Logic Reasoning Under Data Veracity Concerns

Guy De Tré [0000−0002−1283−1915a], Milan Peelman [0000−0003−3104−093Xa], Jozo Dujmović [0000−0002−1715−2700b]

[a] *Dept. of Telecommunications and Information Processing, Ghent University, St.-Pietersnieuwstraat 41, Ghent, B9000, Belgium*
[b] *Dept. of Computer Science, San Francisco State University, San Francisco, CA, USA*

Abstract

Logic reasoning involving big data often requires the proper handling of data veracity. Indeed with data that cannot be trusted to the same extent, users should at least be aware on the trust they can have in the obtained reasoning. In this paper, we propose a novel logic framework that is based on so-called L-grades. L-grades are a special case of Zadeh's Z-numbers, consisting of a pair $(s, c) \in [0, 1]^2$ in which s is a suitability grade (or satisfaction grade) and c is a confidence grade denoting how confident we can be on s. Both grades are further processed using fuzzy logic. Novel logic operators and so-called sibling aggregators for L-grades are proposed and studied in the paper. With this framework we aim to contribute to explainable computational intelligence. The practical use of L-grades is illustrated with criterion evaluation in a decision support application with improved explainablity facilities.

Keywords: computational intelligence, confidence, veracity, fuzzy logic, aggregation, explainability

1. Introduction

In the past decade, data management underwent a considerable change as data get more and more characterized by huge volumes, originating from different sources, and/or being diverse in variety of data formats [16]. Novel data management techniques based on sharding, NoSQL and NewSQL data base systems help to overcome data volume and data variety problems. However, another problem still being subject to research and often caused by using more and larger data sources with a higher variety of data formats is called the data veracity problem [20, 22, 4]. Data veracity refers to the extent that data adequately reflect reality and hence can be trusted. Trust in data sources is a pervasive phenomenon in data management and processing [6]. As veracity propagates from the data sources, through the data processing and data analysis steps, to the computational outputs it is of utmost importance to properly model and handle it. This is a challenge that is hard to meet.

With the research presented in this paper we aim to contribute to veracity handling in criterion evaluation and aggregation, which are important components in, among others, criterion-based flexible query answering and decision support systems.

As first scientific contribution we introduce and study a novel logic reasoning framework, which is based on so-called L-grades. An L-grade is defined by an ordered pair $l = (s, c)$ of two grades and can be seen as a simple form of a Z-number. Z-numbers have been introduced by L.A. Zadeh [29] to better cope with the issue of reliability of (fuzzy) information. The first component s acts as a suitability (or satisfaction) grade and expresses to what extent a data object under consideration satisfies a given criterion or collection of criteria. Like in many conventional flexible criterion evaluation approaches acting on crisply described data, s is considered to be gradual and a number in the unit interval $I = [0, 1]$ [15]. The second component c reflects the confidence, trust, veracity, or strength of belief in s, i.e., c expresses to what extent the computation of s can be trusted. This can encompass the trust in the data object, but also the trust in the evaluation process and the trust in the eventual aggregation. The confidence grade c is also considered to be a number in the unit interval $I = [0, 1]$. It is important to denote that both components of an L-grade are interpreted as truth values and hence should be processed using a logic model.

As second scientific contribution we discuss the comparison of L-grades. Comparing L-grades is important for ranking the results of a (flexible) query or for ordering the different options in decision making. Moreover, it is an indispensable component of many practical applications.

The third scientific contribution is the proposal of novel logic operators that act on L-grades and the study of aggregators for L-grades. These logic operators and aggregators have been specifically designed to support reasoning in criterion handling in a flexible querying or decision support context. Because these operators and aggregators act on both components of an L-grade and the computation of a confidence grade c strongly depends on the computation of its associated satisfaction grade s, we name these operators sibling operators.

L-grades offer a facility to explicitly cope with the veracity of a computed suitability (or satisfaction) grade s. Indeed, their confidence grade c allows to explicitly model the trust we have in s and acts as an indication of the veracity of the data being considered for evaluation, the criterion evaluation and the eventual aggregation in order to obtain an overall evaluation result in the case of multiple criteria. The confidence grade can be communicated as extra information to the users of a query answering or decision support system. This is of pivotal importance in view of (better) interpretable and explainable criterion evaluation and handling, which in its turn is relevant and important for many computational intelligence and explainable artificial intelligence applications [3, 13, 8] and is an important argument for justifying the relevance of the presented research.

This paper is an extended and revised version of our contribution to the IPMU 2022 conference [9]. The remainder is organized as follows. In Section 2 we introduce the notion of an L-grade and discuss its semantics and the ranking of L-grades. Basic definitions of logic operators for negation, conjunction and disjunction of L-grades are presented in Section 3. In Section 4 sibling aggregators for L-grades are proposed and discussed. The use and added value of L-grades in criterion handling is illustrated in Section 5. Next, related work is discussed in Section 6. Finally, in Section 7 we formulate the conclusions of our work and propose some topics for follow-up research.

2. L-grades

Driven by his ambition to model different manifestations of imperfect data in a uniform way, Zadeh [28, 29] in 2011 introduced the notion of a Z-number. A Z-number Z is defined by an ordered pair of fuzzy numbers, i.e., $Z = (A, B)$.

The first fuzzy number A acts as a representation of a fuzzy restriction $R(X)$ on the possible (numerical) values that a linguistic variable X can take, written as $X\ IS\ A$ [27]. Considering that U denotes the universe of discourse consisting of all candidate values that X can take, A is defined to be a fuzzy subset of U playing the role of a possibility distribution, i.e. $R(X) : X\ IS\ A \to Poss(X = u) = \mu_A(u)$ where μ_A is the membership function of A and u is a generic value of U. As such, for each $u \in U$, $\mu_A(u)$ reflects the possibility that u is the actual value of X.

The second fuzzy number B is a measure of reliability (trust, strength of belief) one has in A. More specific B is a fuzzy restriction on the degree of certainty that $X\ IS\ A$. As such, B is usually assumed to be a fuzzy subset of the unit interval $[0, 1]$ modelling a fuzzy restriction on the probability of $X\ IS\ A$, i.e., $Prob(X\ IS\ A)\ IS\ B$, but other approaches and definitions are possible.

Typically, both A and B are described in a natural language [29]. For example, assume that X represents the travel time by car from Ghent to Brussels at 12 am, then $Z = (\text{about 40 minutes}, \text{quite certain})$ can reflect this travel time.

In general Z-numbers lead to complex, calculation-intensive computations (see, e.g., [1, 11, 21]). For that reason, Dubois and Prade studied some simplified cases of Z-numbers (A, B) where one of the fuzzy numbers is crisp and the other is fuzzy [11]. In [21] mixed-discrete Z-numbers are introduced. The simplest forms of Z-numbers, where both components are presented by crisp singletons have been studied in, among others, [2, 19].

Driven by observed needs in criteria handling for flexible database querying and multi-criteria decision support [8], we propose in this work a novel, specific logic interpretation of the simplest form of Z-numbers, which we call L-grades[1].

2.1. Definition and semantics

An *L-grade* l is a Z-number where both components are (crisp) singletons containing a value of the unit interval I. These values are interpreted as grades that will be further processed using fuzzy logic, i.e.

$$l = (\{(s, 1)\}, \{(c, 1)\}), \text{where } s, c \in I. \tag{1}$$

In the remainder, we will use the short notation $l = (s, c)$ for L-grades l.

When criterion evaluation is considered in the context of decision making, the first grade s is interpreted as a *suitability grade*. In the context of flexible querying s is usually called a *satisfaction grade*. Herewith, $s = 1$ denotes full suitability (or satisfaction), $s = 0$ means no suitability (or satisfaction), whereas all other values denote partial suitability (or satisfaction). In the

[1]In our first work, [9], we used the name Z-grade, but since it is of utmost importance to emphasize the logic interpretation, we choose to use the name L-grade from now on (where L stands for logic).

remainder of this work we opt to use the term suitability grade as it denotes how suitable a given object is within a query answer set or as an option in decision making context.

The second grade c reflects the confidence in s, i.e. the veracity of s, and is interpreted as a *confidence grade*. Like with satisfaction grades, $c = 1$ denotes full confidence, $c = 0$ denotes no confidence, whereas all intermediate values denote partial confidence.

As an illustration, consider the L-grades $l_1 = (1, 1)$, $l_2 = (1, 0.7)$ and $l_3 = (1, 0)$, which are obtained by evaluating a criterion C on data of resp. three objects o_1, o_2 and o_3. All three objects fully satisfy the criterion as their corresponding suitability grade equals 1. However, the corresponding confidence grades reveal that there is full confidence in the veracity of the (evaluation of the) data of object o_1, less confidence in object o_2, and no confidence in o_3. The use and applicability of L-grades will be further discussed in Section 5.

Both s and c are graded values, modelling degrees of truth of fuzzy propositions, not degrees of uncertainty [10]. These degrees of truth will further be processed using a logic framework that is truth functional and is described in Sections 3 and 4.

The set of all L-grades will be denoted by $\mathbb{L}$, i.e.

$$\mathbb{L} = I^2. \tag{2}$$

2.2. Comparing L-grades

When using L-grades for criterion handling in querying or decision making processes, (objects with associated) L-grades will have to be compared in order to find those objects that best suit the user's preferences.

Suppose that the L-grade for an object o_i is $l_i = (s_i, c_i)$, and the L-grade for an object o_j is $l_j = (s_j, c_j)$. The objective is the comparison of l_i and l_j in order to find which of o_i and o_j best suits the user's preferences. We may prefer o_i (denoted $o_i \succ o_j$) or prefer o_j (denoted $o_i \prec o_j$). In some special cases we may consider that o_i and o_j are equivalent (denoted $o_i \approx o_j$).

All stakeholders/decision-makers want simultaneously high suitability and high confidence. The preference $o_i \succ o_j$ can be easily assigned in the following cases of full dominance:

$$s_i > s_j, c_i \geq c_j \Rightarrow o_i \succ o_j,$$
$$s_i = s_j, c_i > c_j \Rightarrow o_i \succ o_j.$$

However, the case $s_i > s_j$, $c_i < c_j$ remains problematic. If the confidence c_i is rather low, then the condition $s_i > s_j$ might be insufficient to claim $o_i \succ o_j$. In other words, decision-makers need enough confidence to accept the conclusion that $s_i > s_j$ is sufficient to claim $o_i \succ o_j$. One simple way would be to introduce a minimum threshold value c_{min} so that $s_i > s_j, 0 < c_{min} \leq c_i < c_j \Rightarrow o_i \succ o_j$. Unfortunately, selecting c_{min} is not easy because the selected value depends on the difference $s_i - s_j$. Indeed, if this difference is very small ($s_i - s_j << 1$), then the threshold value c_{min} must be greater than in the case of large differences. In addition, in comparison problems, we are not only interested in a rank order $o_i \succ o_j$, but we also need to know how strong that preference is, considering both s and c values. This is subject to further research.

3. Logic Operators

In this section some basic logic operators for negation, conjunction and disjunction of L-grades are introduced and discussed. The underlying assumption is that suitability grades should be handled as in conventional logic operators for fuzzy criteria handling, while the confidence grades should reflect the impact of the confidence in the suitability grades of the arguments after applying the logic operator.

3.1. Negation operators

Our proposed basic operator for the negation of an L-grade, is based on a strong fuzzy negation, i.e. a function $N : [0,1] \Rightarrow [0,1]$ that satisfies the following properties [24]:

i. N is decreasing,

ii. N is continuous,

iii. $N(N(x)) = x$ holds for every $x \in [0,1]$ (involutiveness).

The standard negation $N : [0,1] \Rightarrow [0,1] : x \mapsto 1 - x$ is an example of a strong fuzzy negation.

With a strong fuzzy negation N, the *negation operator* $\neg$ for L-grades is defined by

$$\forall\, (s,c) \in \mathbb{L} : \neg(s,c) = (N(s), c) \tag{3}$$

This definition reflects that the suitability grade is negated, but the confidence grade remains unchanged. Applying a negation operator does not impact the veracity of the suitability grade. If criterion evaluation resulting in s is based on less trusted data, trust in this data will not increase or decrease by negating s.

The involutiveness property holds for $\neg$, i.e. $\neg\neg(s,c) = \neg(N(s),c) = (N(N(s)),c) = (s,c)$.

3.2. Conjunction and disjunction operators

The proposed basic operators for conjunction and disjunction of two L-grades are resp. based on a t-norm $\top$ and on its dual t-conorm $\perp$ for fuzzy sets [17]. Herewith, the suitability grades of both operands are aggregated using $\top$ in case of conjunction and using $\perp$ in case of disjunction. This is conform to aggregation in conventional fuzzy criteria handling.

The aggregation of the confidence grades of both operands should reflect the impact of conjunction, resp. disjunction on the veracity of the aggregated suitability grades. Only confidence grades of suitability grades that contribute to the aggregated suitability should contribute to the aggregated confidence grade.

The identity law and monotonicity of t-norms and t-conorms imply that $\forall\ x \in [0,1] : \top(0,x) = 0$ and $\perp(1,x) = 1$. So, if exactly one suitability grade equals 0 in case of conjunction or 1 in case of disjunction, then the other suitability grade is absorbed and does not contribute to the aggregated suitability. Hence, its associated confidence grade should neither contribute to the aggregated confidence grade. In all other cases, both associated confidence grades should be taken into account.

Hence, three cases could be considered with respect to the suitability grades of both operands:

1. *Both suitability grades equal the absorbing element of the operator.* In this case, the highest of both associated confidence grades can be assigned to the aggregated suitability grade, because both suitability grades support the suitability aggregation result.
2. *Only one suitability grade equals the absorbing element of the operator.* In this case, the confidence grade associated with this suitability grade reflects the confidence in the suitability aggregation result.
3. *None of the suitability grades equals the absorbing element of the operator.* Here, both suitability grades contribute to the suitability aggregation result. Without further knowledge on which grade contributes

to which extent, the safest strategy is to assign the lowest of both confidence grades to the resulting suitability grade. This is a reserved, rather pessimistic strategy.

Based on the above considerations we propose the following basic operators for conjunction and disjunction.

A basic *conjunction operator* for L-grades based on a t-norm $\top$ is defined by

$$\forall\, (s_1, c_1), (s_2, c_2) \in \mathbb{L} : \\ \top((s_1, c_1), (s_2, c_2)) = (\top(s_1, s_2), a^{conj}((s_1, c_1), (s_2, c_2))) \quad (4)$$

where a^{conj} is a confidence aggregator for conjunction that is defined by

$$a^{conj}((s_1, c_1), (s_2, c_2)) = \begin{cases} \max(c_1, c_2) & \text{if } s_1 = 0 \text{ and } s_2 = 0, \\ c_1 & \text{if } s_1 = 0 \text{ and } s_2 \neq 0, \\ c_2 & \text{if } s_1 \neq 0 \text{ and } s_2 = 0, \\ \min(c_1, c_2) & \text{otherwise.} \end{cases} \quad (5)$$

The dual basic *disjunction operator* for L-grades based on the dual t-conorm $\perp$ is defined by

$$\forall\, (s_1, c_1), (s_2, c_2) \in \mathbb{L} : \\ \perp((s_1, c_1), (s_2, c_2)) = (\perp(s_1, s_2), a^{disj}((s_1, c_1), (s_2, c_2))) \quad (6)$$

where a^{disj} is a confidence aggregator for disjunction that is defined by

$$a^{disj}((s_1, c_1), (s_2, c_2)) = \begin{cases} \max(c_1, c_2) & \text{if } s_1 = 1 \text{ and } s_2 = 1, \\ c_1 & \text{if } s_1 = 1 \text{ and } s_2 \neq 1, \\ c_2 & \text{if } s_1 \neq 1 \text{ and } s_2 = 1, \\ \min(c_1, c_2) & \text{otherwise.} \end{cases} \quad (7)$$

3.3. De Morgan's laws

The fuzzy De Morgan's laws state that for any t-norm $\top$ and strong negation N a corresponding t-conorm $\perp$ can be defined by

$$\forall\, x_1, x_2 \in [0, 1] : \perp(x_1, x_2) = N(\top(N(x_1), N(x_2))). \quad (8)$$

Likewise,

$$\forall\, x_1, x_2 \in [0,1] : \top(x_1, x_2) = \ N(\bot(N(x_1), N(x_2))). \tag{9}$$

If the standard negation $N(s) = 1 - s$ is used, $\bot$ is called a dual t-conorm to $\top$ and reversely, $\top$ is called a dual t-norm to $\bot$.

Theorem 1. *If $\top$, $\bot$ and $\neg$ are resp. defined as in Eq. (4), Eq. (6) and Eq. (3) and if $\top$ and $\bot$ are dual operators, then the triplet $(\top, \bot, \neg)$ satisfies*

$$\forall\, l_1, l_2 \in \mathbb{L} : \top(l_1, l_2) = \neg(\bot(\neg l_1, \neg l_2)).$$

Proof 1. *Let $l_1 = (s_1, c_1)$ and $l_2 = (s_2, c_2)$, then*

$$\begin{aligned}\neg(\bot(\neg(s_1, c_1), \neg(s_2, c_2))) &= \neg(\bot((1 - s_1, c_1), (1 - s_2, c_2))) \\ &= \neg(\bot(1 - s_1, 1 - s_2), a^{disj}((1 - s_1, c_1), (1 - s_2, c_2)))\end{aligned}$$

where

$$\begin{aligned}&a^{disj}((1 - s_1, c_1), (1 - s_2, c_2)) \\ &\qquad = \begin{cases} \max(c_1, c_2) & \textit{if } 1 - s_1 = 1 \textit{ and } 1 - s_2 = 1, \\ c_1 & \textit{if } 1 - s_1 = 1 \textit{ and } 1 - s_2 \neq 1, \\ c_2 & \textit{if } 1 - s_1 \neq 1 \textit{ and } 1 - s_2 = 1, \\ \min(c_1, c_2) & \textit{otherwise.} \end{cases} \\ &\qquad = \begin{cases} \max(c_1, c_2) & \textit{if } s_1 = 0 \textit{ and } s_2 = 0, \\ c_1 & \textit{if } s_1 = 0 \textit{ and } s_2 \neq 0, \\ c_2 & \textit{if } s_1 \neq 0 \textit{ and } s_2 = 0, \\ \min(c_1, c_2) & \textit{otherwise.} \end{cases} \\ &\qquad = a^{conj}((s_1, c_1), (s_2, c_2))\end{aligned}$$

So,

$$\begin{aligned}\neg(\bot(\neg(s_1, c_1), \neg(s_2, c_2))) &= \neg(\bot(1 - s_1, 1 - s_2), a^{conj}((s_1, c_1), (s_2, c_2))) \\ &= (\top(s_1, s_2), a^{conj}((s_1, c_1), (s_2, c_2))) \\ &= \top((s_1, c_1), (s_2, c_2))\end{aligned}$$

Q.E.D.

4. Sibling Aggregators

The basic conjunction and disjunction operators given in Eq. (4) and Eq. (6) are examples of binary aggregators that map $\mathbb{L}^2$ onto $\mathbb{L}$. In this section we study aggregators h of arity n, $n \geq 2$, that map $\mathbb{L}^n$ onto $\mathbb{L}$. More specifically, we propose a weighted mean (WM) and an ordered weighted average (OWA) aggregator for L-grades.

4.1. Weighted mean

Computing the weighted mean of $n \geq 2$ L-grades $l_i = (s_i, c_i)$, $i = 1, \ldots, n$, requires n weights that reflect the relative importance of each L-grade in the aggregation. For that purpose a weight vector $\vec{w} = (w_1, \ldots, w_n) \in [0, 1]^n$, such that $\sum_{i=1}^{n} w_i = 1$, is considered. We propose to aggregate the suitability grades s_i, $i = 1, \ldots, n$, of all operands using a conventional fuzzy weighted mean operator. By doing so, the aggregation of suitability grades is done conform to conventional fuzzy criteria handling.

For the aggregation of the confidence grades c_i, $i = 1, \ldots, n$ we propose to use the same conventional fuzzy weighted mean operator. This is motivated by the requirement that the resulting confidence grade c should reflect the impact of the fuzzy weighted mean operator on the veracity of the aggregated suitability grades s_i, $i = 1, \ldots, n$. Herewith, we consider that each confidence grade c_i should contribute to the same extent to the computation of the aggregated confidence grade c as its corresponding 'sibling' suitability grade s_i contributes to the computation of the aggregated suitability grade s. The impact of each suitability grade s_i in the computation of s is determined by its associated weight w_i, hence the proposal to use w_i also as weight for the 'sibling' confidence grade c_i in the computation of c. This leads to the following definition.

The n-ary *weighted mean* for L-grades is defined by

$$h_{\vec{w}} : \mathbb{L}^n \to \mathbb{L} : ((s_1, c_1), \ldots, (s_n, c_n)) \mapsto (s, c) \tag{10}$$

where $s = \sum_{i=1}^{n} w_i \cdot s_i$ and $c = \sum_{i=1}^{n} w_i \cdot c_i$.

This aggregator is called a sibling aggregator because each confidence grade has a similar impact on the aggregation of the confidence grades, as its sibling suitability grade has on the aggregation of the suitability grades.

4.2. Ordered weighted average

Ordered weighted average (OWA) [25] also works with a weight vector, but a different, dynamic weight assignment is used. The inputs that have to be aggregated are first ranked from largest to smallest, after which the first weight is assigned to the largest input, the second weight is assigned to second largest input, and so on.

Assume again that, to aggregate $n \geq 2$ L-grades $l_i = (s_i, c_i)$, $i = 1, \ldots, n$, a given weight vector $\vec{w} = (w_1, \ldots, w_n) \in [0,1]^n$ with $\sum_{i=1}^{n} w_i = 1$ is used. Then we propose again to aggregate the suitability grades conform to conventional fuzzy criteria handling by using the conventional n-ary OWA operator $h_{\vec{w}}^{OWA}$ that is defined by

$$h_{\vec{w}}^{OWA} : [0,1]^n \to [0,1] : (s_1, \ldots, s_n) \mapsto \sum_{i=1}^{n} w_i \cdot s_{\rho(i)} \tag{11}$$

where $\rho : \{1, \ldots, n\} \to \{1, \ldots, n\}$ is a permutation on the index set satisfying $s_{\rho(1)} \geq s_{\rho(2)} \geq \cdots \geq s_{\rho(n)}$ [25].

The aggregated confidence grade c should reflect the impact of the OWA operator on the veracity of the aggregated suitability grades s_i, $i = 1, \ldots, n$. We propose again that each confidence grade c_i should contribute to the same extent to the computation of c as its corresponding 'sibling' suitability grade s_i contributes to the computation of the aggregated suitability grade s. Hence, for each $i = 1, \ldots, n$, the same weight w_i that has been assigned to $s_{\rho(i)}$ by the OWA operator is also used as weight for $c_{\rho(i)}$, after which c is obtained by computing the weighted mean. This leads to the following definition.

The n-ary *ordered weighted average* for L-grades is defined by

$$h_{\vec{w}}^{OWA} : \mathbb{L}^n \to \mathbb{L} : ((s_1, c_1), \ldots, (s_n, c_n)) \mapsto (s, c) \tag{12}$$

where $s = \sum_{i=1}^{n} w_i \cdot s_{\rho(i)}$ and $c = \sum_{i=1}^{n} w_i \cdot c_{\rho(i)}$.

This aggregator is called a sibling aggregator because each confidence grade has a similar impact on the aggregation of the confidence grades, as its sibling suitability grade has on the aggregation of the suitability grades. Observe that the same permutation ρ that is defined by the ranking of the suitability grades s_i, $i = 1, \ldots, n$ is used for the computation of both s and c. This aggregator is also a sibling aggregator.

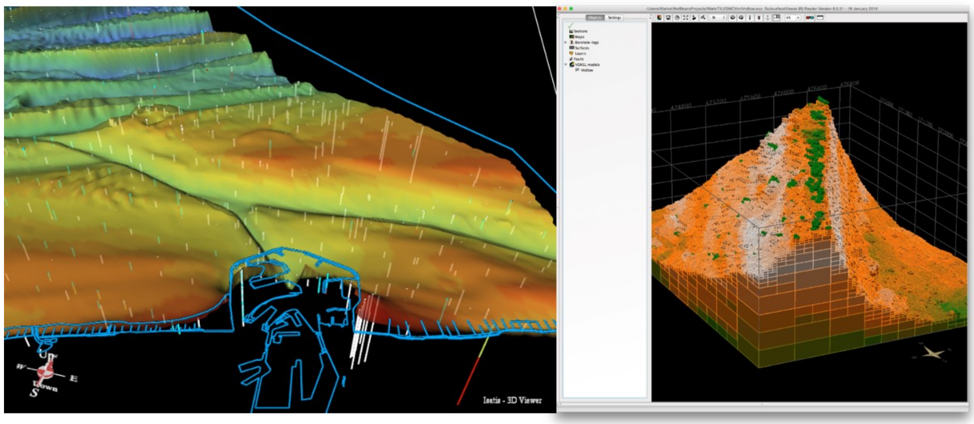


Figure 1: Sand extraction from the North Sea.

5. Applicability

In order to demonstrate the applicability of L-grades in criteria handling for decision making (and querying), consider the evaluation of the soil composition in the Belgian territory of the North Sea for supporting decision making on finding suitable areas for sand extraction [7]. The area under investigation is modelled by a 3D grid model, consisting of cuboids that are called tiles, of which a part representing the Westhinder sandbank is depicted at the right side of Figure 1.

Geological data are obtained from soil samples that are extracted using drilling techniques that are operated from a ship. For sampling locations, the soil composition data can be directly obtained by analysing the samples. For other locations these data have to be approximated by combining geologic domain knowledge with extrapolation techniques. Additionally, there are maps of present infrastructure like pipelines, cables, etc.

For the sake of illustration we consider that candidate tiles should satisfy two criteria. The first criterion, C_1^s, relates to the estimated quantity of fine sand present in a tile, whereas the second criterion, C_2^s, puts a constraint on the minimal distance between a tile and it nearest infrastructure. Both criteria have to be evaluated for each tile in the considered area of investigation. Each criterion is defined by means of a membership functions that reflects the preferences of the decision makers as depicted in Figure 2.

As such, C_1^s determines the desired percentages of fine sand. Tiles with a percentage lower than 70% are not suited and receives a suitability grade

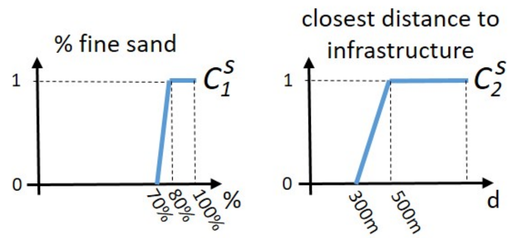


Figure 2: Criteria C_1^s and C_2^s.

0. Tiles with a percentage greater than 80% are fully suited and receive a suitability grade 1. For percentages between 70% and 80% the suitability monotonously increases from 0 to 1. A similar approach is used to define C_2^s. Here, the preferred distance d to the closest infrastructure is larger than 500 metres. Distances closer than 300 metres are unacceptable. Conventional flexible querying [15] and decision support techniques [12], which use fuzzy logic or gradual logic, can handle the suitability grades obtained from this kind of criterion evaluation.

To cope with veracity, trust in data has to be dealt with. Assume that the data for criterion C_1^s are subject to incompleteness and accuracy issues and that the data for criterion C_2^s are vulnerable to currency issues. Due to the sparse distribution of sampling locations, the confidence in the percentage of fine sand present in a given tile depends on the closeness of neighbouring sampling locations. Sampling locations are depicted by vertical lines in the left picture of Figure 1. For the sake of illustration the distance between a candidate tile and its closest sampling location is used to approximately assess the confidence that is related to incompleteness for that tile. More specifically, an approximate confidence grade is obtained by evaluating confidence criterion C_1^c as depicted in Figure 3.

The accuracy of the used sampling method also influences the confidence in the registered percentage of present fine sand. For the sake of illustration, we assume that there are four sampling methods A, B, C and D and only consider the (sampling method that has been used for the) closest sampling location. The accuracy related confidence is then assessed by evaluating criterion C_2^c shown in Figure 3. The confidence in the evaluation of criterion C_2^s is assumed to relate only to the currency of the available data on present infrastructure. For the sake of illustration, this currency related confidence

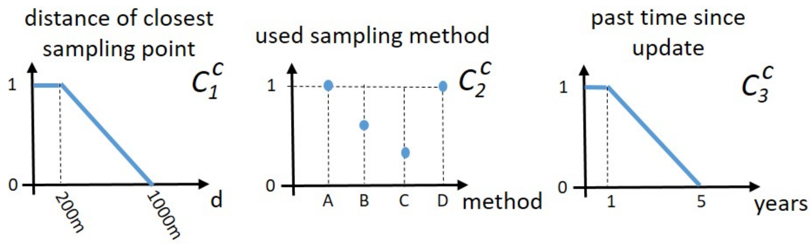


Figure 3: Confidence criteria C_1^c, C_2^c and C_3^c.

is assessed by evaluating criterion C_3^c shown in Figure 3, which is defined on the publication date of the used infrastructure maps for the subarea in which the candidate tile is located. The criterion reflects that if the oldest map used is dated less than 1 year ago, its content is fully truthful. Furthermore, if this map is dated more than 5 years ago the data are considered to be not truthful at all.

Let a_1 =‘% fine sand’ and a_2 =‘closest distance to infrastructure’ be the attributes that are evaluated in criteria C_1^s and C_2^s, and let a_3 =‘distance of closest sampling point’, a_4 =‘used sampling method’ and a_5 =‘past time since update’ be the attributes used in the confidence criteria C_1^c, C_2^c and C_3^c. Moreover, let the actual value of an attribute a_i for a given tile t be denoted by $t[a_i]$. Consider a set $T^c = \{t^1, \ldots, t^n\}$ of candidate tiles.

The evaluation of a tile $t^i \in T^c$, can be done by:

1. Computing the suitability grades $C_1^s(t^i[a_1])$ and $C_2^s(t^i[a_2])$.
2. Computing the confidence grades $C_1^c(t^i[a_3])$, $C_2^c(t^i[a_4])$ and $C_3^c(t^i[a_5])$.
3. Determining the L-grades $l_1^{t^i} = (C_1^s(t^i[a_1]), \top(C_1^c(t^i[a_3]), C_2^c(t^i[a_4])))$ and $l_2^{t^i} = (C_2^s(t^i[a_2]), C_3^c(t^i[a_5]))$, where $\top$ is a t-norm chosen to aggregate the two confidence grades that relate to data used in criterion C_1^s.
4. Compute the overall L-grade $l^{t^i} = \top(l_1^{t^i}, l_1^{t^i})$ using a conjunction operator $\top$ for L-grades as defined by Eq. (4).

If required, more advanced aggregators, as presented in Section 4, can be used.

Using L-grades allows to better inform and support users in their decision making processes. In the case of the above example, tiles with high suitability and high confidence are preferred, but those with high suitability and lower confidence might also be worth further investigation. For example, if

Table 1: Evaluation of some candidate tiles t^i.

t^i	$C_1^s(t^i)$	$C_1^c(t^i)$	$C_2^c(t^i)$	$l_1^{t^i}$	$C_2^s(t^i)$	$C_3^c(t^i)$	$l_2^{t^i}$	l^{t^i}
t^1	1	1	0.2	$(1, 0.2)$	1	1	$(1, 1)$	$(1, 0.2)$
t^2	0.7	1	0.6	$(0.7, 0.6)$	0.8	1	$(0.8, 1)$	$(0.7, 0.6)$
t^3	0.7	0.8	1	$(0.7, 0.8)$	0.8	1	$(0.8, 1)$	$(0.7, 0.8)$
t^4	0	1	1	$(0, 1)$	1	0.6	$(1, 0.6)$	$(0, 1)$

their closest sample location is too far, confidence could be improved by providing new sample locations. This kind of extra information is more difficult to obtain when using conventional logic frameworks. It definitely provides decision makers with a facility to manage data quality and make decisions on which data aspects to prioritize when improving data veracity.

L-grades can be computed and processed at each stage in criteria handling, ranging from criteria evaluation to the aggregation and ranking of the evaluation results. Backtracking to previous stages is always possible. This is illustrated with Table 1. Criteria evaluation results for the suitability criteria $C_1^s(t^i)$ and $C_2^s(t^i)$ were obtained by computing the membership grades of tile t^i's actual attribute values using the membership functions given in Figure 2. Likewise, the evaluation results for the confidence criteria $C_1^c(t^i)$, $C_2^c(t^i)$ and $C_3^c(t^i)$ were computed using the membership functions given in Figure 3. The minimum operator is used as t-norm in the computation of the confidence grade of $l_1^{t^i}$ and is also considered to be the t-norm on which the conjunction operator $\top$, used for the computation of l^{t^i} is based.

Tile t^1 has the highest suitability, but this suitability is least trusted. Tiles t^2 and t^3 are suitable to the same extent, but the suitability for t^2 is less trusted. For tile t^4, the suitability is zero and this can be fully trusted. The reason for the decreased confidence in t^1 is the use of the least trusted sampling method. The used sampling method is also causing the decrease in confidence for t^2. In the case of t^3 the distance to the closest sampling point causes the lack of trust.

Backtracking allows to find the reason of a given suitability or confidence grade, what contributes to the understanding and explainability of criteria handling, and what on its turn is considered as an important step towards better explainable query processing and decision making processes.

6. Related Work

In this work we use L-grades to explicitly cope with the veracity of results in criterion processing. By doing this we opt to use a kind of *truth values* to separately model suitability and confidence and process these values using fuzzy logic.

In other approaches, confidence has been modelled considering a level of *uncertainty* on suitability. As such, intervals [18], type-2 fuzzy sets [5], and R-sets [23] have been used as mathematical tools to impose a kind of upper and lower bound for suitability reflecting uncertainty on it. This uncertainty can be interpreted as reflecting veracity. Yager proposed two approaches for criterion evaluation with imprecise data in [26], the first approach is based on 'containment', the second on 'possibility'.

Data quality has also been studied in the context of skyline querying [14]. In skyline querying, database records are filtered by keeping only those that are not worse than any other. In the proposed approach, each database attribute can be assigned a quality level. These quality levels are then taken into account when checking for record dominance in answer set construction.

An advantage of using L-grades is that these permit to assess and handle data veracity in an explainable way, using (weighted) confidence criteria that reflect the expert knowledge of data managers, as illustrated in Section 5. This is of pivotal importance in view of developing improved interpretable and explainable criterion handling techniques. Moreover, the clear and explicit distinction between suitability and associated confidence is inspired by (and hence in line with) Zadeh's last insights on how to adequately model general numerical data using Z-numbers [29], stating among others that reliability of (uncertain) data should not be implicitly modelled by increasing or decreasing uncertainty, but should be dealt with separately. An idea that also has been shared by Aliev [2] and by Kreinovich [19].

7. Conclusions and Future Work

We proposed a novel logic framework, which is based on so-called L-grades. An L-grade is the simplest form of a Z-number, consisting of a pair of crisp numbers that are both interpreted as truth values. The first truth value reflects a suitability grade (or satisfaction grade), whereas the second truth value is a confidence grade expressing how reliable the suitability grade is. As such, L-grades can be used to model criterion satisfaction and handle criterion processing in flexible querying and decision support. Using explicit confidence grades to handle reliability permits to cope with data veracity, which is considered to be a main issue in many big data applications.

We studied and proposed basic logic operators for the negation, conjunction and disjunction of L-grades. Moreover, we introduced the concept of a sibling aggregator. With a sibling aggregator, suitability grades and confidence grades are aggregated separately. But each confidence grade has a similar impact in the aggregation of the confidence grades, as its corresponding (sibling) suitablity grade has in the aggregation of the suitability grades. As basic sibling aggregators we proposed weighted mean and ordered weighted average for L-grades.

The usability of L-grades has been demonstrated with an illustrative example on evaluating the soil composition in the Belgian territory of the North Sea for supporting decision making on finding suitable areas for sand extraction. L-grades are computed and processed during criterion evaluation and at each aggregation step. These L-grades then explicitly express how good an evaluated object satisfies (each of) the criteria and to what extent the evaluation (and aggregation) results can be trusted. Backtracking to previous criteria handling stages permits to find the reason for a given suitability or confidence grade. This contributes to better explainable criterion handling and could also be useful in view of the development of better interpretable artificial intelligence applications.

As future work, we plan to further investigate the mathematical properties of L-grades. Moreover, we aim to develop more advanced logic operators and sibling aggregators. Another future research topic is the further development of ordering functions for L-grades. We also plan to investigate how L-grades can be fitted within the Logic Scoring of Preference (LSP) framework for decision engineering. Finally, appropriate software tools will be developed in order to encourage the further development of practical applications and experimental validation.

Funding Statement

This research received funding from the Flemish Government under the 'Onderzoeksprogramma Artificiële Intelligentie (AI) Vlaanderen' programme.

References

[1] Aliev, R.A., Huseynov, O.H., Zeinalova, L.M.: The arithmetic of continuous Z-numbers. *Information Sciences*, vol. 373, no. 7, pp. 441–460 (2016).

[2] Aliev, R.A., Kreinovich, V.: Z-Numbers and Type-2 Fuzzy Sets: A Representation Result. *Intelligent Automation and Soft Computing*, vol. 24, no. 1, pp. 205–210 (2018).

[3] Barredo Arrieta, A., Díaz-Rodríguez, N., Del Ser, J., Bennetot, A., Tabik, S., Barbado, A., Garcia, S., Gil-Lopez, S., Molina, D., Benjamins, R., Chatila, R., Herrera, F.: Explainable Artificial Intelligence (XAI): Concepts, taxonomies, opportunities and challenges toward responsible AI. *Information Fusion*, vol. 58, pp. 82–115 (2020).

[4] Berti-Equille, L., Lamine Ba, M.: Veracity of big data: Challenges of cross-modal truth discovery. *ACM Journal of Data and Information Quality*, vol. 7, no. 3, art. 12, 3 pages (2016).

[5] Biglarbegian, M., Melek, W.W., Mendel, J.M.: On the robustness of Type-1 and Interval Type-2 fuzzy logic systems in modeling. *Information Sciences*, vol. 181, no. 7, pp. 1325–1347 (2011).

[6] de Siqueira Braga, D., Niemann, M., Hellingrath, B., Buarque de Lima Neto, F.: Survey on Computational Trust and Reputation Models. *ACM Computing Surveys*, vol. 51, no. 5, art. 101 (2018).

[7] De Tré, G., De Mol, R., van Heteren, S., Stafleu, J., Chademenos, V., Missiaen,, T., Kint, L., Terseleer, N., Van Lancker, V.: Data Quality Assessment in Volunteered Geographic Decision Support. In: Bordogna, G., Carrara, P. (eds.), Mobile Information Systems Leveraging Volunteered Geographic Information for Earth Observation, *Earth Systems Data and Models*, vol. 4, Springer, Switzerland, pp. 173-192, ISBN 978-3-319-70877-5 (2017).

[8] De Tré, G., Dujmović, J.: Dealing with Data Veracity in Multiple Criteria Handling: An LSP-Based Sibling Approach. In: *Proc. of the Flexible Query Answering Systems Conference, FQAS 2021*, *Lecture Notes in Artificial Intelligence*, vol. 12871, pp. 82–96. Bratislava, Slovakia (2021).

[9] De Tré, G., Peelman, M., Dujmović, J.: Logic Operators and Sibling Aggregators for Z-grades. In: *Proc. of the 19th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, IPMU 2022*, *Communications in Computer and Information Science*, vol. 1602, part 2, pp. 572–583. Milan, Italy (2022).

[10] Dubois, D., Prade, H.: The three semantics of fuzzy sets. *Fuzzy Sets and Systems*, vol. 90, pp. 141–150 (1997).

[11] Dubois, D., Prade, H.: A Fresh Look at Z-numbers – Relationships with Belief Functions and p-boxes. *Fuzzy Information and Engineering*, vol. 10, no. 1, pp. 5–18 (2018).

[12] Dujmović, J.: *Soft Computing Evaluation Logic: The LSP Decision Method and Its Applications*, J. Wiley, New Jersey, USA (2018).

[13] Dujmović, J.: Interpretability and Explainability of LSP Evaluation Criteria. In: *Proc. of the 2020 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE)*, pp. 1–8, Glasgow, UK (2020).

[14] Jaudoin, H., Pivert, O., Smits, G., Thion, V.: Data-Quality-Aware Skyline Queries. In: *Proc. of the International Symposium on Methodologies for Intelligent Systems, ISMIS 2014: Foundations of Intelligent Systems*, *Lecture Notes in Computer Science*, vol. 8502, pp. 530–535. Graz, Austria (2014).

[15] Kacprzyk, J., Zadrozny, S., De Tré, G.: Fuzziness in database management systems: Half a century of developments and future prospects. *Fuzzy Sets and Systems*, vol. 218, pp. 300–307 (2015).

[16] Kitchin, R.: Big Data, new epistemologies and paradigm shifts. *Big Data & Society*, vol. 1, no. 1, pp. 1–12 (2014).

[17] Klement, E.P., Mesiar, R., Pap, E.: *Triangular Norms*, Kluwer, Dordrecht, The Netherlands (2000).

[18] Kreinovich, V., Ouncharoen, R.: Fuzzy (and Interval) Techniques in the Age of Big Data: An Overview with Applications to Environmental Science, Geosciences, Engineering, and Medicine. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 23, suppl. 1, pp. 75–89 (2015).

[19] Kreinovich, V., Kosheleva, O., Zakharevich, M.: Z-Numbers: How They Describe Student Confidence and How They Can Explain (and Improve) Laplacian and Schroedinger Eigenmap Dimension Reduction in Data Analysis. In: Marsala, C., Lesot, M.-J. (Eds.), Fuzzy Approaches for Soft Computing and Approximate Reasoning: Theories and Applications, Springer, Cham, Switzerland, pp. 285–297 (2021).

[20] Lukoianova, T., Rubin, V.L.: Veracity Roadmap: Is Big Data Objective, Truthful and Credible? *Advances In Classification Research Online*, Vol. 24, no. 1, pp. 4–15 (2014).

[21] Massanet, S., Riera, J.V., Torrens, J.: A new approach to Zadeh's Z-numbers: Mixed-discrete Z-numbers. *Information Fusion*, vol. 53, pp. 35–42 (2020).

[22] Saha, B., Srivastava, D.: Data quality: The other face of big data. In: *Proc. of the 2014 IEEE 30th International Conference on Data Engineering*, pp. 1294–1297. Chicago, USA (2014).

[23] Seiti, H., Hafezalkotob, A., Martinez, L.: R-Sets, Comprehensive Fuzzy Sets Risk Modeling for Risk-Based Information Fusion and Decision-Making. *IEEE Transactions on Fuzzy Systems*, vol. 29, no. 2, pp. 385–399 (2021).

[24] Trillas, E.: On negation functions in the theory of fuzzy sets. *Stochastica*, vol. 3, no. 1, pp. 47–60 (1979)

[25] Yager, R.R.: On ordered weighted averaging aggregation operators in multi-criteria decision making. *IEEE Trans. on Systems, Man and Cybernetics*, vol. 18, 183–190 (1988).

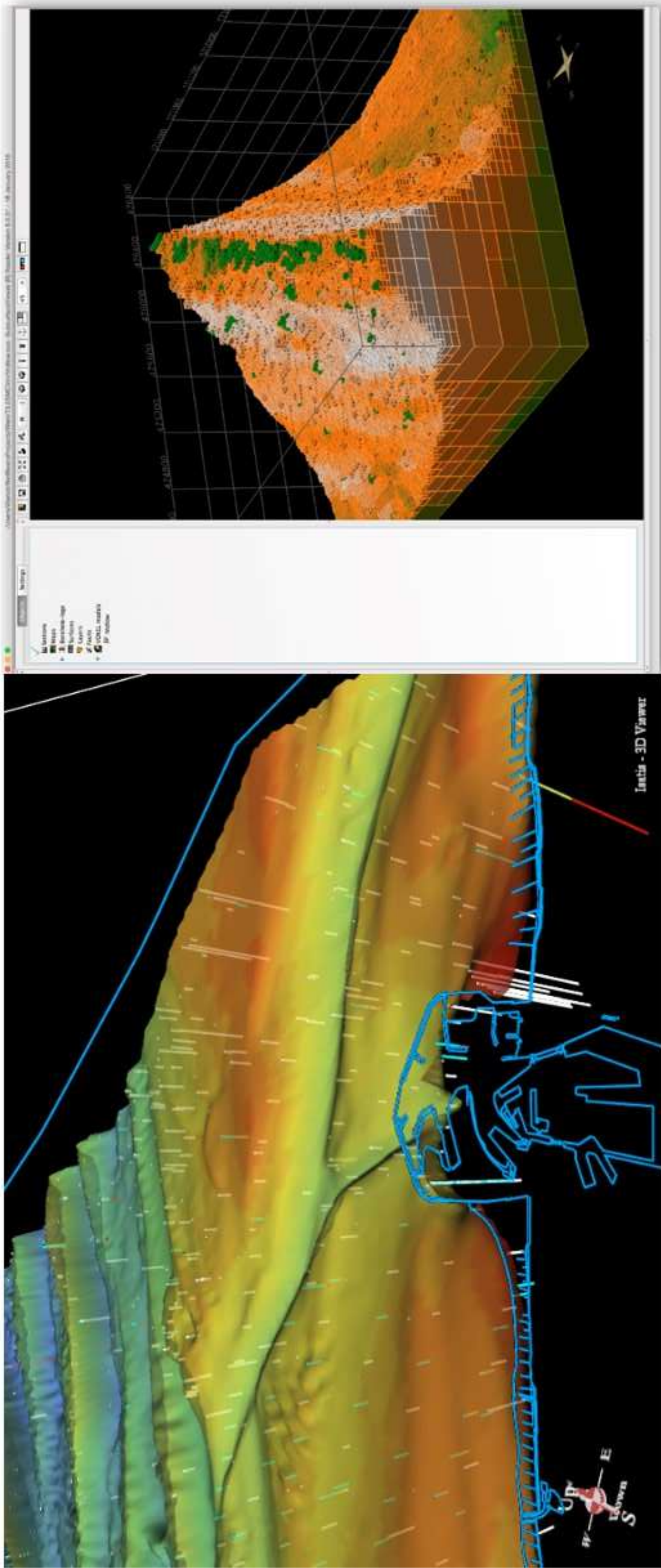
[26] Yager, R.R.: Validating criteria with imprecise data in the case of trapezoidal representations. *Soft Computing*, vol. 15, 601–612 (2011).

[27] Zadeh, L.A.: Calculus of fuzzy restrictions. In: Zadeh, L.A., Fu, K.S., Tanaka, K., Shimura, M. (Eds.), Fuzzy sets and Their Applications to Cognitive and Decision Processes, Academic Press, New York, pp. 1–39 (1975).

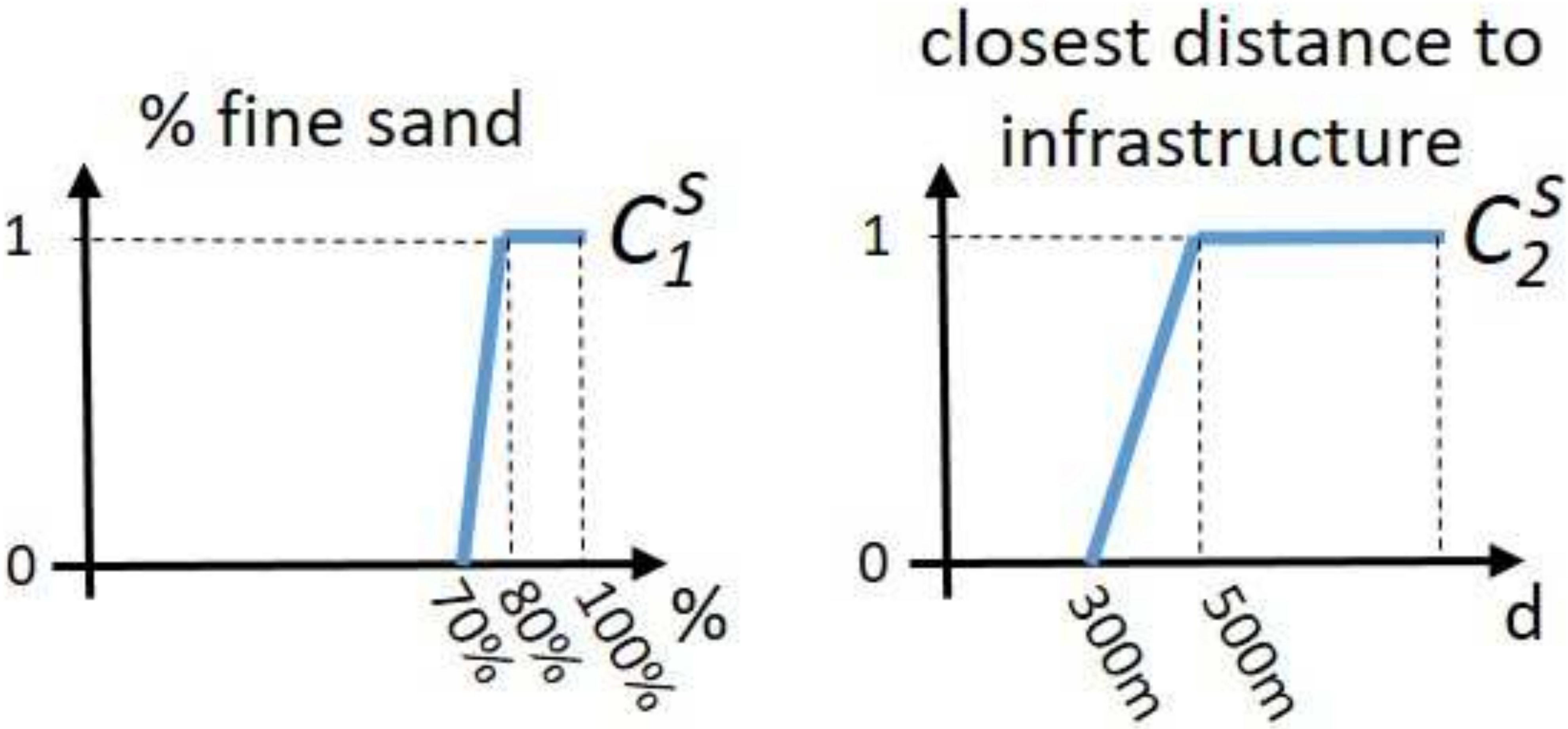
[28] Zadeh, L.A.: From imprecise to granular probabilities. *Fuzzy Sets and Systems*, vol. 154, no. 3, pp. 370–374 (2005).

[29] Zadeh, L.A.: A Note on Z-numbers. *Information Sciences*, vol. 8, no. 3, pp. 2923–2932 (2011).

Figure

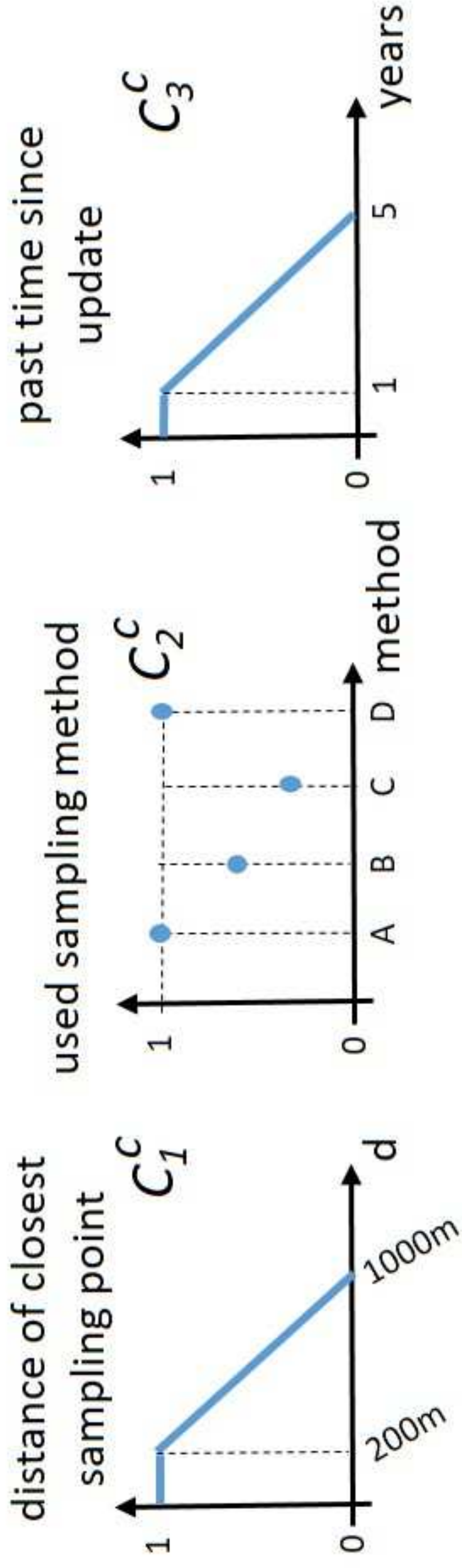


Figure



% fine sand
1
0
C_1^s
70%
80%
100%
%
closest distance to infrastructure
1
0
C_2^s
300m
500m
d

Figure


distance of closest sampling point
C_1^c
1
0
200m
1000m
d
used sampling method
C_2^c
1
0
A
B
C
D
method
past time since update
C_3^c
1
0
1
5
years