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Fractional order MRAC control design for a lightning system based on a fractional order second degree model

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Abstract: This paper presents a new adaptive fractional-order MRAC controller design for the class of non-integer second-order systems. The update of the control law gains follows a derivative of fractional order equal to the order of the controlled system model. The stability analysis of the control system has been performed based on Lyapunov theorem. We apply this control law to an experimental multivariable lightning system. The results obtained show the efficiency and performance improvement of the proposed control solution compared to the classical MRAC.

Keywords: Fractional order system, adaptive control, MRAC, FOMRAC, Lightning system.

1. INTRODUCTION

Since (Vinagre et al., 2002) and (Ladaci & Charef, 2002, 2006) first works on fractional-order adaptive control, and in particular on the fractional MRAC (Model Reference Adaptive Control) approach, a great deal of work has been done on these topics, with increasingly diversified approaches and applications (Ladaci & Charef, 2012).
Most of the research works have focused on the state-space MRAC approach, which can be very efficient and more accurate, and which easily allows the application of Lyapunov's stability theorem for its analysis (Murgaš et al., 1992) because for many systems such as power systems and robot manipulators, all state information can be measured or observed in real time. Moreover, we have become able to ensure the stability of certain complex systems by introducing fractal-order operators for process modeling and/or closed-loop control, while achieving better temporal performance and robustness (Ladaci et al., 2006, 2008; Balaska et al., 2020).
The main reason for this growing interest of the control engineering community for FOMRAC could be justified by the following four facts (Muresan & Ionescu, 2020):

- Many physical systems are described with more accuracy by fractional-order models rather than the classical 'integer-order' ones.
- Fractional order operators are able to improve the closed-loop system dynamics,
- Fractional-order systems are naturally more robust against disturbances and noise.

Many FOMRAC control configuration have been developed so far for different classes of linear, nonlinear, integer and fractional-order systems (Bourouba et al., 2019; Chi et al., 2022; Mukherjee et al., 2022).
These adaptive control schemes have been applied to a wide range of applications like in medicine (Navarro-Guerrero & Tang, 2018), liquid level (Balaska et al., 2018), magnetic levitation system (Tepljakov et al., 2018), vehicle suspension (Khelas et al., 2020), aeronautics (Yynineb & Ladaci, 2022), etc...
In this paper, a direct FOMRAC synthesis is introduced to deal with the class of second degree non-integer order systems represented in the state space domain with a state matrix having different eigenvalues. The proposed adaptive controller is applied to the control of a Laboratory lightning system (Juchem et al., 2019). Event-based control application could save energy since the light is switched on just when someone enters Birs et al. (2020), but in the presence of external light such as sunlight, adaptive control is best. We show that using this FOMRAC controller allow the system stabilization of the lightning system.

The remainder of this paper is organized as follows: In Section 2 basic definitions and concepts of fractional order systems are presented. In Section 3, the main control problem is detailed and the proposed state space FOMRAC scheme is detailed. In Section 4 the proposed FOMRAC controller is applied to the multiple lightning system and the results are discussed. Finally, some concluding comments are given in Section 5.

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2. BASIC CONCEPTS

This section will present some important results on fractional order systems. We will be particularly interested in the definitions of Caputo and Grünwald-Leitnikov for fractional-order derivative which are among the most used as well as the numerical version of Grünwald-Leitnikov which we will use in the implementation of our proposed FOMRAC command. A reminder about relevant stability theory is also included.

2.1 Fractional-order derivative definition

The fractional-order derivative of order η in the sens of Caputo is given by the formula:

$$ {}_0^C D_t^\eta f(t) = \frac{1}{\Gamma(n-\eta)} \int_0^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\eta-n+1}} d(\tau) \tag{1}$$

where the real number η satisfies: $n-1<\eta<n$, $n \in N$ and $\Gamma(\eta)=\int_0^\infty x^{\eta-1}e^{-x}dx$, is the Euler function.
The derivative of order η in the sens of Grünwald-Letnikov is expressed as:

$$ {}_0^{GL} D_t^\eta f(t) = \lim_{h\to 0} \frac{1}{h} \sum_{j=0}^{k} \omega_j^{(\eta)} f(kh-jh) \tag{2}$$

where h is the sampling period, and the coefficients $\omega_j^{(\eta)}$ are as follows:

$$ \omega_j^{(\eta)} = \frac{(-1)^j \Gamma(\eta+1)}{\Gamma(j+1)\Gamma(\eta-j+1)}, \qquad j=0,1,...k \tag{3}$$

2.2 Approximation of Grünwald-Leitnikov derivative

The numerical approximation of the fractional order integral for a causal function $\phi(t)$ and for time operator $t=kh$, is computed as follows (Podlubny, 1999):

$$ \begin{aligned} I_t^\eta \phi(t) &= D_t^{-\eta}\phi(t) \\ &= h^\eta \sum_{j=0}^{k} \omega_j^{(-\eta)} \phi(kh-jh), \qquad j=0,1,...k \end{aligned} \tag{4}$$

where $\omega_j^{(-\eta)}$ is computed from equation (3).

2.3 Stability theorem

Assuming a fractional-order time varying nonlinear system expressed with the Caputo derivative as:

$$ {}_{t_0}^C D_t^\eta f(t) = f(x,t) \tag{5}$$

such that $\eta \in (0,1)$. Then we have a very useful stability result for the system (5) (Duarte-Mermoud et al., 2015):

Definition 2.1. We say that a continuous function $g : [0,t) \to [0,\infty)$ is a class-K function if it is strictly increasing and $g(0)=0$.

And the theorem (Li & Podlubny, 2010),

Theorem 2.2. (Extension of Lyapunov direct method to fractional-order).
Consider a non autonomous fractional order system (5), and let $x=0$ be its equilibrium point. Suppose the existence of a Lyapunov function $V(t,x(t))$ and the class $-K$ functions $g_i (i=1,2,3)$ that satisfy,

$$ g_1(\|\xi\|) \le V(t,\xi(t)) \le g_2(\|\xi\|) \tag{6}$$

$$ {}_{t_0}^C D_t^\beta V(t,\xi(t)) \le -g_3(\|\xi\|) \tag{7}$$

with $\beta \in (0,1)$, then the system (5) is asymptotically stable.

For the stability analysis of the proposed control scheme we need the following lemma (Duarte-Mermoud et al., 2015):

Lemma 2.3. If the function $x(t) \in \Re^n$ is continuous and derivable, with $Q=Q^T \ge 0 \in \Re^{n\times n}$ then, for any time instant t_0

$$ \frac{1}{2} {}_{t_0}^C D_t^\eta x^T(t) Q x(t) \le x(t) Q {}_{t_0}^C D_t^\eta x(t), \qquad \forall \eta \in (0,1) \tag{8}$$

3. FOMRAC CONTROL DESIGN

In this paper, we are studying a fractional order scalar system given by

$$ \begin{cases} D^\alpha x = Ax + Bu \\ y = Cx \end{cases} \tag{9}$$

where, α is a real number lying in the interval $(0,1)$, y is the system output, u is the input, and $A \in \mathsf{R}^{(n\times n)}$ $B \in \mathsf{R}^{(1\times n)}$ and $C \in \mathsf{R}^{(n\times 1)}$ are constant plant parameters that are assumed to be unknown, and are given as follows:

$$ A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -a_0 & -a_1 & \dots & -a_{n-2} & -a_{n-1} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, \tag{10}$$

$$ C = [1\ 0\ \dots\dots\ 0] $$

Remark 1. The state Matrix A is supposed to have eigenvalues that are all different.
This Hypothesis means that it is possible to decouple the system directly to a diagonal (real or complex) one by making use of congruence or strict equivalence transformations to A (Lancastera & Zaballa, 2009).

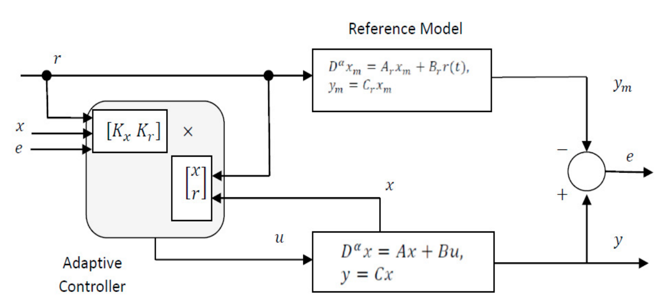

Fig. 1. FOMRAC Adaptive control configuration.

A reference model is chosen as:

$$ \begin{cases} D^\alpha x_m = A_r x_m + B_r r \\ y_m = C_r x_m \end{cases} \tag{11}$$

where $A_r \in \mathsf{R}^{(n\times n)}$ $B_r \in \mathsf{R}^{(1\times n)}$ and $C_r \in \mathsf{R}^{(n\times 1)}$ are constant parameters and $r(t)$ is a bounded external reference signal.
We define the tracking error,

$$e = y - y_m \tag{12}$$

We propose an adaptive feedback control scheme using a fractional adaptation law to make the fractional-order plant (9) track the fractional order reference model (11), as follows (Khelas et al., 2024),

$$u = K_x x + K_r r \tag{13}$$

where $K_x \in \mathsf{R}^{(1\times n)}$ and $K_r \in \mathsf{R}$ are the adaptive gains such that,

$$D^{\alpha} K = -\gamma B^T P e x_a^T \tag{14}$$

where $K = [K_x \ K_r]$, $x_a = \begin{bmatrix} x \\ r \end{bmatrix}$, the matrix P for which the columns are the eigenvectors of the state matrix A and γ, a positive scalar, is the adaptation gain. The overall adaptive controller configuration is represented in Fig. 1.
The stability analysis of this proposed adaptive control scheme is given in the following theorem,

Theorem 3.1. Consider the FOS of (9), and the reference model (11) then the MRAC controller given by (13) and (14) stabilizes the closed loop system and all the error signal (12) converge to zero.

The proof of this theorem is given in (Khelas et al., 2024).

4. APPLICATION TO LIGHTNING SYSTEM CONTROL

The proposed fractional-order adaptive control designed for a class of second degree systems in the state space domain will be applied to the lightning system setup at the Laboratory of Gent University (Chi et al., 2022; Juchem et al., 2018).

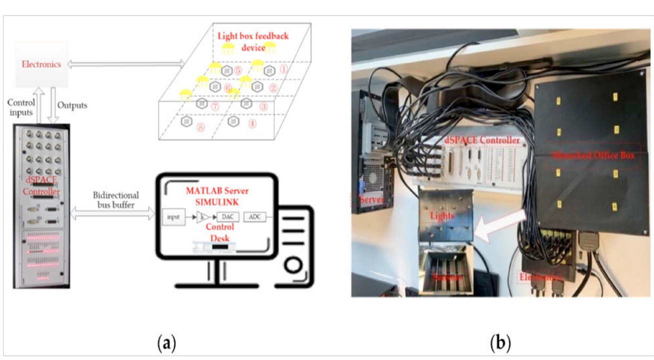

Fig. 2. Lightning system experimental setting.

4.1 Lightning system model

The coupled model with partial interaction (see Fig. 2) is represented by the overall transfer function:

$$G_P(s) = T(s)C_P(s)Q(s) \tag{15}$$

where,

$$Q(s) = 10^{-4}x(s)^4 - 8\,10^{-3}x(s)^3 + 9\,10^{-2}x(s)^2 + 0.2284\,x(s) \tag{16}$$

$$T(s) = \frac{3726}{s^2 + 124.1\,s + 3726} \tag{17}$$

and

$$C_p(s) = \begin{bmatrix} 1.28 & 0.84 & 0.16 & 0.09 & 0.30 & 0.23 & 0.12 & 0.08 \\ 0.35 & 1.13 & 0.39 & 0.10 & 0.15 & 0.24 & 0.18 & 0.07 \\ 0.11 & 0.41 & 1.14 & 0.45 & 0.08 & 0.17 & 0.29 & 0.20 \\ 0.05 & 0.12 & 0.33 & 1.35 & 0.04 & 0.09 & 0.19 & 0.31 \\ 0.06 & 0.47 & 0.11 & 0.07 & 1.18 & 0.69 & 0.14 & 0.07 \\ 0.22 & 0.59 & 0.19 & 0.07 & 0.41 & 1.10 & 0.38 & 0.09 \\ 0.09 & 0.23 & 0.62 & 0.25 & 0.13 & 0.74 & 1.09 & 0.40 \\ 0.05 & 0.09 & 0.27 & 0.77 & 0.05 & 0.12 & 0.37 & 1.24 \end{bmatrix} \tag{18}$$

4.2 Applying the fractional order MRAC on the lightning system model

Reference model Since the lightning system can be approximated into a 1st degree like fractional order system

$$T_a(s) = \frac{1}{(0.035\,s + 1)^{\alpha}} \tag{19}$$

We choose a faster system as the reference model:

$$T_r(s) = \frac{1}{(0.001\,s + 1)^{\alpha}} \tag{20}$$

Introduce the error defined as:

$$e = y - y_m$$

The control law is given by:

$$u = Ky + K_r r$$

where the gain matrices are updated as follows:

$$\begin{cases} K^{\alpha} = -\gamma\, e\, y\, sign(b) \\ K_r^{\alpha} = -\gamma\, e\, r\, sign(b) \end{cases}$$

where γ is the adaptation gain.

4.3 Numerical Simulation results

For the following experiment we choose the fractional order $\alpha = 1.2$ and set the adaptation gain as $\gamma = 1.5$.

Application to the first zone only The FOMRAC adaptive controller is applied to the 1st zone, and the light intensities were as follows:

We maintain all the light intensities per zone on a constant value but not for zone 1 and zone 5. Zone 1 introduces a step to the system at $0.5s$ from 32 % to 34 % and zone 5 sees a step from 32 % to 36 % at $2.5s$. Zone 2, 3 and 6 are maintained at 34 %. Zone 4 and 8 are maintained at 30 %. Zone 7 sees an input of 32 %. The experiment is executed for partial interaction.
The simulation results of the lightning system are presented in Fig. 3 for the outputs and Fig. 4 for the control signals.
The fractional order MRAC results in faster dynamics with an overshoot compared to the conventional MRAC. Looking at the control-effort the fractional order MRAC requires less effort. Overall,The fractional order MRAC performs well with good tracking and minimize the control effort.

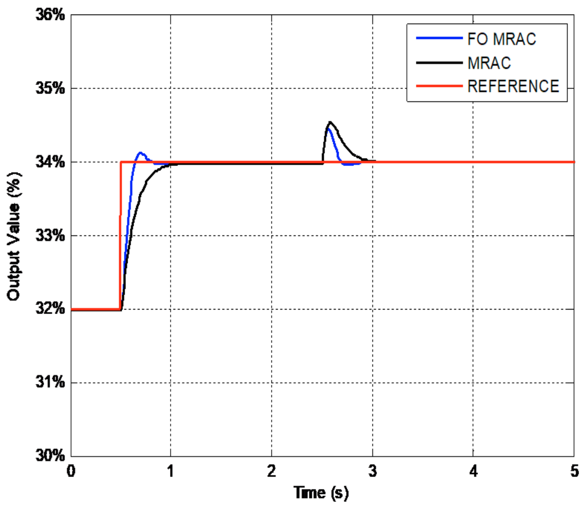

Fig. 3. Comparative responses of the lightning system.

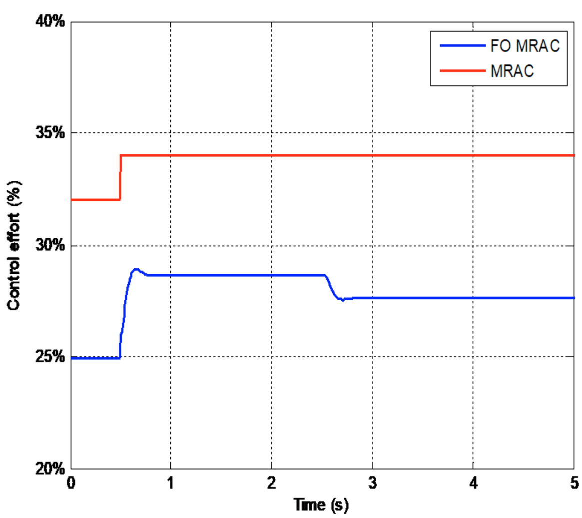

Fig. 4. Comparative control signals of the lightning system.

Application to all the 8 zones Now, the proposed controller is applied on the 8 zones, one FOMRAC controller for each zone and the obtained results are illustrated in Fig. 5 for the outputs and Fig. 6 for the control efforts.

In order to compare the performance of the proposed FOMRAC controller against the classical MRAC, we will compute the ITAE criterion for every zone.
The ITAE objective function is given by the formula,

$$J_{ITAE} = \int_{0}^{\infty} t\,|e(t)|\,dt \tag{21}$$

where $e(t)$ is the error between the set point and the controlled variable.
Table 1 summarizes The obtained results.

Based on these results it is obvious that the proposed control strategy is able to reduce the error between the output and the desired response in all the lightning zones. This means a performance of the MRAC controller is

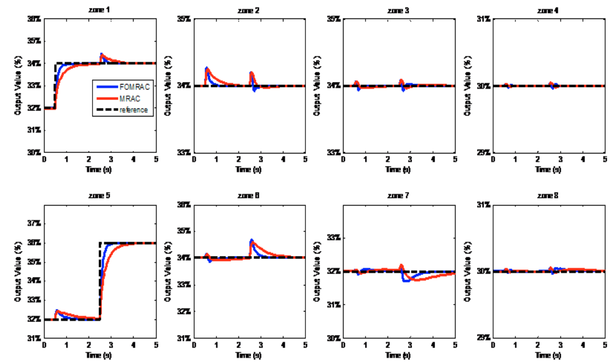

Fig. 5. Output of the eight zones for fractional and integer MRAC.

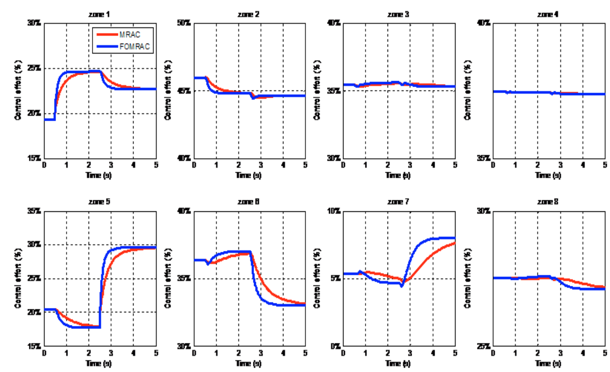

Fig. 6. Control effort of the eight zones for fractional and integer order MRAC.

Table 1. Comparison between ITAE FOMRAC (J_f) and MRAC (J_i) controllers results

	1	2	3	4	5
J_f	11387	1698.1	1041.9	855.63	5215.8
J_i	14318	2462.2	1325.7	1192.9	10087
	6	7	8		
J_f	2536.6	2431	887.6		
J_i	4550.1	5197.5	1230.9		

improved by introducing fractional-order operators in the control configuration.

4.4 Experimental results and discussion

In this section we will present the experimental results obtained by applying the control simulated in the previous subsection to the experimental test bench of the lightning system setup at the Laboratory of Gent University
. For the same parameter values and fractional order, we obtain figure 7 and figure 8 for the output and control efforts of the entire lighting system response.

Figure 9 and figure 10 show the comparison of output responses and control efforts resp. obtained by FOMRAC and conventional MRAC in the zone 1.

We can clearly see from the figures 7 and 9 that the FOMRAC proposed here improves responses compared with the traditional MRAC.
Firstly, FOMRAC demonstrates a superior response in

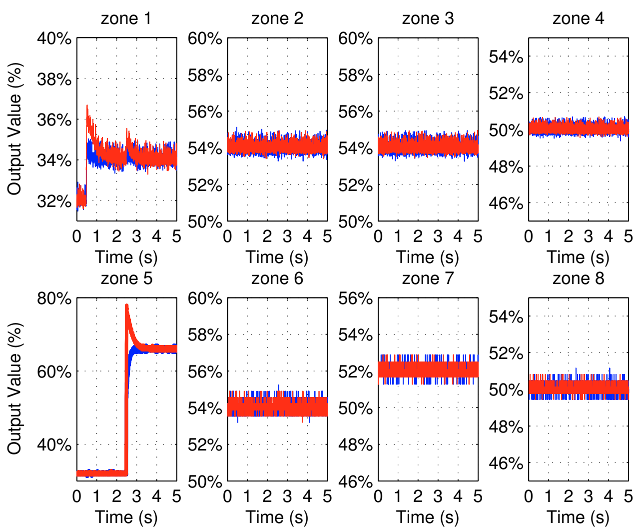

Fig. 7. Experimental output value for all zones controlled with FOMRAC and MRAC.

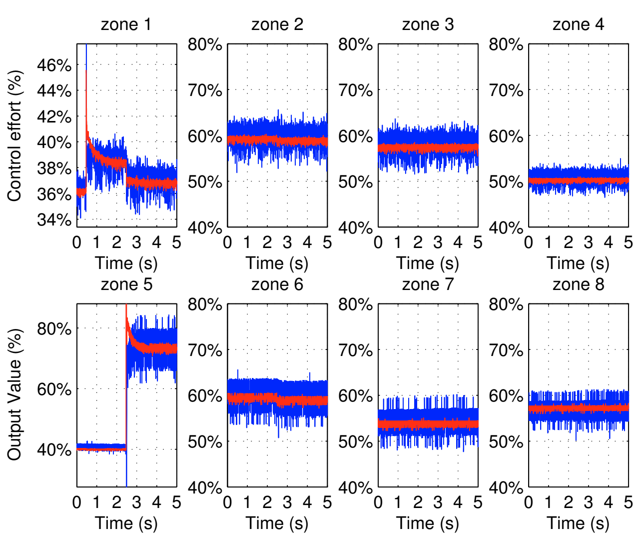

Fig. 8. Control effort for all zones controlled with FOMRAC and MRAC.

terms of speed and overshoot reduction compared to MRAC. The faster response time of FOMRAC ensures quicker adaptation to changing environmental conditions or user preferences, while concurrently minimizing overshoot, thereby enhancing system stability.

Moreover, the capability of FOMRAC to effectively reject disturbances, particularly those arising from inter-zonal interactions within the lighting system, surpasses that of MRAC. This attribute is crucial in maintaining consistent illumination levels across different zones, ensuring optimal functionality despite external influences acting as noise or disturbances (Ionescu et al., 2020).

Interestingly, while both FOMRAC and MRAC exhibit comparable levels of control effort in figure 8 and figure 10, FOMRAC demonstrates a slight advantage in terms of consistency and faster adjustments. Despite minor variations in control effort amplitudes between the two methods, the overall performance of FOMRAC remains

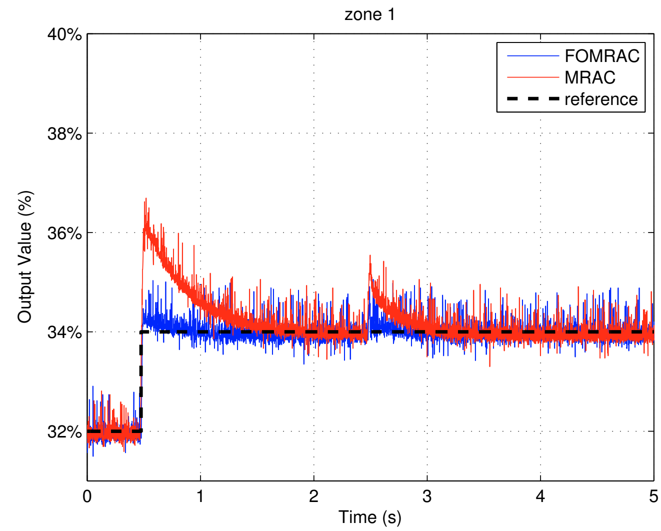

Fig. 9. Experimental output value for zone 1 controlled with FOMRAC and MRAC.

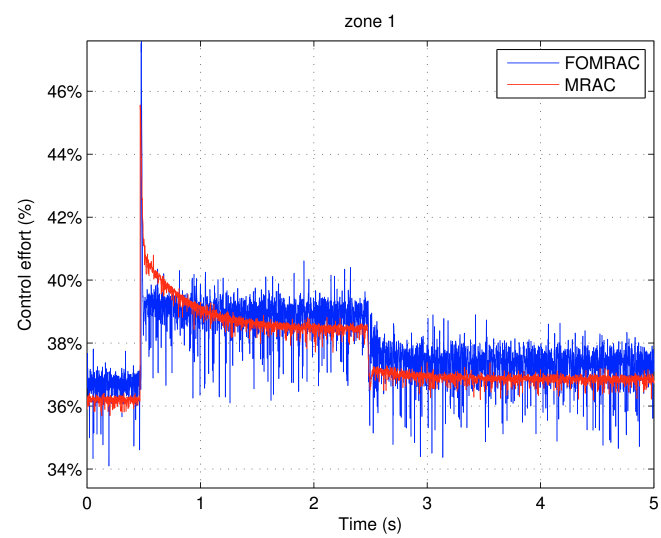

Fig. 10. Control effort for zone 1 controlled with FOMRAC and MRAC.

commendable, with a more rapid plot and minimal fluctuations.

5. CONCLUSION

In conclusion, the application of both FOMRAC and MRAC to the MIMO lighting system has provided valuable insights into their performance characteristics. Through extensive analysis and experimentation, it has become evident that FOMRAC exhibits distinct advantages over MRAC in several key aspects.

In summary, the comparison between FOMRAC and MRAC in the context of the MIMO lighting system underscores the superiority of FOMRAC in providing faster response times, reduced overshoot, superior disturbance rejection, and relatively consistent control efforts. These findings advocate for the adoption of FOMRAC as a more favourable control strategy for MIMO lighting systems, offering improved performance and resilience against disturbances for enhanced operational efficiency and user satisfaction. However, further investigation is needed to fully understand the behaviour of the system and to accurately model the interactions between the different zones.

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