# Mean Value Analysis of the Age of Information in Slotted ALOHA

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*Abstract*—**This letter presents a mean value analysis of the age of information for slotted ALOHA, a widely-used medium access control protocol in communication networks. Unlike previous studies that rely on approximations, we provide exact results for the expected age of information at random slot boundaries and the expected peak age of information. We explore the impact of packet arrival and transmission probabilities on the age of information through a numerical example.**

*Index Terms*—**Age of Information (AoI), Mean Value Analysis, Slotted ALOHA.**

## I. Introduction

Age of Information (AoI) has been recently proposed for the performance evaluation of communication networks as a response to progress in the area of the Internet of Things (IoT) [1]. The primary motivation for the introduction of the AoI is that classic metrics like throughput and delay do not capture *the freshness of the information updates*, although the latter is crucial from the vantage point of a networked control system [2].

In many cyber-physical systems, for example in cooperative autonomous driving based on vehicular networking (V2X), the updates are performed by numerous users over a shared communication channel, with medium access control (MAC) protocols governing the transmissions. For example, these updates are carried by heartbeat messages (also known as Cooperative Awareness Messages or Basic Service Messages) and are relayed to a cooperative adaptive cruise controller (CACC) which performs autonomous longitudinal driving in multi-vehicle formations such as platooning [3]. This case and similar use cases, where the updates' freshness is crucial and numerous users communicate over a broadcast channel, are the main motivation for this work, which investigates slotted ALOHA, a classical random access protocol, from the AoI perspective.

There are some recent results on the AoI in random access channels. The influence of packet retransmission strategies on the peak AoI is analysed in [4]. It is argued in [5] that information freshness improves by resolving the packet collisions. Retransmission scheduling schemes which minimise the average AoI are proposed in [6]. In contrast, we specifically target broadcast communications in this letter, because the key V2X services such as cooperative awareness and collective perception do not adopt retransmissions [7].

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Different variations of slotted ALOHA have been considered from the AoI perspective in the context of IoT. For example, the AoI in frame slotted ALOHA is analysed in [8]. In [9] and [10], the AoI is derived for random access protocols which incorporate successive interference cancellation (SIC). Some authors also include additional protocol features. For instance, when the number of contending users is known and the access probability can be adjusted accordingly, an AoI-related study is presented in [11]. Moreover, the energy-efficiency under AoI constraints is analysed for slotted ALOHA in [12] in the context of low-power IoT applications. Our work is based on traditional random access assumptions: packet collisions result in the impossibility of a reception, the number of users is unknown, and user battery constraints are out of scope.

Furthermore, there are efforts to modify slotted ALOHA to explicitly account for the AoI into the protocol. For instance, in [13] each user suspends its transmission until the AoI of the status update reaches a certain threshold. Further improvements are reported when a short control sequence ahead of the actual data transmission and respective carrier-sensing is applied [14]. The fundamental analysis of the age-efficient transmission policies is performed in [15].

In the V2X context, an approximate analysis of IEEE 802.11p MAC [16] assumes that each channel attempt has a fixed probability of success and a random duration. A similar but somewhat more abstract model is studied in [17], [18], [19], with the added complexity that not all nodes have a packet ready at all times. The model in [18], [19] is more accurate, compared to the one in [17]. At the same time, it is also less complex: the authors exploit that the message generation period is typically much larger than the mean access time for the V2X applications of interest. All these AoI models rely on approximations, what hinders their use for the formal cooperative driving *safety analysis* [20]. Note that the peak AoI is not necessarily enough in this context.

In this letter we provide exact results for the AoI of the slotted ALOHA protocol with a constant access probability. Since IEEE 802.11p adopts random access with *a constant contention window* and it is known to be well approximated by a slotted ALOHA with a respective fixed channel access probability [21], we believe that these results might have further implications in V2X research.

The remainder of this letter is organised as follows. We introduce the AoI model and its analysis in the next section and illustrate our results by a numerical example in section III. Finally, conclusions are drawn in section IV.

## II. Age of Information model

We consider a system with $M+1$ users that send information over a shared channel. Each user has a buffer of size one, which holds the latest information, if any. New packet arrivals overwrite packets that are held in the buffers. Each user with a packet in its buffer accesses the channel with a fixed probability $0 \leq p \leq 1$, and the transmission is successful if no other users send simultaneously. Packets that are not transmitted successfully are lost and are not re-transmitted. New information packets arrive at the buffers of the users in accordance with a Bernoulli process with success probability $\lambda > 0$. We further assume that transmissions at a slot boundary precede arrivals: if a user transmits at a slot boundary and there is a new packet arrival, the old packet is transmitted and the new packet occupies the user's buffer. Moreover, the new packet cannot be transmitted at its arrival slot boundary.

We focus on one (tagged) user in particular. For this user, we track the age of information at the receiver $A_k$ at the beginning of slot $k$, and the age of information at the sender $B_k$ at the beginning of slot $k$. The age is the time since the arrival of the last received packet [2]. If the tagged user has no information, we set $B_k = -1$. For the other users, we only track the number of users $X_k$ with a packet at the beginning of slot $k$.

### A. The number of users with a packet

We now relate these quantities at consecutive slot boundaries. To this end, we first study the evolution of the process $\{X_k\}$. Let $\Lambda_{\ell,k}$ and $T_{\ell,k}$ denote the indicators that there is an arrival and transmission for the $\ell$th user, respectively. With the assumptions above, $\{\Lambda_{\ell,k}\}$ and $\{T_{\ell,k}\}$ are independent doubly indexed sequences of Bernoulli random variables with success probabilities $\lambda$ and $p$, respectively. As the order of the users does not affect the evolution of $X_k$, we can assume that the users with packets have the lowest indices. We can then express $X_{k+1}$ in terms of $X_k$ as follows,

$$X_{k+1} = \sum_{\ell=1}^{X_k} Q_{\ell,k} + \sum_{\ell=X_k+1}^{M} \Lambda_{\ell,k} \,. \tag{1}$$

Here, we introduced the variables $Q_{\ell,k} \doteq 1 - T_{\ell,k} + T_{\ell,k}\Lambda_{\ell,k}$ to simplify the notation. So, a user with a packet at the $k$th boundary has one at the $k+1$st slot boundary if she did not transmit, or if she transmits and there is a new packet arrival. A user without a packet cannot transmit and has a packet at boundary $k+1$ with a new arrival. Clearly, each $Q_{\ell,m}$ is an independent Bernoulli variable, with success probability

$$q \doteq \mathsf{P}[Q_{\ell,m} = 1] = \lambda p + 1 - p \,.$$

The recursion (1) shows the process $\{X_k\}$ constitutes a Markov process. Let $x_{mn}$ denote the transition probability from $X_k = m$ to $X_{k+1} = n$. In view of equation (1), we have,

$$\begin{aligned} x_{mn} &= \mathsf{P}[X_{k+1} = n | X_k = m] \\ &= \mathsf{P}\left[\sum_{\ell=1}^{m} Q_{\ell,k} + \sum_{\ell=m+1}^{M} \Lambda_{\ell,k} = n\right] \\ &= \sum_{i=0}^{n} \mathsf{P}\left[\sum_{\ell=1}^{m} Q_{\ell,k} = i\right] \mathsf{P}\left[\sum_{\ell=m+1}^{M} \Lambda_{\ell,k} = n-i\right] \end{aligned}$$

with $\bar{q} = 1 - q$ and $\bar{\lambda} = 1 - \lambda$, which simplifies to,

$$x_{mn} = \sum_{i=\max(0,n+m-M)}^{\min(n,m)} \binom{m}{i} q^i \bar{q}^{m-i} \binom{M-m}{n-i} \lambda^{n-i} \bar{\lambda}^{M-m-n+i} \,, \tag{2}$$

Here we used the observation that a sum of Bernoulli random variables is a Binomially distributed random variable. We accounted for the range of the Binomial random variables by changing the upper and lower bounds of the summation.

For the evolution of the age of information, we need a more detailed description of the process $\{X_k\}$. To this end, let $Y_k$ denote the indicator that there is a transmission at the $k$th slot boundary. Let $x^{\mathrm{t}}_{mn}$ denote the probability that there is a transition from state $m$ to state $n$ with a transmission, and let $x^{\mathrm{nt}}_{mn}$ denote the probability that there is a transition from state $m$ to state $n$ without transmissions,

$$\begin{aligned} x^{\mathrm{t}}_{mn} &= \mathsf{P}[X_{k+1} = n, Y_k = 1 | X_k = m] \,, \\ x^{\mathrm{nt}}_{mn} &= \mathsf{P}[X_{k+1} = n, Y_k = 0 | X_k = m] \,. \end{aligned}$$

In view of equation (1), and accounting for the absence of transmissions, we have,

$$x^{\mathrm{nt}}_{mn} = \mathsf{P}\left[m + \sum_{\ell=m+1}^{M} \Lambda_{\ell,k} = n, \sum_{\ell=1}^{m} T_{\ell,k} = 0\right]$$

or,

$$x^{\mathrm{nt}}_{mn} = \begin{cases} \bar{p}^m \binom{M-m}{n-m} \lambda^{n-m} \bar{\lambda}^{M-n} & \text{for } m = 0, \ldots, n \,, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

Finally, as there are either transmissions or no transmissions, we have

$$x^{\mathrm{t}}_{mn} = x_{mn} - x^{\mathrm{nt}}_{mn} \,. \tag{4}$$

### B. Age of Information Analysis

With the description of the evolution of the other users, we now turn to the dynamics of the age of information of the tagged user. In line with the notation for the other users, let $\Lambda_k$ denote the indicator that there is an arrival for the tagged user, and let $T_k$ denote the indicator that the tagged user transmits. $\Lambda_k$ and $T_k$ are independent Bernoulli distributed random variables with success probability $\lambda$ and $p$, respectively.

The age at the sender is incremented if there is a packet at the sender and there is neither arrival nor transmission. The age is reset to 0 if there is an arrival. In all other cases, the age is reset to $-1$. Hence, we have,

$$B_{k+1} = \begin{cases} B_k + 1 & \text{if } T_k = \Lambda_k = 0 \,, B_k \geq 0 \,, \\ \Lambda_k - 1 & \text{otherwise.} \end{cases} \tag{5}$$

The age at the receiver is reset to $B_k+1$ if there is a successful transmission and is incremented in all other cases. So, we find,

$$A_{k+1} = \begin{cases} B_k + 1 & \text{if } B_k \geq 0, T_k = 1, Y_k = 0 \,, \\ A_k + 1 & \text{otherwise.} \end{cases} \tag{6}$$

First, consider the state of the other senders. Let $\pi_n$ denote the steady-state probability that $n$ senders have a packet,

$$\pi_n = \lim_{k\to\infty} \mathsf{P}[X_k = n] \,,$$

and let $\boldsymbol{\pi}$ denote the row vector with elements $\pi_n$, $n \in \{0,\dots,M\}$. By conditioning on the state during the preceding slot, the balance equations read,

$$\pi_n = \sum_{m=0}^{M} \pi_m x_{mn} ,$$

or, equivalently, $\boldsymbol{\pi} = \boldsymbol{\pi}\mathcal{X}$. Using the normalisation condition $\boldsymbol{\pi}\mathbf{e}' = 1$, we further find,

$$\boldsymbol{\pi} = \mathbf{e}\,(\mathcal{I} - \mathcal{X} + \mathbf{e}'\mathbf{e})^{-1} . \quad (7)$$

Now, let $\theta_n$ denote the steady-state probability that $n$ other senders have the packet, while the tagged sender has no packet,

$$\theta_n = \lim_{k\to\infty} \mathsf{P}[X_k = n, B_k = -1] .$$

Expressing the probabilities at the $(k+1)$st slot boundary in terms of the probabilities at the $k$th boundary yields,

$$\begin{aligned}\mathsf{P}[X_{k+1} = n, B_{k+1} = -1] &= \mathsf{P}[X_{k+1} = n, B_{k+1} = -1, B_k = -1] \\ &\quad + \mathsf{P}[X_{k+1} = n, B_{k+1} = -1, B_k \geq 0] \\ &= \bar{\lambda} \sum_{m=0}^{\infty} \mathsf{P}[X_k = m, B_k = -1] x_{mn} \\ &\quad + \bar{\lambda} p \sum_{m=0}^{\infty} \mathsf{P}[X_k = m, B_k \geq 0] x_{mn} .\end{aligned}$$

Hence, we obtain the following system of equations for the stationary probabilities $\theta_n$, $n \in \{0,1,\dots,M\}$,

$$\theta_n = \left(\bar{\lambda}\theta_m + \bar{\lambda} p (\pi_m - \theta_m)\right) x_{mn} .$$

Let $\boldsymbol{\theta}$ denote the row vector with elements $\theta_n$, the system of equations above then reads in matrix notation,

$$\boldsymbol{\theta} = \left(\bar{\lambda}\boldsymbol{\theta} + \bar{\lambda} p (\boldsymbol{\pi} - \boldsymbol{\theta})\right) \mathcal{X} .$$

Hence, with $\bar{p} = 1 - p$, we have,

$$\boldsymbol{\theta} = \bar{\lambda} p \boldsymbol{\pi} (\mathcal{I} - \bar{\lambda}\bar{p}\mathcal{X})^{-1} . \quad (8)$$

Note that $\bar{\lambda}\bar{p}\mathcal{X}$ is a sub-stochastic matrix, which ensures that the inverse matrix in the expression above is well defined.

Now consider the steady-state expectation $\beta_n$ of the age at the sender when there are $n$ users with a packet,

$$\beta_n = \lim_{k\to\infty} \mathsf{E}[B_k 1_{\{X_k = n\}}] .$$

By means of the system equation (5), we can express the mean at the $(k+1)$st slot boundary, in terms of probabilities and means at the $k$th slot boundary,

$$\begin{aligned}\mathsf{E}[B_{k+1} 1_{\{X_{k+1}=n\}}] &= -\bar{\lambda} p \sum_{m=0}^{M} \mathsf{P}[X_k = m] x_{mn} \\ &\quad + \bar{\lambda}\bar{p} \sum_{m=0}^{M} \mathsf{E}\left[(B_k + 1) 1_{\{X_k = m, B_k \geq 0\}}\right] x_{mn} \\ &\quad - \bar{\lambda}\bar{p} \sum_{m=0}^{M} \mathsf{P}[X_k = m, B_k = -1] x_{mn} .\end{aligned}$$

Taking the limit $k \to \infty$ in the expression above further yields,

$$\beta_n = -\bar{\lambda} p \sum_{m=0}^{M} \pi_m x_{mn} + \bar{\lambda}\bar{p} \sum_{m=0}^{M} (\beta_m + \pi_m) x_{mn} - \bar{\lambda}\bar{p} \sum_{m=0}^{M} \theta_m x_{mn} ,$$

for $m \in \{0,\dots,M\}$. Let $\boldsymbol{\beta}$ denote the row vector with entries $\beta_n$, we then have in matrix notation,

$$\boldsymbol{\beta} = \left(\bar{\lambda}\bar{p}\boldsymbol{\pi} - \bar{\lambda} p \boldsymbol{\pi} - \bar{\lambda}\bar{p}\boldsymbol{\theta}\right) (\mathcal{I} - \bar{\lambda}\bar{p}\mathcal{X})^{-1} . \quad (9)$$

We finally have collected the necessary results to study the age of information at the receiver. To this end, we now consider the steady state expectations,

$$\zeta_n = \lim_{k\to\infty} \mathsf{E}[A_k 1_{\{B_k = -1, X_k = n\}}] , \quad \alpha_n = \lim_{k\to\infty} \mathsf{E}[A_k 1_{\{X_k = n\}}] ,$$

for $n \in \{0,1,\dots,M\}$. By means of the system equations (5) and (6), we can express the mean values at the $(k+1)$st slot in terms of means and probabilities at the $k$th slot,

$$\begin{aligned}&\mathsf{E}[A_{k+1} 1_{\{B_{k+1} = -1, X_{k+1} = n\}}] = \\ &\qquad \sum_{m=0}^{M} \bar{\lambda} p \mathsf{E}[(B_k + 1) 1_{\{X_k = m\}}] x_{mn}^{\mathrm{nt}} \\ &\qquad + \sum_{m=0}^{M} \bar{\lambda} p \mathsf{E}[(A_k + 1) 1_{\{X_k = m, B_k \geq 0\}}] x_{mn}^{\mathrm{t}} \\ &\qquad + \sum_{m=0}^{M} \bar{\lambda} \mathsf{E}[(A_k + 1) 1_{\{X_k = m, B_k = -1\}}] x_{mn} .\end{aligned}$$

Taking the limit $k \to \infty$ at both sides of the equation then yields the equations for the corresponding steady-state mean,

$$\begin{aligned}\zeta_n = \sum_{m=0}^{M} \bar{\lambda} p (\beta_m + \pi_m) x_{mn}^{\mathrm{nt}} + \sum_{m=0}^{M} \bar{\lambda} p (\alpha_m - \zeta_m + \pi_m - \theta_m) x_{mn}^{\mathrm{t}} \\ + \sum_{m=0}^{M} \bar{\lambda} (\zeta_m + \theta_m) x_{mn} ,\end{aligned}$$

for $n \in \{0,1,\dots,M\}$. Equivalently, in matrix notation, we have,

$$\boldsymbol{\zeta} = \bar{\lambda} p (\boldsymbol{\beta} + \boldsymbol{\pi}) \mathcal{X}^{\mathrm{nt}} + \bar{\lambda} p (\boldsymbol{\alpha} - \boldsymbol{\zeta} + \boldsymbol{\pi} - \boldsymbol{\theta}) \mathcal{X}^{\mathrm{t}} + \bar{\lambda} (\boldsymbol{\zeta} + \boldsymbol{\theta}) \mathcal{X} ,$$

with $\boldsymbol{\zeta}$ the row vector with entries $\zeta_n$. Solving for $\boldsymbol{\zeta}$ yields

$$\boldsymbol{\zeta} = \boldsymbol{\gamma} + \boldsymbol{\alpha}\mathcal{A} , \quad (10)$$

with,

$$\begin{aligned}\boldsymbol{\gamma} &= \left(\bar{\lambda} p (\boldsymbol{\beta} + \boldsymbol{\pi}) \mathcal{X}^{\mathrm{nt}} + \bar{\lambda} p (\boldsymbol{\pi} - \boldsymbol{\theta}) \mathcal{X}^{\mathrm{t}} + \bar{\lambda}\boldsymbol{\theta}\right) \\ &\qquad \times (\mathcal{I} - \bar{\lambda}(\mathcal{X} - p\mathcal{X}^{t}))^{-1} , \quad (11) \\ \mathcal{A} &= \bar{\lambda} p \mathcal{X}^{\mathrm{t}} (\mathcal{I} - \bar{\lambda}(\mathcal{X} - p\mathcal{X}^{t}))^{-1} . \quad (12)\end{aligned}$$

Following a similar approach, the system equations (5) and (6) yield,

$$\begin{aligned}&\mathsf{E}[A_{k+1} 1_{\{X_{k+1} = n\}}] = \\ &\qquad \bar{p} \sum_{m=0}^{M} \mathsf{E}[(A_k + 1) 1_{\{X_k = m\}}] x_{mn}^{\mathrm{nt}} \\ &\qquad + p \sum_{m=0}^{M} \mathsf{E}[(B_k + 1) 1_{\{X_k = m\}}] x_{mn}^{\mathrm{nt}} \\ &\qquad + p \sum_{m=0}^{M} \mathsf{E}[(A_k + 1) 1_{\{X_k = m, B_k = -1\}}] x_{mn}^{\mathrm{nt}} \\ &\qquad + \sum_{m=0}^{M} \mathsf{E}[(A_k + 1) 1_{\{X_k = m\}}] x_{mn}^{\mathrm{t}} ,\end{aligned}$$

while taking the limit $k \to \infty$ leads to,

$$\alpha_n = \sum_{m=0}^{M} (\alpha_m + \pi_m)(\bar{p} x^{\mathrm{nt}}_{mn} + x^{\mathrm{t}}_{mn}) + p \sum_{m=0}^{M} (\beta_m + \pi_m + \zeta_m + \theta_m) x^{\mathrm{nt}}_{mn} .$$

Let $\boldsymbol{\alpha}$ denote the row vector with entries $\alpha_n$, $n \in \{0, 1, \dots, M\}$. The system of equations above is then equivalent to,

$$\boldsymbol{\alpha} = (\boldsymbol{\alpha} + \boldsymbol{\pi})(\bar{p}\mathcal{X}^{\mathrm{nt}} + \mathcal{X}^{\mathrm{t}}) + p(\boldsymbol{\beta} + \boldsymbol{\pi} + \boldsymbol{\zeta} + \boldsymbol{\theta})\mathcal{X}^{\mathrm{nt}} .$$

Finally, plugging (10) in the expression above and solving for $\boldsymbol{\alpha}$ yields,

$$\boldsymbol{\alpha} = \left(\boldsymbol{\pi}(\bar{p}\mathcal{X}^{\mathrm{nt}} + \mathcal{X}^{\mathrm{t}}) + p(\boldsymbol{\beta} + \boldsymbol{\pi} + \boldsymbol{\gamma} + \boldsymbol{\theta})\mathcal{X}^{\mathrm{nt}}\right) \times (\mathcal{I} - (\bar{p}\mathcal{X}^{\mathrm{nt}} + \mathcal{X}^{\mathrm{t}}) - p\mathcal{A}\mathcal{X}^{\mathrm{nt}})^{-1} . \quad (13)$$

### C. Main result

The AoI calculations can be summarised as follows. First, construct the matrices $\mathcal{X}$, $\mathcal{X}^{\mathrm{nt}}$ and $\mathcal{X}^{\mathrm{t}}$ by equations (2), (3) and (4). Then subsequently calculate $\boldsymbol{\pi}$ by equation (7), $\boldsymbol{\theta}$ by (8), $\boldsymbol{\beta}$ by (9), $\boldsymbol{\gamma}$ and $\mathcal{A}$ by (11) and (12), and $\boldsymbol{\alpha}$ by (13). The numerical complexity of the calculations mainly stems from the number of users $M$. Careful evaluation of the expressions shows a overall numerical complexity of $\mathcal{X}$ is $O(M^3)$.

We can now express the mean AoI at random slot boundaries and the mean peak AoI in terms of known vectors. Because of the definition of $\boldsymbol{\alpha}$, the mean age of information at random slot boundaries equals,

$$\mathsf{E}[A_r] = \boldsymbol{\alpha}\mathbf{e}' .$$

For the mean of the peak AoI, we condition on having a successful transmission. We have a successful transmission provided that (i) there is a packet at the sender, (ii) there is a transmission by the sender, and (iii) there are no transmissions by other senders. Therefore, we can express the mean of the peak age in terms of known quantities as follows,

$$\mathsf{E}[A_p] = \mathsf{E}[A_k | B_k \geq 0, T_k = 1, Y_k = 0] = \frac{(\boldsymbol{\alpha} - \boldsymbol{\zeta})\mathcal{X}^{\mathrm{nt}}\mathbf{e}'}{(\boldsymbol{\pi} - \boldsymbol{\theta})\mathcal{X}^{\mathrm{nt}}\mathbf{e}'} .$$

## III. Numerical example

We now illustrate our results with some numerical examples. All results follow from the analytic calculations in the preceding sections and were confirmed by extensive simulations. Figure 1 shows the mean AoI and the mean peak age of information (pAoI) as a function of the transmission probability $p$ for $M = 8$ and $M = 16$ users, and for a low arrival rate $\lambda = 0.05$. Figure 2 shows the same measures but for a higher arrival rate $\lambda = 0.2$. When the load is low, the mean (peak) AoI decreases for increasing $p$. As the load is low, the chance that two transmissions collide is low, and therefore it is best to send the packet once it is available ($p = 1$). For higher load, this is no longer the case, and there is an optimal $p < 1$.

To investigate how the optimal $p$ changes with the packet arrival rate $\lambda$, figures 3 and 4 depict the optimal $p$ which minimises the mean AoI at slot boundaries and the mean pAoI, respectively. When there are but a few packets, it is optimal to immediately send. When the packet arrival rate is higher, the optimal probability drops quickly. The drop is initiated for lower $\lambda$ when the number of users $M+1$ is higher. This is not unexpected, as the chance of contention increases when the number of users increases. Finally, note that optimising the AoI and pAoI yields considerably different $p$, the optimal $p$ for the pAoI is higher than the $p$ optimising the AoI. Hence, there is no single optimal transmission probability $p$, and the choice between optimising for the AoI at slot boundaries or the peak AoI depends on the application at hand. This observation is further confirmed by figure 5 which shows the critical arrival rate $\lambda_c$ where it is no longer optimal to send with $p = 1$, the critical value being higher for the pAoI.

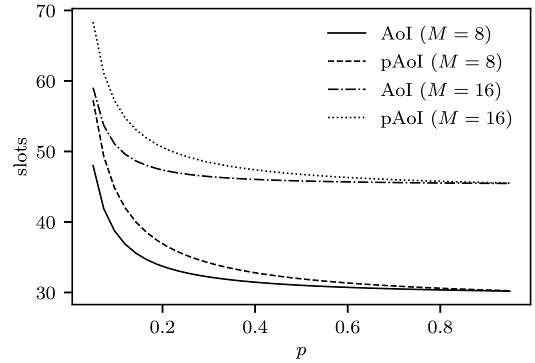


Fig. 1. Mean age of information (AoI) and mean peak Age of Information (pAoI) vs. the transmission probability $p$ for $\lambda = 0.05$ and $M$ as indicated

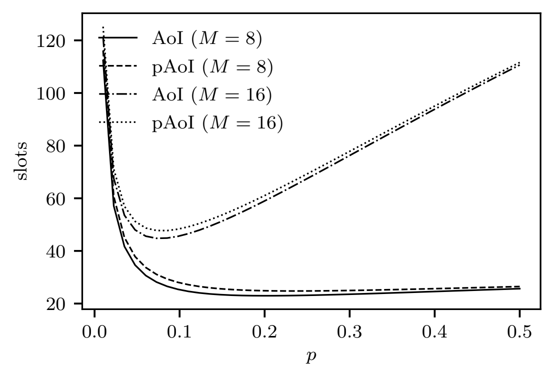


Fig. 2. Mean age of information (AoI) and mean peak Age of Information (pAoI) vs. the transmission probability $p$ for $\lambda = 0.2$ and $M$ as indicated

## IV. Conclusion

We have presented a mean value analysis of the AoI at random slot boundaries and of the peak AoI for slotted ALOHA. Unlike previous studies, our results are exact as we did not rely on approximations of the slotted ALOHA protocol. Through a numerical example, we investigated how the AoI is influenced by the packet arrival and transmission probabilities. Notably, we discovered that the transmission probabilities that optimise the age at slot boundaries and the peak age differ significantly. Our findings have important implications for the design and

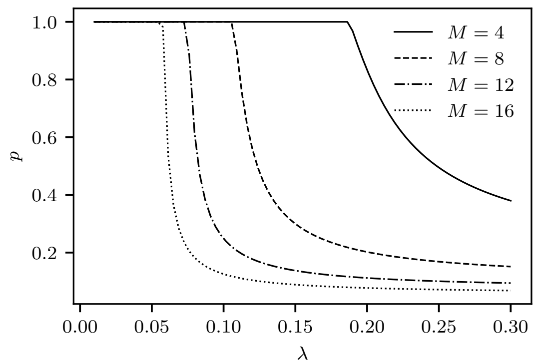


Fig. 3. AoI-optimal transmission probability vs. the arrival probability $\lambda$ for different $M$ as indicated.

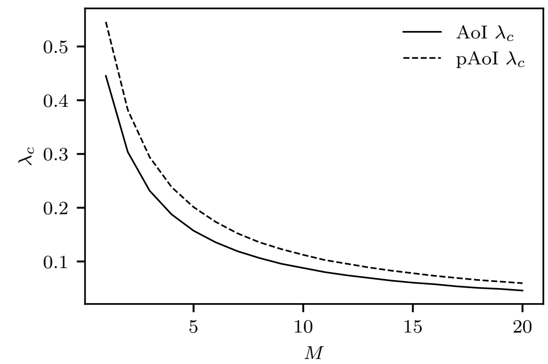


Fig. 5. Critical arrival rate $\lambda_c$ for the AoI and pAoI vs. $M$.

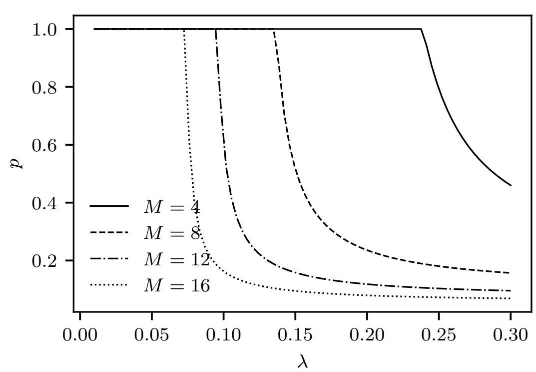


Fig. 4. pAoI-optimal transmission probability vs. the arrival probability $\lambda$ for different $M$ as indicated.

optimisation of slotted ALOHA systems, and we anticipate that they will be of interest to researchers in the field of communication networks in general, and especially in the field of V2X communications.

Future work is needed to incorporate the spatiotemporal dynamics of vehicular networks in the AoI analysis, e.g. by incorporating recent results of [22] and [23] in the model.

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