Int. J. Production Economics 277 (2024) 109354

Contents lists available at ScienceDirect

International Journal of Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Modeling and solving integrated assembly line balancing, assembly line feeding, and facility sizing problems

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A R T I C L E I N F O

Dataset link: Generated assembly line balancing precedence graphs from automotive OEM (Original data)

Keywords:
Assembly system planning
Production
Optimization
Logic-based bender's decomposition

A B S T R A C T

The individual research domains of assembly line balancing and feeding have received considerable attention in recent years, furthered by a continuing trend towards mass customization. This research extends the limited literature on the simultaneous consideration of line balancing and feeding while substantially broadening the scope concerning the number of assembly line feeding policies and the incorporation of facility sizing decisions. This large-scale problem is modeled using mathematical programming techniques and solved using a logic-based Benders' decomposition. We used a combination of real-world and re-engineered data from a car manufacturer to conduct numerical experiments. The major findings reveal that integrated decision-making may lead to a substantial cost reduction of up to 20% in this case. Furthermore, the study explores subsets of feeding policies to reduce the amount of different material flows within the factory while considering their associated costs. Our findings reveal a surprising difference in the importance of individual line feeding policies, specifically identifying boxed-supply as a pivotal policy for ensuring feasibility and reducing costs.

1. Introduction

Over the past decades, economic growth and globalization have promoted a surge in demand for high-priced products such as cars. At the same time, many economies are becoming more saturated. These trends resulted in a shift towards mass customization that considers customers' demands and preferences. Mass customization forces companies to streamline and adjust their manufacturing capabilities. In many companies, such as automobile manufacturers, core activities consist of assembly and in-house logistics that support assembly activities. The planning of assembly activities, known as assembly line balancing problems (ALBP), has been investigated for over half a century (Salveson, 1955). In contrast, assembly line feeding problems (ALFP), i.e., the supporting logistical activities, have started receiving academic attention much later (Bozer and McGinnis, 1992). Nevertheless, both research areas have evolved to support the industry's transition from classical mass production towards mass-customized production.

ALBPs concern the optimal assignment of assembly tasks to workstations while optimizing some metric, e.g., minimizing the number of workstations. To this end, ALBPs consider task time requirements and task precedence relations. When minimizing the number of stations, also known as simple assembly line balancing problem 1 (SALBP-1), the cycle time, i.e., the time available at each workstation, is predetermined, based on product demand, and only time and precedence requirements are considered. The sum of task times assigned to one station must be at most this cycle time. ALFPs are concerned about decisions regarding the provision of assembly materials. Typically, space restrictions and cost considerations heavily impact its decision-making. Five distinct line feeding policies have emerged over the years. In *line stocking*, e.g., a large container or pallet filled with a single type of stock-keeping unit (SKU) is provided. Line stocking tends to consume more space but requires no additional handling. *Sequencing*, in contrast, describes the provision of functionally similar but distinct parts in a single container. These parts are sorted according to the production sequence.

In practice, assembly system planning consists of two phases. First, the assembly line is balanced before the resulting line balance is used to make feeding decisions in the second step. However, both ALFP and ALBP are interdependent for two reasons: (*i*) The assignment of tasks in an ALBP determines the parts needed at each station. Since this serves as a starting point for feeding decisions in a hierarchical approach, it may impose space-saving but expensive feeding policies. When solving both problems simultaneously, one may balance the line

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https://doi.org/10.1016/j.ijpe.2024.109354
Received 5 January 2024; Received in revised form 13 May 2024; Accepted 31 July 2024
Available online 8 August 2024

differently to diminish the use of expensive feeding policies. Therefore, suboptimal balancing solutions may result in optimal global solutions; (ii) Within the ALFP, assembly parts are assigned to feeding policies. Additionally, the positioning of parts at the workstation's border of line, i.e., the associated storage space, may be determined. Both affect a task's duration as some feeding policies may require searching activities, and the walking duration for the operator may vary with a part's placement. Therefore, a joint approach overcomes simplifying assumptions in classic assembly line balancing problems, allowing for more precise and efficient planning. This work aims to incorporate this interdependence by considering both problems simultaneously. To our knowledge, assembly line feeding optimization models always assume a given and static warehouse size. However, when setting up a new production facility, one may define the size of its warehouse based on the actual requirements. This sizing determines transportation distances for part replenishment and overall efforts and costs. Sizing likewise impacts facility rent or investment costs.

With this research, we propose a framework that simultaneously tackles assembly line balancing and feeding. It also considers the sizing of the assembly and logistics facilities, which relies on balancing and feeding decision-making. This decision-making interdependence can be seen in the examples given in Fig. 1: When utilizing only simple line feeding policies such as line stocking, one may require more storage space near the assembly station but very little or no space for preparation activities (as depicted in Fig. 1(a)). However, one may require more workstations due to the space requirements and long walking times. Using a combination of feeding policies may reduce the number of workstations at the cost of additional space requirements and preparation activities taking place in the preparation area (see Fig. 1(b)). This study investigates specific implementations of the balancing and feeding problem and a facility sizing approach. However, one may change the individual components of this work, e.g., by adding additional line feeding policies or balancing considerations such as U-shaped assembly lines or two-sided assembly lines.

While few studies have already approached the integration of assembly line feeding and assembly line balancing (Sternatz, 2015; Battini et al., 2017; Calzavara et al., 2021), this study includes various novelties. (i) We increased the number of line feeding policies under investigation to five; (ii) the proposed planning approach determines the exact positioning of parts at their corresponding workstation, and therefore, exact walking times for balancing the line more efficiently; (iii) facility sizing is incorporated to determine space and transportation costs; and (iv) an exact solution methodology, i.e., a logic-based Benders' Decomposition framework, is proposed to solve this problem.

The remainder of this paper is structured as follows: After motivating the joint optimization of assembly line balancing, feeding, and facility sizing, we discuss the literature on both individual research domains in the following section. Besides, we discuss studies on the integration of both topics. In Section 3, we first explain the planning environment, i.e., the type of assembly system under investigation, and the scope of our investigation before proposing a logic-based Bender's decomposition framework for this problem. To this end, we present cost calculations for this type of problem. Section 5 provides computational and managerial insights. Lastly, we summarize and showcase ideas for further research directions in the last section.

2. Literature review

Within this section, we provide a limited overview of assembly line balancing and feeding problems. Furthermore, studies on the integration of both problems are discussed. We refer the interested reader to Becker and Scholl (2006), Boysen et al. (2007), Battaïa and Dolgui (2013), and Battaïa and Dolgui (2022) for comprehensive reviews of the assembly line balancing literature and to Schmid and Limère (2019) for a review on assembly line feeding literature.

2.1. Assembly line balancing

The origin of modern assembly lines goes back to the assembly line's invention and Ford's Model T at the beginning 20th century. Academics started studying this topic only half a century later (Salveson, 1955). However, interest in this topic is vast. The fundamental problem variants, assigning tasks to stations and minimizing either the number of workstations or cycle time, have been classified as simple assembly line balancing problems (SALBPs) (Baybars, 1986). Other problem types, including various extensions, have been labeled generalized assembly line balancing problems (GALBPs) (Baybars, 1986). Many studies are concerned with the proposal and improvement of exact solution methods for the simple assembly line balancing problem with a fixed cycle time and minimizing the number of stations. Bowman (1960) was the first author to propose a linear programming-based solution approach for assembly line balancing problems. Many studies followed this mathematical programming approach using techniques such as Lagrangian relaxation (Aghezzaf and Artiba, 1995) or formulation improvements (Patterson and Albracht, 1975). Scholl and Klein (1997) proposed a Branch-and-Bound procedure for SALBP-1, which still outperforms many other approaches. The most efficient solution procedure for this problem has been proposed by Sewell and Jacobson (2012), Morrison et al. (2014), using an extended Branch-and-Bound method. Due to the difficulty of this $\mathcal{NP}$-hard problem, many researchers studied heuristic approaches. Fleszar and Hindi (2003) provide enumerative heuristic approaches, i.e., an iterative rule-based assignment that alters some search-determining parameters in each iteration. This heuristics is an extension of the well-known Hoffmann heuristic (Hoffmann, 1963). Recently, Sternatz (2014) extended the heuristic of Fleszar and Hindi (2003) to incorporate more practical considerations such as multiple workers per station or assignment restrictions. Asides from computational aspects, many studies are concerned with the investigation of GALBPs that consider many more balancing aspects. One of these extensions is concerned with the necessity to provide sufficient storage space at the workstations (Bautista and Pereira, 2007; Bautista et al., 2013; Chica et al., 2016). In these studies, each task requires some storage space similar to our study. However, as discussed above, the exact amount of space depends also on other decisions, namely line feeding decisions. Bartholdi (1993) consider two-sided assembly lines, i.e., workers assemble parts on both sides of the product. In reality, the assembly of different products may vary for specific products. Bartholdi III et al. (2001) investigate assembly lines with these properties and propose the use of bucket brigades to balance assembly lines more efficiently. More recently, an increasing number of studies focus on human factors such as ergonomics (Battini et al., 2016) or task complexity (Zeltzer et al., 2017).

2.2. Assembly line feeding

Even though early assembly lines such as the above-mentioned Ford Model T assembly line required various parts, it is unknown when logistics engineers started considering various material supply options. However, the advent of lean manufacturing systems in the 1970s certainly spurred the use of Just-in-Time and Just-in-Sequence part feeding. Nowadays, five line feeding policies are distinguished (Schmid and Limère, 2019): (i) line stocking, i.e., the provision of a container or pallet as received from the supplier or preceding production stage; (ii) boxed-supply (also known as kanban), i.e., a repacking of parts into smaller bins which assembly operators retrieve from flow racks; (iii) sequencing, i.e., the provision of presorted multiple part variants of functionally equivalent but distinct parts (known as part families); (iv) stationary kitting, i.e., the provision of multiple specific parts, required at a specific workstation for the production of a specific product. These parts belong to various part families and (v) traveling kitting. Traveling kitting is almost equivalent to stationary kitting. However, a traveling

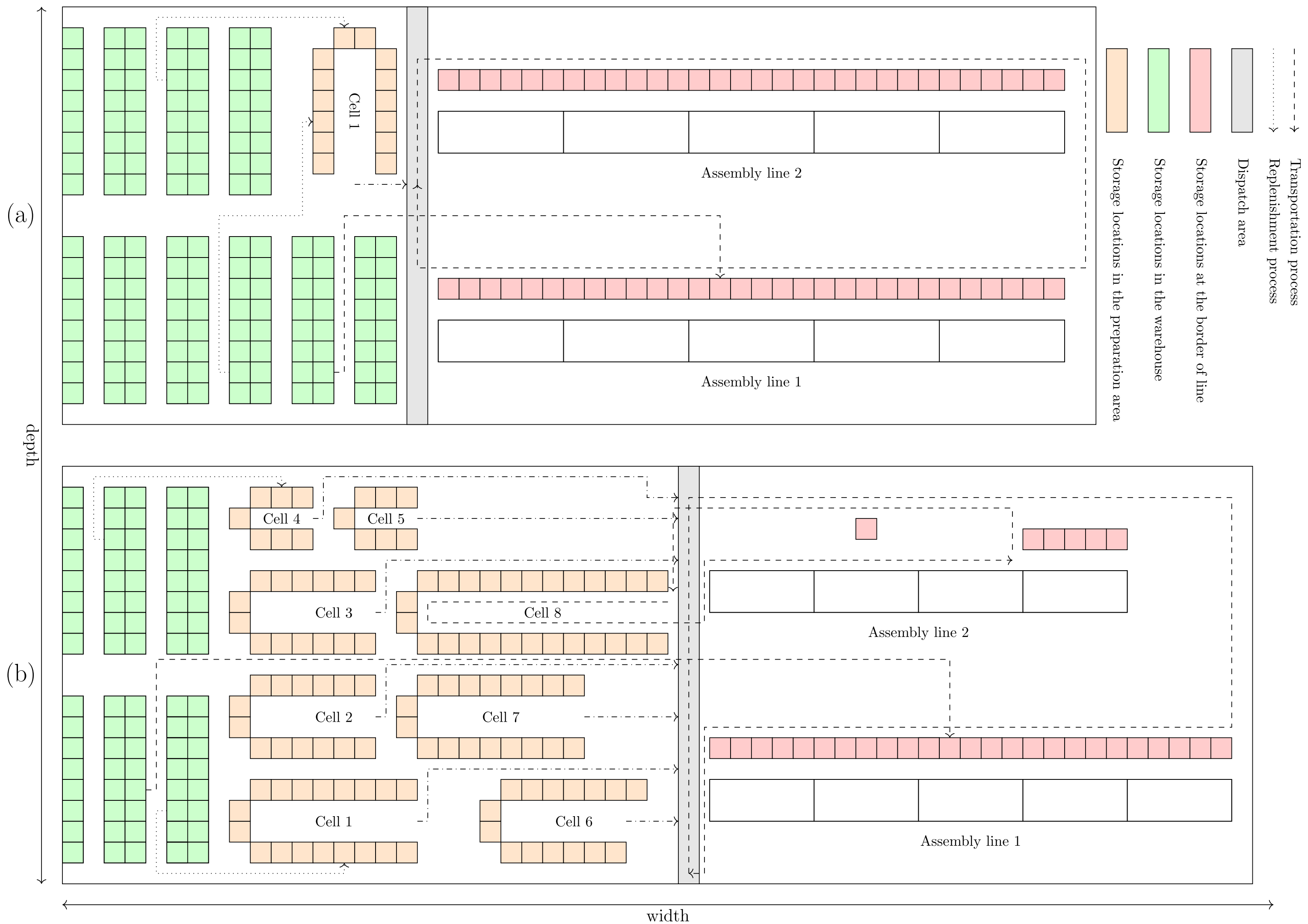

Fig. 1. Research and decision making scope of this study.

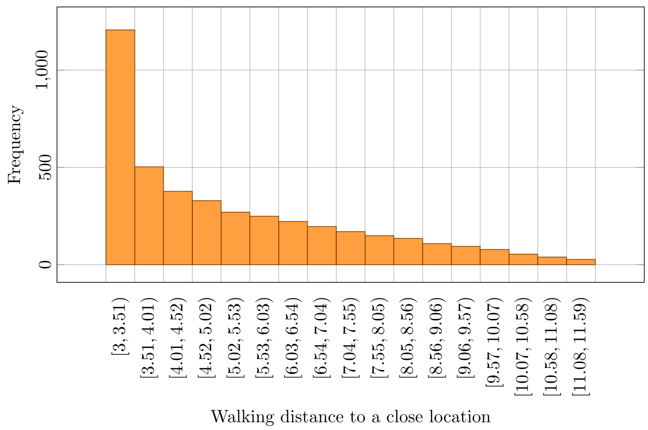

Fig. 2. Probability distribution for walking distances to the border of line starting from a random point at the assembly line.

kit contains parts for multiple stations as the kit is typically attached to the product and travels along the assembly line for multiple stations.

Bozer and McGinnis (1992) formulate and investigate the assembly line feeding problem from an academic perspective. This first research paper investigates the effect of using one or another feeding policy through cost-based comparisons. One major limitation of this work is the application of an *all-or-nothing* approach that assumes a single feeding policy is used to supply all parts. This research has been extended to include additional aspects in terms of costs (Caputo and Pelagagge, 2011) such as error or space costs. Over time, additional feeding policies have been discussed (Battini et al., 2009; Sali et al., 2015). In addition, researchers' focus shifted toward hybrid policy assignment where different feeding policies are assigned to different groups of parts based on specific characteristics. At first, descriptive models were used to investigate the value of these hybrid assignments (Caputo and Pelagagge, 2008, 2011; Limère and Van Landeghem, 2009; Sali et al., 2015) and later on the first optimization-based techniques were developed (Caputo et al., 2010, 2012, 2015; Limère et al., 2011, 2012) allowing to determine feeding policies at an individual part level. The cost-savings of optimal hybrid policies are significant, even when only allowing two line feeding policies. The latter study has been extended to consider the sizing of stations and preparation areas (Limère et al., 2015). Sali and Sahin (2016) proposed an optimization model distinguishing three line feeding policies and ensuring all assembly and logistics activities are executable without violating the cycle time. Caputo et al. (2016, 2018) built on their previous work (Caputo and Pelagagge, 2011) to investigate part features such as volume or weight and provide heuristic rules to select a feeding policy for each part based on such features. Using such part features, Moretti et al. (2021) and Zangaro et al. (2021) study the applicability of machine learning approaches for the assembly line feeding problem. More recently, Schmid et al. (2018, 2021) proposed models that compare all five line feeding policies and minimize the cost of operation. Besides, the latter study finds that flexible use of storage space at the stations can reduce costs by up to 7%. Baller et al. (2020) also distinguish various line feeding policies and even consider different load carrier quantities and sizes for those line feeding policies and report findings consistent with (Schmid et al., 2021). Adenipekun et al. (2022) additionally incorporate route definitions and vehicle selection.

2.3. Integration of both problems

Due to possible cost savings, the integration of line feeding and balancing has been identified a promising problem (Limère et al., 2011; Schmid and Limère, 2019). In a first study, Sternatz (2015) could support this notion by finding cost savings of up to around 20% when integrating feeding and balancing. This study uses a heuristic procedure to solve various cases. However, this study only distinguishes line stocking and stationary kitting. Battini et al. (2017) proposed a MILP model to solve a problem with similar assumptions and limitations complemented by ergonomic considerations. This study was followed-up by Calzavara et al. (2021) in which the problem was extended to include space and operator costs for the preparation area. Cost findings, similar to the ones reported by Sternatz (2015) have been found when applying the model to two case studies. Lastly, Wijnant et al. (2018) investigated the impact of different assembly line balancing decisions on line feeding costs, demonstrating that the balancing objective heavily affects the costs of line feeding. Summarizing the findings of previous works, it is evident that simultaneously optimizing both problems will result in better decision-making and coherent results have been found by Sternatz (2015), Battini et al. (2017) and Calzavara et al. (2021). In addition, it should be noted, that none of those studies incorporate walking-related operation times. However, this may constitute a significant proportion of assembly operator activities (Sedding, 2024) and must, therefore, be considered in the cycle time constraints in addition to assembly times. Fig. 2 illustrates the probability distribution of operator walking distances to fetch a single part from the border of line based on data from the case study investigated in this research. Clearly, the operator may need to fetch multiple parts for the assembly of a single product and walking activities may accumulate to constitute around half of the available time for some workers.

Therefore, the challenge to model the overall problem in more detail by considering additional line feeding policies, the impact of the preparation area's size on transportation efforts, and the exact placement of parts at the border of line and the impact on operation times remain to be studied and constitute the contribution of this paper in conjunction with an optimal solution procedure.

3. Problem definition

As this problem is a combination of both the ALBP and ALFP, each task f is assigned to a station s, and all parts that may be required for that task $i \in \mathcal{I}_f$ are assigned to a feeding policy p. For this work, this collection of parts is denoted as *part family* or *family* and contains different part variants used for the same purpose but vary concerning characteristics such as color or quality. All parts of the same family may be assigned to a single or different feeding policy. However, using different feeding policies is not necessarily advantageous (Schmid et al., 2021) and may confuse the assembly worker. Furthermore, each part is also assigned to a location l at the border of line (BoL), i.e., the part storage area at each assembly station, to determine walking distance and duration. The objective is to minimize the overall costs of the resulting assembly system. Furthermore, the problem includes determining the facility's size, including the logistics facility, comprising a warehouse and preparation area, and the shop floor. The preparation area (in literature, also referred to as *supermarket*) is a logistics area where parts are repacked or sequenced.

The line feeding policies under consideration are:

- Line stocking: Provision of large containers filled with a single type of parts (single SKU).
- Boxed-supply: Provision of smaller boxes filled with a single type of parts (single SKU).
- Sequencing: Provision of presorted interchangeable parts of the same part family, e.g., differently colored parts.
- Stationary kitting: Similar to sequencing but containing presorted parts from multiple part families used at a single station.
- Traveling kitting: Similar to stationary kitting but containing parts for several stations.

3.1. Planning environment

This problem affects multiple physical areas of an assembly system as well as the processes occurring in these areas. For this study, we distinguish three distinct types of storage areas: the warehouse, the preparation area, and the storage at the border of line (BoL). In this study, we excluded the material inbound and product outbound processes .

To define the processes in such a system, we follow the process description of Schmid and Limère (2019), distinguishing replenishment, preparation, transportation, and usage processes. Replenishment describes the transportation of material from the warehouse to the preparation area. The parts are processed there, namely, presorted or repacked into other load carriers. Next, they need to be brought to the assembly stations. In contrast to Schmid and Limère (2019), we separated this process into two stages for this study: dispatch, i.e., the transportation of the load carriers to a pickup area and transportation, i.e., transportation from that pickup point to the assembly stations. Lastly, parts are used by the assembly operator. The exact execution of all these processes varies for the different line feeding policies and is described in greater detail along with their cost calculations in Section 3.3.

3.1.1. Facility layout

The assembly line balancing problem assigns tasks to assembly stations. Therefore, it also assigns parts to workstations. Balancing problems may incorporate determining the stations' size (see, e.g., Chica et al. (2016). In this study, however, the amount of space at the BoL is defined a priori based on product characteristics. Sternatz (2015) showed that assigning tasks (and parts) to stations determines the optimal line feeding. Since the assembly line feeding problem determines the number of parts undergoing preparation, it directly affects the space demand for preparation activities.

We discretized different logistics facility sizes to input them into the model. The warehouse's and preparation area's depth is assumed to be equal to the assembly line's depth while their width varies for different options (see Fig. 1 for an illustration of width and depth). There is a direct link between the warehouse's and the preparation area's size, as parts stored in the preparation area do not require storage space in the warehouse. Therefore, each logistics facility option is associated with a specific warehouse and preparation area size.

3.1.2. Line balancing

As discussed above, assembly line balancing is a central aspect of assembly system planning. In practice, customer demand frequently determines the cycle time for assembly line balancing. As demand-driven balancing is the most prevalent type of balancing, we build upon this type of line balancing, also known as SALBP-1, minimizing the number of workstations. However, in this study, the objective is slightly modified to minimize costs incurred by using those workstations. This alteration does not change the model's output as costs do not vary for different stations in this study. However, this research considers not only balancing costs but also costs incurred through feeding and facility sizing. Therefore, a trade-off between balancing-incurred and feeding-incurred costs arises which generalized the simplifying assumptions of SALBP-1 which simply minimizes the number of stations for a given cycle time.

3.1.3. Line feeding

The BoL describes an area parallel to the assembly line, used to store parts and potentially equipment or tools. However, in this work, it is assumed that the BoL is exclusively used to store parts. Each station's BoL is sized equally. For an accurate determination of walking distances, we discretized the available space by dividing it into equally sized locations. Furthermore, we distinguish two categories of locations: (i) near locations, and (ii) distant locations (One may see the difference in distance to the assembly line in Fig. 3). Using near locations is far more attractive than using distant locations as operators have to walk shorter distances. Furthermore, near and distant locations are mutually exclusive, as can be recognized in Fig. 3 since the use of near locations blocks access to distant locations.

Nevertheless, distant locations may be used under particular circumstances, namely when a kit travels through a station (see Fig. 3a)). Traveling kits travel close to the assembly line such that the operators do not have to walk and fetch parts. Therefore, traveling kits avoid the use of close locations. In those cases, however, distant locations may be used for part storage. Fig. 3 shows a series of stations with a traveling kit (the kits path is represented by an arc), using some distant locations, and a single station without a traveling kit, using only close stations.

The optimization model proposed in this research allows for multiple traveling kits, e.g., one traveling kit may serve stations 1–3, and another one may serve station 4–6. However, it should be noted that traveling kits should never serve less than two stations, as conceptually they would become stationary kits. We discretized all possible traveling kits for the optimization model used. For example, for an assembly line with three stations, the following traveling kits may be used: Tr. kit 1, serving station 1 and 2; Tr. kit 2, serving stations 1, 2, and 3. Tr. kit 3, serving stations 2 and 3.

As described above, walking may take up a significant fraction of the operators' time. In this study, we assume that each part is fetched individually from the border of line. The exact distance depends on where the part is needed for assembly, which we call *demand points*, and its storage location. As represented by the dashed line in Fig. 3, an operator at station $i-1$ may have to walk from various demand points to the corresponding storage location and back.

3.1.4. Problem interdependence

The planning of an assembly system involves the three elements discussed above, i.e., assembly line balancing, assembly line feeding, and facility sizing. While those problems are presented separately above, they are in fact, closely linked to each other as follows:

Assembly line feeding decisions determine the amount of space required in the storage and preparation area of the factory. In the case of line stocking as a single means of feeding, no space is required for picking and repacking. All parts can be stored efficiently in a multi-level warehouse. However, should all parts undergo some transformation, such as repacking, sequencing, or kitting, cells must be installed in the preparation area, requiring significantly more space.

Furthermore, assembly line balancing decisions impact assembly line feeding decisions as follows: Assigning many tasks and parts to a single station often necessitates the preparation and kitting of parts due to space limitations at the border of line while assigning few tasks and parts to a station allows for simpler line feeding policies.

Lastly, assembly line feeding decisions may impact assembly line balancing decisions as follows: When kitting multiple assembly parts into a kit that can be attached to the product or pulled by the operator, the assembly operator does not need to walk a lot to fetch the parts. Therefore, the time required for all tasks diminishes, allowing for a different line balancing. If the operator has to walk frequently to fetch parts, a significant fraction of his available time (cycle time) may be used for walking, necessitating a different line balance.

In summary, one can state that line balancing and feeding affect each other, and feeding affects facility sizing. Therefore, the links depicted in Fig. 4 have hereby been established.

This interdependence of decision-making is also shown by the illustrative example in Fig. 1; balancing and feeding decisions impact the execution of the processes and the required areas for the warehouse, preparation area, and shop floor. For example, considering all parts to be line stocked would require the largest warehouse since no parts are stored in the preparation area. In this scenario, the preparation area is not needed. This scenario also does not include replenishment or preparation processes. However, the assembly stations will be relatively

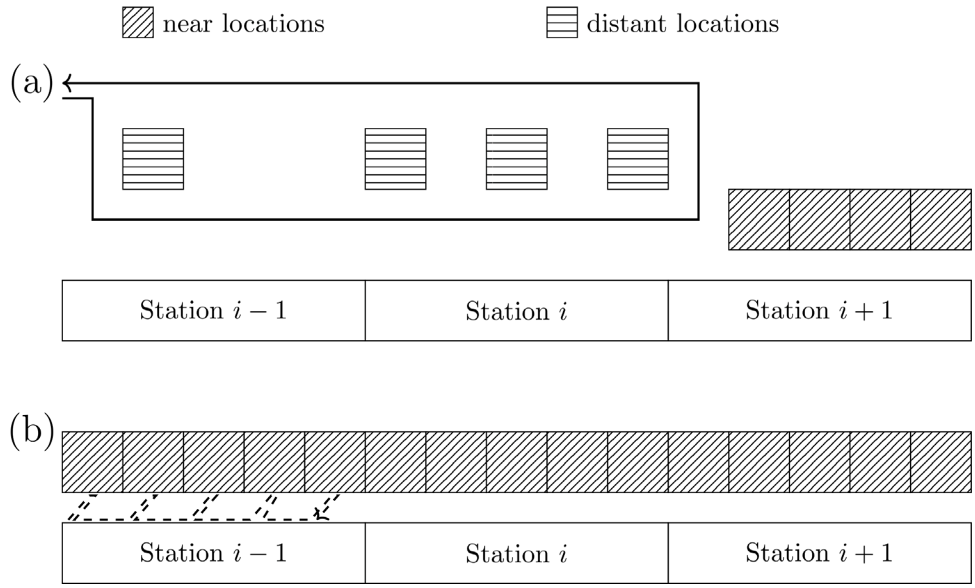

Fig. 3. Use of locations.

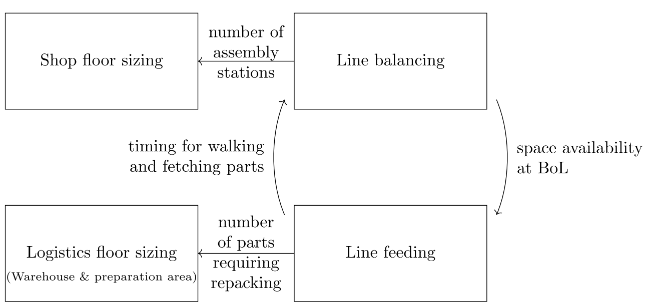

Fig. 4. The inter-dependencies of assembly system planning.

large as load carriers used in line stocking require much space. This would also increase the assembly operators' time for walking and searching the correct part. In the other extreme case, all parts may be fed in traveling kits. In this case, the warehouse area would be smaller as some of the pallets will be stored in the preparation area. At the same time, it would require an extensive preparation area. On the other hand, stations can be relatively small since no space at the BoL is required.

While the above mentioned decision-making processes are inter-related, it may be argued that facility sizing decisions are long term decisions while line balancing and feeding are rather medium term decisions. We chose to consider those problems simultaneously. This is reasoned in discussions with our industrial partner in which case the company seeks to open new facilities with a defined production rate for a given product in mind. This type of facility planning may be considered as green field planning. When companies seek to rearrange their existing production facility, it is considered as brown field planning. In both cases, however, space availability must be taken into account. In green field planning, the determination of the facility's size may be modeled by introducing corresponding decision variables. In brown field planning, knapsack constraints may be introduced to ensure the consideration of space limitations. To the best of our knowledge, previous works in line feeding do not explicitly assume either green or brown field planning. Calzavara et al. (2021) for example impose a cost but neither explicitly model the facility's size nor constrain it. Therefore, we propose a model that can be used for both problems.

3.1.5. Assumptions

To study a problem of the scope described above, several assumptions are needed. These assumptions combine assumptions limiting the scope of this research (indicated by S) and assumptions facilitating the modeling (indicated by F). These are listed hereafter.

1. Demand is assumed to be deterministic. (F)
2. Operation times are assumed to be deterministic. (F)
3. Each part is assumed to be used for one task only. (F)
4. All parts required for a single task are assumed to be fetched individually while no parts for other tasks are fetched in the same process. (S)
5. Costs are optimized over a defined planning horizon constituting a given mode of operation. (S)
6. Transportation and replenishment distances depend on the facility's sizing and are estimated as averages. (F)
7. The logistics facility consists of a single logistics area, including a warehouse and a preparation area. The depth of the logistics facility is assumed to be equal to the depth of the assembly facility. (S)
8. Each line feeding policy uses one type of load carrier with predetermined capacity and predefined costs. (S)
9. Demand points, therefore, walking times of assembly operators depend on the tasks assigned to a station and are calculated based on the duration of preceding tasks at that station where the sequence of tasks at a station follows their indexing. (S)
10. All parts of a part family are assigned to the same feeding policy. (F)
11. The length of a milk run and the length of the cells preparing boxed-supply parts are assumed to be independent of the decisions taken but are approximated by an average value (F)

As both, demand and operation times are assumed to be deterministic, cost calculations and constraint satisfaction do only depend on the parameter values. Relaxing these assumptions required non-deterministic programming approaches. It is assumed that each part is uniquely used for a specific task. This is done to simplify the problem. Otherwise, one would need to calculate the amount of space required in the preparation area in more detail. That is, some parts supplied to different stations by a single traveling kits may be the same, which would only require to store one pallet of this part in the kitting cell instead of multiple ones. The costs are calculated for a fixed time horizon to make the results comparable. It is important to note that in this work, compared to other studies, the replenishment and transportation distances depend on the logistics facility's size. Therefore, the respective costs change whenever a different facility size is chosen. As we do not determine the exact positioning of preparation operations within the preparation area, distances are estimated as average transportation distances. To relax this assumption, more detailed modeling would be required. All load carriers used for the different line feeding policies are assumed to be equally sized to simplify the model. However, this may be relaxed by adding additional load carrier options to each feeding policy. Furthermore, it is assumed that the costs are equal, irrespective of the parts assigned to that load carrier. However, in reality one may not require a stationary kit each takt, if the parts included in this kit are used infrequently. To accommodate for this simplifying assumption, one would need to introduce additional variables for each load carrier to determine their delivery frequency. However, this approach does not remove all inaccuracies since it may not be known to which degree those parts are consumed by the same or different products. When calculating walking distances, it needs to be determined at which point at the line a part is used. To calculate this, we consider the number of tasks per station and their duration. One may also (optimally) sequence the tasks at each station considering their precedence relations. However, this would likely result in either a nonlinear or cumbersome model formulation. Assumption 10 is rooted in findings that this restriction has very limited impact on the objective but simplifies work for the operators (Schmid et al., 2021). The final assumption is taken to facilitate simplify the problem at hand but may be lifted by adding additional variables for greater accuracy.

3.2. Solution approach

The problem discussed in this study is far more complicated than the individual problems of balancing and feeding as it integrates the individual problems and extends their scope, i.e., we added decisions on multiple traveling kits and the facility size in this problem. As discussed in Section 2, both problems are mostly solved sequentially in practice and theory. Therefore, the use of a decomposition technique seems appropriate. Since we are the first to propose a model and a solution approach for this particular problem, we intend to provide an optimal solution approach. Optimal decomposition approaches can be distinguished into row generation and column generation techniques. Both techniques decompose the problem into two problems. One well-studied row generation decomposition approach is Bender's decomposition. Here, the master problem contains some of the difficult decisions, whereas the subproblem solves "lower-level" decisions suiting the solution of the master (Bender, 1962). This approach was often applied to facility location problems, where the opening of facilities are the difficult decisions in the master and the assignment of customers to the opened facilities are the easier decisions taken in the subproblem (see, e.g., Wentges, 1996; Fischetti et al., 2017). In contrast, the problems are transformed through the Dantzig–Wolfe transformation for column generation approaches (Barnhart et al., 1998). This transformation often changes the problem such that the master problem needs to select some variables from a large set of variables. Since the creation of all possible variables is intractable in many practical cases, the variables are created whenever the master found a solution.

We decompose the problem in two stages: (i). A balancing stage, which determines the number of stations, assigns tasks to stations, sizes the warehouse and preparation area, and determines the use of traveling kits. This is also called the master problem. (ii) A feeding stage, which determines the parts' feeding policies and placement at the BoL. The second stage solves the feeding problem and is referred to as the subproblem stage. For this, two types of subproblems are distinguished: (i) subproblems that contain a single station, not served by a traveling kit; and (ii) subproblems that contain multiple stations that are all served by the same traveling kit. The subproblems' solutions determine feeding costs and the amount of space required in the warehouse and preparation area. Therefore, it needs to be verified if a solution to the master problem is feasible concerning the preparation area space. To this end, we solve a *feasibility problem* in a callback procedure that determines the minimum amount of space required in the preparation area and adds lazy constraints accordingly.

Due to the problem's intractability, a linear relaxation of the subproblems may be added (shown in Appendix A). Finding an optimal solution to the master problem in each iteration also slows down computations. Therefore, we utilize an ϵ-based approach (see, e.g., Rahmaniani et al., 2017), which interrupts the master problem when it finds a solution with an optimality gap of ϵ or less. This epsilon is reduced based on a sigmoid function when a non-optimal solution is found repeatedly.

It remains to be stated that, even though a master problem and all subproblems may be feasible, the overall solution might be infeasible. This problem arises when the amount of space required in the preparation area determined in the optimality subproblem is larger than the amount of space designated by the master problem and ensured by the minimum space generated in the callback. In the before-mentioned case, the entire line's feeding needs to be optimized in a single *optimality subproblem*. Fig. 5 shows the proposed solution procedure for this decomposition schematically.

The sequence of solving the *feasibility subproblems* in a callback and the *optimality subproblems* after finding a (sufficiently good) master solution was chosen based on computational pretests.

The notation for master and subproblems is presented in Table 1. The table contains multiple cost parameters, which we discuss in more detail in Section 3.3. For this, we will also introduce additional parameters. The following indices are used when referring to a particular feeding policy: L - line stocking; B boxed-supply; S - sequencing; K - stationary kitting; T - traveling kitting.

3.2.1. Master problem

$$\text{minimize} \sum_{s\in\mathcal{S}} c y_s + \sum_{l\in\mathcal{L}_T} c_l \chi_l + \sum_{a\in\mathcal{A}} c_a v_a + \sum_{b\in\mathcal{B}} z_b \tag{1}$$

$$\text{s.t.} \sum_{s\in\mathcal{S}_f} x_{fs} = 1 \qquad \forall f \in \mathcal{F} \tag{2}$$

$$\sum_{s=1}^{s\leq s'} (x_{f's} - x_{fs}) \geq 0 \qquad \forall s' \in \mathcal{S}\ \forall f \in \mathcal{F}\ \forall f' \in \mathcal{Q}_f \tag{3}$$

$$\sum_{f\in\mathcal{F}_s\cap\mathcal{F}_m} t_f^A x_{fs} \leq ct y_s \qquad \forall s \in \mathcal{S}\ \forall m \in \mathcal{M} \tag{4}$$

$$\sum_{l\in\mathcal{L}_T : s\in\mathcal{S}_l^S} \chi_l \leq y_s \qquad \forall s \in \mathcal{S} \tag{5}$$

$$\sum_{a\in A} v_a = 1 \tag{6}$$

$$\sum_{b\in\mathcal{B}} a_b \leq \sum_{a\in A} a_a v_a \tag{7}$$

$$y_s \in \{0,1\} \qquad \forall s \in \mathcal{S} : s > \underline{s} \tag{8}$$

$$x_{fs} \in \{0,1\} \qquad \forall f \in \mathcal{F}\ \forall s \in \mathcal{S}_f \tag{9}$$

$$\chi_l \in \{0,1\} \qquad \forall l \in \mathcal{L}_T \tag{10}$$

$$v_a \in \{0,1\} \qquad \forall a \in \mathcal{A} \tag{11}$$

$$a_b, z_b \geq 0 \qquad \forall b \in \mathcal{B} \tag{12}$$

As described above, the master problem addresses the balancing of the line (i.e., the assignment of tasks to workstations and the opening of workstations), the use of traveling kits, and the sizing of the preparation area. Those decisions also impact the objective function (see Formula (1)) as each of these decisions incurs specific costs. The fundamental assembly line balancing model formulation follows the formulations proposed by Baybars (1986) extending the formulations of Thangavelu and Shetty (1971), Patterson and Albracht (1975). However, we used

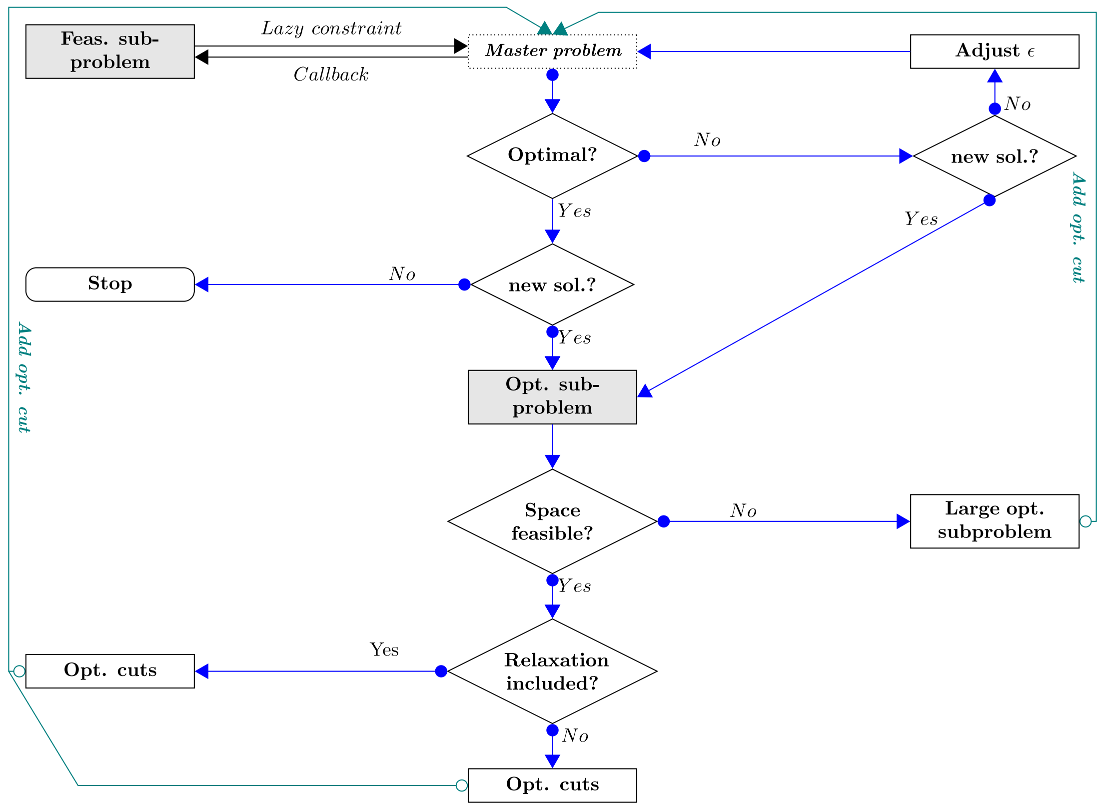

Fig. 5. Algorithmic steps of logic-based Benders' decomposition.

the precedence constraint set proposed by Aghezzaf and Artiba (1995) as it provides a tighter convex hull than previous formulations. In addition, some constraints related to the decisions described previously have been added. Constraints (2) and (3) ensure the assignment of all tasks and adherence to the precedence relations, respectively. Cycle time restrictions are ensured in Constraint (4) for each model of the end-product. In this research, we assume there is space for only one traveling kit at each station and only one traveling kit may traverse a station (see Constraint (5)). Furthermore, this constraint ensures that traveling kits only serve stations that are opened. A third aspect of the master problem is to ensure that preparation space is sufficient. Since the amount of space, however, depends on the solution to the subproblems, additional variables are used to incorporate the information from the lazy callback (see Constraints (28) and (29)). These values are used in Constraint (7) to ensure a proper sizing of the logistics facility. Finally, the master problem considers optimality cuts generated from the *optimality subproblems* and described in Eqs. (34)–(37).

Whenever the master problem is solved, several subproblems are derived. The exact number depends on the number of disconnected and connected stations in the solution of the master, where a set of stations is connected when the same traveling kit is used to supply them with parts. Let $\mathcal{B}_k$ denote the set of subproblems of iteration k. Then, $|\mathcal{B}_k| = \sum_{s\in S} \overline{y}_s^k - \sum_{l\in\mathcal{L}_T} |S_l^S|\overline{\chi}_l^k$ with $\overline{y}_s^k$ representing the y_s decisions variables and $\overline{\chi}_l^k$ representing the χ_l decision variables that have a value of 1 in iteration k, while S_l^S represents the set of stations served by traveling kit l. This translates into two types of subproblems. The first type describes a subproblem for each station not served by any traveling kit. The second type solves a subproblem for each traveling kit and includes all stations served by that traveling kit. For any subproblem of iteration k, the set of stations in that subproblem b is defined as $\mathcal{S}_{kb}$. Similarly, we define the set of locations $\mathcal{L}_{kb}$, the set of families $\mathcal{F}_{kb}$, and the set of parts $\mathcal{I}_{kb}$ for that subproblem.

3.2.2. Feasibility subproblem

This model builds upon the model described in Schmid et al. (2021). However, we adjusted it to incorporate cycle time constraints similar to Sali and Sahin (2016) and to simplify the model's representation. The model described hereafter is solved separately for each iteration k and subproblem b (See Eqs. (14)–(27) in Box I).

$$\text{minimize} \sum_{i\in\mathcal{I}_{kb}} \sum_{p\in\mathcal{P}_i} \sum_{l\in\mathcal{L}_{s_{ik}p}} x_{ipl} a_{ip} \tag{13}$$

This model optimizes the assignment of parts to feeding policies while minimizing the space needed in the preparation area (see Formula (13)). To this end, the model assigns each part to a feeding policy and location (Eq. (14)). It should be noted that a location refers to a discretized location as shown in Fig. 3 for all feeding policies except traveling kits. In contrast, in the case of traveling kits, it refers to a specific traveling kit from the set of traveling kits. Each traveling kit has a specific starting and ending point defined by the entering and leaving locations at the border of line that defines the subproblem. Therefore, each subproblem cannot contain more than one traveling kit. Again, the cycle time must not be violated (Eq. (16)). To this end, accurate task times are calculated in Constraint (15), taking into account not only assembly but also searching and walking times. The walking distances depend on the part positioning at the BoL and the *demand/assembly point* at the line, where the part family is needed. These demand points are calculated before solving this model based on the task's index and the assembly times of all tasks at a station preceding the given task. Next, the available space at each station and its usage is modeled in more detail in Constraints (17)–(22). In Constraint (17), we introduce an indicator variable χ_{pl} that is linked to the assignment variables x_{ipl}, and is used to ensure that no location is used for more than one policy (Constraint (18)). The location(s) l' that may be used depend on the policy. For all line feeding policies except traveling kits $l' = l$. However, for traveling kits l'_{pl} is different from l as it refers to the traveling kit's index instead. Another indicator variable ψ_{fpl} is introduced in

Table 1
Notation for optimization models.

Sets			
$\mathcal{A}$	Set of possible logistics facilities	$\mathcal{B}$	Set of all possible subproblems
$\mathcal{B}_k$	Set of subproblems in callback k	$\mathcal{D}_f^P$	Set of direct predecessors of part family f
$\mathcal{F}$	Set of part families	$\mathcal{F}_{kb}$	Set of part families in subproblem b of callback k
$\mathcal{F}_m$	Set of part families used for model m	$\mathcal{F}_p$	Set of part families assignable to policy p
$\mathcal{F}_s$	Set of part families assignable to station s	$\mathcal{H}$	Set of master iterations
$\mathcal{I}$	Set of parts	$\mathcal{I}_f$	Set of parts in family f
$\mathcal{I}_{kb}$	Set of parts in subproblem b of callback k	$\mathcal{I}_p$	Set of parts assignable to policy p
$\mathcal{I}_s$	Set of parts assignable to station s	$\mathcal{K}$	Set of master callbacks
$\mathcal{L}$	Set of locations and set of tr. kits	$\mathcal{L}_{fp}$	Set of locations/tr. kits that may be used for family f in policy p
$\mathcal{L}_{kb}$	Set of locations and traveling kits that can be used in callback k and subproblem b	$\mathcal{L}_{kb}^A$	Set of locations at the BoL that can be used in callback k and subproblem b
$\mathcal{L}_p$	Set of locations available for policy p; Set of traveling kits in case of $p = T$	$\mathcal{L}_{sp}$	Locations usable by station s and p
$\mathcal{M}$	Set of product models	$\mathcal{P}$	Set of line feeding policies
$\mathcal{P}_i$	Set of line feeding policies for part i	$\mathcal{P}_i$	Set of line feeding policies for part family f
$\mathcal{P}_{kb}$	Set of feeding policies available in callback k and subproblem b	$\mathcal{Q}_f$	Set of predecessors for family f
$\mathcal{R}$	Set of resources under consideration (Weight and Volume)	$\mathcal{S}$	Set of assembly stations
$\mathcal{S}_i$	Set of assembly stations to which part i may be assigned	$\mathcal{S}_f$	Set of assembly stations to which family f may be assigned
$\mathcal{S}_{kb}$	Set of assembly stations in subproblem b of callback k	$\mathcal{S}_l^S$	Set of stations that may be served by traveling kit l
Variables			
a_b	Space variable for subproblem b		
χ_l	Variable indicating whether traveling kit l is used	χ_{pl}	Variable indicating whether location l is used for policy p
ψ_{fpl}	Variable indicating whether family f is assigned to location l and feeding policy p	t_f	Assembly time of family f
v_a	Variable indicating whether logistics facility a is used	x_{ipl}	Variable indicating whether part i is assigned to location l and line feeding policy p
x_{fs}	Assigning family f to station s	y_s	Variable indicating the use of station s
z_b	Optimality variable for subproblem b		
Parameters			
$\bar{a}_k$	index of logistics facility selected in iteration k	a_{kb}^*	Space required in preparation area for subproblem b of iteration k
a_k^*	Space required in the preparation area for iteration k for all subproblems	a_a	Space of preparation area when using logistics facility a
a_{ip}	Space required to prepare part i for policy p	c_a	Costs to use logistics facility a
c_{ipla}	Costs for providing part i to location l with line feeding policy p and logistics facility a	c_{fpla}	Costs to provide part family f to location l using policy p when using logistics facility a
c_l	Costs to use traveling kit l	c_{pla}	Costs to provide use a location l for policy p when using logistics facility a
c	Costs to open a station	ct	Cycle time
l_{kb}	Index of traveling kit used in subproblem b of iteration k	l'_{pl}	Location l used to store a part that is fed with policy p to location/kit k
M	A sufficiently large number	r_{ir}	Resource requirement of resource r for a box of part i
r_{fr}	Resource requirement of resource r for family f	R_{pr}	Resource availability of resource r for policy p
s_{ik}	Station of part i in iteration k	s_{fk}	Station of family f in iteration k
t_f^A	Assembly time of part family f	t_{iplk}	Walking and searching time of the assembly operator if part i is assigned to policy p and location l in iteration h
t_{iplk}	Walking and searching time of the assembly operator if part i is assigned to policy p and location l in iteration k	$\underline{t}_{pl}$	Minimum walking time to location l
z_{kb}^*	Optimal costs for subproblem b of iteration k	z_{kb}^r	Relaxed master costs for subproblem b of iteration k

Constraint (19) to indicate whether any part of family f is assigned to policy p and location l.

Next, the feasibility of part positioning is ensured. To this end, we allow either a single part to be line stocked or a family to be sequenced at a single location (Constraint (20)). Furthermore, it is verified that all boxes assigned to a location fit into a rack (Constraint (21)), and all parts assigned to a kit match its volume and weight capacities (Constraint (22)). To store parts in boxed supply, flow racks are used. These use the entire width of the location and have a specified height. Multiplying the height and width results in a surface area, that can be accessed by the operators, and is used to store the boxes. Similarly to the rack, each box uses a surface area equal to its height multiplied with its width, where the shorter of the two sides of a box is used to determine its width. Lastly, Constraint (23) enforces the assignment of a family's parts to the same feeding policy. This equal assignment has multiple reasons: (i) it simplifies operations for the operator; (ii) possible gains in relaxing this constraint are marginal (Schmid et al., 2021); and (iii) it simplifies the solution of the model significantly.

This model may be feasible or infeasible since the master problem is neglecting information on walking and searching times and on the availability of available space at the BoL. If the master includes the subproblems relaxation, it considers some information but only in a non-integral and simplified manner as the exact walking distances are unknown. Let the optimal solution of subproblem b in iteration k be denoted by a_{kb}^*. After solving each subproblem, lazy constraints of type (28) or (29) will be added for each subproblem solved. The former is for subproblems with a single, and the latter for subproblems with multiple stations. The first multiple lazy constraints are added based on a single

$$\sum_{p\in\mathcal{P}_i\cap\mathcal{P}_{kb}}\sum_{l\in\mathcal{L}_{s_{ik}p}\cap\mathcal{L}_{kb}} x_{ipl} = 1 \qquad \forall i\in\mathcal{I}_{kb} \tag{14}$$

$$\sum_{p\in\mathcal{P}_i\cap\mathcal{P}_{kb}}\sum_{l\in\mathcal{L}_{s_{ik}p}\cap\mathcal{L}_{kb}} t_{iplk}x_{ipl} + t_f^A \le t_f \qquad \forall f\in\mathcal{F}_{kb}\ \forall i\in\mathcal{I}_f \tag{15}$$

$$\sum_{f\in\mathcal{F}_s\cap\mathcal{F}_m:s_{fk}=s} t_f \le ct \qquad \forall s\in\mathcal{S}_{kb}\ \forall m\in\mathcal{M} \tag{16}$$

$$x_{ipl} \le \chi_{pl'_{pl}} \qquad \forall i\in\mathcal{I}_{kb}\ \forall p\in\mathcal{P}_i\cap\mathcal{P}_{kb}\ \forall l\in\mathcal{L}_{s_{ik}p}\cap\mathcal{L}_{kb} \tag{17}$$

$$\sum_{p\in\mathcal{P}} \chi_{pl} \le 1 \qquad \forall s\in\mathcal{S}_{kb}\ \forall l\in\mathcal{L}_{kb} \tag{18}$$

$$x_{ipl} \le \psi_{fpl} \qquad \forall f\in\mathcal{F}_{kb}\ \forall i\in\mathcal{I}_f\ \forall p\in\{S,K,T\}\ \forall l\in\mathcal{L}_{s_{fk}p}\cap\mathcal{L}_{kb} \tag{19}$$

$$\sum_{i\in\mathcal{I}_s\cap\mathcal{I}_{kb}:s_{ik}=s} x_{iLl} + \sum_{f\in\mathcal{F}_s\cap\mathcal{F}_{kb}:s_{fk}=s} \psi_{fSl} + \sum_{p\in\{B,S\}} \chi_{pl} \le 1 \qquad \forall s\in\mathcal{S}_{kb}\ \forall l\in\mathcal{L}_{kb}^A \tag{20}$$

$$\sum_{i\in\mathcal{I}_s\cap\mathcal{I}_{kb}:s_{ik}=s} r_{ir}x_{iBl} \le R_{Br}\chi_{Bl} \qquad \forall s\in\mathcal{S}_{kb}\ \forall l\in\mathcal{L}_{kb}^A\ \forall r\in\mathcal{R} \tag{21}$$

$$\sum_{f\in\mathcal{F}_m\cap\mathcal{F}_p\cap\mathcal{F}_{kb}} r_{fr}\psi_{fpl} \le R_{pr}\chi_{pl} \qquad \forall m\in\mathcal{M}\ \forall p\in\{K,T\}\ \forall l\in\mathcal{L}_{kb} \tag{22}$$

$$\sum_{l\in\mathcal{L}_{bk}\cap\mathcal{L}_{s_{ik}p}} x_{ipl} - \sum_{l\in\mathcal{L}_{bk}\cap\mathcal{L}_{s_{jk}p}} x_{jpl} = 0 \qquad \forall f\in\mathcal{F}_{bk}\ \forall i,j\in\mathcal{I}_f : i<j\ \forall p\in\mathcal{P}_f \tag{23}$$

$$x_{ipl}\in\{0,1\} \qquad \forall i\in\mathcal{I}_{kb}\ \forall p\in\mathcal{P}_i\ \forall l\in\mathcal{L}_{s_{ik}p}\cap\mathcal{L}_{kb} \tag{24}$$

$$\psi_{fpl}\in\{0,1\} \qquad \forall f\in\mathcal{F}_{kb}\ \forall p\in\mathcal{P}_f\ \forall l\in\mathcal{L}_{s_{fk}p}\cap\mathcal{L}_{kb} \tag{25}$$

$$\chi_{pl}\in\{0,1\} \qquad \forall p\in\mathcal{P}\ \forall l\in\mathcal{L}_{kb} \tag{26}$$

$$t_f \ge t_f^A \qquad \forall f\in\mathcal{F} \tag{27}$$

Box I.

subproblem.

$$a_{kb}^* + M\left(\sum_{f\in\mathcal{F}_{kb}}(x_{fs}-1) - \sum_{l\in\mathcal{L}_T:s\in S_l^S}\chi_l\right) \le a_b \qquad \forall k\in\mathcal{K}\ \forall b\in\mathcal{B}_k : |\mathcal{S}_{kb}|=1\ \forall s\in\mathcal{S} : \mathcal{F}_{kb}\subseteq\mathcal{F}_s \tag{28}$$

$$a_{kb}^* + M\left(\sum_{f\in\mathcal{F}_{kb}}(x_{fs_{fk}}-1) + \chi_{l_{kb}} - 1\right) \le a_b \qquad \forall k\in\mathcal{K}\ \forall b\in\mathcal{B}_k : |\mathcal{S}_{kb}|>1 \tag{29}$$

If the subproblem is infeasible, too many families have been assigned to the set of stations in the subproblem. Infeasibility can be rooted in the required operation times, i.e., the worker is not able to perform all tasks within the allowed time (see Constraints (15) and (16)). Another possible cause for infeasibility is insufficient space at the border of line (see Constraints (20)–(23)). Therefore, one of the following combinatorial cuts is added to the master as a lazy constraint. If the subproblem contains a single station, the first cut set is added, whereas the second cut is added if the subproblem contains a traveling kit serving multiple stations.

$$\sum_{f\in\mathcal{F}_{kb}} x_{fs} \le |\mathcal{F}_{kb}| - 1 + \sum_{l\in\mathcal{L}:s_{kb}\in S_l^S}\chi_l \qquad \forall k\in\mathcal{K}\ \forall b\in\mathcal{B}_k : |\mathcal{S}_{kb}|=1\ \forall s\in\mathcal{S} : \mathcal{F}_{kb}\subseteq\mathcal{F}_s \tag{30}$$

$$\sum_{f\in\mathcal{F}_{kb}} x_{fs_{fk}} + \chi_{l_{kb}} \le |\mathcal{F}_{kb}| \qquad \forall k\in\mathcal{K}\ \forall b\in\mathcal{B}_k : |\mathcal{S}_{kb}|>1 \tag{31}$$

These cuts avoid a repetition of the same assignment. However, in case of cut (30), the use of a traveling kit does allow a repetition of the same assignment of tasks to stations because it may reduce operation times or space requirements at the border of line and make the subproblem feasible.

3.2.3. Optimality subproblem

The optimality subproblems start from a feasible master solution where each iteration is labeled h, replacing k from the feasibility subproblems. This model determines the minimum feeding costs for each subproblem given the assignment of families. This includes three cost parameters: (i) c_{ipla} including investment, replenishment, preparation, and transportation costs ; (ii) c_{fpla} including investment, preparation, dispatch, and transportation costs ; and (iii) c_{pla} including, dispatch, and transportation costs. We will discuss the exact cost calculations in the next section.

$$\sum_{i\in\mathcal{I}_{kb}}\sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}_{s_{ikp}}} c_{ipl\bar{a}_k}x_{ipl} + \sum_{f\in\mathcal{F}_{kb}}\sum_{p\in\mathcal{P}_f}\sum_{l\in\mathcal{L}_{s_{fkp}}} c_{fpl\bar{a}_k}\psi_{fpl} + \sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}_p\cap\mathcal{L}_{kb}} c_{pl\bar{a}_k}\chi_{pl} \tag{32}$$

s.t. (14)–(27)

$$\sum_{i\in\mathcal{I}_{kb}}\sum_{p\in\mathcal{P}_i}\sum_{l\in\mathcal{L}_{s_{ikp}}} a_{ip}x_{ipl} \le a_{\bar{a}_k} \tag{33}$$

Due to the feasibility cuts based on the feasibility subproblem added in the callback, the optimality subproblems are always feasible. Their solutions are used to add an optimality cut to the master problem. Similar to the feasibility subproblems, optimality subproblems may be used to generate multiple cuts. Let z_b^* denote the optimal solution value to the subproblem at hand. Then, either an optimality cut of type (34) or (35) is added to the master problem.

In case the master problem contains the subproblems' relaxation, the value of z^*_{kb} must be reduced by the relaxation's objective value z^r_{kb} and the cuts must be adjusted as shown in (36) and (37).

$$z^*_{kb} + M\left(\sum_{f\in\mathcal{F}_{kc}}(x_{fs}-1) + \sum_{a\in\mathcal{A}:a_a\geq a_{\bar{a}_k}} v_a - 1 - \sum_{l\in\mathcal{L}_T:s\in S^S_l}\chi_l\right) \leq z_b \quad \forall k\in\mathcal{K}$$
$$\forall b\in\mathcal{B}_k : |S_{kb}| = 1 \forall s\in\mathcal{S} : s\geq s_{bk} \wedge \mathcal{F}_{kb}\subset\mathcal{F}_s \tag{34}$$

$$z^*_{kb} + M\left(\sum_{f\in\mathcal{F}_{kc}}(x_{fs_{fk}}-1) + \sum_{a\in\mathcal{A}:a_a\geq a_{\bar{a}_k}} +(\chi_{\bar{l}_{kb}}-1)\right) \leq z_b \quad \forall k\in\mathcal{K}$$
$$\forall b\in\mathcal{B}_k : |S_{kb}| > 1 \tag{35}$$

$$z^*_{kb} - z^r_{kb} + M\left(\sum_{f\in\mathcal{F}_{kc}}(x_{fs}-1) + (v_{\bar{a}_k}-1) - \sum_{l\in\mathcal{L}_T:s\in S^S_l}\chi_l - \sum_{f\notin\mathcal{F}_{kc}} x_{fs_{fk}}\right) \leq z_b \quad \forall k\in\mathcal{K}$$
$$\forall b\in\mathcal{B}_k : |S_{kb}| = 1 \forall s\in\mathcal{S} : s\geq s_{bk} \wedge \mathcal{F}_{kb}\subset\mathcal{F}_s \tag{36}$$

$$z^*_{kb} - z^r_{kb} + M\left(\sum_{f\in\mathcal{F}_{kc}}(x_{fs_{fk}}-1) + (v_{\bar{a}_k}-1) + (\chi_{\bar{l}_{kb}}-1) - \sum_{f\notin\mathcal{F}_{kc}} x_{fs_{fk}}\right) \leq z_b \quad \forall k\in\mathcal{K}$$
$$\forall b\in\mathcal{B}_k : |S_{kb}| > 1 \tag{37}$$

3.2.4. Final step

After all the optimality subproblems are solved, the solution must be checked for preparation-space feasibility. For this, the required preparation area space of all optimality subproblems in iteration k is summed up: $a^*_k = \sum_{b\in\mathcal{B}_k} a^*_{kb}$. Then, the algorithm may proceed with one of the following options:

- $a^*_k \leq a_{\bar{a}_k}$: The solution is optimal given the current master solution and feasible. The iteration stops and the algorithm continues with a new iteration until proven optimally.
- $a^*_k \geq a_{\bar{a}_k}$: The solutions obtained from the optimality subproblems do not match the solution of the master problem as preparation area space is insufficient. To obtain a feasible solution, an optimality subproblem, including all stations, must be solved. Afterwards, a single optimality cut is given to the master. For this z^*_{kb} must be reduced by the cost sum of all individual subproblems. Furthermore, one cannot sum over all preparation area variables v_a but needs to consider only the one generated by the solution of the master problem. Then the algorithm continues with a new iteration until proven optimally.

3.3. Cost calculation

The costs considered in this model are split up into multiple blocks as they depend on different decisions and, therefore, on the value of different decision variables. Each of these blocks comprises different costs arising from the processes of replenishment (R), preparation (P), dispatch (D), transportation (T), and usage (U) or investments (I), and space cost (S). The processes and investment or space costs are described by a superscript. It is important to note that even one costs parameter does not necessarily include the same processes when considering different line feeding policies. E.g., transportation costs for line stocking are included in c_{ipla} whereas they are included in c_{pla} for stationary kits. The different cost blocks are $c_{ipla}, c_{fpla}, c_{pla}, cs, c_a, c_l$. The former four consists of costs originating in different processes, whereas the latter only consists of a single origin cost (see Eqs. (38)–(41). The individual cost components will be described in the following subsections. The notation used throughout this section is presented in Table 2. The table introduces the used symbols, described them, and provides some values. However, for values that highly on the problem instance, this is indicated by var.

$$c_{ipla} = c^I_{ispl} + c^R_{ispl} + c^P_{ispl} + c^T_{ispl} \quad \forall i\in\mathcal{I}\ \forall s\in\mathcal{S}_i\ \forall p\in\mathcal{P}\ \forall l\in\mathcal{L}_{sp} \tag{38}$$

$$c_{fpla} = c^I_{fpl} + c^P_{fpl} + c^D_{fpl} + c^T_{fpl} + \quad \forall f\in\mathcal{F}\ \forall p\in\mathcal{P}\ \forall l\in\mathcal{L}_{fp} \tag{39}$$

$$c_{pla} = c^I_{pla} + c^D_{pla} + c^T_{pla} \quad \forall p\in P\ \forall l\in\mathcal{L}_p \tag{40}$$

$$c = c^S + c^U + c^I \tag{41}$$

$$c_a = c^S \tag{42}$$

$$c_l = c^I \tag{43}$$

It is important to remark that this section's cost calculations are not generalizable to any assembly facility. However, we aimed to describe them such that adjustments can easily be made. The following enumeration lists the assumptions for cost calculations described in the following.

1. The same vehicle type transports all parts assigned to a particular line feeding policy. Namely, forklifts are used for line stocking, tugger-trains for boxed-supply, sequencing, and stationary kitting, and AGVs for traveling kitting.
2. The individual preparation cells are spread over the entire preparation area. For the estimation of transportation distances, only the **width** of the preparation area and warehouse are considered (see Fig. 1). For replenishment, the entire warehouse width and half the preparation area width are considered. For dispatching, half the preparation area width is considered.
3. Whenever kits are used, it is assumed that each product requires a kit.

3.3.1. Space costs

As illustrated in Fig. 1, the space required for warehousing and preparation area depends on the decisions made in the feeding systems. If many parts are preprocessed, more space is needed in the preparation area and spatial requirement for the warehouse reduces. Therefore, several possible different sizes for the logistics area are defined of which one will be selected (see Eq. (6)). Each logistics area is defined by a corresponding warehouse size and a preparation area size. The size of the warehouse a^W_a and preparation area a^P_a can be directly linked and, therefore, combined in this study. The costs of every option is simply calculated by the area a_a in space units multiplied with costs for a space unit sc.

$$c_a = (a^W_a + a^P_a)sc \quad \forall \quad a\in\mathcal{A} \tag{44}$$

Space costs also need to be considered for the shop floor since the use of additional stations increases the demand for space. Here, the space of a single station a^S is also multiplied with the costs per space unit sc

$$c^S = a^S sc \tag{45}$$

3.3.2. Investment costs

Investment costs arise at multiple places: The provision of parts in any container requires to buy several containers at a price of cc_p. The numbers of containers for boxed-supply cn_i, sequencing cn_f and both types of kits cn_p (see Eqs. (46)–(51)) may vary. Furthermore, one has to consider the investment in warehouse racks that store pallets of parts. Thus, the number of pallets in racks pn_{ip} is multiplied with the cost for a rack spot cwr (see Eq. (46)). The number of pallets depends on the feeding policy, as line-stocked parts are stored in the warehouse, whereas other feeding policies store parts in the warehouse and the preparation area. Another occurrence of investment costs is the placement of flow racks at the BoL, needed to hold box-supplied parts. In this case, the depreciation or leasing costs of a flow rack cfr need to be considered. Additionally, picking support systems, such as pick-by-light may be leased depreciated with cost cp (see Eq. (48)). Lastly, this research assumes that AGVs transport traveling kits. Therefore, several AGVs na_l need to be leased or depreciated at costs lca. na_l depends on the stations a particular kit is serving and considers that kits need to be refilled. From an assembly line balancing perspective, additional costs arise for the investment in assembly line conveyor technology,

Table 2
Parameters used for cost calculations.

Parameter	Description	Value
AL_B	Aisle length for boxed-supply preparation cells	50 m
ϵ^F	Forklift utilization rate	90%
ϵ^L	Utilization rate logistical operators	80%
ϵ^T	Utilization rate tugger trains	90%
λ	Demand for all products	15 840
λ_f	Normalized demand of part family f	[31–14,436]
λ_m	Demand for model m	[1,584–11,088]
λ_i	Demand for part i	[4–57,810]
a^S	Area of an assembly station	42–69 m^2
a^P_a	Area in preparation area for logistics facility a	*var*
a^W_a	Area in warehouse for logistics facility a	*var*
AL_f	Aisle length for the sequencing cell of part family f	*var*
AL'_i	Incremental aisle length caused by part i	0.8 m
aoc	Assembly operator costs	30 \$/h
bs_p	Preparation batch size for policy p	$\{n.a., 5, 1, 12, 1\}$
cc_p	Depreciation cost for a container used for feeding policy p	$\{n.a., 0.1, 0.3, 8.33, 8.33\}$(\$/month)
cal	Investment cost assembly line	8.33 \$/(month m)
cfr	Depreciation cost for a flow rack	5.33 \$/month
cn_f	Number of containers required for part family f when sequenced	3
cn_i	Number of boxes required for part i when box-supplied	*var*
cn_p	Number of containers required for a kit ($p \in \{K, T\}$)	$\{3, var\}$
cp	Depreciation costs of a pick-by-light system attached to a flow rack	2.08 \$/month
ct	Cycle time	120 s
cwr	Depreciation cost for a rack spot in the warehouse	1.67 \$/month
dd_a	Dispatch distance for logistics facility a	*var*
di_{al}	Transportation distance from warehouse to location l when using logistics facility a	*var*
fv	Forklift velocity	2.8 m/s
lca	Depreciation cost for an AGV	416.66 \$/month
lcf	Lease costs for a forklift	1500 \$/month
lct	Lease costs for a tugger train	500 \$/month
loc	Costs for a logistical operator	20 \$/h
mr_p	Milk run length for feeding policy p	*var*
n_f	Number of parts in a sequencing container for family f	*var*
n_{ip}	Number of parts of type i in a load carrier used for feeding policy p	*var*
na_l	Number of AGVs required for tr. kit l	*var*
ov	Operator Velocity	0.8 m/s
pn_{ip}	Number of pallets of part i stored in the warehouse when fed with policy p	*var*
pt_{ip}	Picking time in preparation area for feeding policy p and part i	*var*
rd_{pa}	Replenishment distance for feeding policy p and logistics area a	*var*
sl	Length of a station	*var*
sc	Space leasing/building depreciation costs	16.67 \$/(month m^2)
st_p	Searching time in preparation for feeding policy p	*var*
t^D	Drop-off time for a pallet	17 s
t^P_{ip}	Pickup time for part i and line feeding policy p	*var*
ttc	Tugger train coupling an loading time	30 s
ttd_p	Tugger train dropoff time for feeding policy p	$\{n.a., 18, 30, 30, n.a.\}s$
ttv	Tugger train velocity	m/s
v	Volume of a kit container	1.92 m^3
v_i	Volume of a box of part i	*var*
v_f	Volume of a sequencing container of family f	*var*
voc	Cost for vehicle operators	25 \$/h
vtt	Volume tugger train	3.84 m^3

depending on the size of the stations and ultimately on the number of stations (see Eq. (52)).

$$c^I_{ipla} = pn_{ip}cwr \qquad \forall i \in \mathcal{I}\ \forall s \in \mathcal{S}_i\ \forall p \in \mathcal{P} \setminus \{B\}\ \forall l \in \mathcal{L}_{sp}\ \forall a \in \mathcal{A} \tag{46}$$

$$c^I_{iBla} = cn_i cc_B + np_{iB}cwr \qquad \forall i \in \mathcal{I}\ \forall s \in \mathcal{S}_i\ \forall l \in \mathcal{L}_{sp}\ \forall a \in \mathcal{A} \tag{47}$$

$$c^I_{fSla} = cn_f cc_S \qquad \forall f \in \mathcal{F}\ \forall l \in \mathcal{L}_p\ \forall a \in \mathcal{A} \tag{48}$$

$$c^I_{Bla} = cfr + cp \qquad \forall l \in \mathcal{L}_B\ \forall a \in \mathcal{A} \tag{49}$$

$$c^I_{Kla} = cn_K cc_K \qquad \forall l \in \mathcal{L}_K\ \forall a \in \mathcal{A} \tag{50}$$

$$c^I_l = cn_T cc_T + na_l lca \qquad \forall l \in \mathcal{L}_T \tag{51}$$

$$c^I = cal\ \ sl \tag{52}$$

3.3.3. Replenishment costs

Replenishment costs occur when parts have to be delivered to the preparation area for boxed-supply, sequencing, or kitting. Replenishment costs depend on the warehouse's and preparation area's size. Additionally, pallet pickup times at the warehouse t^P_{ip} and drop-off times at the preparation area t^D are considered in replenishment cost calculations. Since parts can either be stored (in multiple layers) on the floor or in racks in the warehouse, pickup times may vary. A part's feeding policy determines whether a part is stored on the floor (also known as block stacking) or in a rack. Since the number of pallets stored in the warehouse depends on the feeding policy, we utilize it to determine whether a part is block-stacked or rack-stored. To this end, we defined a threshold on the number of pallets. If the number of pallets is lower than this threshold, it is stored in a rack, otherwise in block-storage.

For the calculation of replenishment costs, we consider (un-)loading and transportation times and multiply it with forklift investment (lcf), and vehicle operator costs voc. Transportation time consider the distance rd_{pa}, forklifts velocity fv, and forklift utilization rate ϵ^F. Lastly, costs are multiplied by the number of pallets transported, which is defined by the part's demand λ_i and the number of parts on a pallet

n_{iL}.

$$c^R_{ipla} = (voc + lcf)((t^P_{ip} + t^D) + \frac{rd_{pa}}{\epsilon_F f v})\frac{\lambda_i}{n_{iL}} \quad \forall \quad i \in \mathcal{I}\ \forall s \in \mathcal{S}_i\ \forall p \in \mathcal{P}_i \setminus \{L\}\ \forall l \in \mathcal{L}_{sp}\ \forall a \in \mathcal{A} \tag{53}$$

3.3.4. Preparation costs

Preparation costs are incurred by the logistics workers labor when picking parts and placing them in a different load carrier such as boxes, sequencing containers, or kitting containers. In sequencing and kitting parts are also presorted. Costs for preparation typically differ for each feeding policy due the batch size, i.e., the number of load carriers that can be prepared simultaneously, the required accuracy (no sequencing vs. sequencing) and the size of the preparation cell. A preparation cell describes a smaller area within the preparation area. In sequencing, e.g., each cell contains all part (variants) of a specific part family. In kitting, all part families, supplied with a certain kit, and their parts are included. Boxed-supply cells contain many different parts from different families. The size of a cell, here measured by the length of an aisle, determines the walking duration.

$$c^P_{iBla} = loc\frac{\lambda_i}{n_{iB}\epsilon^L l}\left[\frac{AL_B}{bs_B ov} + st_B + pt_{iB}n_{iB}\right] \quad \forall i \in \mathcal{I}_B\ \forall l \in \mathcal{L}_B\ \forall a \in \mathcal{A} \tag{54}$$

$$c^P_{fSla} = loc\frac{\lambda_f}{n_{iS}\epsilon^l}\left[\frac{AL_f}{bs_S ov} + (st_S + pt_{iS})n_{iS}\right] \quad \forall f \in \mathcal{F} \cap \mathcal{F}_S\ \forall l \in \mathcal{L}_S\ \forall a \in \mathcal{A} \tag{55}$$

$$c^P_{iKla} = loc(\frac{\lambda_i}{\epsilon^L}(st_K + pt_{iK}) + \frac{\lambda AL'_i}{bs_K ov\epsilon^L}) \quad \forall i \in \mathcal{I}\ \forall l \in L_K\ \forall a \in \mathcal{A} \tag{56}$$

$$c^P_{iTla} = loc(\frac{\lambda_i}{\epsilon^L}(st_T + pt_{iT}) + \frac{\lambda AL'_i}{bs_T ov\epsilon^L}) \quad \forall i \in \mathcal{I}\ \forall l \in \mathcal{L}_{sp}\ \forall a \in \mathcal{A} \tag{57}$$

Eq. (54) covers cost for boxed-supply if parts are loosely provided on a pallet and have to be repacked. Eq. (55) combines the costs for all parts of a family if provided in sequenced containers. Eq. (56) describes costs for parts in stationary kits and preparation costs for parts in traveling kits are calculated in Eq. (57). For kits, the kitting cell's size is unknown. Therefore, an incremental aisle length AL'_i, depending on the pallets width, is used to calculate walking distances.

3.3.5. Dispatch costs

After parts have been prepared, containers need to be placed in a dispatch zone, where they are picked up for final transportation. This concerns only sequenced containers and stationary kits. Line stocked parts do not undergo preparation and, therefore, do not require dispatching. In this study, it is assumed that preparation areas for boxed-supply are placed at the dispatch. Therefore, no dispatching activity is needed. Sequencing containers and traveling kits may be prepared anywhere in the preparation area. Therefore, they need to be dispatched. It is assumed that workers push the container to the dispatch zone. Since the distance depends on the factory's size, calculation is done similar to replenishment costs. In this study, traveling kits are transported by dedicated AGVs. Therefore, dispatch is not needed either.

The dispatch distances dd_a are estimations based on the assumption that the average preparation cell is placed in the center of the preparation area. In practice, one may arrange cells optimally. However, this level of detail is out of scope of this research. Based upon this assumption, the operators have to travel half the width of the preparation area in both directions.

$$c^D_{fSl} = loc\frac{\lambda_f dd_a}{n_f OV\epsilon^L} \quad \forall f \in \mathcal{F}\ \forall p \in \mathcal{P}_i\ \forall l \in \mathcal{L}_p \tag{58}$$

$$c_K l^D = loc\frac{\lambda dd_a}{bs_K OV\epsilon^L} \quad \forall l \in \mathcal{L}_K \tag{59}$$

3.3.6. Transportation costs

During the transportation process, parts are delivered to the BoL. The number of transports depends on part demands divided by the number of parts in a load carrier. The result is multiplied with vehicle operator costs voc and the investment or leasing costs for the forklifts lcf and tugger trains lct, respectively. Lastly, this is multiplied by the time, which is calculated based on transportation distance and (un)loading activities divided by the vehicles' utilization rates (ϵ^F for forklifts and ϵ^T for tugger trains). Transportation distances depend on the direct distances di_l for forklifts and the milk-run lengths for boxed-supply, sequencing, and stationary kitting. Whenever tugger trains are used, only a fraction of their costs is calculated as they can typically hold more than one load carrier. To this end, we divide the load carriers volume (v_i for boxes, v_f for sequenced containers, and v for st. kits) by the tugger train's volume ttv.

$$c^T_{iLla} = (vo + lcf)\frac{\lambda_i}{n_{iL}}(\frac{2di_{al}}{\epsilon^F f v} + \frac{t^P_{iL} + t^D}{\epsilon^F}) \quad \forall i \in \mathcal{I}\ \forall l \in \mathcal{L}_i\ \forall a \in \mathcal{A} \tag{60}$$

$$c^T_{iBla} = (voc + lct)\frac{\lambda_i v_i}{n_{iB} vtt}(\frac{mr_p}{\epsilon^T ttv} + \frac{ttc + ttd_B}{\epsilon^T}) \quad \forall i \in \mathcal{I}\ \forall l \in \mathcal{L}_i\ \forall a \in \mathcal{A} \tag{61}$$

$$c^T_{fSla} = (voc + lct)\frac{\lambda_f v_f}{bs_S n_f ttv}(\frac{mr_S}{\epsilon^T ttv} + \frac{ttc + ttd_S}{\epsilon^T}) \quad \forall f \in \mathcal{F}\ \forall l \in \mathcal{L}_f\ \forall a \in \mathcal{A} \tag{62}$$

$$c^T_{Kla} = (voc + lct)\frac{\lambda v}{ttv}(\frac{mr_K}{\epsilon^T ttv} + \frac{ttc + ttd_K}{\epsilon^T}) \quad \forall l \in \mathcal{L}_K\ \forall a \in \mathcal{A} \tag{63}$$

As transportation activities differ for most line feeding policies, cost calculations need to be adjusted accordingly. Eq. (60) calculates transportation costs for line stocked parts based on the number of pallets required. Similarly, costs for box-supplied parts depend on the number of boxes required (see Eq. (61)). However, transportation is assumed to be conducted by tugger-trains, whereas forklifts transport line stocked parts. Tugger-trains also transport sequencing containers, but the number of transports depends on the families' demand λ_f instead of the individual part demands (Eq. (62)). The transportation costs for stationary kits are calculated in Eq. (63). It is assumed that each final product requires a kit. Therefore, costs are based on the product demand.

3.3.7. Usage costs

Contrary to other studies (Limère et al., 2012; Schmid et al., 2021), usage costs are calculated differently. The usage process consists of various activities such as identifying assembly instructions and required parts, walking to the parts' storage locations, searching for the correct part, fetching and handling the parts, and finally assembling the part. In contrast to calculating all these activities' costs, Constraint (16) ensures the execution of all activities within the cycle time. If that cycle time is not sufficient, some activities may be shifted to another station. Therefore, usage costs consist of the assembly operator costs per second multiplying by the number of products λ and cycle time ct.

$$c^U = aoc\ ct\lambda \tag{64}$$

4. Case study

We applied the approach described above to optimize three assembly lines of an automobile manufacturer. While we obtained feeding-related data, balancing-related data was unavailable. Therefore, we reverse-engineered missing data based upon available information. Therefore, the case does not represent real-life data exactly. However, we created multiple instances with varying parameters to mitigate this problem.

4.1. Assembly line feeding data

Data related to assembly line feeding contains information about the demands of parts, delivery quantities, physical characteristics, or costs for investment, space, and operations. Table 3 summarized some characteristics of the assembly lines under investigation. It contains the number of different product models assembled (#Models), the overlap of parts, i.e., the percentage of parts that are used in at least two product models, the number of parts (#Parts), and part families (#Families). Lastly, the range of the number of parts per family and the average part demand, and its standard deviation are presented.

Table 3
Instance characteristics w.r.t. feeding.

Lines	#Models	Overlap	#Parts	#Families	Parts/Family	Avg. daily demand (± std. dev.)
Trim3L	3	56	265	138	[1–16]	148 ± 240
Final2EL	3	89	75	47	[1–8]	195 ± 367
Final4L	3	85	104	55	[1–20]	236 ± 422

Table 4
Instance characteristics w.r.t. balancing.

Metric/Lines	Scholl (1993)	Otto et al. (2013)	Final2EL	Final4L	Trim3L
Instances	269	7350	16	16	16
Tasks	7–297	20–1000	47	55	138
# Is. tasks	0–4	0–8	0–0	0–0	0–0
Average immediate pred.	0.86–1.93	0.65–2.81	1.21–1.89	1.2–1.89	1.20–2.23
Tasks w/o predecessors	1–26	1–30	5–28	7–16	8–51
Tasks w/o successors	1–34	1–373	1–6	2–14	7–20
Order strength	22.49–83.82	14.21–90.45	20–80	20–80	20–80
Min. task degree	0–2	0–2	1–1	1–1	1–1
Max. task degree	3–23	2–47	4–10	5–8	9–25
% Bottleneck tasks	0–11.11	4.5–5	0	0	0.72-0.72
% Chain tasks	0–57.14	0–95	0-17.02	0–36.36	0–38.40
Avg. Chain length	0–4.3	0–4.98	0–4	0–4	0–5.3
Convergence	51.09–90.91	34.89–1	49.47–80.70	46.61–80.88	42.86–77.97
Divergence	51.38–1	35.47–1	46.53–73.43	45.83–75.34	42.46–67.65
$\frac{T_{min}}{c}$ [%]	0.06–17	2-41.8	2.5–2.5	2.5–2.5	1.66–1.66
$\frac{T_{max}}{c}$ [%]	30–100	19–99.7	93.33–93.33	53.33–75.16	93.33–93.33
Std. dev. time	0.05–1	0.04–0.19	0.16–0.18	0.12–0.14	0.17–0.17

4.2. Assembly line balancing data

Since the company could not provide balancing-associated data, we reverse-engineered it based on available information. For this problem, balancing data consists of assembly times and precedence constraints. We create assembly times for each task, which we assume to be equal to a part family. We are aware of two dataset generators in the literature (Serrano et al., 2014; Otto et al., 2013) describing a simulation-based approach to generate precedence graphs and assembly times jointly. The generation in both cases is based on probability distributions and randomized constructive procedures. Contrary, we aim to reverse-engineer this data using real-world information such as the number of tasks, the available cycle time, and the number of tasks per station. In Appendices B and C, we propose methods to generate precedence data and assembly times separately, allowing for combinatorial combination. Table 4 shows the data obtained from these methods and compares it to real-life datasets from literature.

For this comparison, we utilize various metrics summarized by Otto et al. (2013). Amongst them are number of tasks (Elmaghraby and Herroelen, 1980), order strength (Mastor, 1970), average number of direct predecessors (Rosenberg and Ziegler, 1992), maximum task degree (Baybars, 1986), and the number of isolated tasks. Table 4 shows different sets of instances per column and the corresponding range of values for any given metric. For example, the assembly line found in the case study, named Final2EL has been reengineered such that 16 instances have been created. One metric that can be observed is that over these 16 instances, the maximum task degree ranges in between 4 and 10. Based on the value dispersion, one can conclude that our approach produces realistic data.

5. Results

This section will first provide an overview of the type of experiments conducted and their managerial implications. Afterwards, we will report on the solution quality regarding optimality gaps and computation times for the different experiments. The experiments are based on the problem instances described in Tables 3 and 4. Costs are minimized for a planning horizon of one month. Four different assembly time distributions and precedence graphs have been created for each assembly line according to the methods described in Appendices B and C. Those were factorially combined to create 16 instances for each assembly line.

5.1. Comparison of hierarchical and simultaneous optimization

This section investigates the monetary value of an integrated planning approach for line balancing and feeding while considering facility sizing. To this end, we consider and optimize various settings.

- **Fixed balance, fixed feeding (FBFF):** The assembly line is optimally balanced, similar to the current company setup. It assumes that all parts are supplied in boxes, with exceptions for oversized or heavy parts, which are either line stocked or sequenced based on the part family size, namely families with a single part are line stocked and others are sequenced. This reflects a practical decision rule.
- **Fixed balance, optimized feeding (FBOF):** The assembly line is balanced exactly as in the previous setting. However, part feeding is optimized.
- **Re-balance by 1 station, optimized feeding (RB1OF):** The assembly line may undergo slightly re-balancing, shifting tasks by one station in each direction while maintaining precedence relations. Part feeding is optimized.
- **Re-balance by 2 stations, optimized feeding (RB2OF):** Similar to Re-balance by 1 station, optimized feeding (RB1OF), but with tasks potentially shifted by up to two stations in each direction. Part feeding is optimized as well.
- **Optimized balance, optimized feeding (OBOF):** In this setting, the assembly line is balanced optimally, while considering precedence relations. At the same time, part feeding is optimized.

The results for those different settings and the different assembly lines are summarized in Table 5. Most importantly, it can be seen that each setting outperforms the previous, more restricted, setting as one would expect. The table shows information on the different cost elements in different solutions, and provides the number of stations and the size of the preparation area averaged over all 16 instances for each assembly line. The rightmost column reports on the range of improvement for each individual instance. In most cases without any

Table 5
Influence of balancing decisions on overall costs.

Setting	Line	Final2EL	Final4L	Trim3L	Change interval compared to prev. setting over all instances [%]
	# Inst.	16	16	16	
FBFF	Avg. total cost	129 020.75	136 805	413 093.5	
	Avg. logistics costs	9739.02	34 563.41	88 366	
	Avg. assembly costs	119 281.73	102 241.59	324 727.5	
	Avg. number stations	7	6	18	
	Avg. logistics facility space [m^2]	374.4	528	1296	
	Avg. number of tr. kits	0	0	0	
FBOF	Avg. total cost	126 959.75	117 009.75	380 212	[−14.47;−1.5]
	Avg. logistics costs	7677.9	14 767.94	73 487.9	
	Avg. assembly costs	119 281.85	102 241.81	306 724.1	
	Avg. number stations	7	6	18	
	Avg. logistics facility space [m^2]	278.4	432	1296	
	Avg. number of tr. kits	0	0	0	
RB1OF	Avg. total cost	113 843.31	115 797.19	370 706.63	[−13.46;0]
	Avg. logistics costs	10 517.97	17 815.41	67 155.76	
	Avg. assembly costs	103 325.34	97 981.78	303 550.86	
	Avg. number stations	6.06	5.75	17.81	
	Avg. logistics facility space [m^2]	287.4	444	1002	
	Avg. number of tr. kits	1	1	3.56	
RB2OF	Avg. total cost	112 114.38	113 241.56	356 218.13	[−8.7;0]
	Avg. logistics costs	9872.68	19 516.93	69 716.66	
	Avg. assembly costs	102 241.70	93 724.63	286 501.47	
	Avg. number stations	6	5.5	16.81	
	Avg. logistics facility space [m^2]	299.4	456	1107	
	Avg. number of tr. kits	0.69	1.5	0.81	
OBOF	Avg. total cost	109 741.25	108 667.44	348 343	[−10.84;0]
	Avg. logistics costs	11 753.62	21 320.76	69 293.49	
	Avg. assembly costs	97 987.63	87 346.68	279 049.51	
	Avg. number stations	5.75	5.125	16.38	
	Avg. logistics facility space [m^2]	305.4	462	1113	
	Avg. number of tr. kits	1.13	1.63	0.56	

improvement, this can likely be attributed to computational difficulties in finding better solutions.

In conclusion, simultaneous optimization of both assembly line balancing and feeding problems results in an average reduction of 10.13% in overall costs compared to individually optimized solutions (FBOF). When applying simple decision rules for line feeding (FBFF), the joint optimization of balancing and feeding (OBOF) leads to an average cost reduction of 18.8%. It is important to note that these findings may vary depending on the specifics of cost calculations and the assembly task times and precedence graphs.

As shown in Table 5, feeding costs can be minimized for a given balance. However, the simultaneous optimization of (re)balancing and feeding may reduce the number of stations but increase feeding costs.

Fig. 6 provides an example of how different settings impact an assembly line. While actual assembly facilities may have different spatial organizations, decisions, and costs are expected to align with the assumptions made in this study.

A key observation from Fig. 6 is that the FBFF setting does not utilize the entire space available at the BoL. When optimizing feeding decisions, the utilization of BoL-space almost doubles. Those optimal feeding decisions reduce the space requirements in both preparation area and warehouse and, thus, reduce transportation distances. While minimal re-balancing (RB1OF) redistributes task to minimize feeding costs only, a more comprehensive rebalancing reduces the number of stations. At the same time, feeding efforts are strongly increasing as more space-saving but costlier policies need to be chosen. Simultaneously, the logistics facility's size is increasing to facilitate the required preparation efforts.

5.2. Line feeding policy selection

In the preceding section, we demonstrated the adverse economic consequences of hierarchical decision-making compared to a simultaneous approach, while this section examines the impact of selecting subsets of feeding policies.

The number of feeding policies used in any assembly system increases its organizational complexity, both for underlying software systems and operators on the shop floor executing the corresponding activities. Therefore, a solution that utilizes fewer line feeding policies is preferable from a managerial perspective. Therefore, we conducted a series of experiments similar to those described in the previous section but allowing only the use of a subset of feeding policies, represented in Table 6. We iteratively selected different feeding policy subsets. We removed a single line feeding policy in ($\mathcal{P}^1 - \mathcal{P}^5$). Based on the corresponding findings, we removed two ($\mathcal{P}^6 - \mathcal{P}^{11}$) and three line feeding policies ($\mathcal{P}^{13} - \mathcal{P}^{16}$) but always retained boxed-supply due to the findings explained in the next paragraph. In addition, we also tested setting $\mathcal{P}^{12}$, only allowing line stocking and stationary kitting, to compare our results to similar studies such as Limère et al. (2012), Sternatz (2014), or Calzavara et al. (2021).

A noteworthy finding concerns the feasibility of the solutions as reported in Table 7. Certain subsets of line feeding policies such as $\mathcal{P}^{13}$ and $\mathcal{P}^{14}$ never resulted in a feasible solution, even when allowing the rebalancing of the line. In practice, one may find solutions to accommodate these policy subsets, but it will likely require significant adaptations to the working mode. Other policy subsets, namely $\mathcal{P}^2$, $\mathcal{P}^6$, and $\mathcal{P}^{12}$ exhibit similar problems. However, in these cases, rebalancing the line resolves some infeasibilities. Similar practical concerns as mentioned above hold for those instances. Due to the infeasibilities encountered when removing boxed-supply, we retained this feeding policy in policy subsets $\mathcal{P}^6$-$\mathcal{P}^{11}$ and $\mathcal{P}^{13}$-$\mathcal{P}^{16}$.

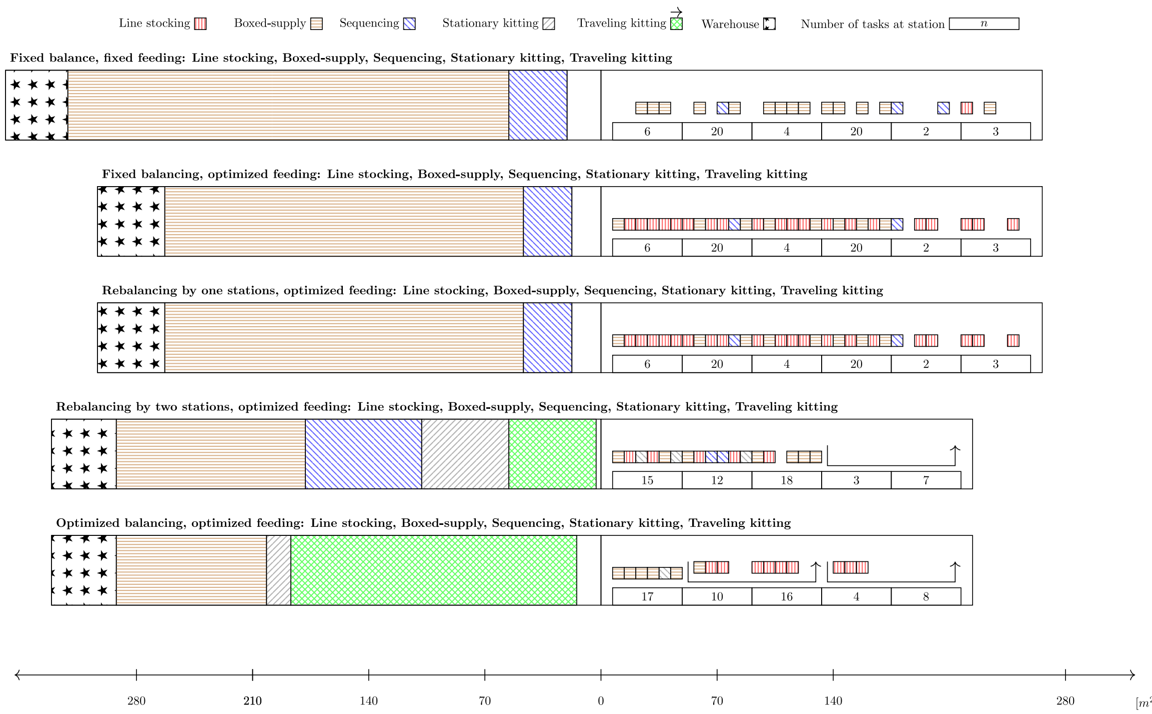

Fig. 6. Change of line organization for one instance of assembly line 'Final4L'.

Table 6
Line feeding policy subsets.

Setting	$\mathcal{P}^0$	$\mathcal{P}^1$	$\mathcal{P}^2$	$\mathcal{P}^3$	$\mathcal{P}^4$	$\mathcal{P}^5$	$\mathcal{P}^6$	$\mathcal{P}^7$	$\mathcal{P}^8$	$\mathcal{P}^9$	$\mathcal{P}^{10}$	$\mathcal{P}^{11}$	$\mathcal{P}^{12}$	$\mathcal{P}^{13}$	$\mathcal{P}^{14}$	$\mathcal{P}^{15}$	$\mathcal{P}^{16}$
Line stocking	✓	✗	✓	✓	✓	✓	✓	✓	✓	✗	✗	✗	✓	✓	✗	✗	✗
Boxed-supply	✓	✓	✗	✓	✓	✓	✓	✓	✓	✓	✓	✓	✗	✓	✓	✓	✓
Sequencing	✓	✓	✓	✗	✓	✓	✓	✗	✗	✓	✓	✗	✗	✗	✓	✗	✗
Stationary kitting	✓	✓	✓	✓	✗	✓	✗	✓	✗	✓	✗	✓	✓	✗	✗	✓	✗
Traveling kitting	✓	✓	✓	✓	✓	✗	✗	✗	✓	✗	✓	✓	✗	✗	✗	✗	✓

Table 7
Number of infeasible instances considering various line feeding policies and balancing approaches.

Balancing setting	$\mathcal{P}^2$	$\mathcal{P}^6$	$\mathcal{P}^{12}$	$\mathcal{P}^{13}$	$\mathcal{P}^{14}$
FBOF	16	16	48	48	48
RB1OF	9	14	27	48	48
RB2OF	4	6	20	48	48
OBOF	3	6	16	48	48

In addition, we analyzed the cost changes for the different lines compared to the lowest cost solutions found in Tables 8–10. Slightly reducing the number of possible line feeding policies, rarely causes significant cost increases for any assembly line. Conversely, removing boxed-supply as an option increases costs by at least 17.78%

The setting $\mathcal{P}^{12}$, studied most frequently in literature (Limère et al., 2012; Sternatz, 2015; Calzavara et al., 2021), is consistently performing much worse than any other setting while additionally suffering from infeasibility in some cases. In contrast, $\mathcal{P}^{16}$, another setting with only two line feeding policies, shows more consistent and better results. This stark discrepancy may initially seem surprising since the above-mentioned studies found feasible solutions using only feeding policy subset $\mathcal{P}^{12}$. However, an explanation may be found in the fact that the company under investigation in this study had already implemented all five line feeding policies, likely due to its necessity.

Table 8
Cost increase considering various line feeding policies and balancing approaches for assembly line Final2EL [%] compared to the lowest cost achievable.

Policy subset	OBOF	RB2OF	RB1OF	FBOF
$\mathcal{P}^0$	0	4.19	6.25	19.07
$\mathcal{P}^4$	0.08	4.32	6.37	19.07
$\mathcal{P}^3$	0.51	4.69	6.83	19.54
$\mathcal{P}^8$	0.7	5.71	8.11	20.26
$\mathcal{P}^1$	1.69	7.1	8.9	22.55
$\mathcal{P}^{10}$	2.3	7.24	9.7	22.55
$\mathcal{P}^{11}$	3.35	7.38	9.49	23.12
$\mathcal{P}^{16}$	3.95	8.29	10.28	23.12
$\mathcal{P}^5$	6.9	7.03	8.97	19.07
$\mathcal{P}^6$	6.9	7.03	9.84	19.07
$\mathcal{P}^7$	7.78	7.9	9.92	19.54
$\mathcal{P}^9$	10.85	11.01	12.92	23.54
$\mathcal{P}^{15}$	12.06	12.22	14.38	24.68
$\mathcal{P}^2$	17.78	19.73	20.74	27.52
$\mathcal{P}^{12}$	37.22	43.89	48.74	NA

5.3. Computational results

The Bender's decomposition approach described in Section 3.2 is applied to solve all instances in all the scenarios described above. We implemented this approach in C++, using Gurobi 9.1 as a solver. A High-Performance-Computer with 64 GB of RAM and six CPU cores of an Intel(R) Xeon(R) Gold 6130 computed each instance. Since this study deals with medium-to-long-term decisions, the computation time

Table 9
Cost increase considering various line feeding policies and balancing approaches for assembly line Final4L [%] compared to the lowest cost achievable.

Policy subset	OBOF	RB2OF	RB1OF	FBOF
$\mathcal{P}^0$	0	3.7	6.94	9.89
$\mathcal{P}^4$	0.46	4.02	7.15	9.89
$\mathcal{P}^3$	1.24	5.17	8.4	11.79
$\mathcal{P}^8$	1.97	5.86	9.59	13.13
$\mathcal{P}^1$	2.36	5.87	10.25	19.07
$\mathcal{P}^{10}$	2.48	6.11	10.25	19.12
$\mathcal{P}^{11}$	2.58	6.68	10.98	19.27
$\mathcal{P}^{16}$	2.89	6.83	10.98	19.37
$\mathcal{P}^5$	3.87	8.6	9.73	9.89
$\mathcal{P}^6$	6.89	9.65	9.75	9.89
$\mathcal{P}^7$	7.66	11.6	11.69	12.01
$\mathcal{P}^9$	12.09	17.61	19.85	21.45
$\mathcal{P}^{15}$	15.74	20.98	22.32	23.85
$\mathcal{P}^2$	36.35	37.83	39.55	NA
$\mathcal{P}^{12}$	54.75	54.13	58.54	NA

Table 10
Cost increase considering various line feeding policies and balancing approaches for assembly line Trim3L [%] compared to the lowest cost achievable.

Policy subset	OBOF	RB2OF	RB1OF	FBOF
$\mathcal{P}^0$	0	0.6	5.33	10.46
$\mathcal{P}^5$	0.3	1.72	5.66	11.44
$\mathcal{P}^4$	1.44	1.62	6.21	10.46
$\mathcal{P}^1$	1.54	2.31	6.76	11.93
$\mathcal{P}^9$	2.27	4.1	8.34	12.76
$\mathcal{P}^3$	3.02	3.19	7.68	13.3
$\mathcal{P}^6$	3.03	3.96	4.26	NA
$\mathcal{P}^{10}$	3.52	3.52	6.78	12.18
$\mathcal{P}^8$	4.02	4.02	7.97	13.3
$\mathcal{P}^{11}$	4.85	5.52	8.16	13.79
$\mathcal{P}^7$	5.51	7.09	10.37	14.92
$\mathcal{P}^{16}$	5.7	5.7	8.21	13.79
$\mathcal{P}^{15}$	8.52	11.81	16.35	19
$\mathcal{P}^2$	20.81	20.81	23.18	31.67
$\mathcal{P}^{12}$	28.23	29.05	29.5	NA

was set to three hours, i.e., 10,800 s. This truncation was done due to computational resource availability even though, in practice, more time is justifiable. The relaxation, described in Appendix A, is included for all tests. The ϵ, i.e., the allowed master optimality gap, was set to 0, 0, 0.25, 0.3, and 0.35 for the different balancing settings FBFF, FBOF, RB1OF, RB2OF, OBOF, respectively.

Table 11 shows the computational results of optimizing the various settings described in Sections 5.1 and 5.2. To this end, the table shows the number of instances (# inst.) for each setting. For each setting, the table shows the number of instances that could be solved optimally (#opt.), remained unsolved due to infeasibility or intractability (#uns.), and could be solved suboptimally (#subopt.). In addition, we report on the average solution time of all instances (avg. time all) and the computation time for instances that solved optimality (avg. time opt.). Similarly, the average gaps over all instances (avg. gap all) are compared to those of suboptimally solved instances (avg. gap).

The results indicate that, as expected, additional decisions complicate the solution procedure. However, gaps for rebalancing settings seem reasonably small. The impact of the number of line feeding policies on computational tractability does not reveal any recognizable patterns.

6. Conclusion and future research

6.1. Key contributions and findings

This study proposes a comprehensive framework for concurrently addressing assembly line balancing, assembly line feeding, and facility sizing. Our study introduces several novel aspects:

1. The assembly line balancing and feeding integration, considering five distinct line feeding policies.
2. Accurate consideration of walking and searching times at the assembly line.
3. Modeling of multiple traveling kits in a discretized manner.
4. Incorporation of assembly facility design in the model.
5. Introduction of a logic-based Bender's decomposition scheme to solve this complex problem.
6. Extensive testing of various feeding policy subsets.

Our findings reveal that simultaneous consideration of both problems leads to an average cost reduction of around 10% compared to individually optimized solutions (FBOF). For approaches employing simple decision rules for line feeding (FBFF), costs can be reduced by around 20% on average (see Table 5). These results confirm earlier findings (Sternatz, 2015; Battini et al., 2017) regarding the benefits of integration. The magnitude of cost reductions due to an integrated planning approach in this study is lower than in previous studies. This discrepancy in cost reductions may be due to the introduction of additional line feeding policies, reducing the initial cost FBOF more profoundly, and the broader scope of our research, considering the logistics facilities sizing and the associated costs.

In the selection of feeding policy subsets, our findings carry practical implications. We discovered that opting for specific feeding policy subsets may increase cost insignificantly while significantly reducing logistical complexity. Notably, considering only four of the five feeding policies under investigation has a negligible effect on cost increase. However, removing boxed-supply as a feeding option leads to far more substantial cost increases.

While exact cost calculations may vary across companies, a critical insight from our study is the importance of striking a balance between reducing the number of line feeding policies without encountering infeasibilities or a sharp increase in costs. Our research underscores the need for a nuanced approach to finding this balance. Based on our results, it seems impossible to generalize the findings and we conjecture that depending on the cost structure present in a company, the demand data, and assembly data other feeding policies may turn out to be dispensable. Therefore, we encourage practitioners to use a similar approach to this study: it is recommended to model or assess multiple different feeding policies. Based on the results, one can gradually remove feeding policies that have been used infrequently in the previous iteration until a satisfactory degree of complexity and cost has been obtained.

Lastly, our study corroborates previous findings on the benefits of jointly optimizing balancing and feeding and provides an example of incorporating walking times that depend on feeding decisions.

6.2. Limitation

This research focuses on large product assembly, which researchers may adapt to smaller-scale products. The primary trade-off, choosing between cost-effective line feeding policies or space-saving alternatives, remains consistent, though transportation and facility sizing will likely lose importance.

A fundamental limitation is the model's applicability to an industrial context. This difficulty is primarily due to the need for detailed data collection and the complexity of the models and corresponding solution procedures. Sales-related data from ERP systems is easily accessible in most firms, and part-related data, such as volume, size, or weight, should be readily available based on technical drawings and documentation from upstream production stages or suppliers. Obtaining other data, such as walking, waiting, or picking times, may be time-consuming. Simple estimations can mitigate this challenge by using part weights and volume and time-estimation concepts like VEWF or MTM.

Table 11
Solving times and gaps when applying solving procedure.

Setting	# inst.	#opt.	#uns.	#subopt.	avg. time opt. (s)	avg. gap	avg. gap all	avg. time all (s)
FBFF $\mathcal{P}^0$	48	48	0	0	6.66	–	0	6.66
FBOF $\mathcal{P}^0$	48	32	0	16	16.94	0.05	0.02	3615.6
FBOF $\mathcal{P}^1$	48	48	0	0	760.32	–	0	760.32
FBOF $\mathcal{P}^2$	48	16	16	16	85.08	0.39	–	3650.94
FBOF $\mathcal{P}^3$	48	32	0	16	16.93	0.06	0.02	3612.32
FBOF $\mathcal{P}^4$	48	32	0	16	15.02	0.03	0.01	3610.87
FBOF $\mathcal{P}^5$	48	32	0	16	8.42	0.06	0.02	3606.22
FBOF $\mathcal{P}^6$	48	32	16	0	4.84	–	–	3.67
FBOF $\mathcal{P}^7$	48	32	0	16	5.61	0.07	0.02	3604.7
FBOF $\mathcal{P}^8$	48	32	0	16	15.81	0.03	0.01	3614.42
FBOF $\mathcal{P}^9$	48	48	0	0	24.66	–	0	24.66
FBOF $\mathcal{P}^{10}$	48	48	0	0	1058.05	–	0	1058.05
FBOF $\mathcal{P}^{11}$	48	48	0	0	119.3	–	0	119.3
FBOF $\mathcal{P}^{12}$	48	0	48	0	–	–	–	0.58
FBOF $\mathcal{P}^{13}$	48	0	48	0	–	–	–	0.01
FBOF $\mathcal{P}^{14}$	48	0	48	0	–	–	–	0.01
FBOF $\mathcal{P}^{15}$	48	48	0	0	16.02	–	0	16.02
FBOF $\mathcal{P}^{16}$	48	48	0	0	31.92	–	0	31.92
RB1OF $\mathcal{P}^0$	48	3	0	45	4945.08	0.1	0.09	10 435.28
RB1OF $\mathcal{P}^1$	48	2	0	46	1246.1	0.1	0.1	10 402.86
RB1OF $\mathcal{P}^2$	48	0	9	39	–	0.19	0.34	10 801.82
RB1OF $\mathcal{P}^3$	48	2	0	46	722.47	0.1	0.09	10 381.57
RB1OF $\mathcal{P}^4$	48	3	0	45	4215.35	0.1	0.1	10 390.43
RB1OF $\mathcal{P}^5$	48	0	0	48	–	0.06	0.06	10 801.43
RB1OF $\mathcal{P}^6$	48	1	14	33	3759.3	0.05	0.32	10 655.09
RB1OF $\mathcal{P}^7$	48	2	0	46	7638.08	0.08	0.07	10 669.66
RB1OF $\mathcal{P}^8$	48	2	0	46	494.01	0.1	0.09	10 372.94
RB1OF $\mathcal{P}^9$	48	1	0	47	1871.29	0.09	0.09	10 614.95
RB1OF $\mathcal{P}^{10}$	48	2	0	46	615.49	0.1	0.1	10 376.76
RB1OF $\mathcal{P}^{11}$	48	2	0	46	1170.7	0.1	0.09	10 399.74
RB1OF $\mathcal{P}^{12}$	48	0	27	21	–	0.18	–	10 350.95
RB1OF $\mathcal{P}^{13}$	48	0	48	0	–	–	–	0
RB1OF $\mathcal{P}^{14}$	48	0	48	0	–	–	–	0
RB1OF $\mathcal{P}^{15}$	48	2	0	46	3308.07	0.12	0.11	10 488.79
RB1OF $\mathcal{P}^{16}$	48	3	0	45	2506.31	0.08	0.07	10 282.63
RB2OF $\mathcal{P}^0$	48	1	0	47	6936.52	0.12	0.12	10 723.17
RB2OF $\mathcal{P}^1$	48	0	0	48	–	0.13	0.13	10 801
RB2OF $\mathcal{P}^2$	48	0	4	44	–	0.26	0.32	10 801.47
RB2OF $\mathcal{P}^3$	48	2	0	46	5640.15	0.13	0.12	10 586.24
RB2OF $\mathcal{P}^4$	48	4	0	44	4272.28	0.12	0.12	10 257.71
RB2OF $\mathcal{P}^5$	48	3	0	45	3683.44	0.11	0.1	10 357.07
RB2OF $\mathcal{P}^6$	48	3	6	39	417.62	0.1	0.21	10 152.54
RB2OF $\mathcal{P}^7$	48	2	0	46	1527.09	0.12	0.12	10 415.81
RB2OF $\mathcal{P}^8$	48	3	0	45	1003.89	0.12	0.11	10 189.4
RB2OF $\mathcal{P}^9$	48	3	0	45	424.55	0.13	0.12	10 152.49
RB2OF $\mathcal{P}^{10}$	48	0	0	48	–	0.11	0.11	10 801
RB2OF $\mathcal{P}^{11}$	48	0	0	48	–	0.13	0.13	10 801.1
RB2OF $\mathcal{P}^{12}$	48	0	20	28	–	0.25	–	10 576.93
RB2OF $\mathcal{P}^{13}$	48	0	48	0	–	–	–	0
RB2OF $\mathcal{P}^{14}$	48	0	48	0	–	–	–	0
RB2OF $\mathcal{P}^{15}$	48	3	0	45	338.84	0.16	0.15	10 147.12
RB2OF $\mathcal{P}^{16}$	48	2	0	46	447.9	0.11	0.1	10 369.7
OBOF $\mathcal{P}^0$	48	0	0	48	–	0.18	0.18	10 801.04
OBOF $\mathcal{P}^1$	48	0	0	48	–	0.18	0.18	10 801
OBOF $\mathcal{P}^2$	48	0	3	45	–	0.32	0.37	10 801
OBOF $\mathcal{P}^3$	48	0	0	48	–	0.18	0.18	10 800.98
OBOF $\mathcal{P}^4$	48	0	0	48	–	0.18	0.18	10 801
OBOF $\mathcal{P}^5$	48	4	0	44	3490.69	0.19	0.18	10 191.94
OBOF $\mathcal{P}^6$	48	0	6	42	–	0.17	0.27	10 801
OBOF $\mathcal{P}^7$	48	2	0	46	1442.24	0.2	0.19	10 411.3
OBOF $\mathcal{P}^8$	48	0	0	48	–	0.17	0.17	10 806.09
OBOF $\mathcal{P}^9$	48	3	0	45	2139.03	0.22	0.2	10 259.65
OBOF $\mathcal{P}^{10}$	48	0	0	48	–	0.18	0.18	10 801
OBOF $\mathcal{P}^{11}$	48	0	0	48	–	0.18	0.18	10 801
OBOF $\mathcal{P}^{12}$	48	0	16	32	–	0.3	0.53	10 801.24
OBOF $\mathcal{P}^{13}$	48	0	48	0	–	–	–	0
OBOF $\mathcal{P}^{14}$	48	0	48	0	–	–	–	0
OBOF $\mathcal{P}^{15}$	48	1	0	47	652.11	0.22	0.21	10 589.58
OBOF $\mathcal{P}^{16}$	48	0	0	48	–	0.16	0.16	10 801

Concerning the implementation of sophisticated algorithms and models, the development of dedicated assembly system planning software within ERP or other solutions is recommended. This research and similar studies lay the groundwork for such software developments.

6.3. Future research

Future research may build upon this study to investigate the effect of multiple versions of each line feeding policy. For example, different kit sizes could be relevant. Similarly, one may provide sequenced parts in smaller boxes and store them in the same flow racks as boxed-supplied parts. Furthermore, applying this planning approach to a multi-model setting, where larger batches of different products are produced on the same assembly line, seems interesting because multi-model lines require an exchange of parts at the BoL between any two batches. In addition, it may be relevant for decision-makers to examine solutions proposed in this and similar research by stochastic simulation runs, incorporating stochastic demand and logistics and production task duration. Similarly, it could be particularly relevant to investigate a change in production rate or product specification. By doing so, the implications of facility sizing could be better estimated, and decision makers could choose a facility size that is more robust to product or process changes. Lastly, improving the solution approach is vital to optimize multiple (dis)connected assembly lines. The proposed solution approach may be a benchmark for (meta-)heuristic approaches.

CRediT authorship contribution statement

Nico André Schmid: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Benoit Montreuil:** Supervision, Resources, Project administration, Conceptualization. **Veronique Limère:** Writing – review & editing, Supervision, Project administration, Conceptualization.

Data availability

Data will be made available on request.

Generated assembly line balancing precedence graphs from automotive OEM (Original data) (Mendeley Data)

Acknowledgments

The first author received support from the Research Foundation Flanders (FWO) under grant number FWO18/ASP/198. This research was also made possible by the patronage of the National Bank of Belgium (NBB).

Appendix A

As discussed in Section 3.2, one may include a relaxation of the optimality subproblems into the master problem. In this case, the domain of the variables x_{ipl}, ψ_{fpl}, and t_f need to be extended by the set of stations. Furthermore, the following terms need to be added to the objective function. Lastly, the constraints listed below need to be considered (See Eqs. (66)–(83) in Box II).

$$\sum_{i\in\mathcal{I}}\sum_{s\in\mathcal{S}_i}\sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}_{sp}} c_{ipl}x_{ispl} + \sum_{f\in\mathcal{F}}\sum_{s\in\mathcal{S}_f}\sum_{p\in\mathcal{P}_f}\sum_{l\in\mathcal{L}_{sp}} c_{fpl}\psi_{fspl} + \sum_{p\in\mathcal{P}}\sum_{l\in\mathcal{L}_p} c_{pl}\chi_{pl} + \sum_{i\in\mathcal{I}} c_i^R d_i^R + \sum_{f\in\mathcal{F}} c_f^D d_f^D + \sum_{l\in\mathcal{L}} c^D d_l^D \tag{65}$$

Cost in the relaxation need to be calculated differently than for the optimality subproblem. While replenishment costs were calculated as in Eq. (53) for the optimality subproblem, they need to be set to 0 and c_i^R needs to be introduced to calculate costs as in Eq. (84).

$$c_i^R = (voc + lcf)\frac{\lambda_i}{n_{iL}\epsilon_L f v} \quad \forall \quad i\in\mathcal{I}\ \forall p\in\mathcal{P} \tag{84}$$

Similar adjustments need to be made to the dispatch costs, presented in Eqs. (58) and (59). Both equations need to be set to 0, but the following cost calculations need to be added:

$$c_f^D = loc\frac{\lambda_f}{n_f OV\epsilon^l} \qquad \forall f\in\mathcal{F} \tag{85}$$

$$c^D = loc\frac{\lambda}{bs_K OV\epsilon^l} \tag{86}$$

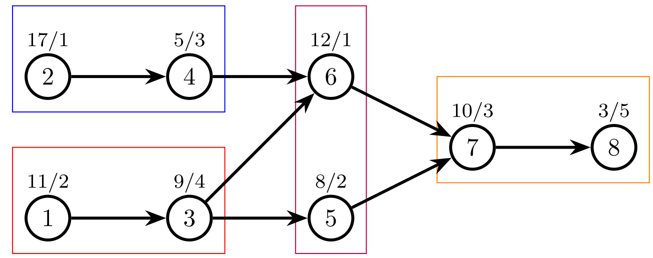

Fig. 7. Precedence graph of assembly tasks described in Bowman (1960).

Appendix B

A precedence graph links some or all of the tasks under consideration with each other. The links between those are typically distinguished in direct and indirect precedence relations. A classical and easy to explain example is the precedence graph described by Bowman (1960) (see Fig. 7). Here, task 1 is a direct predecessor of task 3, whereas it is an indirect predecessor of task 5 since task 3 indirectly connects tasks 1 and 5. More generally, one could state that every two tasks that are linked without a task in between them are direct predecessors/successors. Tasks that are linked through several intermediate tasks are called indirect predecessors/successors.

In addition to the metrics used in Table 4, we propose a new metric called station distance (SD) for reverse-engineered precedence graphs. With a given line balance, precedence graphs could be created somewhat arbitrarily. The only restriction is that a task assigned to a later station cannot be a predecessor of a task assigned to an earlier station. To diminish this arbitrariness, we propose to link at least one task of each station to any task of a station with a maximum distance. This maximum distance defines the SD metric:

$$SD_s = \max_{f,g\in\mathcal{F}:x_{fs}=1\wedge g\in\mathcal{D}_f^P\cup\mathcal{D}_f^S} |sx_{fs} - sx_{gs}| \quad \forall \quad s\in\mathcal{S} \tag{87}$$

We define the desired order strength and input the current assignment to a constraint programming model to reverse-engineer precedence graphs. This model does not allow isolated tasks, i.e., tasks without any predecessor or successor. This assumption is made because isolated tasks occur rather seldom in real life (Sternatz, 2014; Otto et al., 2013). Before running the model described in the following, tasks are given a number in increasing order such that tasks at earlier stations have lower numbers than tasks at later stations.

$$\text{s. t.:} \quad p_{fg} + p_{gk} = 2 \Rightarrow p_{fk} = 1 \qquad \forall f,g,k\in\mathcal{F} : f<g<k \tag{88}$$

$$\sum_{f\in\mathcal{F}:f<k} p_{fk} + \sum_{f\in\mathcal{F}:k<f} p_{kf} \geq 1 \qquad \forall k\in\mathcal{F} \tag{89}$$

$$\sum_{f\in\mathcal{F}}\sum_{k\in\mathcal{F}:f<k} p_{fk} = \frac{|\mathcal{F}|^2 - |\mathcal{F}|}{2}OS \tag{90}$$

$$\sum_{f\in\mathcal{F}:s_f=s}\ \sum_{t\in\mathcal{S}:t<s\wedge t\geq s-SD}\ \sum_{g\in\mathcal{F}:s_g=t} p_{gf} \geq 1 \qquad \forall s\in\mathcal{S} \tag{91}$$

$$\sum_{f\in\mathcal{F}:s_f=s}\ \sum_{t\in\mathcal{S}:t>s\wedge t\leq s+SD}\ \sum_{g\in\mathcal{F}:s_g=t} p_{gf} \geq 1 \qquad \forall s\in\mathcal{S} \tag{92}$$

$$p_{fg} + p_{gk} = 2 \Rightarrow d_{fk} = 0 \qquad \forall f\in\mathcal{F} + \forall g\in\mathcal{F} : f<g \quad \forall k\in\mathcal{F} : j<k \tag{93}$$

$$p_{fg} = 1 \Rightarrow d_{fg} = 1 \qquad \forall f\in\mathcal{F} + \forall g\in\mathcal{F} : g = f+1 \tag{94}$$

$$d_{fg} \leq p_{fg} \qquad \forall f\in\mathcal{F} + \forall g\in\mathcal{F} \tag{95}$$

$$p_{fg} = 1 \wedge \sum_{k\in\mathcal{F}:f<k<g} p_{fk}p_{kg} = 0 \Rightarrow d_{fg} = 1 \qquad \forall f\in\mathcal{F} + \forall g\in\mathcal{F} \tag{96}$$

$$\sum_{f\in D_j^P} d_{fg} \leq \overline{D} \qquad \forall f\in\mathcal{F} \tag{97}$$

$$\sum_{p\in\mathcal{P}_i}\sum_{l\in\mathcal{L}_{sp}} x_{ispl} = x_{fs} \qquad \forall s\in\mathcal{S}\ \forall f\in\mathcal{F}_s\ \forall i\in\mathcal{I}_f \tag{66}$$

$$\sum_{p\in\mathcal{P}_i}\sum_{l\in\mathcal{L}_{sp}} \underline{t}_{pl} x_{ispl} + t^A x_{fs} \le t_{fs} \qquad \forall s\in\mathcal{S}\ \forall f\in\mathcal{F}_s\ \forall i\in\mathcal{I}_f \tag{67}$$

$$\sum_{f\in\mathcal{F}_m\cap\mathcal{F}_s} t_{fs} \le c \qquad \forall s\in\mathcal{S}\ \forall m\in\mathcal{M} \tag{68}$$

$$\sum_{i\in\mathcal{I}_s\cap\mathcal{I}_p} x_{ispl} \le M\chi_{pl'_{pl}} \qquad \forall s\in\mathcal{S}\ \forall p\in\mathcal{P}\ \forall l\in\mathcal{L}_{sp} \tag{69}$$

$$\sum_{i\in\mathcal{I}_f} x_{ispl} \le M\psi_{fspl} \qquad \forall f\in\mathcal{F}\ \forall s\in\mathcal{S}_f\ \forall p\in\mathcal{P}_f\setminus\{L,B\}\ \forall l\in\mathcal{L}_{sp}\cap\underline{\mathcal{L}} \tag{70}$$

$$\sum_{i\in\mathcal{I}_s} x_{isLl} + \sum_{f\in\mathcal{F}_s}\psi_{fsSl} + \chi_{Bl} + \chi_{Kl} + \chi_{Tl} \le 1 \qquad \forall s\in\mathcal{S}\ \forall l\in\mathcal{L}_s^B \tag{71}$$

$$\sum_{i\in\mathcal{I}_s\cap\mathcal{I}_B} r_{ir} x_{isBl} \le R_{Br} \qquad \forall s\in\mathcal{S}\ \forall l\in\mathcal{L}_{sB} \tag{72}$$

$$\sum_{f\in\mathcal{F}_s\cap\mathcal{F}_p} r_{fr}\psi_{fspl} \le R_{pr} \qquad \forall s\in\mathcal{S}\ \forall p\in\{K,T\}\ \forall l\in\mathcal{L}_{sp} \tag{73}$$

$$\sum_{i\in\mathcal{I}}\sum_{s\in\mathcal{S}_i}\sum_{l\in\mathcal{L}_{sp}} x_{ispl} = \sum_{s\in\mathcal{S}_i}\sum_{l\in\mathcal{L}_{sp}} x_{jspl} \qquad \forall f\in\mathcal{F}\ \forall p\in\mathcal{P}_f\ \forall j\in\mathcal{I}_f \tag{74}$$

$$\sum_{s\in\mathcal{S}_i}\sum_{l\in\mathcal{L}_{sp}} d^R_{pa} x_{ispl} + M(v_a - 1) \le d^R_i \qquad \forall i\in\mathcal{I}\ \forall a\in\mathcal{A}\ \forall p\in\mathcal{P}_i \tag{75}$$

$$\sum_{s\in\mathcal{S}_i}\sum_{l\in\mathcal{L}_{sS}} d^D_a \psi_{fsSl} + M(v_a - 1) \le d^D_f \qquad \forall f\in\mathcal{F}_S\ \forall a\in\mathcal{A} \tag{76}$$

$$d^D_a \chi_{Kl} + M(v_a - 1) \le d^D_l \qquad \forall s\in\mathcal{S}\ \forall l\in\mathcal{L}_{sK}\ \forall a\in\mathcal{A} \tag{77}$$

$$x_{ispl} \ge 0 \qquad \forall i\in\mathcal{I}\ \forall s\in\mathcal{S}_i\ \forall p\in\mathcal{P}_i\ \forall l\in\mathcal{L}_{sp}\cap\underline{\mathcal{L}} \tag{78}$$

$$\psi_{fspl} \ge 0 \qquad \forall f\in\mathcal{F}\ \forall s\in\mathcal{S}_f\ \forall p\in\mathcal{P}_f\setminus\{L,B\}\ \forall l\in\mathcal{L}_{sp}\cap\underline{\mathcal{L}} \tag{79}$$

$$t_{fs} \ge 0 \qquad \forall f\in\mathcal{F}\ \forall s\in\mathcal{S}_f \tag{80}$$

$$d^R_i \ge 0 \qquad \forall i\in\mathcal{I} \tag{81}$$

$$d^S_f \ge 0 \qquad \forall f\in\mathcal{F} \tag{82}$$

$$d^K_l \ge 0 \qquad \forall l\in\underline{\mathcal{L}} \tag{83}$$

Box II.

$$\sum_{f\in D_j^P} d_{fg} \ge \underline{D} \qquad \forall g\in\mathcal{F} \tag{98}$$

$$d_{fg}, p_{fg} \in \{0,1\} \qquad \forall f\in\mathcal{F}\ \forall g\in\mathcal{F} : f \le g \tag{99}$$

This constraint programming model does not have an objective function since it only needs to create a precedence graph with predefined properties. It will create a precedence graph with $|\mathcal{F}|$ tasks by deciding for all task pairs f and g whether task f is a predecessor of task g, indicated by the binary variable p_{fg}. If $f < g$, g is either assembled at the same or a later station in the current solution. Therefore, we exclude this precedence link. As stated above, this model generates graphs that do not have isolated tasks, ensured by Constraint (89). Constraint (88) ensures consistency in the links between tasks: If a task f is a predecessor of task g, which in turn is a predecessor of task k, then f is an indirect predecessor of task k. Constraint (90) ensures that this model will create a precedence graph with the desired order strength OS. Constraints (91) and (92) represent the consideration of the station distance (forwards and backwards). Constraints (93)–(96) calculate the number of direct predecessors. The binary variable d_{fg} indicates if task f is a direct predecessor of task g. Constraints (97) and (98) enforce upper $\overline{D}$ and lower bounds $\underline{D}$ on the number of direct predecessors for each task.

We used this model to generate five different precedence graphs for each assembly line with different order strengths, i.e., 60, 65, 70, 75, and 80%.

Appendix C

For the generation of assembly times, we utilized the cycle time and the number of tasks at each station. For simplicity and without loss of much accuracy, we restrict ourselves to integer assembly times. In this work, assembly time exclusively describes the actual assembly and the handling of the part but does not contain times for searching and walking activities. To estimate reasonable task times, we assume a specific cycle time and a reasonably efficient line balance. Next, we generate a station time t_s for each station by deducting some time for non-assembly activities from the cycle time. These activities may include walking, searching, or idling. The time deducted depends on the number of tasks at that station.

For the generation of task times, we propose a stochastic (linear) goal programming model. This model assigns assembly times to tasks based on the station time and the tasks at that station. For each station, multiple scenarios C are created. Those can be seen as multi-stage stochastic programs with the execution of each task representing a stage. Each task's probability depends on the number of executions over a number of cycles. For clarity, consider a station with two tasks. Task 1 needs to be executed 1000 out of 1000 times, whereas task 2 is only executed 700 out of 1000 times. With this information, two scenarios are created: (i) Only task 1 is executed with probability 0.3 ($1 \cdot (1-0.7)$); and (ii) Both tasks need to be executed with probability 0.7 ($1 \cdot 0.7$).

The program assigns task times to all tasks t_f^A while calculating the deviations from the goal, i.e., the stations time t_s, and minimizes the sum of deviations over all scenarios u_c weighted with the scenarios' probabilities p_c. Only negative deviations are allowed since positive deviations would violate the cycle time constraint. Also, each task is given an upper $\bar{t}_f$ and a lower bound $\underline{t}_f$ to increase the variability of task times. These bounds are determined by drawing a number from a trimodal distribution. When all tasks are assigned a value, these values are normalized based on the station time t_s. A deviation of, e.g., 50% from that value in both directions determines the bounds.

$$\text{Minimize:} \quad \sum_{c \in C} p_c u_c \tag{100}$$

$$\text{subject to:} \quad \sum_{f \in \mathcal{F}_c} t_f^A + u_c = t_s \qquad \forall s \in \mathcal{S}\ \forall c \in \mathcal{C} : s_c = s \tag{101}$$

$$t_f^A \le \bar{t}_f \qquad \forall f \in \mathcal{F} \tag{102}$$

$$t_f^A \ge \underline{t}_f \qquad \forall f \in \mathcal{F} \tag{103}$$

$$t_f^A \in \mathbb{Z}^+ \qquad \forall f \in \mathcal{F} \tag{104}$$

$$u_c \ge 0 \qquad \forall c \in \mathcal{C} \tag{105}$$

This model requires the number of tasks, the line's balance, and the tasks' probabilities of execution as an input. Table 4 summarizes the model's results for the different assembly lines. They are highly compliant with the values described by Sternatz (2014) for another automotive company.

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