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Galilean Challenges

Jochen Büttner, *Swinging and Rolling: Unveiling Galileo's Unorthodox Path from a Challenging Problem to a New Science*, Dordrecht, The Netherlands: Springer, 2019, xv + 472 pp., ISBN: 9789402415926.

This is a brilliant book. It revisits what used to be a vexed issue in the historiography of the Scientific Revolution, Galileo's use of experiments and their role in the profound conceptual shifts in his analyses of motion. Based on a meticulous study of the manuscript material gathered in the famous Codex 72 of the Galilean collection at the Bibliotheca Nazionale di Firenze, it offers an utterly convincing and fascinating reconstruction of Galileo's research in the pivotal years 1602–1604. But given the task that Jochen Büttner has set himself, to fully justify the details of his reconstruction, his book offers no light reading. Three years after its publication, only one review has been published, as far as I can tell. There seems to be a danger that its importance will remain undervalued. In this review essay, I seek to summarize its main findings in a forthright and accessible way, and indicate their importance with respect to the existing literature on Galileo.

The Challenge of Codex 72

Codex 72 consists of more than 200 folios, the bulk of which contain scattered fragments related to Galileo's investigations into local motion. Many of these are sketches of demonstrations of propositions contained in Galileo's *Discorsi* from 1638, in which he finally presented his new science of motion. Other fragments consist of diagrams, with or without added material, or seemingly isolated numerical calculations. The existence of the material had been long known, as Favaro's monumental edition of Galileo's work contained a selection of the fragments (in a volume that was first published in 1898), but their potential importance for understanding Galileo's thinking was most forcefully stressed by Stillman Drake in the 1970s and 1980s. Drake identified some folios that seemed to show clear signs of Galileo's experimental activity, which he saw as invalidating Koyré's famous claims about the lack of significance of experiments for Galileo's innovations. Drake kept on revisiting the codex, trying out different reconstructions of the meaning and temporal order of what he

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saw as key fragments.[1] The importance of Drake's work cannot be denied, but it also shows the hazards involved in interpreting the material gathered in Codex 72. Fragments taken in isolation can sometimes be given wildly diverging interpretations, and any proposal about their temporal order will quickly depend on choices that cannot be anchored in the material itself and that can border on the outright arbitrary. Some other authors followed Drake's lead (most notably David K. Hill and Ronald Naylor), but it is fair to state that these contributions remained plagued by the same problems. Undoubtedly the most solid attempt was Winifred Wisan's impressive PhD dissertation, published in 1974, which offered a detailed reconstruction of the path that could have led Galileo towards the final publication in 1638.[2] Both in level of detail and thoroughness, Wisan's work remained unparalleled until the publication of Büttner's book (even if the latter limits his temporal scope to the first stage of the development of Galileo's science of motion). After the initial wave of the much publicized studies from the 1970s, attention diminished and literature on the topic probably never again reached an audience outside of the small number of scholars studying Galileo. A beautiful article by the mathematician Alexander J. Hahn on the experiment recorded on fol. 116v is worth mentioning for its careful analysis.[3] Finally, Jürgen Renn, Büttner's PhD supervisor, has admirably shown that much can still be learned from the manuscript by asking the right kind of questions.[4]

Büttner's book finds its origin in his work contributing to the digital representation of the contents of Codex 72.[5] An early example of a successful digitization project (it was launched at the end of the 1990s), it has provided scholars with an important research tool. As with all tools, its specific instrumental modality has also shaped the use that is made of it. Whereas Drake's work can be seen as characterized by a "cut-and-paste" attitude (he published in 1979 a "facsimile" edition of the Codex that actually consisted of reproductions of fragments cut out of their original context on the folios and "arranged in probable order of composition"), Büttner's methodology seems closer to the experience of online browsing: he goes back and forth between the folios, as if attaching multiple hyperlinks that connect fragments on different folios without removing them from their original context.[6] Adding these different layers of depth to the relations between different fragments enables him to bypass some of the serious shortcomings of earlier interpretations. The book contains many useful flow-charts, laying out the topical relations between material on different folios, with some folios reoccurring in different charts, which allows Büttner to build up a set of cross-references between different issues simultaneously treated in the manuscript. Paying attention to the layout of fragments on the folios, we can follow Galileo as he turns a folio already in use 90° clockwise and adds new notes related to a

1 See Drake (1999) for a collection of the relevant papers.
2 Wisan (1974).
3 Hahn (2002).
4 See especially Damerow, Freudenthal, McLaughlin, & Renn (2004), which mainly discusses Galileo's treatment of projectile motion in the manuscript, a topic not treated by Büttner.
5 See the project online (https://www.mpiwg-berlin.mpg.de/Galileo_Prototype/INDEX.HTM).
6 See Drake (1979).

sketch of a proof that he had already started on another folio. At times, Büttner can even give informed guesses about which folios would probably have been lying on Galileo's desk as he was working more or less contemporaneously on different issues. While these glimpses of Galileo at work are fascinating in their own right, their real importance lies in the way in which this network of relations seriously constrains the interpretative freedom with regards to the contents of the notes.

The book is divided in two parts of unequal length. In the first and longer part, Büttner follows Galileo as he sets up an experiment and explores its implications. The special significance of this experiment has been missed in all earlier literature (although it was discussed by Hill), but Büttner builds a convincing case that it is absolutely crucial to understand the genesis of Galileo's science of motion, even if his initial research predicated on its outcome ended in an impasse.[7] The second part describes Galileo's attempts at reorganizing the results he had reached before the impasse, with the aim of presenting an axiomatically structured science of motion, which would ultimately result in Day 3 of the *Discorsi*. The analyses in both parts can each be anchored in two important letters that allow us to unequivocally date some key results reached by Galileo: a letter to Guidobaldo del Monte from November 1602, and a letter to Paolo Sarpi from October 1604. It is generally acknowledged that these letters are central documents for any attempt to interpret the developments in Galileo's thinking, but never before have they been so convincingly tied to the manuscript material gathered in Codex 72.

The Challenge of the Pendulum

The experiment analyzed in the first part of the book is the following. Galileo took a pendulum of about 2 m length and determined how long it took to swing towards its lowest point from a set height, by using a water clock to measure the time for eight full swings and dividing this time by 32. He then compared this time with the time that it took for a ball to descend along an inclined plane that started from the point at which the pendulum had been released and that ended at its lowest point—that is, a trajectory along a chord inscribed in the circle defined by the pendulum's motion. To determine the latter time, Galileo actually timed the motion along a gently inclined plane of about 6.5 m in length and then transformed the result in the time for motion along the inscribed chord in a few steps, as follows. He first assumed that a freely falling body would have reached the lowest point in the same time if it would have started falling from a height given by the length of the inclined plane (that is, assuming what is commonly called the law of chords, which states that motion along chords inscribed in a circle that end in the lowest point of the circle always take the same time—in this case the vertical diameter and the long inclined plane, respectively). He then used the law of fall to calculate the time it would have taken if it had rather fallen from the highest point of the circle defined by the pendulum.

7 See Hill (1994).

Finally, he used the law of chords again to conclude that this time equaled the time taken for motion along the chord inscribed in the pendulum's circle that he wanted to determine. Comparing the time for swinging and rolling (hence the book's title), Galileo found out that the pendulum's motion was considerably swifter.

Two important points can immediately be made, one more philosophical, the other historical. First, one can notice how comfortable Galileo was in using theory-mediated measures (a feature of Galileo's experimental work that is also made clear by Hahn's analysis of fol. 116v). Rather than trying to directly measure the time for one quarter-swing, he sensibly chose to measure a larger number of swings and average out the result; but this procedure only makes sense if one can assume that all swings in principle take the same time, even if the motion is noticeable dampened after just a few swings. His measurement procedure thus depended on the validity of the claim that pendulum motion is isochronous. And rather than trying to directly measure the very short time span of the motion along the rather short and steeply inclined plane inscribed in the pendulum's circle, he chose to measure a much slower motion and theoretically transform its result in the value he sought. Second, this procedure shows that at the time of the experiment, which Büttner dates to 1602, not long before the letter to Guidobaldo del Monte, Galileo already felt confident enough about the law of fall, the law of chords, and the isochrony of the pendulum to depend on them in setting up further experiments. It is a corollary of Büttner's analyses that anyone looking for an answer to the question of how Galileo first became convinced of the law of fall will not find the answer in Codex 72![8] The inclined plane he used to time the motion along the inscribed chord does perfectly match the one with which he claimed in the *Discorsi* to have tested the law of fall, though, and given the partly uncertain history of the material in the Codex, it is certainly possible that it originally contained folios which would have contained more information on the matter.

Sometime in 1602 Galileo must have been struck by the remarkable analogy that could be noticed between pendular motion and motion along inclined places inscribed in the pendulum's circle: both showed a property of isochronity. Quite probably, he would also have noticed that both showed a quadratic relation between distances and times (in the case of the pendular motion between its length and period). The latter similarity does not figure explicitly in his investigations, though, and it would only be made explicit later in his life as a planned addition to the *Discorsi*, as shown by Büttner in one of the chapters in his book. The letter to Guidobaldo del Monte makes explicit that, at this point in time, Galileo's ultimate goal was to provide a demonstration of the pendulum's isochronous motion, and he clearly expected to base this demonstration on what he knew about motion along inclined planes. Büttner suggests that the experiment was set up as a test of the simplest hypothesis that could provide the scaffolding for such a demonstration: that pendular motion and motion along the associated chords were equal in time. To what extent Galileo actually would have seen such a hypothesis as plausible is impossible to ascertain, but

8 See Renn, Damerow, & Rieger (2000) and Van Dyck (2021) for discussion and some suggestions about the provenance of the law.

what really matters is how he proceeded on finding out that the pendulum's motion was about 30% faster.

If the properties of pendular motion were to be demonstrated from motion along inclined planes, the sensible next step was to see whether the experimentally established time difference could be recovered by constructing a trajectory that more closely approximated the pendular motion. Accordingly, Galileo tried to find out what could be demonstrated about a body that first descended along an inclined plane finishing in a point on the circular arc in between the starting point and the lowest point, and that was then diverted along a second inclined plane that did end in the lowest point. As Galileo communicated in his letter to Guidobaldo, and as Codex 72 testifies, he was able to prove what Büttner calls the law of the broken chord, that is, that the body's motion along this trajectory was swifter than along the single chord. The demonstration on which Galileo settled would be taken over without significant changes in the *Discorsi*. But the next steps taken in the manuscript would leave no immediate traces in Galileo's published work, and these throw important new light on Galileo's multifaceted approach to the study of motion: on the one hand, he searched for further theoretical demonstrations; on the other hand, he engaged in extensive computational explorations.

In one group of notes, Galileo tried to find out whether he could give a general demonstration concerning the ratio in which the time diminished when the motion more closely approached the circular arc, rather than merely proving that it did diminish. He explored a few properties of the relevant diagrams to this end, but reached no useful results. In another, more extended group of notes, Galileo calculated the times taken along particular trajectories that could be constructed by adding more broken chords between the starting point and the end point of the initial, "non-broken" chord. He laboriously calculated the times taken along trajectories consisting of two, four, and eight chords. The results not only showed that the times become progressively shorter, but also that the difference between the four-chord and eight-chord trajectories was so small that they could be assumed to converge very quickly. Using the calculated values, he tried to see whether a general rule could be hypothesized about how the times diminished. Guided by the geometry of the situation, Galileo identified a few candidate parameters that could possibly be related to the ratios of the times he had established. At this point, he was using a kind of calculational trial-and-error method, playing with the geometrical and numerical information at his disposal to see whether he could hit upon a plausible regularity that fulfilled a number of established conditions. But this approach reached a dead end: no sustainable regularity emerged. Even worse, his calculations also showed some worrisome tendencies, such as the fact that the times of motion along two broken chords spanning a different angle in the same circle were not equal—whereas the overall goal was to reach a demonstration that the times of motion along the circular path did not depend on the angle.

By unearthing the central role of the pendulum-plane experiment, Büttner's analyses offer a refreshing take on the old question on the role of experiments in Galileo's work. The experiment was not so much testing a theory as it was setting a challenge—hence Büttner's characterization of the pendulum as a “challenging object.” When treating pendular motion as a problem, Galileo was first and foremost a mathematician in the 16th-century sense of the category.[9] He was exploring mathematical constructions that could represent different relations between quantities that could in principle be empirically measured. His notes testify to a search for a way to simultaneously meet the double mathematical constraints of geometrical constructability and empirical validity. This heuristically oriented approach resembles neither the inductive nor the hypothetico-deductive ways of proceeding that framed much of the earlier discussions on Galileo. While the search did not reach its intended goal (and we now know that it could not have, as explained by Büttner in some technical asides), the process itself was highly fruitful. Galileo developed a new technique to represent time in his geometrical diagrams (by making the geometry itself encode ratios of times) and he established partial results on the properties of accelerated motion down inclined planes that could stand on their own. Almost all of the propositions that Galileo would publish decades later in the third day of his *Discorsi* as the first part of the new science of motion find their immediate origin in his unsuccessful attempt at directly relating the properties of a swinging body to that of bodies rolling down inclined planes. Already starting with Descartes, some earlier researchers had surmised the importance of the pendulum for the development of Galileo's science of motion, but none had been able to show the full extent of its role as the central guiding object. There is only one place where this is made partly visible in the *Discorsi*, in Galileo's flawed brachistochrone argument at the end of the third day, and for that reason Wisan's earlier analysis had reconstructed Galileo's research path as one that started from his attempt to construe that argument. Büttner establishes that it is only a by-product of the more fundamental challenge that was set by the pendulum-plane experiment, but which up till now had remained hidden in the labyrinth of Codex 72.

The Challenge of Foundations

The second part of the book follows Galileo as he tried to integrate his results in a mathematically articulated structure that could form the basis for a treatise on the new science of motion. Büttner usefully distinguishes two related challenges that Galileo faced: the search for an axiomatic foundation and the search for an analytical foundation. A proper mathematical science needed to start from a minimal set of propositions that could serve as axiomatic basis from which to derive all other propositions; and these axioms should be analytically privileged over other candidates by a property that could ground their special status as not in need of further mathematical demonstration. Galileo's notes show that he must have very quickly realized that he could identify three propositions as possible axioms, but that any two of these sufficed since the third could always be derived from the other two: the law of fall,

9 See Van Dyck (2022) for more on this.

the law of chords, and the length-time proportionality which had emerged as a central proposition in the search for the pendulum's isochrony. From a purely deductive point of view, selecting any two of these as axioms was an arbitrary choice, but Galileo tried to find further grounds that could single out the proper candidates to ground his new science.

Büttner lays bare two stages in Galileo's search for the analytical foundation. The first stage can be dramatically located at the moment in which Galileo crossed out a demonstration of the law of chords that was based on the law of fall and the length-time proportionality, and replaced it with an alternative "mechanical" proof. This proof is well-known in the literature, as it would later recur in the *Discorsi* as one of the alternative proofs that Galileo introduced there for the law of chords. It is based on the mechanical law of the inclined plane, which determines the effective weight of a body on an inclined plane, and which Galileo had first successfully established in his *De motu antiquiora* (ca. 1589–1592) and repeated in his *Meccaniche* (ca. 1592–1600). By setting the speeds of bodies on inclined planes proportional to their effective weights, the law of chords immediately follows. Büttner retraces the path towards this proof in the manuscript in three steps: 1) The starting point is a note in which Galileo explicitly formulated the goal that he should be able to find such mechanical proofs for what could consequently become his basic axioms. This idea was based on his observation that bodies descending on differently inclined planes with the same height had velocities inversely proportional to the lengths of the inclined planes, which implied that (at least in this case) their velocities were indeed proportional to their effective weights. 2) Not long after having formulated this note, he made a further note on another folio (which shared the same watermark as the folio containing the initial note) showing that he must have recognized the structure of the diagram that he had earlier used in the proof of the mechanical law of the inclined plane in the diagram on that folio, in which he was exploring ways to meet the challenge set by the pendulum-plane experiment. This must have allowed him to see a direct relation between the law of the inclined plane and the law of chords that played a central role in the analysis on that folio. 3) Based on this basic recognition, he drafted the mechanical proof of the law of chords in two consecutive phases, the first of which happens on a folio that carried other content directly related to the folio on which he had recognized the diagram. I will come back to the further significance of Büttner's tightly knit reconstruction of the provenance of the proof in the next section, but for now we will follow his reconstruction of what happened after Galileo had inserted this mechanical proof in the projected mathematical structure—giving at least one of the axioms the independent grounding he was looking for.

To understand the next stage, it is important to stress the extent to which Galileo's analyses in which he tackled the implications of the pendulum-plane experiment were exclusively carried out in terms of distances and times. The velocity or speed of these motions was not an independent object of analysis, aside from the proportions between distances and times that were directly studied. It is only the search for an analytical foundation that brought the concept to the fore. Starting from the assumption that the proportions studied were the result of underlying properties of the motion of the bodies, it was a natural move to see velocity as the crucial bridge: as the general measure of a body's motion, it could be related as an effect to hypothesized causes (effective weights), while it simultaneously translated in the proportions between distances and times. Büttner follows the groundbreaking analyses by Pierre Souffrin that reconstructed the historical meaning of Galileo's concept of velocity, and according to which *velocitas* was a holistic measure of motion, characterized by a set of kinematic propositions that allowed one to relate ratios and equalities of velocities to ratios of distances and times.[10] Importantly, as a holistic measure it was indifferent to the fine-grained details of the motion, such as being uniform or accelerated. But once Galileo had established velocity as a foundational concept, the intricacies of accelerated motion started to put considerable stress on its further applicability in that role.

Earlier literature has paid extensive attention to the so-called "mirandum paradox," a brief note contained in the Codex where Galileo struggles with the question of how to correctly use the concept of velocity in the context of his newly developing science. In the note, he first writes that motion of free fall along a vertical direction is faster than motion along an inclined plane, and then points out that (using the length-time proportionality and the traditional definition of velocity) a distance along the vertical can be chosen such that the free-fall motion has the same velocity as motion for another distance along the inclined plane. Büttner offers a convincing reading of Galileo's note that superbly shows the importance of firmly situating notes such as these within their immediate context in Codex 72. On Büttner's reconstruction it is the final note in a series of propositions drafted on a number of consecutive folios with an eye to a possible treatise. On the first of these folios, Galileo had given the mechanical proof of the law of chords its foundational role by putting it in the place of the earlier demonstration based on the law of fall and the length-time proportionality. The mirandum paradox ends this series and thus signals the breakdown of this attempt at foundation. The problem that Galileo had come across is the following. The mechanical proof depended on setting velocity proportional to effective weight, and thus made velocity a property of motion that was dependent only on the inclination of the plane on which a body was moving. On the other hand, for the accelerated motions that Galileo was considering, the kinematical proportions that defined the application of velocity implied that the ratios between bodies' velocities depended on the distances of motion chosen. Importantly, at no point in his later work did Galileo suggest that for this reason the holistic concept of velocity was not applicable to accelerated motions; rather, he concluded that it could not play its foundational role as a direct effect of a body's effective weight.

Just as with the search for the isochrony of a pendulum, Galileo's notes show how he reached a dead end in grappling with the challenge that he had set himself. Here, too, the dead end eventually bore fruit. In 1604, Galileo wrote a letter to his friend Paolo Sarpi that shows that he had returned to the challenge of finding an analytical foundation, and that he was hopeful that he had found a way out. Rather

10 See, among others, Souffrin (1992).

than treating the letter as the first documented instance of Galileo's adherence to the law of fall, which it no longer is, Büttner stresses how Galileo's enunciation of the ill-fated principle that speeds grow with distances in free fall was meant to solve the question of finding proper foundations for his new science, the mathematical core of which had already been established in 1602. Crucially, the mirandum paradox had shown him that accelerated motion could only be analytically comprehended by paying attention to how velocities changed during the motion. It is at this point that Galileo turned to another conceptual tradition for thinking about the relation between a motion's "extensive" and "intensive" properties: the framework for the configurations of motion as developed by the medieval *calculatores*. Büttner shows how Galileo had already used some of its resources in 1602 to calculate the time taken by motion down an inclined plane and the time taken by motion along the horizontal with the speed acquired during the first motion, establishing a version of what is commonly called the double distance rule. At that point, it mainly had functioned as a toolbox enabling Galileo to carry out some calculations, probably first prompted by his attempt to formulate a version of his cosmogonical hypothesis according to which the planets moved at the speeds acquired during free fall from a common point of departure, but its assumptions were not spelled out in any meaningful way. The letter to Sarpi shows how Galileo had now fastened upon the idea of using the graphical representation of the changes in degrees of velocity that characterized accelerated motions as a way to find an alternative analytical foundation; assuming these changes to be characterized by a basic regularity could enable the construction of a proof of both the law of fall and the length-time proportionality, and sketches of such proofs are indeed contained in Codex 72. This is the point at which Büttner's story stops, a point at which Galileo's new science had found the identity it would retain on publication, even if much conceptual work remained to be done.[11]

Revolutionary Challenges

Büttner's book offers more than a fascinating insight in the details of Galileo's research path. Just as in the case of Frederic Holmes' celebrated reconstruction of Lavoisier's experimental research practice based on a close study of his laboratory notebook, this reconstruction has profound implications for how to think about revolutionary changes in ways of doing science.[12] One cannot understand the ways in which Galileo was brought to rethink the ideals of his new science without paying close attention to the details of his mathematical explorations as brought to light by Büttner.

One story, popularized by Drake and recently revived by John Henry, stresses the supposedly kinematical nature of Galileo's science, and sees this as a necessary step in

11 Damerow et al. (2004) and Palmerino (2010) offer useful outlines of this work.
12 Holmes (1997).

leaving behind occult notions of causality.[13] Büttner fruitfully complicates discussions of Galileo's shift towards a purely kinematical analysis of accelerated motion in two related ways: he brings out important subtleties in the category of "kinematics," and he forces us to rethink the chronological evolutions of Galileo's work.

As we saw, the initial stage of Galileo's research path as documented in Codex 72 was kinematical in the sense that it remained silent on causes and forces, but Büttner suggests calling it "pheno-kinematics," to stress its direct dependence on empirical input. Galileo was investigating spatio-temporal proportions that were themselves grounded in empirical findings, and which were nowhere related to the kinematical properties of velocities. These proportions were only presented as consequences of how velocities change in uniformly accelerated motion in the very last stage of his research, at which point some of the empirical results could be introduced as a test of these consequences, confirming the thesis that free fall occurring in nature is uniformly accelerated. The typically kinematical relationship between the proportions and the definition of accelerated motion was only added as a foundational layer after the initial foundational attempt based on the effective weights on inclined planes had foundered; and even then, the main part of the third day of the *Discorsi* could proceed completely independently from this kinematical grounding.

This has some important consequences. First, the strikingly hypothetico-deductive presentation that Galileo offers in the *Discorsi* was the outcome of a very specific and highly contingent development. Second, what has struck most commentators is how Galileo's new science erased the search for causes, but Büttner's reconstruction throws a somewhat different light on this decision. Galileo's search for a causal foundation came only after the body of interrelated and empirically grounded mathematical propositions had already been established, so he clearly did not see causal questions as incompatible with that specific form of mathematical progress. And he abandoned this search not because he thought the notion of force was unintelligible or obscure, but because he could not find a coherent place for velocity as proportional to effective weight within these mathematically established results. The primary stumbling block that led Galileo to abandon his causal endeavor was not questions about the nature of force, but the conceptual intractability of acceleration as an independent mathematical quantity within his proportional framework.

It is clear that we should not read too much of a principled stance into the way in which Galileo presented his results in the *Discorsi*. But it is equally true that Drake's general interpretation of the nature of Galileo's innovations can find some confirmation in how Galileo approached the foundational challenge more as a mathematician than as a natural philosopher. He gathered a body of mathematically interrelated propositions, and was trying to decide which of these could be put forward as an axiomatic basis for the rest, more or less along the lines of what was known in the period as a reduction to art or, relatedly, the analytic step preceding a synthetic presentation according to the outline of mathematical reasoning as given

13 Henry (2011).

by Pappus; and to this end he needed to find a way to show their privileged status.[14] Undoubtedly, some scholars will be tempted to translate this search for foundations in the Aristotelian language provided by the late Renaissance method of *regressus*, and this might indeed be a fruitful move to see how Galileo could try to inscribe his new mathematical science within established philosophical ideals—even if he was less explicit on this score than one would expect from this perspective.[15] But it must be stressed that we will probably not understand why Galileo saw the move towards non-causal foundations (uniform accelerations having a particular configuration of velocities) as a possible way out of the conundrum created by the mirandum paradox if we do not realize that the foundational challenge arose as part of a mathematical endeavor in the first place. Ideas about reduction to art and Pappus's analytic method were firmly inscribed in a discourse that was primarily devoted to articulating a systematic basis for the constructive solution of problems, with Aristotelean causal considerations taking a back seat at best. Büttner's reconstruction of the set of challenges that shaped the work of Galileo documented in Codex 72 clearly shows him as a mathematician engaged in sustained efforts at this kind of problem solving.

This stress on the essentially mathematical nature of Galileo's explorations raises the question of its relation to the explicitly philosophical program set out in his youthful *De motu antiquioria*. It is clear that the specific kind of causal grounding that Galileo introduced was firmly anchored in that earlier work, as is his general ambition to offer a mathematical treatment of problems involving free fall, a topic traditionally belonging to natural philosophy. The attempt to reintegrate the results stemming from the research documented in Codex 72 with more explicitly philosophical concerns only happened later in the *Dialogo* and the *Discorsi*, though, and is not the topic of Büttner's book. But there is one point at which he puts this material at a considerable distance from *De motu antiquioria*. Indeed, one of the more surprising results of his analysis is the fact that the mechanical proof of the law of chords was a relative late-comer in Galileo's work. Most earlier analyses have assumed that it was actually among the very first results Galileo reached, as a direct result of the conceptualization of motion presented in *De motu antiquioria*. The main reason for this assumption is the fact that the mechanical law of the inclined plane was already present in that work, while he also explicitly held on to the idea that velocities should be proportional to effective weights. An additional reason for ascribing an early date to the mechanical proof of the law of chords was the assumption that it was only formally valid for uniform velocities, but since the afore-mentioned analyses by Souffrin we know this to be false. Büttner's reconstruction of the steps leading to the formulation of the proof in the folios of Codex 72 is convincing, but it does leave us with the question of how Galileo first came to the law of the chords, since we saw that it was already assumed by him in setting up the pendulum-plane experiment. By the time of that experiment, he would have probably carried out experimental

14 See Van Dyck (2022) for the relation of Galileo's work to reduction to art.

15 See Miller (2018) for an interesting, recent example of this line of interpretation with respect to Galileo's interpretation of his telescopic observations.

tests comparable to the ones with which he established the law of fall, but it is highly improbable that he would have set up these experiments without having an idea of the proportions he was looking for.

Büttner offers a speculative reconstruction of the kind of question that could have led Galileo to the hypothesis expressed in the law of chords, which he would then have corroborated experimentally.[16] But given its purely speculative character, it must be considered equivalent to the following (equally speculative) suggestion. As suggested by older scholarship, Galileo may very well have first come to the law of chords based on the conceptual framework presented in *De motu antiquiora*, probably at more or less the same time as he first became interested in the properties of pendular motion—maybe due to the expanded treatment of the mechanical law of the inclined plane in the longer version of his *Meccaniche* (usually dated to the period shortly before the work documented in Codex 72), where he had interpreted the central diagram of his proof in a way that suggested its possible relation to pendular motion.[17] He would have proceeded to test the law experimentally, and the empirical result would have heightened his interest in pendular motion, given that it seemed to confirm the idea that there was a non-accidental relation between motion down inclined planes and pendular motion, both of which seemed to show a property of isochronism. But given that the properties of pendular motion also forced him to treat downwards motion as essentially accelerated (how else to understand isochrony, if not by assuming that there was a strict mathematical regularity in how the pendulum bob accelerated and decelerated over different distances?), the law of chords quickly took on the status of a primarily empirical law (given that he no longer could no longer unproblematically rely on the foundational framework of *De motu antiquiora*, in which he had assumed uniform velocities). At this point, it would have had more or less the same epistemic status as the law of fall, and accordingly he used both as premises in his mathematical investigation of the pheno-kinematics of pendular motion and accelerated motion down inclined planes. Searching for an analytical foundation for his mathematical propositions, he would then have noticed that (surprisingly) his "velocity is proportional to effective weight" framework appeared to remain valid (the first step in Büttner's reconstruction of Galileo's path towards the mechanical of the law of chords, as discussed above). The subsequent steps in sketching out a full proof would thus be directed towards the integration of an old idea into a new context.

The possibility of offering multiple reconstructions of the possible origin of the law of chords highlights the fact that the documentary evidence starts *in medias res*, at a point where Galileo was already in the possession of some fundamental results

16 On Büttner's reconstruction of how Galileo could have come to the law of chords: it was a heuristic assumption triggered by a presumed interest on Galileo's part in investigating isochrone curves, which are "defined by the points reached by a given type of motion from a given starting position in the same time" (p. 234). This mathematical challenge would have arisen once Galileo started considering natural motion as essentially accelerated, but not initially related to either the mechanical law of the inclined plane or an interest in pendular motion.

17 See Wisan (1974, pp. 160–162).

that were not present in *De Motu Antiquiora*, which leaves us with considerable interpretive freedom in how to fill in the gaps left open. But even on the alternative reconstruction suggested here, the distance from Galileo's earliest attempt at treating free fall is significant in a way that only Büttner's work allows us to fully appreciate. While Galileo could have established something like the law of chords based on his earliest ideas, this would never have set him on the path towards the new science as presented in the *Discorsi* as long as this law was not put at the service of the analysis of pendulum motion. In the context of the *De motu antiquioria* framework, it would have remained an isolated statement without further implications or deeper interest.[18] To put it as strongly as possible: it is not general questions concerning the nature of free fall that led Galileo towards his groundbreaking results, but a very specific mathematical challenge ("mathematical" in the expansive 16th-century meaning of the term) in which the behavior of a simple object such as a pendulum was put in relation with another kind of motion, that of balls rolling down inclined planes. Significantly, neither of these had any place in natural philosophical debates on motion.[19] The kind of questions he was asking about them at this point were not primarily driven by specifically philosophical concerns, apart from the mere fact that he was interested in the treatment of motion—even if he would return to some of these later, when he tried to reassess the status of concepts like "natural motion" in the context of his novel mathematical discoveries.

There is no need to rehash old debates about the existence of an ill-defined entity such as the Scientific Revolution, since hopefully we no longer feel the urge to present "ourselves" as essentially modern in opposition to what happened either before or elsewhere. This should also free scholarship on Galileo from a burden that it was never able to carry. Galileo was not the first modern scientist, but he did belong to a group of early modern mathematicians with striking natural philosophical ambitions

18 Maybe it can be counted as a point in favor of my alternative reconstruction that it provides a common genesis for Galileo's interest in the law of chords and in pendular motion, while Büttner's suggestion needs a more or less fortuitous meeting of two different questions: one concerning isochrone curves and one concerning pendulum motion.

19 Galileo had only included the inclined plane in *De motu antiquioria* because his treatment of free fall in a medium on the model of Archimedean hydrostatics made motion on the inclined plane an analogous problem, given that both involved a diminishment in effective weight (see Souffrin, 2001)—but the provenance of the problem itself was squarely within the tradition of mechanical problems. The pendulum was sometimes mentioned in scholastic debates on impetus, and reoccurred in that context in *De motu antiquioria* (see Hall, 1978), but this was always as an example of other, more central phenomena, not as an object of attention in its own right. In an earlier publication, Büttner already introduced the idea of the pendulum as a "challenging object" that found its origin primarily in a technical rather than a natural philosophical context: Büttner (2008).

and this brought him into uncharted territory. Büttner has offered us a compelling picture of the tortuous path that resulted: Galileo's new science of motion was born from some very specific mathematical challenges and the ways in which he tried to circumvent his inability to fully meet them.

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