = x - TN_a \\ TP_b = 1 - y - FN_b \\ FP_b = y - FN_b \\ TN_b = FN_b \wedge x = y \\ FN_a = TN_a \end{cases}$$

A.19.1 Added Constraints through Inequalities

$$\begin{cases} TN_b \geq FN_b \\ y \geq x \\ \frac{x - TN_b}{y - TN_b} \leq \frac{TN_b}{FN_b} \\ \frac{1 - x - FN_b}{1 - y - FN_b} \geq \frac{FN_b}{TN_b} \end{cases} \vee \begin{cases} TN_b \leq FN_b \\ y \leq x \\ \frac{x - TN_b}{y - TN_b} \geq \frac{TN_b}{FN_b} \\ \frac{1 - x - FN_b}{1 - y - FN_b} \leq \frac{FN_b}{TN_b} \end{cases}$$

A.20 False Omission Rate Parity and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a * TN_b = FN_b * TN_a \\ \frac{FN_a}{FP_a} = \frac{FN_b}{FP_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{FN_b * TN_a}{TN_b} \\ FN_a = \frac{FN_b * FP_a}{FP_b} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ \frac{FN_b * FP_a}{FP_b} = \frac{FN_b * TN_a}{TN_b} \\ FN_a = \frac{FN_b * TN_a}{TN_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a * FP_b = TN_b * FP_a \vee FN_b = 0 \\ FN_a = \frac{FN_b * TN_a}{TN_b} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a(y - TN_b) = TN_b(x - TN_a) \vee FN_b = 0 \\ FN_a = \frac{FN_b * TN_a}{TN_b} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \vee FN_b = 0 \\ FN_a = \frac{FN_b * TN_a}{TN_b} \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - \frac{x}{y} FN_b \\ FP_a = x - \frac{x}{y} TN_b \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ TN_a = \frac{x}{y} TN_b \\ FN_a = \frac{x}{y} FN_b \end{cases} \vee \begin{cases} TP_a = 1 - x \\ TN_a = x - FP_a \\ TP_b = 1 - y \\ TN_b = y - FP_b \\ FN_b = 0 \\ FN_a = 0 \end{cases}$$

A.20.1 Added Constraints through Inequalities

$$\frac{y}{x}(1 - x) \geq TN_b$$

A.21 Overall accuracy Equality and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a + FN_a = FP_b + FN_b \\ FN_a = \frac{FN_b * FP_a}{FP_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a + \frac{FN_b * FP_a}{FP_b} = FP_b + FN_b \\ FN_a = \frac{FN_b * FP_a}{FP_b} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a \frac{FP_b + FN_b}{FP_b} = FP_b + FN_b \\ FN_a = \frac{FN_b * FP_a}{FP_b} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b \vee FP_b = FN_b = 0 \\ FN_a = \frac{FN_b * FP_a}{FP_b} \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_b \\ TN_a = x - FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FP_a = FP_b \\ FN_a = FN_b \end{cases} \vee \begin{cases} TP_a = 1 - x \\ TN_a = x - FP_a \\ TP_b = 1 - y \\ TN_b = y \\ FP_b = FN_b = 0 \\ FN_a = 0 \end{cases}$$

A.21.1 Added Constraints through Inequalities

$$\begin{cases} x \geq FP_b \\ 1 - x \geq FN_b \end{cases}$$

Appendix B. Combining Three Definitions

B.1 Demographic Parity and Equalised Odds

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{x}{y} FP_b = \frac{x-y}{1-y} TP_b + FP_b \\ \frac{1-x}{1-y} TP_b = \frac{y-x}{y} FP_b + TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{x-y}{y} FP_b = \frac{x-y}{1-y} TP_b \\ \frac{1-x-1+y}{1-y} TP_b = \frac{y-x}{y} FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b = \frac{y}{1-y} TP_b \vee x = y \\ \frac{y-x}{1-y} TP_b = \frac{y-x}{y} FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b = \frac{y}{1-y} TP_b \\ \frac{y-x}{1-y} TP_b = \frac{y-x}{y} \frac{y}{1-y} TP_b \end{cases} \vee \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \\ 0 = 0 \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{x}{1-y} TP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - \frac{y}{1-y} TP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{1-y} TP_b \\ FP_b = \frac{y}{1-y} TP_b \\ TP_b = TP_b \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \\ 0 = 0 \end{cases}$$

B.2 Demographic Parity, Equal Opportunity and Predictive Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_a \\ TP_a = TP_b \\ FP_a = FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{x-y}{1-y} TP_b = 0 \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ x - y = 0 \vee TP_b = 0 \\ TP_a = TP_b \\ FP_a = FP_b \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ x = y \\ TP_a = TP_b \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \vee \begin{cases} FN_a = 1 - x \\ TN_a = x - FP_b \\ FN_b = 1 - y \\ TN_b = y - FP_b \\ TP_b = 0 \\ TP_a = 0 \\ TP_a = 0 \\ FP_a = FP_b \end{cases}$$

B.3 Demographic Parity, Equal Opportunity and False Omission Rate Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ TN_a = TN_b \\ FN_a = FN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ -FP_a = -x + y - FP_b \\ -TP_a = -1 + x + 1 - y - TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ TP_a = -x + y + TP_b \\ -x + y + TP_b = \frac{1-x}{1-y} TP_b \\ x - y + FP_b = \frac{x-y}{1-y} TP_b + FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ TP_a = -x + y + TP_b \\ -x + y = \frac{y-x}{1-y} TP_b \\ x - y = \frac{x-y}{1-y} TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ TP_a = -x + y + TP_b \\ 1 = \frac{1}{1-y} TP_b \lor x = y \\ 1 = \frac{1}{1-y} TP_b \lor x = y \end{cases}$$

$$\iff \begin{cases} FN_a = 0 \\ TN_a = y - FP_b \\ FN_b = 0 \\ TN_b = y - FP_b \\ FP_a = x - y + FP_b \\ TP_a = 1 - x \\ TP_b = 1 - y \end{cases} \lor \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ FP_a = FP_b \\ TP_a = TP_b \\ x = y \end{cases}$$

B.4 Demographic Parity, Equal Opportunity and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ FP_a = FP_b + \frac{x-y}{2} \\ TP_a = TP_b + \frac{y-x}{2} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ \frac{x-y}{1-y} TP_b + FP_b = FP_b + \frac{x-y}{2} \\ \frac{1-x}{1-y} TP_b = TP_b + \frac{y-x}{2} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ \frac{x-y}{1-y} TP_b = \frac{x-y}{2} \\ \frac{y-x}{1-y} TP_b = \frac{y-x}{2} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ \frac{x-y}{1-y} TP_b = \frac{x-y}{2} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ TP_b = \frac{1-y}{2} \vee x = y \end{cases}$$

$$\iff \begin{cases} FN_a = \frac{1-x}{2} \\ TN_a = x - \frac{x-y}{2} - FP_b \\ FN_b = \frac{1-y}{2} \\ TN_b = y - FP_b \\ TP_a = \frac{1-x}{2} \\ FP_a = \frac{x-y}{2} + FP_b \\ TP_b = \frac{1-y}{2} \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \end{cases}$$

B.5 Demographic Parity, Equal Opportunity and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x-y}{1-y} TP_b + FP_b \\ FN_a = FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ FP_a = FP_b(\frac{x-y}{FP_b - FN_b} + 1) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = \frac{x-y}{1-y}(1 - y - FN_b) + FP_b \\ FN_a = FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ FP_a = FP_b(\frac{x-y}{FP_b - FN_b} + 1) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = x - y - \frac{x-y}{1-y} FN_b + FP_b \\ \frac{1-x}{1-y} FN_b = FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ x - y - \frac{x-y}{1-y} FN_b + FP_b = FP_b(\frac{x-y}{FP_b - FN_b} + 1) \end{cases}$$

$$
\iff \begin{cases}
TP_a + FN_a = 1 - x \\
TN_a + FP_a = x \\
TP_b + FN_b = 1 - y \\
TN_b + FP_b = y \\
FN_a = \frac{1-x}{1-y} FN_b \\
FP_a = x - y - \frac{x-y}{1-y} FN_b + FP_b \\
\frac{1-x}{1-y} = (\frac{x-y}{FP_b - FN_b} + 1) \vee FN_b = 0 \\
x - y - \frac{x-y}{1-y} FN_b = FP_b (\frac{x-y}{FP_b - FN_b})
\end{cases}
$$

$$
\iff \begin{cases}
TP_a + FN_a = 1 - x \\
TN_a + FP_a = x \\
TP_b + FN_b = 1 - y \\
TN_b + FP_b = y \\
FN_a = \frac{1-x}{1-y} FN_b \\
FP_a = x - y - \frac{x-y}{1-y} FN_b + FP_b \\
-\frac{1}{1-y} = \frac{1}{FP_b - FN_b} \vee FN_b = 0 \vee x = y \\
(x - y)(1 - \frac{1}{1-y} FN_b) = FP_b \frac{x-y}{FP_b - FN_b}
\end{cases}
$$

$$
\iff \begin{cases}
TP_a + FN_a = 1 - x \\
TN_a + FP_a = x \\
TP_b + FN_b = 1 - y \\
TN_b + FP_b = y \\
FN_a = \frac{1-x}{1-y} FN_b \\
FP_a = x - y - \frac{x-y}{1-y} FN_b + FP_b \\
-1 + y = FP_b - FN_b \vee FN_b = 0 \vee x = y \\
1 - \frac{1}{1-y} FN_b = \frac{FP_b}{FP_b - FN_b} \vee x = y
\end{cases}
$$

$$
\iff \begin{cases}
TP_a + FN_a = 1 - x \\
TN_a + FP_a = x \\
TP_b + FN_b = 1 - y \\
TN_b + FP_b = y \\
FN_a = \frac{1-x}{1-y} FN_b \\
FP_a = x - y - \frac{x-y}{1-y} FN_b + FP_b \\
-1 + y = FP_b - FN_b \vee FN_b = 0 \vee x = y \\
FP_b - FN_b - \frac{1}{1-y}(FP_b - FN_b) FN_b = FP_b \vee x = y
\end{cases}
$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = x - y - \frac{x-y}{1-y} FN_b + FP_b \\ -1 + y = FP_b - FN_b \vee FN_b = 0 \vee x = y \\ -1 - \frac{1}{1-y}(FP_b - FN_b) = 0 \vee x = y \vee FN_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = x - y - \frac{x-y}{1-y} FN_b + FP_b \\ -1 + y = FP_b - FN_b \vee FN_b = 0 \vee x = y \\ FP_b - FN_b = -1 + y \vee x = y \vee FN_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = x - y - \frac{x-y}{1-y} FN_b + FP_b \\ FP_b = -1 + y + FN_b \vee FN_b = 0 \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - \frac{1-x}{1-y} FN_b \\ TN_a = y + \frac{x-y}{1-y} FN_b \\ FN_b = 1 - y \\ TN_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = x - y - \frac{x-y}{1-y} FN_b \\ FP_b = -TP_b = 0 \end{cases} \vee \begin{cases} TP_a = 1 - x \\ TN_a = y + FP_b \\ TP_b = 1 - y \\ TN_b = y - FP_b \\ FN_a = 0 \\ FP_a = x - y - FP_b \\ FN_b = 0 \end{cases} \vee \begin{cases} TP_a = 1 - x - FN_b \\ TN_a = x - FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FN_a = FN_b \\ FP_a = FP_b \\ x = y \end{cases}$$

B.6 Demographic Parity, Predictive Equality and Predictive Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_b = \frac{y-x}{y} FP_b + TP_b \\ FP_b = \frac{x}{y} FP_b \\ TP_a = TP_b \\ FP_a = FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ 0 = \frac{y-x}{y} FP_b \\ 1 = \frac{x}{y} \vee FP_b = 0 \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ x = y \vee FP_b = 0 \\ x = y \vee FP_b = 0 \\ TP_a = TP_b \\ FP_a = FP_b \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ x = y \\ TP_a = TP_b \\ FP_a = FP_b \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x \\ FN_b = 1 - y - TP_b \\ TN_b = y \\ FP_b = 0 \\ TP_a = TP_b \\ FP_a = 0 \end{cases}$$

B.7 Demographic Parity, Predictive Equality and False Omission Rate Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ TN_a = \frac{x}{y} TN_b \\ TN_a = TN_b \\ FN_a = FN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ TN_b = \frac{x}{y} TN_b \\ TN_a = TN_b \\ -TP_a = -1 + x + 1 - y - TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ 1 = \frac{x}{y} \vee TN_b = 0 \\ TN_a = TN_b \\ TP_a = -x + y + TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ -x + y + TP_b = \frac{y-x}{y} FP_b + TP_b \\ x = y \vee TN_b = 0 \\ TN_a = TN_b \\ TP_a = -x + y + TP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ -x + y = \frac{y-x}{y} FP_b \\ x = y \vee TN_b = 0 \\ TN_a = TN_b \\ TP_a = -x + y + TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ 1 = \frac{1}{y} FP_b \vee x = y \\ x = y \vee TN_b = 0 \\ TN_a = TN_b \\ TP_a = -x + y + TP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ y = FP_b \vee x = y \\ x = y \vee TN_b = 0 \\ TN_a = TN_b \\ TP_a = -x + y + TP_b \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - TP_b \\ FP_a = x - TN_b \\ FN_b = 1 - y - TP_b \\ FP_b = y - TN_b \\ x = y \\ x = y \\ TN_a = TN_b \\ TP_a = TP_b \end{cases} \vee \begin{cases} FN_a = 1 - y - TP_b \\ FP_a = x \\ FN_b = 1 - y - TP_b \\ 0 = 0 \\ FP_b = y \\ TN_b = 0 \\ TN_a = 0 \\ TP_a = -x + y + TP_b \end{cases}$$

B.8 Demographic Parity, Predictive Equality and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \\ TP_a = TP_b + \frac{y-x}{2} \\ FP_a = FP_b + \frac{x-y}{2} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{y-x}{y} FP_b + TP_b = TP_b + \frac{y-x}{2} \\ \frac{x}{y} FP_b = FP_b + \frac{x-y}{2} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{y-x}{y} FP_b = \frac{y-x}{2} \\ \frac{x-y}{y} FP_b = \frac{x-y}{2} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{1}{y} FP_b = \frac{1}{2} \vee x = y \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b = \frac{y}{2} \vee x = y \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - \frac{y-x}{2} - TP_b \\ TN_a = \frac{x}{2} \\ FN_b = 1 - y - TP_b \\ TN_b = \frac{y}{2} \\ TP_a = \frac{y-x}{2} + TP_b \\ FP_a = \frac{x}{2} \\ FP_b = \frac{y}{2} \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \end{cases}$$

B.9 Demographic Parity, Predictive Equality and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{y-x}{y} FP_b + TP_b \\ FP_a = \frac{x}{y} FP_b \\ FN_a = FN_b * (\frac{x-y}{FP_b - FN_b} + 1) \\ FP_a = FP_b * (\frac{x-y}{FP_b - FN_b} + 1) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ -FN_a = x - y - FN_b + \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{x}{y} FP_b = FP_b * (\frac{x-y}{FP_b - FN_b} + 1) \\ FN_a = FN_b * (\frac{x-y}{FP_b - FN_b} + 1) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{x}{y} = (\frac{x-y}{FP_b - FN_b} + 1) \vee FP_b = 0 \\ y - x + FN_b - \frac{y-x}{y} FP_b = FN_b * (\frac{x-y}{FP_b - FN_b} + 1) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{x-y}{y} = \frac{x-y}{FP_b - FN_b} \vee FP_b = 0 \\ y - x - \frac{y-x}{y} FP_b = FN_b * (\frac{x-y}{FP_b - FN_b}) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b - FN_b = y \vee FP_b = 0 \vee x - y = 0 \\ 1 - \frac{1}{y} FP_b = -FN_b * \frac{1}{FP_b - FN_b} \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b - FN_b = y \vee FP_b = 0 \vee x - y = 0 \\ y - FP_b = -yFN_b * \frac{1}{FP_b - FN_b} \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b - FN_b = y \vee FP_b = 0 \vee x - y = 0 \\ yFP_b - yFN_b - FP_b^2 + FN_b FP_b = -yFN_b \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b - FN_b = y \vee FP_b = 0 \vee x - y = 0 \\ yFP_b - FP_b^2 + FN_b FP_b = 0 \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b - FN_b = y \vee FP_b = 0 \vee x = y \\ y - FP_b + FN_b = 0 \vee x = y \vee FP_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_b - FN_b = y \lor FP_b = 0 \lor x = y \\ FP_b - FN_b = y \lor x = y \lor FP_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ FN_b = FP_b - y \lor FP_b = 0 \lor x = y \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_a \\ TN_a = x - FP_a \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FN_b = FP_b - y \\ FN_a = -x(1 - \frac{1}{y} FP_b) \\ FP_a = \frac{x}{y} FP_b \end{cases} \lor \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b - \frac{y-x}{y} FP_b \\ FP_a = \frac{x}{y} FP_b \\ FP_a = 0 \lor FP_a = FP_b \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x \\ TN_a = 0 \\ TP_b = 1 - y \\ TN_b = 0 \\ FN_b = 0 \\ FN_a = 0, FP_b = y \\ FP_a = x \end{cases} \lor \begin{cases} TP_a = 1 - y - FN_b \\ TN_a = x \\ TP_b = 1 - y - FN_b \\ TN_b = y \\ FP_b = 0 \\ FN_a = y - x + FN_b \\ FP_a = 0 \end{cases} \lor \begin{cases} TP_a = 1 - x - FN_b \\ TN_a = x - FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ x = y \\ FN_a = FN_b \\ FP_a = FP_b \end{cases}$$

B.10 Demographic Parity, Predictive Parity and False Omission Rate Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ TN_a = TN_b \\ FN_a = FN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ x - FP_a = y - FP_b \\ 1 - x - TP_a = 1 - y - TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ x - FP_a = y - FP_a \\ -x - TP_a = -y - TP_a \end{cases} \iff \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \\ x = y \end{cases}$$

B.11 Demographic Parity, Predictive Parity and Overall Accuracy equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ FP_a = FP_b + \frac{x-y}{2} \\ TP_a = TP_b + \frac{y-x}{2} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ FP_a = FP_a + \frac{x-y}{2} \\ TP_a = TP_a + \frac{y-x}{2} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ 0 = \frac{x-y}{2} \\ 0 = \frac{y-x}{2} \end{cases} \iff \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \\ x = y \end{cases}$$

B.12 Demographic Parity, Predictive Parity and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b \\ FP_a = FP_b \\ FN_a = FN_b * (\frac{x-y}{FP_b - FN_b} + 1) \\ FP_a = FP_b * (\frac{x-y}{FP_b - FN_b} + 1) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ 1 - x - FN_a = 1 - y - FN_b \\ FP_a = FP_b \\ FN_a = FN_b * (\frac{x-y}{FP_b - FN_b} + 1) \\ FP_b = FP_b * (\frac{x-y}{FP_b - FN_b} + 1) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b \\ FP_a = FP_b \\ y - x + FN_b = FN_b * (\frac{x-y}{FP_b - FN_b} + 1) \\ 1 = \frac{x-y}{FP_b - FN_b} + 1 \vee FP_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = y - x + FN_b \\ FP_a = FP_b \\ y - x = FN_b * (\frac{x-y}{FP_b - FN_b}) \\ x = y \vee FP_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_b \\ TN_a = x - FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FN_a = FN_b \\ FP_a = FP_b \\ 0 = 0 \\ x = y \end{cases} \vee \begin{cases} TP_a = 1 - y - FN_b \\ TN_a = x \\ TP_b = 1 - y - FN_b \\ TN_b = y \\ FN_a = y - x + FN_b \\ FP_a = FP_b = 0 \\ y - x = -(x - y) \\ FP_b = 0 \end{cases}$$

B.13 Demographic Parity, False Omission Rate Parity and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = TN_b \\ FN_a = FN_b \\ FP_a = FP_b + \frac{x-y}{2} \\ TP_a = TP_b + \frac{y-x}{2} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ x - FP_a = y - FP_b \\ 1 - x - TP_a = 1 - y - TP_b \\ FP_a = FP_b + \frac{x-y}{2} \\ TP_a = TP_b + \frac{y-x}{2} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ TP_a = y - x + TP_b \\ x - y + FP_b = FP_b + \frac{x-y}{2} \\ y - x + TP_b = TP_b + \frac{y-x}{2} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ TP_a = y - x + TP_b \\ x - y = \frac{x-y}{2} \\ y - x = \frac{y-x}{2} \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ TP_a = y - x + TP_b \\ x = y \\ x = y \end{cases} \iff \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ FP_a = FP_b \\ TP_a = TP_b \\ x = y \end{cases}$$

B.14 Demographic Parity, False Omission Rate Parity and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ FN_a = FN_b \\ FN_a = FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ FP_a = FP_b(\frac{x-y}{FP_b - FN_b} + 1) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ FN_a = FN_b \\ FN_b = FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ x - y + FP_b = FP_b(\frac{x-y}{FP_b - FN_b} + 1) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ FN_a = FN_b \\ 1 = \frac{x-y}{FP_b - FN_b} + 1 \vee FN_b = 0 \\ x - y = FP_b \frac{x-y}{FP_b - FN_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ FN_a = FN_b \\ 0 = \frac{x-y}{FP_b - FN_b} \vee FN_b = 0 \\ 1 = \frac{FP_b}{FP_b - FN_b} \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ FN_a = FN_b \\ x = y \vee FN_b = 0 \\ FP_b - FN_b = FP_b \vee x = y \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = x - y + FP_b \\ FN_a = FN_b \\ x = y \vee FN_b = 0 \\ FN_b = 0 \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_b \\ TN_a = x - FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FP_a = FP_b \\ FN_a = FN_b \\ x = y \end{cases} \vee \begin{cases} TP_a = 1 - x \\ TN_a = y - FP_b \\ TP_b = 1 - y \\ TN_b = y - FP_b \\ FP_a = x - y + FP_b \\ FN_a = 0 \\ FN_b = 0 \end{cases}$$

B.15 Demographic Parity, Overall accuracy Equality and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b + \frac{x-y}{2} \\ TP_a = TP_b + \frac{y-x}{2} \\ FN_a = FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ FP_a = FP_b(\frac{x-y}{FP_b - FN_b} + 1) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b + \frac{x-y}{2} \\ FN_a = y - x + FN_b - \frac{y-x}{2} \\ y - x + FN_b - \frac{y-x}{2} = FN_b(\frac{x-y}{FP_b - FN_b} + 1) \\ FP_b + \frac{x-y}{2} = FP_b(\frac{x-y}{FP_b - FN_b} + 1) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b + \frac{x-y}{2} \\ FN_a = y - x + FN_b - \frac{y-x}{2} \\ \frac{y-x}{2} = FN_b \frac{x-y}{FP_b - FN_b} \\ \frac{x-y}{2} = FP_b \frac{x-y}{FP_b - FN_b} \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b + \frac{x-y}{2} \\ FN_a = y - x + FN_b - \frac{y-x}{2} \\ -\frac{1}{2} = FN_b \frac{1}{FP_b - FN_b} \vee x = y \\ \frac{1}{2} = FP_b \frac{1}{FP_b - FN_b} \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b + \frac{x-y}{2} \\ FN_a = y - x + FN_b - \frac{y-x}{2} \\ \frac{-FP_b + FN_b}{2} = FN_b \vee x = y \\ \frac{FP_b - FN_b}{2} = FP_b \vee x = y \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b + \frac{x-y}{2} \\ FN_a = FN_b + \frac{y-x}{2} \\ \frac{-FP_b}{2} = \frac{FN_b}{2} \vee x = y \\ \frac{-FN_b}{2} = \frac{FP_b}{2} \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b + \frac{x-y}{2} \\ FN_a = FN_b + \frac{y-x}{2} \\ -FP_b = FN_b = 0 \vee x = y \\ -FN_b = FP_b = 0 \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - \frac{y-x}{2} \\ TN_a = x - \frac{x-y}{2} \\ TP_b = 1 - y \\ TN_b = y \\ FP_a = \frac{x-y}{2} \\ FN_a = \frac{y-x}{2} \\ -FP_b = FN_b = 0 \end{cases} \vee \begin{cases} TP_a = 1 - x - FN_a \\ TN_a = x - FP_a \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ FP_a = FP_b \\ FN_a = FN_b \\ x = y \end{cases}$$

B.16 Equalised odds and Predictive Parity/Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{1-x}{1-y} FP_b = \frac{x}{y} FP_b \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{1-x}{1-y} = \frac{x}{y} \vee FP_b = 0 \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ y - xy = x - xy \vee FP_b = 0 \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ y = x \vee FP_b = 0 \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ y = x \\ FP_a = FP_b \end{cases} \vee \begin{cases} FN_a = 1 - x - \frac{1-x}{1-y} TP_b \\ TN_a = x \\ FN_b = 1 - y - TP_b \\ TN_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_b = 0 \\ FP_a = 0 \end{cases}$$

B.17 Equalised odds and False Omission Rate Parity

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{x}{y} TN_b \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{1-x}{1-y} TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{x}{y} TN_b \\ \frac{x}{y} TN_b = \frac{1-x}{1-y} TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{x}{y} TN_b \\ \frac{x}{y} = \frac{1-x}{1-y} \lor TN_b = 0 \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{x}{y} TN_b \\ x = y \lor TN_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_b \\ FP_a = x - TN_b \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ FN_a = FN_b \\ TN_a = TN_b \\ x = y \end{cases} \lor \begin{cases} TP_a = 1 - x - \frac{1-x}{1-y} FN_b \\ FP_a = x \\ TP_b = 1 - y - FN_b \\ FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = 0 \\ TN_b = 0 \end{cases}$$

B.18 Equalised odds and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ \frac{y-x}{1-y} TP_b = TN_b - TN_a \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ \frac{1-x}{1-y} TP_b = \frac{1-x}{1-y} TP_b \\ \frac{y-x}{1-y} TP_b = \frac{y-x}{y} TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ TP_b = \frac{1-y}{y} TN_b \vee x = y \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - \frac{1-x}{y} TN_b \\ TN_a = x - \frac{x}{y} FP_b \\ FN_b = 1 - y - \frac{1-y}{y} TN_b \\ TN_b = y - FP_b \\ TP_a = \frac{1-x}{y} TN_b \\ FP_a = \frac{x}{y} FP_b \\ TP_b = \frac{1-y}{y} TN_b \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ FP_a = FP_b \\ x = y \end{cases}$$

B.19 Equal Opportunity, False Omission Rate Parity and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{1-x}{1-y} TN_b \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = TN_b - \frac{y-x}{1-y} TP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{1-x}{1-y} TN_b \\ \frac{1-x}{1-y} TN_b = TN_b - \frac{y-x}{1-y} TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{1-x}{1-y} TN_b \\ \frac{1-x-1+y}{1-y} TN_b = -\frac{y-x}{1-y} TP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{1-x}{1-y} TN_b \\ \frac{y-x}{1-y} TN_b = -\frac{y-x}{1-y} TP_b \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_a \\ FP_a = x - TN_a \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{1-x}{1-y} TN_b \\ TN_b = -TP_b \vee x = y \end{cases} \iff \begin{cases} TP_a = 1 - x - \frac{1-x}{1-y} FN_b \\ FP_a = x - \frac{1-x}{1-y} TN_b \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = \frac{1-x}{1-y} TN_b \\ TN_b = -TP_b = 0 \vee x = y \end{cases}$$

B.20 Equal Opportunity, False Omission Rate Parity and Predictive Parity/Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{1-x}{1-y} FP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = \frac{1-x}{1-y} TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ x - TN_a = \frac{1-x}{1-y}(y - TN_b) \\ TN_a = \frac{1-x}{1-y} TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ x - \frac{1-x}{1-y}(y - TN_b) = \frac{1-x}{1-y} TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ x = \frac{1-x}{1-y} y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ x = y \end{cases} \iff \begin{cases} FN_a = 1 - x - TP_b \\ FP_a = x - TN_b \\ FN_b = 1 - y - TP_b \\ FP_b = y - TN_b \\ TP_a = TP_b \\ TN_a = TN_b \\ x = y \end{cases}$$

B.21 Equal Opportunity, Overall accuracy Equality and Predictive Parity/Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ FP_a = \frac{1-x}{1-y} FP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = TN_b - \frac{y-x}{1-y} TP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ x - TN_a = \frac{1-x}{1-y}(y - TN_b) \\ TN_a = TN_b - \frac{y-x}{1-y} TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ x - \frac{1-x}{1-y} y + \frac{1-x}{1-y} TN_b = TN_b - \frac{y-x}{1-y} TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ -TN_b + \frac{1-x}{1-y} TN_b = -\frac{y-x}{1-y} TP_b - x + \frac{1-x}{1-y} y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ \frac{y-x}{1-y} TN_b = -\frac{y-x}{1-y} TP_b - x + \frac{1-x}{1-y} y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ TN_b = -TP_b - \frac{1-y}{y-x}x + \frac{1-x}{1-y}\frac{1-y}{y-x}y \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ TN_b = -TP_b - \frac{1-y}{y-x}x + \frac{1-x}{y-x}y \vee x = y \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = x - \frac{1-x}{1-y}(y - TN_b) \\ TN_b = -TP_b + \frac{1}{y-x}(-x + y) \vee x = y \end{cases}$$

$$\iff \begin{cases} FN_a = 1 - x - TP_a \\ TN_a = x - FP_a \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = \frac{1-x}{1-y} TP_b \\ TN_a = 1 - \frac{1-x}{1-y} TP_b \\ TN_b = 1 - TP_b \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_a \\ TN_a = x - FP_a \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ TN_a = TN_b \\ x = y \end{cases}$$

$$\iff \begin{cases} FN_a = 0 \\ FP_a = 0 \\ FN_b = 0 \\ TN_b = 0 \\ TP_a = 1 - x \\ TN_a = x \\ TN_b = y \\ TP_b = 1 - y \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ TP_a = TP_b \\ TN_a = TN_b \\ x = y \end{cases}$$

B.22 Predictive Equality, Predictive Parity and False Omission Rate Parity/Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ TP_a = \frac{x}{y} TP_b \\ FP_a = \frac{x}{y} FP_b \\ FN_a = \frac{x}{y} FN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ FN_a = \frac{x}{y} FN_b \\ 1 - x - FN_a = \frac{x}{y}(1 - y - FN_b) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ FN_a = \frac{x}{y} FN_b \\ 1 - x - \frac{x}{y} FN_b = \frac{x}{y} - x - \frac{x}{y} FN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = \frac{x}{y} FP_b \\ FN_a = \frac{x}{y} FN_b \\ 1 = \frac{x}{y} \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_b \\ TN_a = x - FP_b \\ TP_b = 1 - y - FN_b \\ TN_b = y - FP_b \\ y = x \\ FP_a = FP_b \\ FN_a = FN_b \end{cases}$$

B.23 Predictive Equality, Predictive Parity and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a = \frac{x}{y} TP_b \\ TN_a = \frac{x}{x-y}(TP_b - TP_a) \\ TN_b = \frac{y}{x-y}(TP_b - TP_a) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a = \frac{x}{y} TP_b \\ TN_a = \frac{x}{x-y}(TP_b - \frac{x}{y} TP_b) \\ TN_b = \frac{y}{x-y}(TP_b - \frac{x}{y} TP_b) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a = \frac{x}{y} TP_b \\ TN_a = \frac{x}{x-y}(\frac{y-x}{y} TP_b) \\ TN_b = \frac{y}{x-y}(\frac{y-x}{y} TP_b) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a = \frac{x}{y} TP_b \\ TN_a = -\frac{x}{y} TP_b \\ TN_b = -TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{x}{y} TP_b \\ TN_a = -\frac{x}{y} TP_b \\ TN_b = -TP_b \\ -\frac{x}{y} TP_b = -\frac{x}{y} TP_b \end{cases} \iff \begin{cases} FN_a = 1 - x \\ FP_a = x \\ FN_b = 1 - y \\ FP_b = y \\ TP_a = \frac{x}{y} TP_b = 0 \\ TN_a = -\frac{x}{y} TP_b = 0 \\ TN_b = -TP_b = 0 \\ 1 = 1 \end{cases}$$

B.24 Predictive Equality, Overall accuracy Equality and False Omission Rate Parity/Treatment equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ FN_a = \frac{x}{y} FN_b \\ TN_a = \frac{x}{x-y}(TP_b - TP_a) \\ TN_b = \frac{y}{x-y}(TP_b - TP_a) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a = 1 - \frac{x}{y} + \frac{x}{y} TP_b \\ TN_a = \frac{x}{x-y}(TP_b - 1 + \frac{x}{y} - \frac{x}{y} TP_b) \\ TN_b = \frac{y}{x-y}(TP_b - 1 + \frac{x}{y} - \frac{x}{y} TP_b) \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a = 1 - \frac{x}{y} + \frac{x}{y} TP_b \\ TN_a = \frac{x}{x-y}(\frac{x-y}{y} + \frac{y-x}{y} TP_b) \\ TN_b = \frac{y}{x-y}(\frac{x-y}{y} + \frac{y-x}{y} TP_b) \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{x}{y} TN_b \\ TP_a = 1 - \frac{x}{y} + \frac{x}{y} TP_b \\ TN_a = \frac{x}{y} - \frac{x}{y} TP_b \\ TN_b = 1 - TP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = 1 - \frac{x}{y} + \frac{x}{y}TP_b \\ TN_a = \frac{x}{y} - \frac{x}{y}TP_b \\ TN_b = 1 - TP_b \\ \frac{x}{y} - \frac{x}{y}TP_b = \frac{x}{y}(1 - TP_b) \end{cases} \iff \begin{cases} FN_a = 1 - x - TP_a \\ FP_a = x - TN_a \\ FN_b = 1 - y - TP_b \\ FP_b = y - TN_b \\ TP_a = 1 - \frac{x}{y} + \frac{x}{y}TP_b \\ TN_a = \frac{x}{y} - \frac{x}{y}TP_b \\ TN_b = 1 - TP_b \\ 1 = 1 \end{cases}$$

$$\iff \begin{cases} FN_a = 0 \\ FP_a = 0 \\ FN_b = 0 \\ FP_b = 0 \\ TP_a = 1 - x \\ TN_a = x \\ TN_b = y \\ TP_b = 1 - y \\ 1 = 1 \end{cases}$$

B.25 Predictive Parity, False Omission Rate Parity and Overall accuracy Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b * FP_a}{FP_b} \\ FN_a = \frac{FN_b * TN_a}{TN_b} \\ TP_a + TN_a = TP_b + TN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a * (y - TN_b) = TP_b * (x - TN_a) \\ (1 - x - TP_a) = (1 - y - TP_b) * \frac{TN_a}{TN_b} \\ TP_a + TN_a = TP_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b * \frac{x - TN_a}{y - TN_b} \\ (1 - x - TP_b * \frac{x - TN_a}{y - TN_b}) * TN_b = (1 - y - TP_b) * TN_a \\ TP_a + TN_a = TP_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b * \frac{x - TN_a}{y - TN_b} \\ -TP_b TN_b \frac{x - TN_a}{y - TN_b} + TP_b TN_a = -TN_b + xTN_b + TN_a - yTN_a \\ TP_a + TN_a = TP_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b * \frac{x - TN_a}{y - TN_b} \\ TP_b(TN_a - TN_b * \frac{x - TN_a}{y - TN_b}) = (x-1)TN_b + (1-y)TN_a \\ TP_a + TN_a = TP_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b * \frac{x - TN_a}{y - TN_b} \\ TP_b(\frac{yTN_a - TN_a TN_b - xTN_b + TN_b TN_a}{y - TN_b}) = (x-1)TN_b + (1-y)TN_a \\ TP_a + TN_a = TP_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = TP_b * \frac{x - TN_a}{y - TN_b} \\ TP_b(\frac{yTN_a - xTN_b}{y - TN_b}) = (x-1)TN_b + (1-y)TN_a \\ TP_a + TN_a = TP_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{(x-1)(x - TN_a)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(x - TN_a)}{yTN_a - xTN_b} TN_a \\ TP_b = \frac{(x-1)(y - TN_b)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(y - TN_b)}{yTN_a - xTN_b} TN_a \\ TP_a + TN_a = TP_b + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{(x-1)(x-TN_a)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(x-TN_a)}{yTN_a - xTN_b} TN_a \\ TP_b = \frac{(x-1)(y-TN_b)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(y-TN_b)}{yTN_a - xTN_b} TN_a \\ \frac{(x-1)(x-TN_a)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(x-TN_a)}{yTN_a - xTN_b} TN_a + TN_a = \\ \frac{(x-1)(y-TN_b)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(y-TN_b)}{yTN_a - xTN_b} TN_a + TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{(1-x)(x-TN_a)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(x-TN_a)}{yTN_a - xTN_b} TN_a \\ TP_b = \frac{(1-x)(y-TN_b)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(y-TN_b)}{yTN_a - xTN_b} TN_a \\ (x^2 - xTN_a - x + TN_a)TN_b + (x - xy - TN_a + yTN_a)TN_a + \\ (yTN_a - xTN_b)TN_a = (xy - xTN_b - y + < tn - b)TN_b + \\ (y - TN_b - y^2 + yTN_b)TN_a + (yTN_a - xTN_b)TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{(1-x)(x-TN_a)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(x-TN_a)}{yTN_a - xTN_b} TN_a \\ TP_b = \frac{(1-x)(y-TN_b)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(y-TN_b)}{yTN_a - xTN_b} TN_a \\ (xy - 2xTN_b - y + TN_b + yTN_a - x^2 + xTN_a + x - TN_a)TN_b = \\ (x - xy - TN_a + 2yTN_a - xTN_b - y + TN_b + y^2 - yTN_b)TN_a \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{(1-x)(x-TN_a)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(x-TN_a)}{yTN_a - xTN_b} TN_a \\ TP_b = \frac{(1-x)(y-TN_b)}{yTN_a - xTN_b} TN_b + \frac{(1-y)(y-TN_b)}{yTN_a - xTN_b} TN_a \\ (-2x + 1)TN_b^2 + (xy - y + x + 2yTN_a - x^2 + 2xTN_a - 2TN_a)TN_b \\ +(xy - x + TN_a - 2yTN_a + y - y^2)TN_a = 0 \end{cases}$$

B.26 Predictive Parity, False Omission Rate Parity and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TP_a = \frac{TP_b FP_a}{FP_b} \\ FN_a = \frac{FN_b TN_a}{TN_b} \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ \frac{1-x}{1-y} TP_b = \frac{TP_b FP_a}{FP_b} \\ \frac{1-x}{1-y} FN_b = \frac{FN_b TN_a}{TN_b} \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ \frac{1-x}{1-y} FP_b = FP_a \vee TP_b = 0 \\ TN_a = \frac{1-x}{1-y} TN_b \vee FN_b = 0 \\ FN_a = \frac{1-x}{1-y} FN_b \\ FP_a = \frac{1-x}{1-y} FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = \frac{1-x}{1-y} TN_b \vee FN_b = 0 \\ FN_a = \frac{1-x}{1-y} FN_b \\ TN_a = x - \frac{1-x}{1-y} y + \frac{1-x}{1-y} TN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = x - \frac{1-x}{1-y} y + \frac{1-x}{1-y} TN_b \\ TN_a = \frac{1-x}{1-y} TN_b \\ FN_a = \frac{1-x}{1-y} FN_b \end{cases} \vee \begin{cases} TP_a = 1 - x \\ FP_a = x - TN_a \\ TP_b = 1 - y \\ FP_b = y - TN_b \\ TN_a = x - \frac{1-x}{1-y} y + \frac{1-x}{1-y} TN_b \\ FN_b = 0 \\ FN_a = 0 \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ \frac{1-x}{1-y} TN_b = x - \frac{1-x}{1-y} y + \frac{1-x}{1-y} TN_b \\ TN_a = \frac{1-x}{1-y} TN_b \\ FN_a = \frac{1-x}{1-y} FN_b \end{cases} \vee \begin{cases} TP_a = 1 - x \\ FP_a = x - TN_a \\ TP_b = 1 - y \\ FP_b = y - TN_b \\ TN_a = x - \frac{1-x}{1-y} y + \frac{1-x}{1-y} TN_b \\ FN_b = 0 \\ FN_a = 0 \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x - FN_b \\ FP_a = x - TN_b \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ x = y \\ TN_a = TN_b \\ FN_a = FN_b \end{cases} \vee \begin{cases} TP_a = 1 - x \\ FP_a = \frac{1-x}{1-y}y - \frac{1-x}{1-y}TN_b \\ TP_b = 1 - y \\ FP_b = y - TN_b \\ TN_a = x - \frac{1-x}{1-y}y + \frac{1-x}{1-y}TN_b \\ FN_b = 0 \\ FN_a = 0 \end{cases}$$

B.27 Predictive Parity, Overall accuracy Equality and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b \\ FN_a = FN_b \\ TP_a = \frac{1-x}{1-y}TP_b \\ FP_a = \frac{1-x}{1-y}FP_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b \\ 1 - x - TP_a = 1 - y - TP_b \\ TP_a = \frac{1-x}{1-y}TP_b \\ FP_b = \frac{1-x}{1-y}FP_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b \\ TP_a = y - x + TP_b \\ y - x - TP_b = \frac{1-x}{1-y}TP_b \\ x = y \vee FP_b = 0 \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b \\ TP_a = y - x + TP_b \\ y - x = \frac{y-x}{1-y}TP_b \\ x = y \vee FP_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b \\ TP_a = y - x + TP_b \\ TP_b = 1 - y \vee x = y \\ x = y \vee FP_b = 0 \end{cases} \iff \begin{cases} FN_a = 0 \\ TN_a = x \\ FN_b = 0 \\ TN_b = y \\ FP_a = 0 \\ TP_a = 1 - x \\ TP_b = 1 - y \\ FP_b = 0 \end{cases} \vee \begin{cases} FN_a = 1 - x - TP_b \\ TN_a = x - FP_b \\ FN_b = 1 - y - TP_b \\ TN_b = y - FP_b \\ FP_a = FP_b \\ TP_a = TP_b \\ x = y \end{cases}$$

B.28 False Omission Rate Parity, Overall accuracy Equality and Treatment Equality

$$\begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ FP_a = FP_b \\ FN_a = FN_b \\ TN_a = \frac{x}{y} TN_b \\ FN_a = \frac{x}{y} FN_b \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ x - TN_a = y - TN_b \\ FN_a = FN_b \\ TN_a = \frac{x}{y} TN_b \\ FN_b = \frac{x}{y} FN_b \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = x - y + TN_b \\ FN_a = FN_b \\ x - y + TN_b = \frac{x}{y} TN_b \\ 1 = \frac{x}{y} \vee FN_b = 0 \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = x - y + TN_b \\ FN_a = FN_b \\ x - y = \frac{x-y}{y} TN_b \\ x = y \vee FN_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = x - y + TN_b \\ FN_a = FN_b \\ 1 = \frac{1}{y} TN_b \vee x = y \\ x = y \vee FN_b = 0 \end{cases} \iff \begin{cases} TP_a + FN_a = 1 - x \\ TN_a + FP_a = x \\ TP_b + FN_b = 1 - y \\ TN_b + FP_b = y \\ TN_a = x - y + TN_b \\ FN_a = FN_b \\ TN_b = y \vee x = y \\ x = y \vee FN_b = 0 \end{cases}$$

$$\iff \begin{cases} TP_a = 1 - x \\ FP_a = 0 \\ TP_b = 1 - y \\ FP_b = 0 \\ TN_a = x \\ FN_a = 0 \\ TN_b = y \\ FN_b = 0 \end{cases} \vee \begin{cases} TP_a = 1 - x - FN_b \\ FP_a = x - TN_b \\ TP_b = 1 - y - FN_b \\ FP_b = y - TN_b \\ TN_a = TN_b \\ FN_a = FN_b \\ x = y \end{cases}$$

Appendix C. p%-rule Feasibility

C.1 Equalised Odds

$$\frac{1-x}{1-y}TP_b + \frac{x}{y}FP_b \geq pTP_b + pFP_b$$

$$TP_b + FP_b \geq p\frac{1-x}{1-y}TP_b + p\frac{x}{y}FP_b$$

Case 1: $x = y$

$$TP_b \geq -FP_b$$

This is always true, thus for $x = y$ equalised odds satisfies the p%-rule for all p.

Case 2: $p < \frac{1-x}{1-y}$ and $p < \frac{1-y}{1-x}$

$$TP_b \geq \frac{1-y}{y}\frac{yp-x}{1-x-p+yp}FP_b$$

$$TP_b \geq \frac{1-y}{y}\frac{xp-y}{1-y-p+xp}FP_b$$

Case 3: $p > \frac{1-x}{1-y}$

From this condition follows that $x > y$ as $p \leq 1$ and $x \neq y$. Furthermore this means that $p < \frac{1-y}{1-x}$.

$$TP_b \leq \frac{1-y}{y}\frac{yp-x}{1-x-p+yp}FP_b$$

$$TP_b \geq \frac{1-y}{y}\frac{xp-y}{1-y-p+xp}FP_b$$

Case 4: $p > \frac{1-y}{1-x}$

Similar to case 3, we can derive that $y > x$ and $p < \frac{1-x}{1-y}$.

$$TP_b \geq \frac{1-y}{y}\frac{yp-x}{1-x-p+yp}FP_b$$

$$TP_b \leq \frac{1-y}{y}\frac{xp-y}{1-y-p+xp}FP_b$$

Case 5: $p = \frac{1-x}{1-y}$

From this condition we can derive that $x \geq y$ and thus $p < \frac{1-y}{1-x}$.

$$TP_b \geq \frac{1-y}{y}\frac{xp-y}{1-y-p+xp}FP_b$$

Case 6: $p = \frac{1-y}{1-x}$

From this we can derive that $x \leq y$ and thus $p < \frac{1-x}{1-y}$.

$$TP_b \geq \frac{1-y}{y}\frac{yp-x}{1-x-p+yp}FP_b$$

C.2 Equal Opportunity and Predictive Parity

$$\frac{1-x}{1-y}TP_b + \frac{1-x}{1-y}FP_b \geq pTP_b + pFP_b$$

$$TP_b + FP_b \geq p\frac{1-x}{1-y}TP_b + p\frac{1-x}{1-y}FP_b$$

Case 1: $x = y$

$$TP_b \geq -FP_b$$

This is always true, thus for $x = y$ all p values are satisfied.

Case 2: $p \leq \frac{1-x}{1-y}$ and $p \leq \frac{1-y}{1-x}$

$$TP_b \geq -FP_b$$

This is always for all p values that fall within these constraints.

Case 3: $p > \frac{1-x}{1-y}$ or $p > \frac{1-y}{1-x}$

$$TP_b \leq -FP_b$$

This can only be satisfied if both variables are 0, which we do not considered as an acceptable constraint, thus these p-values cannot be satisfied.

C.3 Equal Opportunity and False Omission Rate Parity

$$1 - p + pFN_b + pTN_b \geq \frac{1-x}{1-y}FN_b + \frac{1-x}{1-y}TN_b$$

$$1 - p + \frac{1-x}{1-y}pFN_b + \frac{1-x}{1-y}pTN_b \geq FN_b + TN_b$$

Case 1: $x = y$

$$1 - FN_b \geq TN_b$$

This condition is always satisfied, thus for equal base rates this combination of fairness constraints will also satisfy demographic parity.

Case 2: $p < \frac{1-x}{1-y}$ and $p < \frac{1-y}{1-x}$

$$\frac{(1-p)(1-y)}{p - yp - 1 + x} + FN_b \leq -TN_b$$

$$\frac{(1-p)(1-y)}{p - xp - 1 + y} + FN_b \leq -TN_b$$

Case 3: $p > \frac{1-x}{1-y}$ and thus $p < \frac{1-y}{1-x}$

$$\frac{(1-p)(1-y)}{p - yp - 1 + x} + FN_b \geq -TN_b$$

$$\frac{(1-p)(1-y)}{p - xp - 1 + y} + FN_b \leq -TN_b$$

If $p = 1$ than the fraction in the second equation becomes zero. In order to satisfy that equation under the condition $p = 1$ then it requires $FN_b = TN_b = 0$, which is not acceptable. Thus the equation is not possible but for larger p, the ranges for FN_b and TN_b become undesirable.

Case 4: $p > \frac{1-y}{1-x}$ and thus $p < \frac{1-x}{1-y}$

$$\frac{(1-p)(1-y)}{p-yp-1+x} + FN_b \leq -TN_b$$

$$\frac{(1-p)(1-y)}{p-xp-1+y} + FN_b \geq -TN_b$$

If $p = 1$ than the fraction in the first equation becomes zero. In order to satisfy that equation under the condition $p = 1$ then it requires $FN_b = TN_b = 0$, which is not acceptable. Thus the equation is not possible but for larger p, the ranges for FN_b and TN_b become undesirable.

Case 5: $p = \frac{1-x}{1-y}$

$$\frac{(1-p)(1-y)}{p-xp-1+y} + FN_b \leq -TN_b$$

Case 6: $p = \frac{1-y}{1-x}$

$$\frac{(1-p)(1-y)}{p-yp-1+x} + FN_b \leq -TN_b$$

C.4 Equalised Opportunity and Overall accuracy Equality

$$x - py + TN_b(\frac{1-x-p+2py-xp}{y-x}) \geq TN_a(\frac{1-2x-p+yp+y}{y-x})$$

$$-(y-xp) + TN_a(\frac{1-y-p+2xp-yp}{y-x}) \leq TN_b(\frac{1-2y-p+xp+x}{y-x})$$

Case 1: $x = y$

$$x + TP_b \geq TN_b$$

This statement is always true, thus in this case the p%-rule will always be satisfied for every p.

Case 2: $y > x$

Case 2a: $p \leq \frac{1-2x+y}{1-y} \wedge p \leq \frac{1-2y+x}{1-x}$

$$\frac{(x-py)(y-x)}{1-2x-p+yp+y} + \frac{1-x-p+2py-xp}{1-2x-p+yp+y} TN_b \geq TN_a$$

$$-\frac{(y-xp)(y-x)}{1-2y-p+xp+x} + \frac{1-y-p+2xp-yp}{1-2y-p+xp+x} TN_a \leq TN_b$$

Case 2b: $p \leq \frac{1-2x+y}{1-y} \wedge p > \frac{1-2y+x}{1-x}$

$$\frac{(x-py)(y-x)}{1-2x-p+yp+y} + \frac{1-x-p+2py-xp}{1-2x-p+yp+y} TN_b \geq TN_a$$

$$-\frac{(y-xp)(y-x)}{1-2y-p+xp+x} + \frac{1-y-p+2xp-yp}{1-2y-p+xp+x} TN_a \geq TN_b$$

In the case that $p=1$ then these equations would simplify to $TN_a = TN_b + \frac{x-y}{2}$. While this equation does not violate this relationship will make $TP_a = \frac{1-x}{2}$ and $TP_b = \frac{1-y}{2}$, which is a set value for these variables, making that for $p=1$ this combination is not possible.

Case 3: $y < x$
Case 3a: $p \leq \frac{1-2x+y}{1-y} \wedge p \leq \frac{1-2y+x}{1-x}$

$$\frac{(x-py)(y-x)}{1-2x-p+yp+y} + \frac{1-x-p+2py-xp}{1-2x-p+yp+y} TN_b \leq TN_a$$

$$-\frac{(y-xp)(y-x)}{1-2y-p+xp+x} + \frac{1-y-p+2xp-yp}{1-2y-p+xp+x} TN_a \geq TN_b$$

Case 3b: $p > \frac{1-2x+y}{1-y} \wedge p \leq \frac{1-2y+x}{1-x}$

$$\frac{(x-py)(y-x)}{1-2x-p+yp+y} + \frac{1-x-p+2py-xp}{1-2x-p+yp+y} TN_b \geq TN_a$$

$$-\frac{(y-xp)(y-x)}{1-2y-p+xp+x} + \frac{1-y-p+2xp-yp}{1-2y-p+xp+x} TN_a \geq TN_b$$

In the case that $p=1$ then these equations would simplify to $TN_a = TN_b + \frac{x-y}{2}$. While this equation does not violate this relationship will make $TP_a = \frac{1-x}{2}$ and $TP_b = \frac{1-y}{2}$, which is a set value for these variables, making that for $p=1$ this combination is not possible.

C.5 Equal Opportunity and Treatment Equality

$$1-x-\frac{1-x}{1-y}FN_b + \frac{1-x}{1-y}FP_b \geq p(1-y-FN_b) + pFP_b$$

$$1-y-FN_b+FP_b \geq p(1-x-\frac{1-x}{1-y}FN_b) + p\frac{1-x}{1-y}FP_b$$

Case 1: $p > \frac{1-x}{1-y} \vee p > \frac{1-y}{1-x}$

$$-(1-y)+FN_b \geq FP_b$$

This is impossible to satisfy, so it is not possible to satisfy for $p > \frac{1-x}{1-y}$.
Case 2: $p \leq \frac{1-x}{1-y} \wedge p \leq \frac{1-y}{1-x}$

$$(1-y)+FN_b \leq FP_b$$

This always holds true, so for $p \leq \frac{1-x}{1-y} \wedge p \leq \frac{1-y}{1-x}$ this sets of constraints always satisfies the p%-rule.

C.6 Predictive Equality and Predictive Parity

$$\frac{x}{y}TP_b + \frac{x}{y} \geq pTP_b + pFP_b$$

$$TP_b + FP_b \geq p\frac{x}{y}TP_b + p\frac{x}{y}FP_b$$

Case 1: $p > \frac{x}{y} \vee p > \frac{y}{x}$

$$TP_b \leq -FP_b$$

This is impossible to satisfy, thus $p > \frac{x}{y} \vee p > \frac{y}{x}$ is not possible.

Case 2: $p \leq \frac{x}{y} \wedge p \leq \frac{y}{x}$

$$TP_b \geq -FP_b$$

This always hold true, thus this combinations for constraints always satisfies for $p \leq \frac{x}{y} \wedge p \leq \frac{y}{x}$.

C.7 Predictive Equality and False Omission Rate Parity

$$1 - p + pFN_b + pTN_b \geq \frac{x}{y}FN_b + \frac{x}{y}TN_b$$

$$1 - p + p\frac{x}{y}FN_b + p\frac{x}{y}TN_b \geq FN_b + TN_b$$

Case 1: $p \geq \frac{x}{y} \wedge p < \frac{y}{x}$

$$\frac{y(1-p)}{py - x} + FN_b \geq -TN_B$$

$$\frac{y - yp}{y - xp} - FN_b \geq TN_b$$

The first equation is always simplified so this simplifies to:

$$\frac{y - yp}{y - xp} - FN_b \geq TN_b$$

This equation can only be satisfied for $p = 1$ if $FN_b = 0$ and $TN_b = 0$.

Case 2: $p < \frac{x}{y} \wedge p \geq \frac{y}{x}$

$$\frac{y(1-p)}{py - x} + FN_b \leq -TN_B$$

$$\frac{y - yp}{y - xp} - FN_b \leq TN_b$$

This first equation cannot be satisfied for $p = 1$, unless $FN_b = 0$ and $TN_b = 0$.

Case 3: $p < \frac{x}{y} \wedge p < \frac{y}{x}$

$$\frac{y(1-p)}{py - x} + FN_b \leq -TN_B$$

$$\frac{y - yp}{y - xp} - FN_b \geq TN_b$$

C.8 Predictive Equality and Treatment Equality

$$1 - x - \frac{x}{y}FN_b + \frac{x}{y}FP_b \geq p(1 - y - FN_b) + pFP_b$$

$$1 - y - FN_b + FP_b \geq p(1 - x - \frac{x}{y}FN_b) + p\frac{x}{y}FP_b$$

Case 1: $p \geq \frac{x}{y} \wedge p < \frac{y}{x} \Longrightarrow y \geq x$

$$y - y(\frac{1-p}{x - py}) + FN_b \geq FP_b$$

$$y - y(\frac{1-p}{y-px}) + FN_b \leq FP_b$$

The first equation is always satisfied. The second equation cannot be satisfied for $p = 1$ unless $FN_b = 0$ and $FP_b = y$

Case 2: $p \geq \frac{y}{x} \wedge p < \frac{x}{y} \implies x \geq y$

$$y - y(\frac{1-p}{x-py}) + FN_b \leq FP_b$$

$$y - y(\frac{1-p}{y-px}) + FN_b \geq FP_b$$

The second equation is always satisfied. The first equation cannot be satisfied for $p = 1$ unless $FN_b = 0$ and $FP_b = y$

Case 3: $p < \frac{y}{x} \wedge p < \frac{x}{y}$

$$y - y(\frac{1-p}{x-py}) + FN_b \leq FP_b$$

$$y - y(\frac{1-p}{y-px}) + FN_b \leq FP_b$$

C.9 Predictive Parity and Overall accuracy Equality

$$(FP_a - pFP_b)(\frac{y-x}{FP_a - FP_b}) \geq -2(FP_a - pFP_b)$$

$$(FP_b - pFP_a)(\frac{y-x}{FP_a - FP_b}) \geq -2(FP_b - pFP_a)$$

Case 1: $FP_a \geq pFP_b \wedge FP_a \geq FP_b \wedge FP_b \geq pFP_a$

$$\frac{y-x}{2} + FP_a \geq FP_b$$

If $y \geq x$ then this statement is always true. For $p = 1$ the constraints would require that $FP_a = FP_b$.

Case 2: $FP_a \geq pFP_b \wedge FP_a \geq FP_b \wedge FP_b \leq pFP_a$

$$\frac{y-x}{2} + FP_a \geq FP_b$$

$$\frac{y-x}{2} + FP_a \leq FP_b$$

This simplifies to:

$$\frac{y-x}{2} + FP_a = FP_b$$

Case 3: $FP_a \geq pFP_b \wedge FP_a \leq FP_b \wedge FP_b \geq pFP_a$

$$\frac{y-x}{2} + FP_a \leq FP_b$$

Case 4: $FP_a \leq pFP_b \wedge FP_a \leq FP_b \wedge FP_b \geq pFP_a$

$$\frac{y-x}{2} + FP_a \geq FP_b$$

$$\frac{y-x}{2} + FP_a \leq FP_b$$

This simplifies to:

$$\frac{y-x}{2} + FP_a = FP_b$$

C.10 Predictive Parity and Treatment Equality

$$1 - x - p + px + (p - \frac{1-x}{1-y})FN_b \geq (p - \frac{1-x}{1-y})FP_b$$

$$1 - y - p + xp + (p\frac{1-x}{1-y} - 1)FN_b \geq (p\frac{1-x}{1-y} - 1)FP_b$$

Case 1: $p > \frac{1-x}{1-y} \wedge p \leq \frac{1-y}{1-x}$

$$-(1-y) + FN_b \geq FP_b$$

$$-(1-y) + FN_b \leq FP_b$$

The first equation cannot be satisfied, it is thus impossible to satisfy the p%-rule in these conditions.

Case 2: $p \leq \frac{1-x}{1-y} \wedge p > \frac{1-y}{1-x}$

$$-(1-y) + FN_b \leq FP_b$$

$$-(1-y) + FN_b \geq FP_b$$

The second equation cannot be satisfied, it is thus impossible to satisfy the p%-rule in these conditions.

Case 3: $p \leq \frac{1-x}{1-y} \wedge p \leq \frac{1-y}{1-x}$

$$-(1-y) + FN_b \leq FP_b$$

This equation is always satisfied for all values for p.

C.11 False Omission Rate Parity and Overall Accuracy Equality

$$\frac{1}{1 - FN_b - TN_b} - (\frac{FN_b + TN_b}{1 - FN_b - TN_b})(1 + \frac{x+y}{TN_b - FN_b}) \geq p$$

$$1 - FN_b - TN_b \geq p(1 - (FN_b + TN_b)(1 + \frac{x-y}{TN_b - FN_b})$$

In the case that $p = 1$ this equation will simplify to:

$$-TN_b \geq FN_b$$

$$FN_b \geq -TN_b$$

These equations can only be satisfied for $TN_b = 0$ and $FN_b = 0$, however this is not possible due to the original formulation of the equations.

C.12 False Omission Rate Parity and Treatment Equality

$$1 - p + (p - \frac{x}{y})FN_b \geq -(p - \frac{x}{y})TN_b$$

$$1 - p + (p\frac{x}{y} - 1)FN_b \geq -(p\frac{x}{y} - 1)TN_b$$

Case 1: $p \geq \frac{x}{y} \land p < \frac{y}{x}$

$$-\frac{y - yp}{x - yp} + FN_b \geq -TN_b$$

$$-\frac{y - yp}{y - xp} + FN_b \leq -TN_b$$

This first equation is always satisfied as $p > \frac{x}{y}$, the second equation however cannot be satisfied for $p = 1$, unless $FN_b = 0$ and $TN_b = 0$.

Case 2: $p < \frac{x}{y} \land p \geq \frac{y}{x}$

$$-\frac{y - yp}{x - yp} + FN_b \leq -TN_b$$

$$-\frac{y - yp}{y - xp} + FN_b \geq -TN_b$$

This second equation is always satisfied as $p > \frac{y}{x}$, the first equation however cannot be satisfied for $p = 1$, unless $FN_b = 0$ and $TN_b = 0$.

Case 3: $p < \frac{x}{y} \land p < \frac{y}{x}$

$$-\frac{y - yp}{x - yp} + FN_b \leq -TN_b$$

$$-\frac{y - yp}{y - xp} + FN_b \leq -TN_b$$

C.13 Overall accuracy Equality and Treatment Equality

$$\frac{1 - x - p + yp}{1 - p} + FP_b \geq FN_b$$

$$\frac{1 - y - p + xp}{1 - p} + FP_b \geq FN_b$$

These equations hold for all cases.

In the special case that p goes to 1, then the first equation requires $x \geq y$ for the inequality to hold, however the second equation would require $y \geq x$. Thus these equations can only hold for $p = 1$ if $x = y$.

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