# DMD-T: Thermographic Inspection of Composites using Dynamic Mode Decomposition


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***Abstract*—In the realm of Non-Destructive Testing (NDT) of fibre reinforced polymers, InfraRed Thermography (IRT) serves as a valuable tool for diagnosis. Various processing techniques, such as principal component analysis, Fourier transformation, and thermographic signal reconstruction, are commonly utilized to improve the identification of defects. However, the practical application of infrared thermography is limited by the need for expert operator experience, which hampers its broader adoption in industrial settings. This paper presents a preliminary examination of the application of data-driven based Dynamic Mode Decomposition (DMD) to study the thermal dynamics of composites and evaluate its potential for defect detection. The new method is known as the DMD for Thermography (DMD-T). The performance has been validated via an experimental dataset of a Carbon Fibre Reinforced Polymer (CFRP) plate with an impact damage.**



***Keywords—Non-Destructive Testing (NDT); Dynamic Mode Decomposition (DMD); Thermographic analysis; Data-driven; Impact damage***


## I. INTRODUCTION[1]

Carbon Fibre Reinforced Polymer (CFRP) and Glass Fibre Reinforced Polymer (GFRP) composites play a significant role as structural materials in various industries, including aerospace and automotive [1-2]. However, these composites are prone to defects like delamination and porosity, resulting from manufacturing processes and excessive loads. Therefore, Non-Destructive Testing (NDT) has become crucial in ensuring the integrity and reliability of these materials.

Active InfraRed Thermography (IRT) is an effective NDT method that utilizes an InfraRed (IR) camera to detect and quantify defects by measuring thermal responses [3]. However, the recorded data from the IR camera is often affected by various sources of noise, including external reflections, variations in specimen optical properties, non-uniform heating, and instrumental noise. These noise sources can compromise the accuracy of defect detection. To overcome these challenges and improve defect detectability in composite materials, several post-processing techniques have been developed in the last few decades [4-7].

One such technique is the Thermographic Signal Reconstruction (TSR) method [4], which utilizes low-order polynomial functions to fit surface temperature evolution. These polynomial coefficients are represented by multiple images for a comprehensive defect evaluation. Another approach is the Principal Component Thermography (PCT) [5], which projects 3D thermal response data into an orthogonal space using Principal Component Analysis. The resulting Principal Component images capture the variability in the thermal response, but identifying defect-specific components can be challenging. Additionally, the Phase Pulse Thermography (PPT) method [6] extracts harmonic components through Fourier decomposition, and optimal defect detectability depends on the evaluation frequencies tailored to the defect type and depth. Nevertheless, many existing methods primarily focus on either spatial or temporal feature representations, which may not fully capture the intrinsic characteristics of thermal datasets.

This paper investigates the use of Dynamic Mode Decomposition (DMD) [8-9], specifically DMD for Thermography (DMD-T), to analyse thermal datasets and capture their spatial patterns and temporal dynamics. DMD, originating in fluid dynamics, is a powerful method for analysing time-resolved data by extending Proper Orthogonal Decomposition (POD) [10]. By approximating the Koopman operator, which characterizes system evolution, DMD constructs a finite-dimensional linear operator. It decomposes data into dynamic modes, revealing essential features like coherent structures and frequencies. Unlike empirical-based Model Of Reduction (MOR) methods such as PCT and Independent Component Thermography (ICT) [11], DMD-T relies on snapshot measurements of thermal dynamics, obtained from numerical simulations or experiments. Applying DMD-T to temporal-spatial thermal datasets identifies dominant thermal modes, including static modes and eigenvalue modes.

## II. DYNAMIC MODE DECOMPOSITION FOR THERMOGRAPHY (DMD-T)

### A. Dynamic Mode Decomposition (DMD)

The Fourier's law of heat conduction describes the heat conduction in a discretised structure as [12]:

$$\mathbf{C}\dot{\mathbf{X}}(t)+\mathbf{K}\mathbf{X}(t)=\mathbf{q}(t) \tag{1}$$

where $\mathbf{X}(t)\in \mathrm{R}^{N}$ is a vector of the temperature values of each element at time $t$, $\mathbf{C}\in \mathrm{R}^{N\times N}$ is the capacity matrix, $N$ are the number of discrete elements in the structure, $\mathbf{K}\in \mathrm{R}^{N\times N}$ is the conductivity matrix and $\mathbf{q}(t)\in \mathrm{R}^{N}$ represents the heat loads applied to each element.

Considering that velocity is the first derivative of displacement, the following equation (2) can be obtained by applying the forward difference technique to equation (1).

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$$\dot{\mathbf{X}}(t) = \frac{\mathbf{X}(t+\Delta t) - \mathbf{X}(t)}{\Delta t} \quad (2)$$

where $\Delta t$ is the discrete sampling interval. Then equation (1) becomes

$$\mathbf{X}(t+\Delta t) = (\mathbf{I} - \mathbf{C}^{-1}\mathbf{K}\Delta t)\mathbf{X}(t) + \mathbf{C}^{-1}\mathbf{K}\Delta t\mathbf{q}(t) \quad (3)$$

Equation (3) can be further simplified considering pulsed thermography,

$$\mathbf{X}(t+\Delta t) = (\mathbf{I} - \mathbf{C}^{-1}\mathbf{K}\Delta t)\mathbf{X}(t) \quad (4)$$

The mathematical definition of the DMD is expressed as follows: The time intervals between consecutive snapshots are denoted as $t_{k+1} = t_k + \Delta t$. We consider a matrix $\mathbf{X}$ that that contains snapshots of the generalized coordinates $\mathbf{x}_j$ representing the $\mathbf{x}(t)$ at the time $t_k$ for $k = 1, \ldots, K$ as its column vectors. Consequently, we can represent the first snapshot matrix as $\mathbf{X}_1 = [\mathbf{x}_1, \ldots, \mathbf{x}_K]$. Additionally, we define a time-shifted snapshot matrix $\mathbf{X}_2 = [\mathbf{x}_2, \ldots, \mathbf{x}_{K+1}]$. Under the assumption of a linear transformation between $\mathbf{x}_k$ and $\mathbf{x}_{k+1}$, we represent this transformation by a matrix $\mathbf{A}$. Thus, we have

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k \quad (5)$$

which leads to

$$\mathbf{x}_2 = \mathbf{A}\mathbf{x}_1 \quad (6)$$

The solution to equation (5) is

$$\mathbf{x}_k = \sum_{j=1}^{N} \phi_j \lambda_j^{\,k-1} b_j = \mathbf{\Phi}\mathbf{\Lambda}^{k-1}\mathbf{b} \quad (7)$$

in the discrete time domain, or

$$\mathbf{x}(t) = \sum_{j=1}^{N} \phi_j \exp(\omega_j t) b_j = \mathbf{\Phi}\exp(\mathbf{\Omega}t)\mathbf{b} \quad (8)$$

in the continuous time domain, where $\mathbf{\Omega} = \ln(\mathbf{\Lambda})/\Delta t$ are continuous eigenvalues.

The matrix $\mathbf{\Phi} \in \mathrm{R}^{N\times N}$ comprises the eigen column vectors or modes of the transition matrix. The vector $\mathbf{b} \in \mathrm{R}^{N}$ scales the modes and represents the initial conditions of the system, where $\mathbf{b} = \mathbf{\Phi}^{-1}\mathbf{x}_1$. The matrix $\mathbf{\Lambda}^k$ is a diagonal matrix consisting of eigenvalues of $\mathbf{A}$, which indicates how the modes evolve at discrete time $k$. By comparing equation (4) and (5), discretized thermal snapshots can be formulated into the DMD form as equation (5) to investigate the linear transformation matrix $(\mathbf{I} - \mathbf{C}^{-1}\mathbf{K}\Delta t)$ and $\mathbf{A} = \mathbf{X}_2\mathbf{X}_1^{\dagger}$, $\mathbf{X}_1^{\dagger}$ denotes the Moore-Penrose pseudo inverse matrix of $\mathbf{X}_1$. The DMD modes and eigenvalues are defined as the eigenvectors and eigenvalues of $\mathbf{A}$ by apply Singular Value Decomposition (SVD). The well-known SVD of a matrix $\mathbf{X}_2$, truncated at $r$ singular values, is given by the following:

$$\mathbf{X}_2 \approx \mathbf{U}_r \mathbf{\Sigma}_r \mathbf{V}_r^{*} \quad (9)$$

where $\mathbf{U}_r \in \mathrm{R}^{N\times r}$, $\mathbf{\Sigma}_r \in \mathrm{R}^{r\times r}$, $\mathbf{V}_r \in \mathrm{R}^{r\times M-1}$ and * denotes the complex conjugate transpose. The SVD provides a principled method for reducing the dimension of the data matrix. The following is the reduced order operator:

$$\tilde{\mathbf{A}} \approx \mathbf{U}_r^{*}\mathbf{X}_2\mathbf{V}_r\mathbf{\Sigma}_r^{-1} \quad (10)$$

The dynamic properties can be determined using the widely-used eigen decomposition: $\tilde{\mathbf{A}}\mathbf{W} \approx \mathbf{W}\mathbf{\Lambda}$, where $\mathbf{W}$ contains the eigenvectors and $\mathbf{\Lambda}$ represents the eigenvalues.

To conclude, the continuous-time model of the system, expressed in equation (1), can be obtained by reconstructing the eigenvalues and eigenvectors of $\mathbf{A}$ (referred to as the DMD modes) in (8) using

$$\mathbf{\Phi} = \mathbf{X}_2\mathbf{V}_r\mathbf{\Sigma}_r^{-1}\mathbf{W} \quad (11)$$

$$\mathbf{\Omega} = \ln(\mathbf{\Lambda})/\Delta t \quad (12)$$

$$\mathbf{b} = \mathbf{\Phi}^{-1}\mathbf{x}_1 \quad (13)$$

## *B. Material and methods*

In Fig.1 (a), a visual representation of the thermographic setup is presented. The input of optical energy is provided by a Hensel linear flash lamp, which delivers 6 kJ of energy over a 5 ms flash duration. To capture the surface temperature of the sample, an infrared camera, specifically the FLIR A6750sc, is utilized. This camera is equipped with a focal plane array consisting of cryo-cooled InSb detectors with dimensions of $640\times512$. It possesses a Noise-Equivalent Differential Temperature (NEDT) of less than or equal to ≤ 20 mK and a bit depth of 14 bits. The camera operates within the mid-infrared wavelength range of 3-5 μm. Additionally, it comes with a 25 mm focal length lens, and the inspected samples are positioned at a distance that adequately fills the camera's field of view.

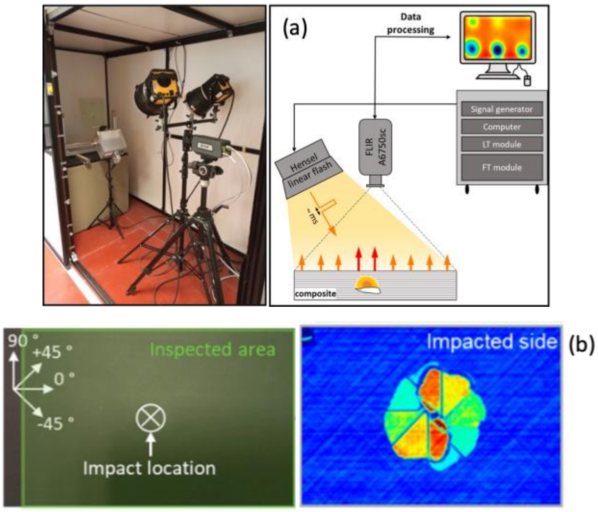


Fig.1 (a) Schematic of the experimental setup for flash thermography; (b) Photographs of CFRPBVID coupon, and the corresponding ultrasonic C-scan results [13]

In the experimental dataset, the sample under investigation is a carbon fiber reinforced polymer (CFRP) plate manufactured as an autoclave with a thickness of 5.5 mm. It possesses a quasi-isotropic layup of [(+45/0/-45/90)3]s. The sample is subjected to a low-energy impact event. The impactor used weighs 7.72 kg and is equipped with an Endevco Isotron 23-1 load cell, which features a 16 mm diameter hemispherical hardened solid steel impact tip. The CFRP sample, referred to as CFRPBVID, experiences an impact energy of 6 J when the impactor falls from a height of 0.1 m. This impact leads to the formation of barely visible impact damage (BVID) in the CFRP laminate [13]. The $CFRP_{BVID}$ sample is positioned approximately 500 mm away from the flash optical excitation. Cooling measurements are recorded for 120 s at a framerate of 30 Hz, with a distance of around 1050 mm between the IR camera and the $CFRP_{BVID}$ sample. To determine the size of the impact-induced damage and obtain an accurate image of the damage, a 5 MHz ultrasonic C-scan in transmission is conducted. Figure 1(b) presents a photograph of the sample, displaying the amplitude C-scan data. The impacted $CFRP_{BVID}$ sample, captured from the impacted side (front view), is recorded for 50 s with a sampling rate of 50 Hz.

## III. Results and Discussion

In flash thermography, the excitation conditions commonly result in a pronounced non-uniform background. Unfortunately, this non-uniformity often hampers the effectiveness of post-processing techniques that rely on temperature magnitude for defect detection. The raw thermal response in the time domain clearly reveals this excitation non-uniformity, as depicted in Fig.2.

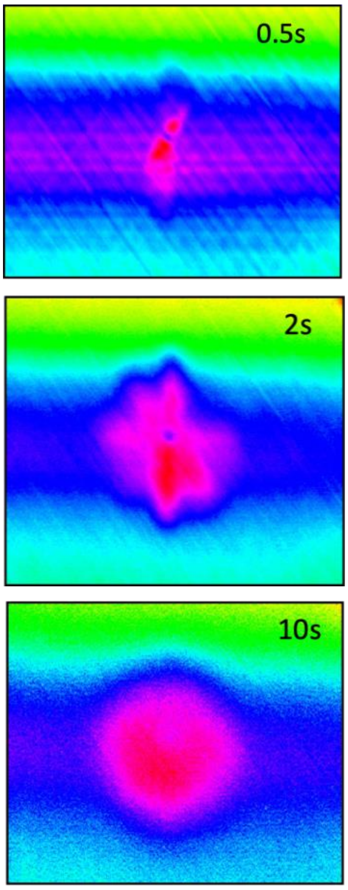


Fig.2 Thermogram of impacted side of the CFRPBVID coupon at 0.5s, 2s and 10s, respectively

The following paragraph will delve into the integration of DMD (Dynamic Mode Decomposition) into this thermal analysis, with the goal of exploring the thermal dynamics and potentially detecting defects by examining the dominant thermal modes. By employing DMD on the thermal raw data snapshots, we can identify dynamic modes with single frequencies that exhibit damped behaviour characterized by negative decay rates. This finding confirms the transient nature of pulsed thermography. Among these modes, the static mode represents the mean (background) of the snapshots, as depicted in Figure 3, with an eigenvalue of 1.0+0.0j (a phase of 0). Additionally, the amplitude of the static mode provides the information regarding the impact-induced defects as well as the non-uniformity of the background.

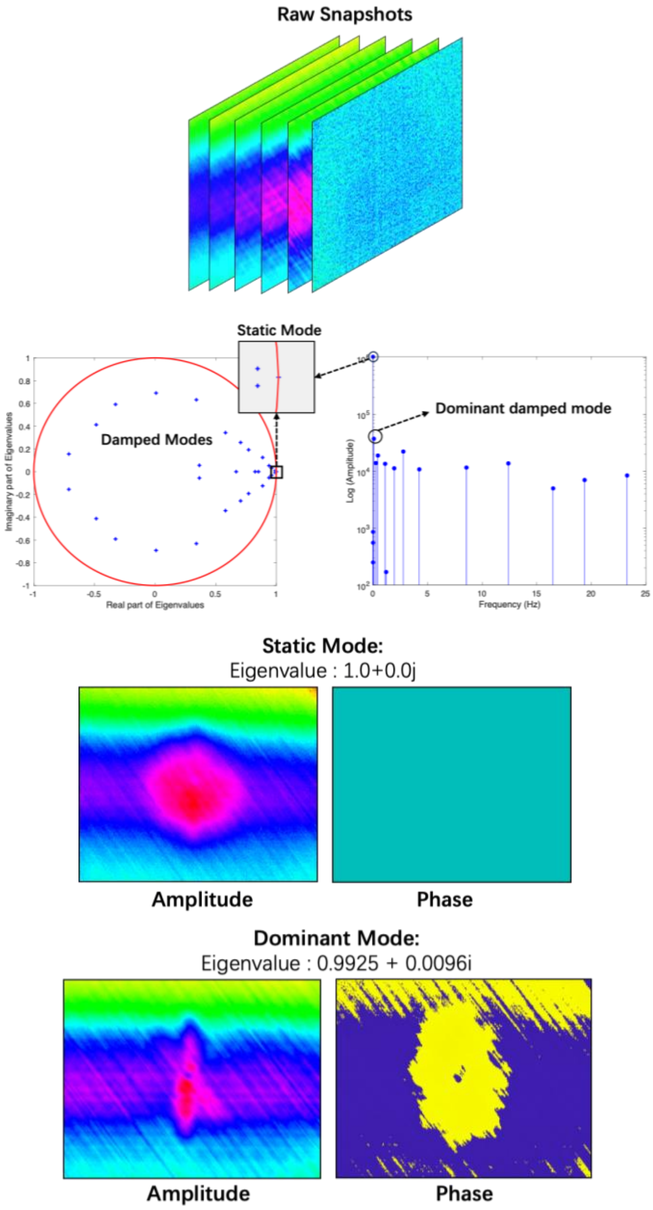


Fig.3. Thermal modes extracted by DMD

In addition to the static mode, a dominant damped mode with an eigenvalue of 0.9925+0.0096i has been identified, in terms of the most significant contribution from thermal mode amplitude. By visually examining the amplitude and phase of the corresponding eigenvectors, defects can be effectively identified, particularly through the phase information contained within the eigenvectors. This verifies the underlying principle of PPT. Nonetheless, unlike PCT or PPT, the thermal modes detected by DMD-T have the capability to capture the prominent tempo-spatial dynamics of the sample, thereby aiding in the improved identification of concealed defects.

However, the objective of this study is to utilize DMD for extracting thermal modes and detecting defects in pulsed thermography with a transient nature as a preliminary study.

## IV. CONCLUSION

In this paper, a data-driven method, termed DMD-T, was proposed for conducting preliminary case studies on a $CFRP_{BVID}$ coupon. By analysing the extracted static and dynamic thermal modes, it was observed that the dominant dynamic mode at a low frequency effectively indicated the location of impact damage, primarily based on the phase information of its eigenvector. While DMD-T demonstrated promising detectability of impact damage, it requires thorough theoretical and experimental validation, including the exploration of different excitation-induced thermography methods such as lock-in and step-wise techniques, in future work.

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