Modelling and probabilistic evaluation of repair strategies in the initiation phase of chloride-induced corrosion of the reinforcement of concrete structures

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ABSTRACT

Chloride ingress is a common issue in concrete structures that can result in reinforcement corrosion. In order to extend the lifetime of a structure, several preventive repair methodologies are available which reduce the ingress of chlorides. In this work, the computational efficiency of the analytical model for a two-material diffusion problem has been improved to model the effect of a mortar overlay or partial cover replacement, and is further implemented to model the influence of a coating. This enables the assessment of each repair method in a probabilistic framework using Latin Hypercube Samples within an acceptable timeframe. A parameter study executed in both a deterministic and probabilistic framework demonstrated the model efficiency and the importance of the timing and quality of the intervention for successful repair when applying a repair intervention in the initiation phase.

1 INTRODUCTION

In concrete structures, chloride ingress can result in reinforcement corrosion, leading to problems concerning both serviceability and safety. In order to quantify the initiation phase of corrosion (depassivation of reinforcement), prediction models for chloride ingress in uncracked [1], [2] and cracked [3] concrete have been developed, of which the former are considered widely accepted. Chloride ingress in concrete is a complex process and significant uncertainties are associated with both the model and its input parameters. Therefore, a probabilistic approach is most often used to quantify the probability of depassivation of the reinforcement. This probability of depassivation enables to assess the potential need for a repair strategy, as depassivation of the reinforcing steel is often considered as the end of the design service life of a structure [4], [5]. In this work, models developed for the prediction of chloride ingress after the application of a particular repair strategy are considered, and their computational efficiency is improved to allow for a probabilistic analysis within a practical timeframe. Using these models, an objective comparison between different repair strategies can be made, enabling decision making in relation to an optimized repair campaign.

Several repair options are available for concrete structures subjected to chloride ingress, as reported in *fib* Bulletin 102 [5]. Generally, the different repair strategies can be categorized according to their working principle: delaying the ingress of chlorides (e.g. coating or the use of a mortar overlay), reducing the chloride concentration (e.g. chloride extraction or recasting), reducing the corrosion rate (e.g. using corrosion inhibitors) and reducing the corrosion level (e.g. removing and replacing corroded reinforcement). In this work, that gives focus to interventions during the initiation phase of chloride ingress, three repair strategies are considered which correspond to the first two categories, i.e. the application of a coating, the application of a mortar overlay and the partial removal and replacement of the concrete cover by repair mortar. The latter two interventions can be analyzed by solving the chloride ingress problem in two materials with a different diffusion coefficient. Although available literature is scarce, this has been modelled for the so-called skin effect at the concrete surface [6], [7]. The models in [6], [7] assume that there are no chlorides present at the start of the diffusion process. However, in practical situations, when a repair mortar is applied, chlorides have likely already penetrated the concrete. The latter has been modelled analytically in the work of [8] and numerically in the work of [9]–[11], which propose a two-material diffusion model that is able to account for an initial chloride profile. The existing models consider time-dependent properties of e.g. the diffusion coefficient requiring a step-wise evaluation of the chloride profile during which the time-dependent properties are altered, which significantly reduces the computational efficiency of the calculation and makes a probabilistic calculation cumbersome. In this work, this aspect is improved by working with an averaged constant diffusion coefficient, which is shown to have no significant influence on the results.

Next to the mortar overlay and replacement of the concrete cover by mortar material, the application of a coating is considered. Similar to the other repair methods, a coating forms a continuous protective layer on the concrete substrate and counters the ingress of new chlorides. However, it is not able to re-absorb already integrated chlorides. The influence of a coating has been studied in various experimental campaigns [12]–[15] and several models have been developed in order to predict the measured chloride profiles after application of the treatment. The most common ways to model the effect of a coating are through an equivalent reduction of the surface concentration, an additional diffusive material layer with a low diffusion coefficient [16], or an additional convection layer [17]. Apart from applying a coating, hydrophobic impregnation such as a silane treatment [18], [19] can also be considered, but this is not explicitly covered in this work.

In the next section, the general solution for the one-material diffusion problem (as is the case for concrete subjected to chloride ingress without surface treatments) is briefly revisited, including the typical simplifications. In the third section, the two-material diffusion solution as proposed in [8] is improved in order to more effectively facilitate a probabilistic calculation of the chloride ingress after either the application of a mortar overlay, or the replacement of part of the contaminated concrete by a mortar layer. This is done by implementing a constant averaged diffusion coefficient, similar to an equivalent diffusion coefficient used in the widely accepted one-material diffusion solution. As a result, the computational efficiency of the models is significantly increased. In the next section, this two-material diffusion model is extended to predict the chloride ingress after application of a coating layer and is fitted to experimental data from literature.

A quantitative comparison between the effect of the different repair strategies is made through a deterministic calculation to assess the impact of each strategy, and a sensitivity study is executed to detect the most important variables in relation to the effectiveness of the repair method. Afterwards, the developed models are incorporated in a probabilistic framework and the influence of the most important parameters is investigated to have more insight into the governing factors for a successful repair. Finally, the results are discussed and some challenges for future research are presented.

2 MODELLING CHLORIDE DIFFUSION IN A SINGLE MATERIAL

The one-dimensional transport of chlorides through the concrete substrate is generally described by Fick's second law of diffusion [20]:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} D \frac{\partial C}{\partial x} \tag{1}$$

where C *[m%cement]* is the chloride concentration at time t *[y]* and location x *[m]* and D *[m²/y]* is the diffusion coefficient of the substrate material. Under the assumptions that chlorides are diffusing through a single material with a time-invariant diffusion coefficient, no initial chlorides are present in the substrate and a constant chloride concentration is maintained at the surface, the solution of the differential equation in Eq. (1) is described by Eq. (2):

$$C(x,t) = C_s \left[1 - erf\left(\frac{x}{2\sqrt{Dt}}\right)\right] \tag{2}$$

where C_s *[m%cement]* is the surface chloride concentration which results from the environmental conditions, and *erf(x) is* the Gauss error function. When the diffusion coefficient and surface concentration are known, this equation allows the determination of the chloride concentration at any time and location. However, some important aspects should be considered when modelling the chloride ingress by means of Eq. (2).

First of all, it is widely known that the surface concentration is in most cases not constant over time and chloride transport at the surface is not solely due to diffusion, but also due to convection. In order to maintain the practical functionality of Eq. (2), these problems are solved by replacing the convection zone with an equivalent surface concentration $C_{s,\Delta x}$ at a depth Δx equal to the thickness of the convection zone [2]. In this work, this convection layer is not considered, but can be added to the models that are developed.

Further, the diffusion coefficient of concrete reduces in time due to a decrease in porosity caused by ageing of the concrete and binding of the chlorides to form salts [21]. As a consequence, Eq. (2) for the single material diffusion problem is in principle no longer applicable. To overcome this issue, an averaged equivalent diffusion coefficient $D_{app}(t)$ is commonly defined. This so-called apparent diffusion coefficient can be related to the results of migration tests through the following equation (*fib* Bulletin 34 [1]):

$$D_{app}(t) = k_e \cdot D_{RCM,0} \cdot k_t \cdot \left[\frac{t_0}{t}\right]^a \tag{3}$$

where a *[-]* is the ageing exponent describing the time-dependent behaviour of the diffusion coefficient, generally considered to be between 0.2 and 0.8 [2]. k_e *[-]* is the environmental transfer variable considering the outside temperature. This value is assumed to be equal to 1 in the subsequent analyses in this article, i.e. the temperature of the concrete element under consideration is equal to a reference temperature of 20°C. $D_{RCM,0}$ is the migration coefficient at a reference time t_0 of 28 days measured by the Rapid Chloride Migration (RCM) method. k_t *[-]* is the transfer parameter that relates $D_{RCM,0}$ to D_{app} and depends on the concrete composition, which is in this work considered equal to 1.

3 MODELLING OF CHLORIDE INGRESS FOR A TWO-MATERIAL DIFFUSION PROBLEM

In case a repair mortar is applied to the concrete surface, an additional layer of a new material with a different diffusion coefficient is introduced. Consequently, the diffusion coefficient in Eq. (1) varies with the depth of the substrate, and the single material solution given by Eq. (2) does not apply anymore. To solve the chloride migration problem, the differential equation can be solved in two ways: (1) discretization, e.g. using the finite-difference method, or (2) deriving an (approximate) analytical solution for the differential equation. These two methods are described in the next sections. The properties of the mortar layer are labelled with subscript 1, whereas the properties of the concrete are labelled with subscript 2.

3.1 *Discretization of the diffusion equation*

The first approach, proposed in the work of Petcherdchoo [10], is to discretize the diffusion problem in time and space using the finite-difference principle of Crank-Nicolson [22]:

$$\frac{c_{i,j+1}-c_{i,j}}{\Delta t}=\frac{1}{2}\left(\frac{D_{i+1/2}\left(c_{i+1,j+1}-c_{i,j+1}\right)-D_{i-1/2}\left(c_{i,j+1}-c_{i-1,j+1}\right)}{(\Delta x)^2}+\frac{D_{i+1/2}\left(c_{i+1,j}-c_{i,j}\right)-D_{i-1/2}\left(c_{i,j}-c_{i-1,j}\right)}{(\Delta x)^2}\right) \qquad (4)$$

where $c_{i,j}$ is the chloride concentration at mesh point i and timestep j as illustrated in Figure 1. Diffusion coefficients $D_{i+1/2}$ and $D_{i-1/2}$ are located in between two mesh points and determined by taking the average of the diffusion coefficients of their neighbors.

Figure 1: Discretization scheme finite-difference method

To numerically approximate the multi-material diffusion process using Eq. (4), a discretization in time and space is required. Therefore, the mesh size (Δx) and size of the time increments (Δt) need to be determined. Furthermore, the truncation limit of the semi-infinite region of the 1D space (x_{limit}) needs to be decided. Increasing Δx and Δt and decreasing x_{limit} reduces the computation time, but at the expense of numerical accuracy and therefore an appropriate choice should be made. Based on previous work, the optimal value of these parameters has been derived [23], which are verified depending on the problem-specific case. As a starting point, Δx can be taken equal to 1 mm. Secondly, the considered depth x_{limit} of the 1D space should be large enough to ensure that the semi-infinite region boundary condition $C(x=+\infty;t)=0$ for general concrete structures with considerable thickness is satisfied. As investigated in [23], a value x_{limit} equal to 300 mm leads to an acceptable prediction of the chloride concentration at the level of the reinforcement. Finally, Δt is an important parameter because it has a significant influence on the computation time. This is because the chloride concentration needs to be consecutively determined for every time step, compared to the spatial characteristics which can be calculated simultaneously using matrix calculations. In [10], Δt is chosen equal to 0.08 y (1 month) to ensure a smooth transition of the chlorides into the two-material substrate. However as discussed in [23], the value Δt can be set equal to one year to increase the speed of the calculation and still maintain a reasonable approximation of the chloride concentration at the point of interest.

3.2 *Analytical solution*

Alternative to the finite-difference approach, the diffusion equation can be solved analytically using the principle of separation of variables [20]:

$$C(x,t) = \sum_{n=1}^{\infty}[A_n \sin(\lambda_n x) + B_n \cos(\lambda_n x)] \exp(-\lambda_n^2 D t) \tag{5}$$

where A_n, B_n and λ_n are unknown constants, and D is the constant diffusion coefficient. Taking into account the appropriate boundary conditions and initial condition, the following analytical solution for the two-material problem is obtained [8]:

$$C(x,t) = C_s + \sum_{n=1}^{\infty} A_n{}' f_n(x,\lambda_n)\exp(-\lambda_n^2 t) \tag{6}$$

with:

$$f_n(x,\lambda_n) = \begin{cases} \dfrac{\cot(\lambda_n h_2/\sqrt{D_2})}{\sin(\lambda_n h_1/\sqrt{D_1})} \sin\left(\dfrac{\lambda_n x}{\sqrt{D_1}}\right), & 0 \le x \le h_1 \\ \dfrac{1}{\sin\left(\frac{\lambda_n h_2}{\sqrt{D_2}}\right)} \cos\left[\lambda_n (x - h_1 - h_2)/\sqrt{D_2}\right], & h_1 \le x < h_1 + h_2 \end{cases} \tag{7}$$

$$A_n' = \frac{\int_0^{h_1+h_2} [\bar{C}(x) - C_s] f_n(x,\lambda_n) dx}{\int_0^{h_1+h_2} f_n(x,\lambda_n)^2 dx} \tag{8}$$

and λ_n governed by equation:

$$\tan\left(\frac{\lambda_n h_1}{\sqrt{D_1}}\right)\tan\left(\frac{\lambda_n h_2}{\sqrt{D_2}}\right) = \sqrt{D_1/D_2} \qquad (9)$$

where $\bar{C}(x)$ is the initial chloride concentration, C_s the surface concentration (assumed to be constant in time), h_1 and h_2 the thickness of respectively the first (i.e. closest to the surface) and second layer and D_1 and D_2 of their corresponding constant diffusion coefficient. The calculation can be performed considering a relevant depth of the concrete substrate $h_2 = x_{\text{lim}}$, as discussed before.

For practical applications, the summation in Eq. (6) is limited to a maximum value n_{max}, which is in literature suggested to be taken equal to 10 [8]. The value of n_{max} should be kept small in view of the computation time but high enough to accurately represent the chloride profile. This is investigated in Figure 2 for an example case where a repair mortar layer of 20 mm is applied to a concrete substrate after a period of 50 years. In the period shortly after application, the chloride profile is the most irregular and therefore more terms, i.e. higher values of n, are required for an accurate prediction of the chloride profile. As illustrated in Figure 2, a total number of terms n_{max} equal to 5 is not able to represent the more complex chloride profile, and larger values are needed. In contrast to the advised value in [9], a value n_{max} of 20 is chosen so that the chloride profile is described well for the first years after mortar application and coincides with the finite-difference solution, using Δt = 0.1y. It is noted that when the profile is predicted sufficiently accurately in the first years after mortar application, this will also be the case for the following years, as the curve becomes generally less complex.

Figure 2: Influence of n_{max} on the analytical solution of the two-material diffusion problem (C_s = 2 m%cement, h_1 = 20 mm, h_2 = 300 mm, D_1 = 0.1e-12 m^2/s, D_2 = 0.5e-12 m^2/s)

The computation time of the analytical method depends heavily on the way Eq. (9) is solved to find the values of λ_n. As described by [8], an effective method is to first determine the varying intervals in which there exists a solution for λ_n and where the function in Eq. (9) is monotonic.

Afterwards, the bisection method can be used to determine the solution in each interval [24]. The amount of bisections taken is by default equal to 10, which is considered broadly sufficient for the examples in this work. The considered bisection method can be executed for multiple intervals at the same time. This means the different considered values of λ_n (and every sample considered) are determined simultaneously, which drastically improves the calculation time.

3.3 *Comparison discretization and analytical approach*

One of the main disadvantages of the finite-difference method (discretization) is that the chloride content levels in the next time step require the results of the previous time step, which is not the case for the analytical method. As a result, the calculation time of the finite-difference method increases with increasing time after repair application, while the calculation time of the analytical method remains constant. This is illustrated in Figure 3 by considering the same example as considered in Figure 2, with $\Delta t = 1y$, Δx = 1 mm for the finite-difference method and n_{max} equal to 20 for the analytical method. The results show the analytical method is up to 10 times faster than the finite-difference method.

Figure 3: Calculation time of the finite-difference method and the analytical method for the two-material diffusion problem example (IntelI CITM i7, 3.00GHz , 32 GB)

The benefit of the analytical method is that the chloride profile can be determined at any time instant without increasing the computation time. However, this is only possible under the assumption of a constant diffusion coefficient and surface concentration over time. If this is not the case, a stepwise approach can be followed where each output of the previous step is used as input for the next one, in which the diffusion coefficient and/or surface concentration is changed. However, this decreases the computational advantage of the analytical method compared to the finite-difference method. As stated previously, the surface concentration is generally considered to be constant, but the diffusion coefficient varies over time. In the next section, the time dependency of the diffusion coefficient is discussed and the potential use of an averaged constant diffusion coefficient is investigated for each of the two layers.

3.4 *Averaged diffusion coefficients in the two-material diffusion process*

Typically, a repair intervention is applied to a concrete structure of considerable age, for which measurements indicate a later risk of corrosion. Before the repair intervention, the chloride profile is described by Eq. (2), using the apparent diffusion coefficient defined in Eq. (3). This apparent diffusion coefficient decreases with increasing time as illustrated in Figure 4 and stabilizes in due time.

Figure 4: Time-dependent behaviour of the diffusion coefficient for different values of the ageing coefficient

It can be assumed that in practical situations, where the repair intervention is applied to an existing structure of considerable age, the diffusion coefficient of the concrete substrate remains almost constant after most of the repair interventions. In case the age of the structure or the ageing coefficient is low such that the diffusion coefficient of the concrete still varies significantly over time, this assumption is not valid and a more detailed analysis is needed. However, in this work, the diffusion coefficient of the concrete substrate after the repair intervention is considered constant and equal to the average value of the instantaneous diffusion coefficient in the time period which spans from the instant of repair t_1 to the instant t_2 at which the chloride profile is predicted. The instantaneous diffusion coefficient at a certain moment in time is defined to have the following exponential behaviour [25]:

$$D(t) = D_0 \left[\frac{t_0}{t}\right]^b \qquad (10)$$

where D_0 is the diffusion coefficient at reference time t_0 (28 days) and b [-] is the instantaneous ageing exponent. Using this relationship, the average diffusion coefficient of the concrete substrate after the repair intervention can be determined as:

$$D_c = \frac{1}{t_2-t_1}\int_{t_1}^{t_2} D(t)dt = \frac{D_0}{t_2-t_1}\int_{t_1}^{t_2}\left[\frac{t_0}{t}\right]^b dt = \frac{D_0}{t_2-t_1}\frac{t_0^b}{1-b}\left[t_2^{1-b} - t_1^{1-b}\right] \qquad (11)$$

Note that the instantaneous ageing coefficient is generally not known, as only the apparent diffusion coefficient and its corresponding ageing coefficient are determined by fitting the one-material solution of Eq. (2) to chloride measurements over time. The relation between both ageing coefficients can be determined by considering that $D_{app}(t)$ is the averaged value of $D(t)$ and combining Eq. (3) and (11):

$$D_{app}(t) = \frac{1}{t-t_0}\int_{t_0}^{t} D(\tau)d\tau$$

$$\leftrightarrow \left(\frac{t_0}{t}\right)^{a} = \frac{t_0^{b}}{t-t_0}\frac{1}{1-b}\left[t^{-b+1} - t_0^{-b+1}\right] \qquad (12)$$

where a and b are the ageing coefficient of respectively the equivalent and instantaneous diffusion coefficient.

The diffusion coefficient of the repair mortar can evolve with time as well and this evolution can have a substantial effect on the ingress of chlorides. As both the concrete and repair mortar are assumed to be cementitious materials, the time-dependent behaviour of the diffusion coefficient is considered to be similar and hence follows Eq. (10). This is in line with the standardized tests for chloride resistance in concrete (NBN EN 12390-11 [26]), which is generally applied to test repair mortars as well. However, one should note that additional experimental data on repair mortars is required to validate this assumption.

Considering a constant diffusion coefficient, the computationally beneficial analytical approach can be applied. To investigate the influence of the assumption of a constant (averaged) diffusion coefficient, the chloride profile is determined for a reference case where the diffusion coefficient of the repair mortar is considered as time-variant (using the finite-difference method) versus the situation where a constant diffusion coefficient is applied (using the analytical method). In both cases, the diffusion coefficient of concrete is assumed to be constant, but has a different value before and after the application time. Before the application time, the diffusion coefficient of the concrete is equal to the apparent diffusion coefficient $D_{app}(t_{repair})$, while after application of the repair the diffusion coefficient is equal to the averaged value between the time instant considered and $t = t_{repair}$.

The result is given in Figure 5 for an example set of variables where the mortar is applied on concrete with an age of 50 years. Considering the constant (averaged) diffusion coefficient for the repair mortar instead of the time-dependent behaviour leads to very similar results in the region located at 20 to 50 mm from the original concrete surface, i.e. the typical location of the reinforcement.

Figure 5: Effect of a time-dependent vs. time-independent diffusion coefficient on chloride profile (T_{rep} = 50 y; h_1 = 20 mm; h_2 = 300 mm; $D_{0,1}$ = 2e-12 m^2/s; $D_{0,2}$ = 10e-12 m^2/s and b_1 = b_2 = 0.5)

In the example considered, a constant, averaged diffusion coefficient leads to accurate results. However, to extend this conclusion more generally, a larger set of examples is investigated. Therefore, five parameters are varied by generating 10,000 samples according to a uniform distribution within the following intervals, which represent realistic parameter values:

- $\mathrm{D}_{0,1} \in [0.5 \cdot \frac{10^{-12}\mathrm{m}^2}{s};\ 2.5 \cdot \frac{10^{-12}\mathrm{m}^2}{s}]$
- $\mathrm{D}_{0,2} \in [2 \cdot \frac{10^{-12}\mathrm{m}^2}{s};\ 20 \cdot \frac{10^{-12}\mathrm{m}^2}{s}]$
- $\mathrm{h}_1 \in [10mm;\ 30mm]$
- $\mathrm{b}_1 \in [0.4;\ 0.6]$
- $\mathrm{b}_2 \in [0.4;\ 0.6]$

For each sample, the chloride concentrations are determined at a depth of 20 mm in the original concrete substrate and 10 years after the application of a repair mortar to a 50-year-old structure. This is done for both for the original time-variant mortar diffusion coefficient, and the approximation by an equivalent time-invariant diffusion coefficient. The relative difference between both outcomes is given by the histogram in Figure 6. It is shown that in 89% of the cases, using the averaged constant diffusion coefficient leads to conservative results, predicting a larger chloride content, which deviates only to a small extent from the actual chloride profile obtained with a time-dependent diffusion coefficient for the repair mortar. The error has a mean value of 0.78%

and standard deviation of 1.58% and hence it is possible to use the analytical method in combination with the constant (averaged) diffusion coefficient. This results in a much more efficient model than the finite-difference method, while maintaining acceptable accuracy.

Figure 6: Histogram of overestimation chloride level due to averaged diffusion coefficient of the repair mortar

4 MODELLING OF CHLORIDE INGRESS AFTER COATING APPLICATION

Another repair method considered in this work is the application of a coating to the concrete surface. As this coating is a very thin layer, modelling its effect as a convection-diffusion problem seems more appropriate compared to a two-material diffusion problem. In this section, a novel analytical solution for this problem is proposed to predict the ingress of chlorides after application of the coating.

4.1 *Modelling*

Different approaches have been proposed to predict the chloride ingress after application of a coating. The model of [16] suggests to reduce the surface concentration over time, and in particular cases add an additional second layer with a low diffusion coefficient at the surface. Furthermore, to predict the chloride ingress under this time-variant boundary condition, the finite-difference method was used, which requires a significant computational effort as discussed in the previous part of this article. Moreover, a large number of parameters need to be fitted to the experimental data, i.e. the surface concentration over time, and the thickness and diffusion coefficient of the second layer. This causes the model to be less suitable for practical applications.

A different approach has been proposed by [17], which considered the coating as a convective layer. This method succeeds in predicting the chloride profile, starting from an initial situation with zero initial chloride concentration, i.e. $C(x,0)=0$:

$$C(x,t) = C_s\left[1-\mathrm{erf}\left(\frac{x}{2\sqrt{Dt}}\right)\right] - C_s\exp\left(\frac{h_s x + h_s^2 t}{D}\right)\left[1-\mathrm{erf}\left(\frac{x+2h_s t}{2\sqrt{Dt}}\right)\right] \tag{13}$$

where C_s is the constant surface concentration, D the constant diffusion coefficient of the concrete substrate and h_s the convective coefficient describing the effect of the coating.

The original model [17] does not allow to take into account an initial chloride profile, which is present in case a treatment is applied to an existing concrete structure. Therefore, a novel approach is presented in this work, which is an extension of the convective layer approach that accounts for an arbitrary initial chloride distribution. The following boundary conditions and initial conditions are used to solve the general diffusion equation defined in Eq. (1):

$$-D\frac{\partial C}{\partial x}(x=0,t) = h_s[C_s - C(x=0,t)] \tag{14}$$

$$C(x,t=0) = \bar{C}(x) \tag{15}$$

$$\frac{\partial C}{\partial x}(x=h_1,t) = 0 \tag{16}$$

where $\bar{C}(x)$ is the initial chloride distribution and h_1 the distance from the surface at which the semi-infinite region of the 1D space is truncated, equivalent to the parameter x_{lim} previously defined (section 3.1) which is considered at 300 mm. The solution of this problem can be written in a similar form as the analytical solution of the two-material diffusion solution:

$$C(x,t) = C_s + \sum_{n=1}^{\infty} B_n{}' h_n(x,\lambda_n)\exp(-\lambda_n^2 t) \tag{17}$$

with:

$$h_n(x,\lambda_n) = \sin\left(\frac{\lambda_n x}{\sqrt{D}}\right) + \frac{\sqrt{D}}{h_s}\lambda_n \cos\left(\frac{\lambda_n x}{\sqrt{D}}\right) \tag{18}$$

$$B_n' = \frac{\int_0^{h_1}[\bar{C}(x) - C_s]h_n(x,\lambda_n)dx}{\int_0^{h_1} h_n(x,\lambda_n)^2 dx} \tag{19}$$

and λ_n governed by equation:

$$\frac{1}{\sqrt{D}}\cos\left(\frac{\lambda_n h_1}{\sqrt{D}}\right) = \frac{\lambda_n}{h_s}\sin\left(\frac{\lambda_n h_1}{\sqrt{D}}\right) \tag{20}$$

where C_s represents the surface concentration at the outside of the coating and $\bar{C}(x)$ the initial chloride content at the time the coating is applied. To solve Eq. (20) in order to get the values of λ_n, the bisection approach can be used as discussed previously (section 3.2), taking into account the fact that there is one solution in each (monotonically increasing) interval:

$$\lambda_n \in \left] \frac{\frac{\pi}{2}+(n-1)\pi}{\frac{h_1}{\sqrt{D}}} ; \frac{\frac{\pi}{2}+n\pi}{\frac{h_1}{\sqrt{D}}} \right] \qquad \text{with } n = 1, 2, \ldots \tag{21}$$

By default, the number of terms n_{max} is considered as 20.

In Figure 7, the proposed model is compared to the model in Eq. (13) for a situation where the initial chloride concentration is zero. Both models correspond very well, except for a small deviation right after the application of the coating, which can be attributed to the number of terms considered in Eq. (17).

Figure 7: Validation proposed model for coating in absence of an initial chloride profile ($C_s = 2m\%conc$, $h_s = 1.4 \times 10^{-10} m/s$, $D = 3.5 \times 10^{-12} m^2/s$)

4.2 *Fitting of the proposed model based on measurement data*

A limited number of experiments that investigate the influence of a coating on the ingress of chlorides have been performed [12], [27]. The data recorded in [27] is considered as the most extensive and covers different types of coatings. In these tests, the chloride profile of different specimens placed in a tidal zone was measured at several time instants. The data is compared to the previously developed model of [16] and the model proposed in this work, in due consideration of the following three aspects. Firstly, as measurements were executed after respectively 9, 36 and 60 months of application, a period during which the time-dependent behaviour of the diffusion coefficient of concrete D is substantial, a time-variant diffusion coefficient is considered. Secondly, a time dependent surface concentration C_s is considered to take into account the effect of the tidal environment [28]. Thirdly, also a time dependent convective coefficient is considered as research has shown coatings might have a limited lifetime after which their performance in reducing the ingress of chlorides rapidly decreases [29]. Therefore, the convective coefficient h_s is considered to increase linearly in time by an ageing factor R $[mm/y^2]$.

The previous aspects show the complexity of the situation at hand, which involves several time-dependent effects taking place simultaneously. To take these aspects into account using the proposed model in Eq. (17), h_s, D and C_s are modelled as time-dependent, considering a constant value per time step of a year. The result of the previous year is used as initial chloride concentration for the next year and this way the time-dependent properties can be taken into account, as illustrated in Figure 8. Note that in this case, the benefit of the analytical method is less pronounced as it requires consecutive timesteps where Eq. (20) needs to be solved each time, similar to the finite-difference approach. However, on a longer time span, the time-dependent effects will reduce and average values of the time-dependent parameters could be used, as discussed in section 3.4.

The parameters h_s and R are fitted to the measured data using the least-squares method and compared to the fitted model of [16] which uses up to four fitting parameters. The results are given in Figure 9 and Table 1. Despite the complexity of the situation, it can be seen that the data are well fitted by the model with tuned parameters. Moreover, the proposed model is able to fit the measured data better compared to the model proposed in [15], leading to a lower Mean Squared Error (MSE), especially for the polyurethane (PU) coating. Besides the better fit by the proposed model, also only two fitting parameters (h_s and R) are required, compared to up to a maximum of four parameters in the model of [16].

Figure 8: Framework to incorporate time-dependent properties in the analytical coating model

Figure 9: Validation of the developed model and comparison with the model by Petcherdchoo [16]

Table 1: Fitted parameters for the developed model

Coating [16]	h_s $[10^{-10} m/s]$	R $[10^{-18} m/s^2]$
Aliphatic acrylic solvent (AA)	0.95	1.25
Styrene acrylate solvent (SA)	1.24	0.00
Polyurethane coating (PU)	0.89	0.00
Acrylic modified cementitious coating (CPE)	1.24	0.00
Cementitious coating (CE)	2.19	2.10

It is important to note that three of the five coatings have a value of R equal to zero, indicating that the convective coefficient does not increase in time. Consequently, the effectiveness of the coating does not change in the timeframe of the experiments. This is in contrast to previous conclusions [27], which argued that the increasing concentration below the treatment was caused by deterioration of the treatment. However, as was found here, this is in most cases the direct result of continuous convection through the coating layer.

It must be noted that the available data is limited and considers a complex situation due to the increasing surface concentration in the tidal area. There is a clear need for more experimental data to confirm the behaviour of the coatings, validate the proposed model and investigate the evolution in time of coatings.

5 DETERMINISTIC SENSITIVITY STUDY OF REPAIR STRATEGIES

The influence of different repair solutions can now be compared using the analytical methods discussed previously. In this section, the comparison is made between (i) a mortar overlay, (ii) a partial replacement of concrete cover with mortar and (iii) coating, which are applied to an existing concrete structure. The comparison is done for a reference case with values of the input parameters as given in Table 2. The structure is considered to have reached the end of its lifetime when the chloride concentration at the reinforcement reaches a critical value. According to [2], the critical chloride content is on average around 0.5 $m\%cem$. In order to take into account the variability on this parameter, a conservative value of 0.4 $m\%cem$ is applied in this work which is consistent with the values described in [30].

Before any intervention, the chloride ingress is found by solving the one-material diffusion problem. For this particular example, this corresponds to an age $T_{LT,0}$ of 52 years. It is chosen to repair the structure at time T_{int}, which is in the reference case equal to half the predicted time to depassivation without repair $T_{LT,0}$ (instant at which depassivation would occur without repair intervention). The resulting chloride content over time at the location of the reinforcement surface (i.e. the cover depth) is shown in Figure 10. Moreover, it is noticed that the influence of the repair intervention on the chloride concentration becomes only visible after ca. 10 years. In this case, using a repair mortar leads to better results compared to the coating considered in this work, where the effect on the chloride transport remains limited. However, it must be stressed that the findings cannot be generalized for all coatings and repair mortars as it depends on the specific material properties of the products.

Nevertheless, applying a repair mortar results in additional benefits besides creating a barrier for new chlorides. For a mortar overlay, this is an increased cover for the reinforcement. For a partial cover replacement, part of the contaminated concrete is removed. Moreover, the uncontaminated layer of repair mortar will take up part of the already integrated chlorides by means of so-called 'reverse diffusion'.

Figure 10: Influence of repair strategies on chloride content at reinforcement, based on reference values from Table 2.

Table 2: Reference values deterministic example and parameter range for Sobol indices

Variable	**Reference value**	**Minimum value**	**Maximum value**
Concrete cover $[mm]$	40	30	50
Initial diff. coeff. concrete $D_{RCM,0,c}$ $[10^{-12} m^2/s]$	5.0	3.0	7.0
Averaged ageing coeff. concrete a_c $[-]$	0.4	0.3	0.5
Surface chloride content C_s $[m\%cem]$	2	1.5	2.5
Intervention time relative to original time to depassivation $T_{int}/T_{LT,0}$ $[-]$	0.5	0.25	0.75
Thickness mortar layer $[mm]$	10	5	20
Initial diff. coeff. mortar $D_{RCM,0,m}$ $[10^{-12} m^2/s]$	1.27	0.5	1.5
Averaged ageing coeff. mortar a_m $[-]$	0.5	0.3	0.5
Convective coeff. coating h_s $[10^{-10} m/s]$	0.89	0.5	2
Ageing factor coating R $[mm/y^2]$	0	0	0

Due to the intervention, the lifetime of the concrete structure, i.e. the time at which the threshold of 0.4 $m\%cem$ is exceeded, is extended. In order to investigate which parameters have the largest influence on this lifetime extension, a sensitivity analysis is performed. This is done using the principle of Sobol indices [31]–[33], which is an indicator of the sensitivity of a variable based on a variance-based analysis, making use of the python package SALib [34]. The Sobol index represents the importance of the corresponding parameter in the total variability of the model output, which is determined by keeping one or more variables fixed and assessing the variation of the output. The total Sobol indices [33] are determined by considering a uniform distribution of the parameters in the range given in Table 2. The resulting Sobol indices are given in Figure 11 for each model.

Figure 11: Sobol indices for different repair strategies

For the coating, the time of application is the most important parameter as it has the largest influence on the final lifetime extension in the range of the parameter values considered. In case of the repair mortar applications, several governing parameters are important. The properties of the concrete have an effect on both the overlay and replacement method because the lower the diffusion through the original concrete substrate, the less substantial the effect of a repair will be. In contrast to the mortar overlay, the partial cover replacement is less dependent on the time of intervention. This is because the concrete replacement can significantly reduce the chloride level for a larger window of time (compared to the mortar overlay), which makes the time of application less important. Additionally, the influence of the surface concentration is of minor importance. This is explained by the fact that having a low diffusion coefficient or thicker repair mortar can reduce the ingress of new chloride more effectively than reducing the surface concentration.

6 PROBABILISTIC FRAMEWORK

Values of parameters that influence the chloride ingress over time might be subjected to a certain variability. Therefore, it is important to consider the parameters as probabilistic, taking into account the uncertainty in material properties and variability of the environmental conditions governing the chloride concentration at the surface. In this section, the probabilistic models are discussed and the influence of the most dominant parameters is investigated in a probabilistic analysis.

6.1 *Probabilistic models for the input variables*

In the *fib* Model Code for Service Life Design [1] and *fib* Bulletin 76 [2], the probabilistic characteristics of the parameters describing the diffusion process have been characterised. In Table 3, the probabilistic properties are given, where the mean values corresponding to the reference values of the case previously considered in section 5. Note that the parameters that were defined in literature as normally distributed are defined in this work as lognormally distributed in order to exclude negative values.

The coefficient of variation of the diffusion coefficient $D_{RCM,0,m}$ and ageing exponent a_m of the mortar is considered to be equal to the corresponding coefficient of variation of the original concrete. The characteristics of the coating are those of the polyurethane coating (PU) considered in section 4, with the convective coefficient considered lognormally distributed. The ageing factor of the coating R is thus considered to be equal to zero, and it is assumed that the properties of the coating stay constant in due time. In reality, the coating would need to be replaced every ca. 20 years to maintain its functionality.

Table 3: Probabilistic properties of chloride ingress parameters for default example

Variable	**Distribution**	$\boldsymbol{\mu}$	$\boldsymbol{V}$	$\boldsymbol{a}^{*}$	$\boldsymbol{b}^{**}$
Concrete cover c $[mm]$	Lognormal	40	$8/\mu$	/	/

Initial diff. coeff. concrete $D_{RCM,0,c}$ $[10^{-12} m^2/s]$	Lognormal	5.0	0.2	/	/
Averaged ageing exponent concrete a_c $[-]$	Beta	0.4	0.4	0	1
Surface chloride content C_s $[m\%cement]$	Lognormal	2	0.75	/	/
Intervention time $T_{int}/T_{LT,0}$ $[-]$	Deterministic	0.5	/	/	/
Thickness mortar layer $[mm]$	Deterministic	10	/	/	/
Initial diff. coeff. mortar $D_{RCM,0,m}$ $[10^{-12} m^2/s]$	Lognormal	1.27	0.2	/	/
Averaged ageing exponent mortar a_m $[-]$	Beta	0.5	0.4	0	1
Convective coefficient coating h_s $[10^{-10} m/s]$	Lognormal	0.89	0.2	/	/
Critical concentration C_{crit} $[m\%cement]$	Beta	0.6	0.25	0.2	2.0

a^*: lower bound of Beta distribution

b^{**}: upper bound of Beta distribution

6.2 *Probabilistic analysis related to different repair strategies*

The probability that the chloride content at the location of the reinforcement exceeds the critical chloride content C_{crit} is determined using 10 000 Latin Hypercube Samples [35], where the number of samples was based on convergence of the results. This so-called probability of depassivation increases with time as chlorides enter the concrete and can be used as an indicator for the service life of reinforced concrete structures. In the design of concrete structures, the service life of the structure is typically considered to correspond to the instant in time where the probability of depassivation reaches ca. 10% [2], [30].

The influence of each repair strategy on the probability of depassivation is investigated in Figure 12, considering the previously described parameters. The time until the depassivation limit is reached without intervention is equal to 18 years when incorporating a 10% probability and variability of parameters. Different repair strategies are applied at 9 years after construction. As was the case for the deterministic situation considered in the previous section, the coating is able to slow down ingress of chlorides (flatten the curve), extending the lifetime of the structure by a couple of years. However, both the mortar overlay and replacement are able to lower the probability of depassivation more pronouncedly. As a result, it would take more than 50 and 100 years, respectively, until the threshold of 10% is reached.

Figure 12: Influence of repair strategies on probability of depassivation

In order to investigate the influence of the repair solution on the extension of a structure's lifetime, the input parameters defining the influence of the repair strategy are varied in Figure 13, by considering different mean values of a_m, D_m and h_s, mortar thickness and intervention time. Each time, only one variable varied at the time, and the values are further indicated in Figure 13.

For the mortar overlay and concrete replacement, a higher ageing coefficient, lower diffusion coefficient and higher mortar thickness all improve the effect of the repair intervention and have a significant effect on the efficiency. These findings are in line with the sensitivity analysis given by Figure 11, although the ageing coefficient of the mortar seems to play a larger role than found in the previous deterministic sensitivity study.

The quality of the coating is found to have a less pronounced influence on the reduction of the probability of depassivation. However, the timing of the intervention is very important which shows that an early application of the repair or treatment has a beneficial effect. This was generally also concluded from the previous sensitivity study.

Additionally, the ageing coefficient of the concrete is altered as well, as it was found in the previous parameter study that this parameter can have a significant influence on the efficiency of the repair solution. Also in the probabilistic framework, the ageing coefficient of the concrete substrate has a significant influence on the effectiveness of the repair strategy. However, the effect seems to be less pronounced than e.g. changing the repair mortar properties.

7 DISCUSSION

Because of the variability in the chloride ingress parameters, it is advised to assess the probability

Figure 13: Parameter study of repair properties on probability of depassivation

of depassivation within a probabilistic framework. On the basis of the investigated cases and parameter ranges it is concluded that the probability of depassivation is generally most effectively reduced by the (partial) replacement of concrete by a repair mortar, followed by a mortar overlay and lastly a coating, considering all three options would be applied at the same time instant in the life of the structure. The effectiveness of the repair methodology is in line with the application effort in practice, which means the largest reduction in probability of depassivation will also result in the most extensive repair intervention.

What is the best choice of repair intervention is also largely dependent on the costs involved, which are generally very case-specific. However, with the models derived in this work, it is possible to optimize the long-term planning of interventions, taking into account the case-specific costs of each intervention, in order to schedule the preventive maintenance operations as efficiently as possible [36], [37].

The general lack of (long-term) data remains an issue in the prediction of chloride ingress after (preventive) repair. For the properties of the repair mortar, some data is available in the technical documents of manufacturers, however there is in general no indication of the uncertainties involved. In relation to the coatings, some experimental data is available which can be used to calibrate the proposed model. Nevertheless, the experimental data includes a large amount of scatter and there is not enough data to predict the long-term behaviour of the treatment. There is a need for more experimental data to determine the accuracy of the proposed models and determine a model error.

8 CONCLUSIONS

In this work, analytical prediction models for the ingress of chlorides after three different concrete repair strategies were considered, targeting preventive interventions which take place before depassivation of the reinforcement steel. The analytical models proposed in this study can be applied both in a deterministic and probabilistic framework, for which the latter is advised to account for variability of parameters and estimate the lifetime with respect to an allowable probability of depassivation. The first two strategies are the application of a mortar overlay or a partial replacement of contaminated concrete with a repair mortar, which are governed by the two-material diffusion problem for which an analytical model has been derived previously. In this work, the efficiency of this model was improved using an equivalent time-independent diffusion coefficient, which allows to execute a probabilistic analysis within a limited timeframe. Moreover, a new analytical model was proposed to predict the chloride ingress after application of a coating, which is able to account of an initial chloride profile at the time of coating application.

Due to the simple analytical nature of the models, a probabilistic framework becomes feasible, allowing the consideration of uncertainties which are inherently present in the parameters considered. Generally, on the basis of the considered cases and parameter ranges, the most effective strategy is found to be the replacement of partially contaminated concrete, followed by a mortar overlay and lastly the application of a coating. However, it should be emphasized that this conclusion holds for the specific material properties (e.g. diffusion coefficient and ageing coefficient) considered in this work, and e.g. coatings have been successfully applied to supress chloride ingress in the past [38]. This is also in line with the impact of the application, and therefore the appropriate choice will depend on a cost-benefit analysis. With the models proposed in this work, a probabilistic analysis becomes feasible and comparing different strategies can be done efficiently, which enables to develop the most cost-optimal solutions.

Future work involves the development of prediction models for other (preventive) repair methodologies, such as the application of a hydrophobic treatment (e.g. silane) [19], [39]–[41] or the extraction of chlorides [42], [43]. Moreover, the effect of essential maintenance strategies (e.g. impressed current cathodic protection) on the corrosion process should be studied in more detail. This would finally enable to evaluate the range of all possible repair possibilities and optimize the repair strategy of a concrete structure over its remaining lifetime.

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