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Estimation of FOPDT and SOPDT Models from Noisy Step Response Data

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Summary

This paper proposes the estimation of first-order-plus-dead-time (FOPDT) and second-order-plus-dead-time (SOPDT) models from noisy step response data. The model parameters are estimated by computation of areas, which makes it robust in the presence of stochastic disturbances in the step response data. The efficiency of the methodology is extensively tested in various numerical examples as well as in real-life experimental tests. The results - comparing our proposed estimation method with some other methods - suggest that the novel algorithm can be used with noisy step response data and adequately approximates high order systems. Moreover, it does not require any system identification expertise, making it readily accessible for the non-experienced user in industrial practice. The method is successfully validated for overdamped, reasonably underdamped, as well as highly oscillatory processes, hence offering a comprehensive estimation method.

KEYWORDS:
FOPDT model, SOPDT model, model estimation, step response data, stochastic disturbances, quasi-automatic estimation procedure.

1 | INTRODUCTION

Process identification is the starting point in control design and it received great attention among the control community, being in top three items relevant to industry [13]. Model reduction techniques are usually performed prior to the design and tuning of controller parameters stages. Hence, the complex dynamics of an industrial process can be simplified by a model reduction to a first-order-plus-dead-time (FOPDT) or second-order-plus-dead-time (SOPDT) model. Consequently, the efficiency of model-based control strategies is highly dependent to the derivation of such simplified process models. The rationale consists in the proofs that such continuous-time transfer functions give a relatively good approximated step response, when compared to the measured data. Even for higher-order systems, SOPDT models characterize the essential dynamics of a process for further use in controller's tuning [1]. In practice, such low-order models make the controller design and online tuning more understandable and friendly for the system engineer.

Since an oscillatory response commonly occurs in the practical industrial processes, several techniques to estimate the parameters of an underdamped SOPDT model are currently available. The majority of those can be classified based on the applied methodology in: *i)* open-loop techniques, using the step response data [3, 6, 15] and *ii)* closed-loop algorithms, which employ a relay feedback system to ensure an oscillatory response [2, 10]. In the former, in order to derive the parameters of a SOPDT model, a step response experiment is performed in open-loop, whereas in the latter, the key idea is to use a relay with or without hysteresis in a control loop to ensure an oscillatory system response [14].

Since our proposed estimation method uses open-loop step response data to provide an accurate SOPDT model, in the following several available results are briefly presented. In [11], a tutorial review on process identification from step and relay feedback tests is presented. The reviewed identification methods using an open-loop step response test are based on the following: model fitting using key points from the transient response, integral methods, frequency response estimation, or least squared algorithm, among others. In [3], a comparison between different identification methods using step response data is provided. Hence, an integral equation method, a non-linear curve fitting available in Matlab® Optimization Toolbox and a particle swarm optimization technique using noise-free data are evaluated. The results are promising but limited to noise-free data, thus encouraging us to propose a method suitable for noisy step response data. A set of identification procedures using step response information is presented in both [6, 15]. These methods are based on estimating specific points of the step response (e.g., peak and valley points on an oscillatory response and their corresponding time instants). However, the location of these points is sensitive to stochastic disturbances and it can be difficult to select from a noisy signal. In [4], a SOPDT estimation method based on step response data is given, in which the model parameters are obtained by computing areas instead of points, which makes it less sensitive to noise. The method is successfully validated for overdamped and reasonably underdamped systems, but fails to provide accurate results for highly oscillatory systems (with a small damping factor). In [7], a process identification for a SOPDT model using rectangular pulse input is presented. The model parameters are estimated starting from an open-loop test, by means of minimizing the sum of modelling errors using the least square method. The solution of the minimization problem is obtained using a specialized optimization toolbox from Matlab®, and requires specific expertise in the identification field. Moreover, the method is tested only on overdamped and reasonably damped processes, without testing its validity on highly oscillatory dynamics. A real-coded genetic algorithm used for identification of FOPDT and SOPDT models from open-loop step response is given in [16]. The method uses a complex optimization problem to search for the model parameters and was not tested for poorly damped step responses. In [17], a direct identification method from step response data is provided. The procedure derives linear regression equations directly from the process differential equations, and is validated in simulation on a SOPDT model with a zero. In [18], a robust method is proposed for identification of linear continuous systems using linear regression equations based on least squared method. Both methods require dedicated tools and understanding. In [9], a frequency domain approach for estimating FOPDT and SOPDT models from step response data, using the least squares fitting algorithm is provided, which also assumes previous specialized knowledge. An indirect identification method of continuous-time delay systems from step responses is given in [5]. The method firstly uses the discretized continuous-time data to compute a discrete model, which is then converted to a continuous-time one. In [8], an analytical method based on process moments for estimating a second-order system with zero plus time delay is presented. The identification can be performed starting from either closed-loop or open-loop time response data. Nevertheless, the burgeoning demand of simple, yet robust tools in industrial systems practices increases the ongoing research interest on them.

In this paper, a comprehensive estimation methodology for FOPDT and SOPDT models is provided. Starting from available step response data, the model parameters are estimated using computation of areas. For validation purposes, the results were successfully compared to existing estimation methods.

The novelty and valuable contributions of this work can be summarized as follows:

- it provides a quasi-automatic estimation method, which does not require specialized expertise on classical system identification concepts and methods.
- the methodology is extensive and suitable for the entire range of typical second-order process dynamics. Hence, a two folded algorithm offers an estimation solution for either:
  - *i)* overdamped and reasonably underdamped processes - in practice this will result in a SOPDT model with damping factor > 0.5, or
  - *ii)* highly oscillatory processes - in practice this will result in a SOPDT model with damping factor < 0.5.
- the algorithm is highly reliable in realistic situations, when noisy signals are used as entry data in the estimation procedure.

The remaining of this paper is structured as follows. In Section 2 the description of our novel estimation method in given. The method is validated on numerical examples in Section 3 and experimental tests in Section 4. The conclusions and future perspectives are presented in Section 5. The theoretical proofs and Matlab® software are given in the Appendix.

2 | NOVEL ESTIMATION ALGORITHM FOR SOPDT MODELS

Let us consider a SOPDT model defined as:

$$\frac{K}{(\tau_1 s+1)(\tau_2 s+1)}e^{-\tau_d s} \tag{1}$$

in which τ_1 and τ_2 are the unknown process time constants and τ_d is the unknown dead-time value of the process. A reasonable estimate $\hat{K}$ of the gain can readily be obtained by averaging the steady-state values of the step response.

The remainder of this section is organized as follows:

1. Subsection 2.1 presents the method for estimating a second-order model without delay, suitable for overdamped, and reasonably underdamped processes.
2. Subsection 2.2 explains the robustness of the proposed method.
3. Subsection 2.3 presents an extension of the proposed method for estimating a second-order model without delay, suitable for highly oscillatory processes.
4. Subsection 2.4 considers the dead time influence, thus presenting the full estimation algorithm for a SOPDT model.

2.1 | Estimation of a SO model from an overdamped or a reasonably underdamped step response

The second-order (SO) model has the generic form:

$$\frac{K}{(\tau_1 s+1)(\tau_2 s+1)} = \frac{K}{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1} = \frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{K}{\frac{1}{\omega_n^2}s^2+\frac{2\zeta}{\omega_n}s+1} \tag{2}$$

in which τ_1 and τ_2 may be real (overdamped) or complex conjugated (underdamped) values, and with ζ the damping factor and ω_n the natural frequency (rad/s).

In this section, the procedure to estimate the two SO parameters (i.e., ζ and ω_n) is tailored for the particular case of overdamped, as well as reasonably underdamped processes. In practice this will result in a SO model with ζ in the range $\{\zeta_{\min}\dots\zeta_{\max}\}$, with e.g., $\zeta_{\min}=0.5$ and $\zeta_{\max}=3$. Notice that, for $\zeta>3$, the model is practically a FO. Here it is assumed that the dead time is zero, then in Section 2.4 it is explained how to apply the procedure when the dead time is non zero. For a simplified exposition, all the proofs are given in the Appendix A.1.

Step 1. Referring to Fig. 1, using the measured step response $y(t)$, $0\le t\le T_m$, calculate the area:

$$A_1=\int_0^{T_m}[\hat{K}-y(t)]dt \tag{3}$$

In the discrete-time domain, the area is easily calculated as:

$$A_1\cong T_s\sum_{k=0}^{N_s}[\hat{K}-y(k)] \tag{4}$$

with T_s the sampling period and N_s+1 the number of samples in the measurement period $T_m=N_sT_s$.

Step 2. Construct the signal $x(t)$ as the step response of the first-order (FO) system $\frac{\hat{K}}{1+\tau s}$, with the time constant τ calculated as:

$$\tau=\frac{A_1}{\hat{K}} \tag{5}$$

Referring again to Fig. 1 the following relations are proven in the Appendix A.1:

$$A_2=A_1\ (\overset{\Delta}{=}A) \tag{6}$$

$$\text{Area } S_1=\text{Area } S_2 \tag{7}$$

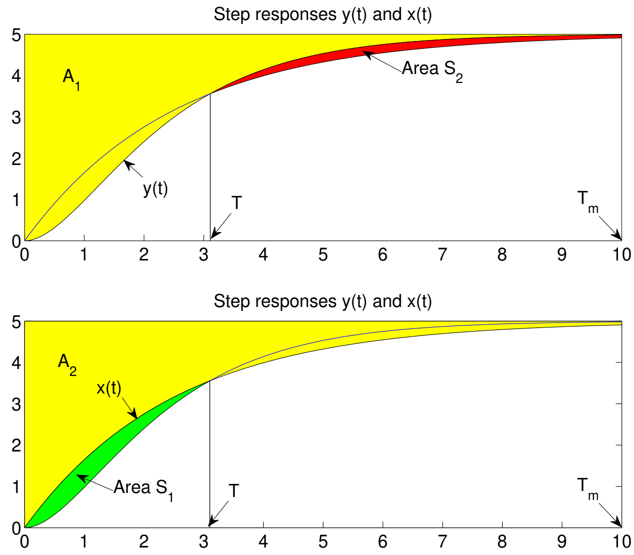

FIGURE 1 Definition of step responses and corresponding areas as used in the algorithm

Step 3. Define now the signal $\mathcal{A}(t)$ as the (signed) area between $x(t)$ and $y(t)$, $0 \leq t \leq T_m$ as:

$$\mathcal{A}(t) = \int_0^t [x(\nu) - y(\nu)] d\nu \tag{8}$$

In discrete-time, $\mathcal{A}(t)$ becomes:

$$\mathcal{A}(k) = \mathcal{A}(k-1) + T_s[x(k) - y(k)] \text{ for } k = 1...N_s \tag{9}$$

Notice that $\mathcal{A}(0) = 0$; moreover, in the Appendix it is proven that $\mathcal{A}(T_m) \approx 0$. It follows that $\mathcal{A}(t)$ must have an extreme in the range $0 \leq t \leq T_m$.

Step 4. Find the time instant T when $|\mathcal{A}(t)|$ reaches its extreme:

$$T = \arg \max_t |\mathcal{A}(t)| \tag{10}$$

and define the area $S \overset{\Delta}{=} \mathcal{A}(T)$. In Fig. 1 S corresponds to Area S_1.

Step 5. Defining $\alpha \overset{\Delta}{=} \frac{T}{\tau}$ and $Z \overset{\Delta}{=} (\zeta - \sqrt{\zeta^2 - 1})^2$, calculate the following ζ-dependent function, with ζ in the range $\{\zeta_{\min} \dots \zeta_{\max}\}$, with e.g. $\zeta_{\min} = 0.5$ and $\zeta_{\max} = 3$ (if $\zeta > 3$, the system is practically first order):

$$F(\zeta) = \frac{S}{A} - e^{-\alpha} \left[1 - 2\mathrm{Re} \left\{ \frac{e^{-\alpha Z}}{1 - Z^2} \right\} \right], \text{if } 0.5 < \zeta < 1 \tag{11}$$

$$F(\zeta) = \frac{S}{A} - e^{-\alpha} \left[1 - \frac{e^{-\alpha Z}}{1 - Z^2} - \frac{e^{-\alpha Z^{-1}}}{1 - Z^{-2}} \right], \text{if } 1 < \zeta < 3 \tag{12}$$

Step 6. The solution is then the value of ζ which fulfils:

$$F(\zeta) = 0 \tag{13}$$

with the corresponding value

$$\omega_n = \frac{2\zeta}{\tau}. \tag{14}$$

This solution is unique (Ref. Appendix A.1).

2.2 | Robustness of the proposed estimation method

Obviously, the method is robust w.r.t. stochastic disturbances on the measured step response (because it is solely based on the calculation of areas A and S, and because integration has an an averaging effect on the noise).

The method is also robust w.r.t. the estimated gain, $\widehat{K}$, except in the case of a very poorly damped (highly oscillatory) step response. In this section we give an explanation which will lead to a remedy.

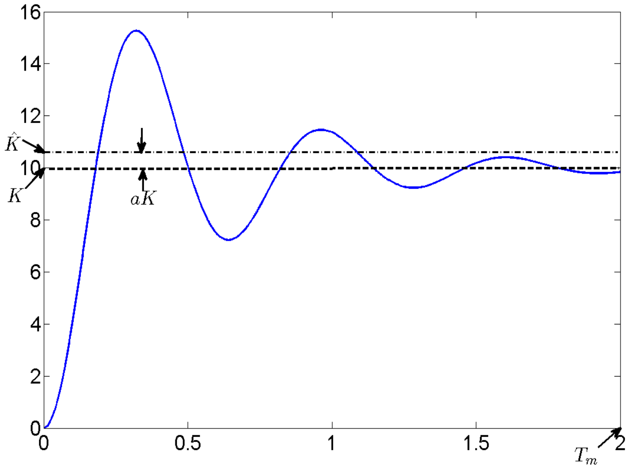

FIGURE 2 Support figure depicting an error in the static gain estimation

Referring to Fig. 2, it can be observed that an error $\Delta K = \widehat{K} - K$ in the estimated static gain is equivalent to an error $\Delta A = \widehat{A} - A = T_m \Delta K$ in the calculated area A.

From Fig. 2 it follows that $\widehat{A} = A + aKT_m$, with a the relative error on the static gain: $a = \frac{\hat{K}-K}{K} = \frac{\Delta K}{K}$.

Then

$$\frac{\widehat{A}}{\widehat{K}} = \frac{A + aKT_m}{(1+a)K} \cong \frac{A}{K} + aT_m \tag{15}$$

for a small relative error (i.e., $a \cong 0$).

With the measurement time T_m being about equal to the settling time for a SO system (i.e., $T_m = \frac{4}{\zeta\omega_n}$), the combination of (15),(14),(5),(6) results in:

$$\frac{\widehat{A}}{\widehat{K}} \cong \frac{A}{K} + \frac{4a}{\zeta\omega_n} = \frac{A}{K}\left(1 + \frac{2a}{\zeta^2}\right) \tag{16}$$

In conclusion, from (16) it results that a small relative error a on the static gain K corresponds to a relative error $\frac{2a}{\zeta^2}$ on $\frac{A}{K}$.

Considering (5),(6), this error is also the relative error in the estimation of τ, which is a key parameter in the proposed method.

For example, consider $a = 0.01$ which is a small error of 1% and a reasonably damped system with $\zeta = 0.7$. It results a relative error of $\frac{\Delta\tau}{\tau} = \frac{2a}{\zeta^2} = \frac{2*0.01}{0.7^2} = 0.04$ in (16), which is a negligible 4% error. However, in the case of a highly oscillatory system with $\zeta = 0.2$, this relative error becomes quite large (e.g. $\frac{\Delta\tau}{\tau} = \frac{2a}{\zeta^2} = \frac{2*0.01}{0.2^2} = 0.50 = 50\%$) and has major influence in the overall performance of the algorithm.

Hence, the method from Section 2.1 is not suitable for highly oscillatory systems (say $\zeta < 0.5$). From an engineering practice point of view, a highly oscillatory process has more than 1 oscillation period in the step response. A typical step response for such systems is provided in Fig. 3.

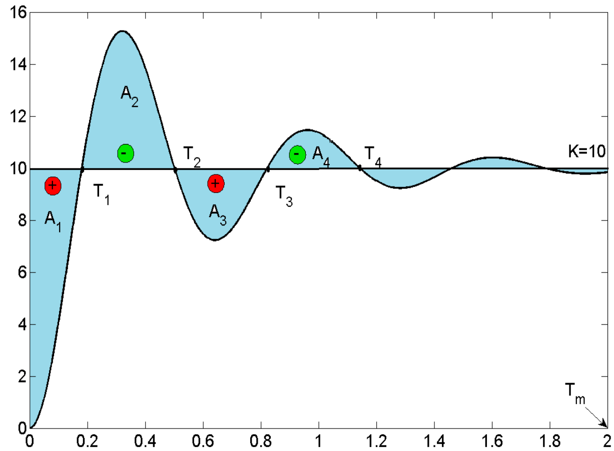

FIGURE 3 Useful step response points and areas for highly oscillatory system

Referring to (3), in the calculation of the area $A = \int_0^\infty [K - y(t)]dt = A_1 + A_2 + A_3 + A_4 \cdots$ there is a positive contribution from A_1, A_3, A_5, $\cdots$ and a negative contribution from A_2, A_4, A_6, $\cdots$. This can make the area A quite small - thus becoming prone to errors - in case of a highly oscillatory step response.

Referring to (5) and (14), area $A = K\tau = K2\frac{\zeta}{\omega_n} = K2\frac{\zeta}{\omega_n}T_m\frac{\zeta\omega_n}{4} = 0.5\zeta^2 KT_m$, with KT_m the area under 'the static gain line'. Thus using the previous example, for a reasonably damped system with $\zeta = 0.7$, the area is $A = 0.25KT_m$, whereas in the case of a highly oscillatory system with $\zeta = 0.2$, the area $A = 0.02KT_m$ becomes very small and unreliable to calculate τ.

Hence, for highly oscillatory step responses, an alternative area $\underline{A} = \int_0^\infty |K - y(t)|dt = A_1 - A_2 + A_3 - A_4 \cdots$ is introduced, which never becomes small.

2.3 | Estimation of a SO model from a highly oscillatory step response

Hereafter, the algorithm to estimate the SO model parameters from (2) (i.e., ζ and ω_n) from highly oscillatory step response data is provided. In practice this will result in a SO model with ζ in the range $\{\zeta_{\min} \ldots \zeta_{\max}\}$, with e.g., $\zeta_{\min} = 0.01$ and $\zeta_{\max} = 0.5$. Similar with the description from Section 2.1, here it is assumed that the dead time is zero, which will be followed in Section 2.4 by the description in the general case with dead time different from zero. For the interested reader, all the proofs are given in the Appendix A.2.

Step 1. Using the measured step response $y(t)$ calculate the alternative area:

$$\underline{A} = \int_0^{T_m} |\hat{K} - y(t)|dt \tag{17}$$

In the Appendix A.2 it has been proven that the area $\underline{A}$ from (17) can be analytically obtained as:

$$\underline{A} = 2K\mathrm{Re}\left\{\frac{\overline{p}/p}{\overline{p} - p}\left(-1 + 2\frac{e^{pT_1}}{1 + e^{0.5pT_p}}\right)\right\} \tag{18}$$

where p and $\overline{p}$ are the complex conjugated poles for the SO model (2) (i.e. $p = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$ with $\omega_n = \frac{2\pi}{T_p\sqrt{1-\zeta^2}}$) and $T_1 = \frac{\arcsin(\zeta)+\frac{\pi}{2}}{\omega_n\sqrt{1-\zeta^2}}$.

The value T_p is the period of oscillations; this period can be well estimated from the measured step response (averaging over multiple periods is indeed possible because of the very poor damping).

Notice that the right hand side of (18) is a function of the damping factor ζ only, and is further used to estimate the damping factor value.

Step 2. Referring to expression (18), define two functions in the variable ζ as:

$$f(\zeta) = \frac{\overline{p}/p}{\overline{p} - p}\left(-1 + 2\frac{e^{pT_1}}{1 + e^{0.5pT_p}}\right) \tag{19}$$

and

$$F(\zeta) = \underline{A} - 2\widehat{K}\mathrm{Re}\{f(\zeta)\} \tag{20}$$

Step 3. From (18), rewritten as (20), the solution is then the value of ζ which fulfils:

$$F(\zeta) = 0 \tag{21}$$

with the corresponding value

$$\omega_n = \frac{2\pi}{T_p\sqrt{1-\zeta^2}} \tag{22}$$

This solution is unique (Ref. Appendix A.2).

2.4 | Generic procedure to obtain the SOPDT model

In this section, it is explained how to take into account the dead time, when estimating a SOPDT model from step response data, with ζ in the range $\{\zeta_{\min} \dots \zeta_{\max}\}$, where $\zeta_{\min} = 0.01$ and $\zeta_{\max} = 3$.

Given the input parameters:
1) measured step response data $y(k)$, $k = 0 \dots N_s$, sampled with period T_s during the time-range $0 \dots T_m$, with $T_m = N_s T_s$;
2) an estimate of the static gain K and of the period of oscillations T_p (in case of highly oscillatory step responses); note that these values are easy to estimate from the measured step response;
3) a minimum value $\underline{\tau_d} = \underline{d}T_s$ and a maximum value $\overline{\tau_d} = \overline{d}T_s$ for the dead-time τ_d, with $\{\underline{d},\ \overline{d}\}$ integer numbers. It is noteworthy to mention that these values are easy to estimate from the measured step response. Moreover, these values are not critical, as long as the optimal τ_d is in the range $\{\underline{\tau_d} \dots \overline{\tau_d}\}$. Default values could be $\underline{\tau_d} = 0$ and $\overline{\tau_d} = T_m$.

In order to estimate the SOPDT model parameters (i.e. ω_n, ζ and τ_d), the following procedure is provided.
For each $d = \underline{d} : \overline{d}$ do the following:

Step 1. Replace the measured step response $y(k)$ by a 'delay-free' step response $s(k)$. This is simply done by skipping the first d samples (i.e. $s(k-d) = y(k)$, with $k = d \dots N_s$).

Step 2. Use the shifted step response $s(k)$ to estimate the parameters ζ and ω_n
As a rule of thumb, if there is more than one oscillation period in the step response $y(k)$, then use the extension for the method suitable for highly oscillatory systems provided in Section 2.3, else use the algorithm from Section 2.1.

Step 3. Calculate the step response $\widehat{y}(k)$ of the estimated SOPDT model $\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}e^{-\tau_d s}$ with $\tau_d = dT_s$.

Step 4. As an error criterion use for example the Sum of Squared Errors $SSE = \sum_{k=0}^{N_s}[y(k) - \widehat{y}(k)]^2$.
Note that SSE is a commonly used error criterion, but any other criterion which involves the error between actual and estimated step responses would do.

Step 5. The optimal value of τ_d is then $\tau_d^* = \arg\min_{\tau_d} SSE(\tau_d)$.

3 | NUMERICAL EXAMPLES

Example 1: Overdamped system
Consider the system: $P(s) = \dfrac{100(s+1.5)(s+3)(s+6)}{(s+1)(s+2)(s+4)(s+8)(s+16)}e^{-0.5s}$, $\hat{K} = 2.6$.
Our estimation: $\hat{P}_{RDKAM}(s) = \dfrac{2.6}{0.015s^2 + 0.735s + 1}e^{-0.54s}$, hereafter referred with subscript 'RDKAM'.
Reference model: $\hat{P}_{TFEST}(s) = \dfrac{2.6}{0.010s^2 + 0.754s + 1}e^{-0.50s}$, hereafter referred with subscript 'TFEST'.
It is noteworthy to mention that the latter was obtained using the Matlab® System Identification Toolbox, i.e., the *tfest* function.

The estimation using the proposed method is acceptable, from a practical engineering p.o.v.; ref. Fig. 4, right subplot.

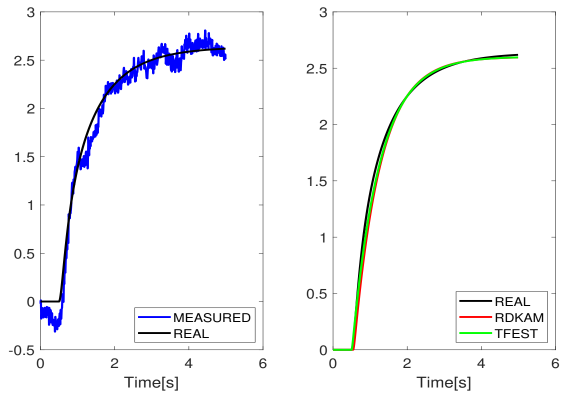

FIGURE 4 MEASURED, REAL, RDKAM and TFEST step responses for Ex. 1

Due to severe low and high frequency stochastic disturbances, the measured step response deviates from the real step response; for this reason, the value for estimated gain $\hat{K}$ was chosen as an average value over the last 0.5 seconds from the measured step response, ref. Fig. 4, left subplot.
In the experiment, we injected stochastic disturbance at the input, with the standard deviation value $\sigma_{in} = 0.472$, and at output of the process, with $\sigma_{out} = 0.052$. For comparison, we have also computed the Mean Squared Error, obtaining the values: $MSE_{RDKAM} = 0.0051$, and $MSE_{TFEST} = 0.0020$, where $MSE_{RDKAM} = \dfrac{1}{N_s+1}\sum_{k=0}^{N_s}[y_{REAL}(k) - y_{RDKAM}(k)]^2$ with y_{REAL} the step response of $P(s)$ and y_{RDKAM} the step response of $\hat{P}_{RDKAM}(s)$ estimated using our proposed method (and similar for the definition of MSE_{TFEST}); $N_s + 1$ is the number of samples in the data set.

Example 2: Underdamped system
Consider the system: $P(s) = \dfrac{1000}{s^2 + 10s + 100}e^{-0.1s}$, $\hat{K} = 10.22$, $\hat{T}_p = 0.7$.
Our estimation: $\hat{P}_{RDKAM}(s) = \dfrac{1098}{s^2 + 10.36s + 107.4}e^{-0.115s}$.
TFEST estimation: $\hat{P}_{TFEST}(s) = \dfrac{937}{s^2 + 11.79s + 92.49}e^{-0.075s}$.

The estimation using the proposed method is acceptable, from a practical engineering p.o.v.; ref. Fig. 5, right subplot.
The injected stochastic disturbances have the following values: $\sigma_{in} = 0.284$, and $\sigma_{out} = 0.566$.
For comparison, we have also computed the value MSE, obtaining: $MSE_{RDKAM} = 0.064$, and $MES_{TFEST} = 0.162$.

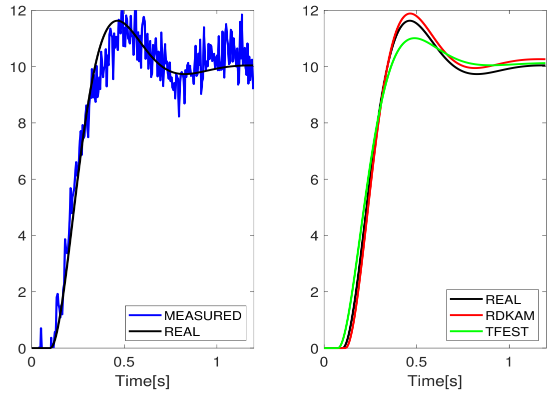

FIGURE 5 MEASURED, REAL, RDKAM and TFEST step responses for Ex. 2

Example 3: Highly oscillatory system

Consider the system: $P(s) = \dfrac{1000}{(s^2+4s+100)(0.05s+1)}$, $\widehat{K} = 10$, $\widehat{T}_p = 0.63$.

Our estimation: $\widehat{P}_{RDKAM}(s) = \dfrac{10}{0.0097s^2 + 0.0374s + 1}e^{-0.04s}$.

TFEST estimation: $\widehat{P}_{TFEST}(s) = \dfrac{10.10}{0.0105s^2 + 0.0503s + 1}$.

The estimation using the proposed method is acceptable, from a practical engineering p.o.v.; ref. Fig. 6, right subplot.

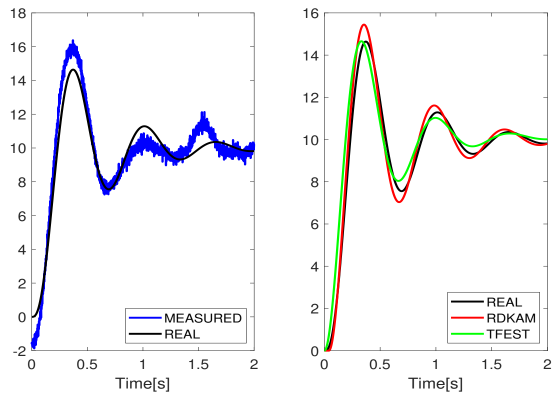

FIGURE 6 MEASURED, REAL, RDKAM and TFEST step responses for Ex. 3

The injected stochastic disturbances have the following values: $\sigma_{in} = 1.00$, and $\sigma_{out} = 0.20$.
For comparison, we have also computed the value MSE, obtaining: $MSE_{RDKAM} = 0.21$, and $MES_{TFEST} = 0.73$.

Example 4: High-order monotonic process. Comparative evaluation with existing methods. Our proposed method was compared with two methods from the literature. The first method uses a three-point fitting estimation method [15], hereafter will be referred with the subscript 'GPR'. The second method is based on least squares regression equations [18], hereafter will be referred with the subscript 'QGW'. As the reference for comparison we also employed the estimation method *tfest*.
Consider the system [18]: $P(s) = \dfrac{1}{(s+1)^5}$, $\widehat{K} = 1$.

Our estimation: $\widehat{P}_{RDKAM}(s) = \dfrac{1}{4.04s^2 + 3.58s + 1}e^{-1.45s}$.
GPR estimation: $\widehat{P}_{GPR}(s) = \dfrac{1}{4.71s^2 + 3.58s + 1}e^{-1.35s}$.
QGW estimation: $\widehat{P}_{QGW}(s) = \dfrac{1}{4.40s^2 + 3.45s + 1}e^{-1.45s}$.
TFEST estimation: $\widehat{P}_{TFEST}(s) = \dfrac{0.98}{4.37s^2 + 3.40s + 1}e^{-1.50s}$.

The estimations are perfect from a practical engineering p.o.v. (ref. Fig. 7, where for simplicity we only show the comparison results with QGW method. The graphical results obtained with the other methods are basically identical, and for simplicity were not plotted).
For comparison, we have also computed the value MSE, obtaining: $MSE_{RDKAM} = 7.35 \times 10^{-5}$, $MSE_{GPR} = 9.31 \times 10^{-5}$, $MSE_{QGW} = 13.29 \times 10^{-5}$, and $MSE_{TFEST} = 9.12 \times 10^{-5}$.

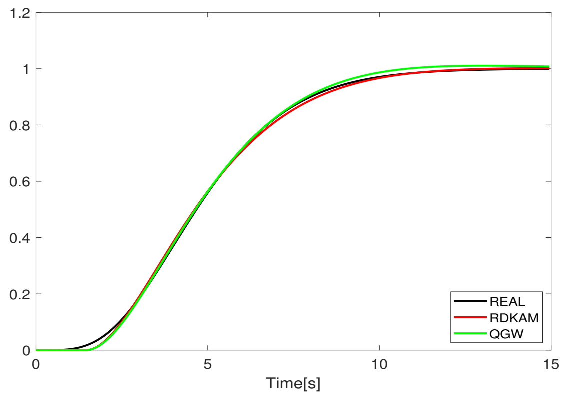

FIGURE 7 REAL, RDKAM and QGW step responses for Ex. 4

Example 5: High-order system with delay. Comparative evaluation with existing estimation methods.
Consider the system [18]: $P(s) = \dfrac{1.08}{(s+1)^2(2s+1)^3}e^{-10s}$, $\widehat{K} = 1.08$.
Our estimation: $\widehat{P}_{RDKAM}(s) = \dfrac{1.08}{10.87s^2 + 5.78s + 1}e^{-12.1s}$.
GPR estimation: $\widehat{P}_{GPR}(s) = \dfrac{1.08}{11.76s^2 + 5.64s + 1}e^{-12.17s}$.
QGW estimation: $\widehat{P}_{QGW}(s) = \dfrac{1.08}{11.90s^2 + 5.87s + 1}e^{-12.06s}$.
TFEST estimation: $\widehat{P}_{TFEST}(s) = \dfrac{1.06}{11.89s^2 + 5.71s + 1}e^{-12.0s}$.

The estimations are perfect from a practical engineering p.o.v. (ref. Fig. 8, where for simplicity we only show the comparison results with GPR method. The graphical results obtained with the other methods are very similar and were omitted).
For comparison, we have also computed the value MSE, obtaining: $MSE_{RDKAM} = 3.93 \times 10^{-5}$, $MSE_{GPR} = 13.85 \times 10^{-5}$, $MSE_{QGW} = 5.17 \times 10^{-5}$, and $MSE_{TFEST} = 2.42 \times 10^{-5}$.

In this section, several numerical examples were used to test the efficiency of the proposed estimation method in various situations:

i) noisy overdamped, underdamped and highly oscillatory system dynamics, i.e., entry data subject to severe stochastic disturbances (ref. Example 1, 2 and 3, respectively).

ii) noise-free system dynamics for comparative evaluation with existing estimation methods from the literature, (ref. Examples 5 and 6).

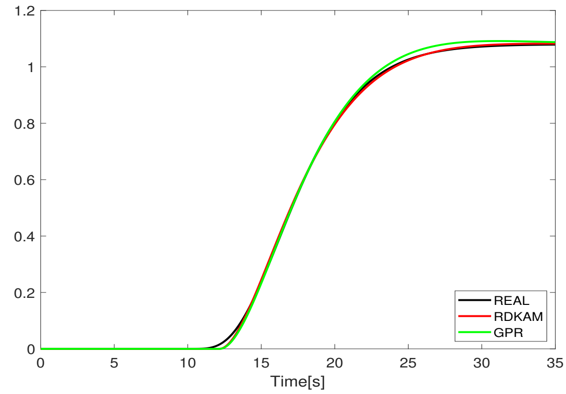

FIGURE 8 REAL, RDKAM and GPR step responses for Ex. 5

In all the above mentioned examples, the simulation results clearly indicated that our proposed estimation method is reliable when tested in reasonable cases.

4 | EXPERIMENTAL TESTS

In this section, our novel estimation method is tested on two real-time benchmarks, which are representative for manifold control engineering processes. As such, the six tanks process from Quanser can be viewed as a dynamics simulator for chemical plants, such as distillation columns from petrochemical industry, pharmaceutical industry or water treatment plants. On the other hand, the Mass-Spring-Damper (MSD) system is illustrative for mechatronics engineering applications, more specifically in vibration control problems. Thus, it can be used to simulate the active suspension system of a car, the train with multiple coaches, the chain of cars in a highway, or the damping system in tall buildings or the movement in air-plane wings.

Real-time Example 1: The six tanks process
In the six tanks process from Quanser® (Fig. 9) described in [12], the input variable is the water flow (i.e., the voltage [V] of the pump) for the 1^{st} tank, while the output variable is the water level [cm] of the 6^{th} tank. All thanks are in series, which makes the system a 6^{th} order.
The measured data was sampled with $T_s = 1[s]$ and K is specified as 0.82.
Our estimation: $\widehat{P}_{RDKAM}(s) = \frac{0.0011}{s^2+0.0646s+0.0013}e^{-18s}$, with $MSE_{RDKAM} = 7.00 \times 10^-5$..
TFEST estimation: $\widehat{P}_{TFEST}(s) = \frac{0.0010}{s^2+0.0624s+0.0013}e^{-19s}$, with $MSE_{TFEST} = 8.16 \times 10^-5$..
Figure 10 denotes a good approximation of the measured step response.

FIGURE 9 Overview of the six tanks process from Quanser®

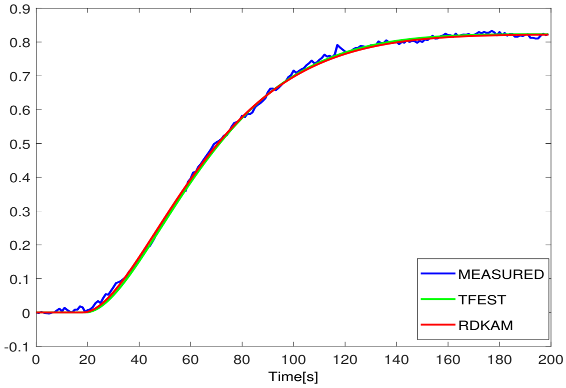

FIGURE 10 Measured, TFEST and RDKAM step responses for the Quanser® six tanks process from real-time Example 1

Real-time Example 2: The Mass-Spring-Damper (MSD) system
In the Mass-Spring-Damper (MSD) system from ECP® depicted in Fig. 11, the input variable is the voltage [V] of motor, while the output variables are the positions [mm] of the two masses. For our test, only the model for the second mass is of interest, which is a 4th order system with two resonance peaks (i.e., 4 complex conjugated poles).
The measured data was sampled with $T_s = 0.010[s]$.
Our estimation: $\widehat{P}_{RDKAM}(s) = \frac{736e^{-0.11s}}{s^2+2.01s+404}$, with $MSE_{RDKAM} = 0.042$.
TFEST estimation: $\widehat{P}_{TFEST}(s) = \frac{729e^{-0.15s}}{s^2+2.72s+400}$, with $MSE_{TFEST} = 0.095$.
The estimation successfully approximates the measured data; ref. Fig. 12.

5 | CONCLUSIONS

In this paper a novel method has been presented to estimate a FOPDT or SOPDT model from noisy step response data. It can be used with non-oscillatory as well as with highly oscillatory step responses. The key idea is the calculation of areas of interest, rather than using specific points of the step response, for estimating the model parameters. The method is quasi-automatic and does not require specialized expertise in system identification techniques. Moreover its validation (both with numerical and experimental examples) showed increased reliability w.r.t. entry data corrupted with stochastic disturbances, thanks to the averaging effect of the integration, as opposed of needing to detect interest points in a noisy signal.

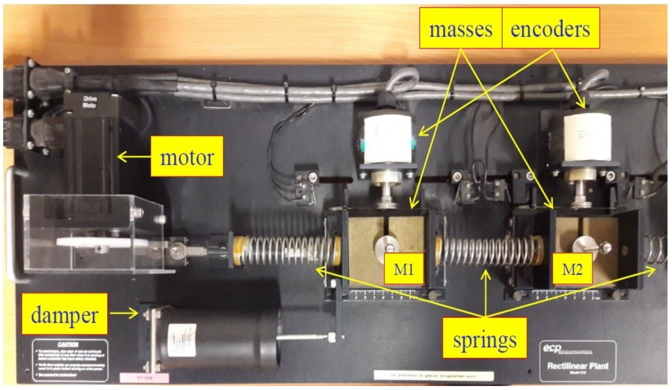

FIGURE 11 Overview of the MSD system from ECP®

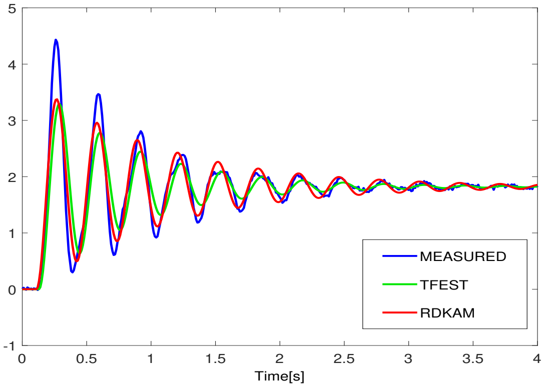

FIGURE 12 Measured, TFEST and RDKAM step responses for the ECP® MSD system from real-time Example 2

As a reference, our estimation results were compared with a model obtained using the Matlab® System Identification Toolbox. Moreover, we also compared our method, with two methods from the literature, which use different estimation approaches, i.e., a three-point fitting method and a method using least squares regression equations. The results show the efficiency of the proposed estimation method, which has the benefit of being extremely easy to use, and does not requisite identification proficiency. Our aim was to develop a simple and convenient estimation method, especially dedicated to industry practitioners.

Future work will focus on performing an extensive robustness validation for our proposed method, using for example Monte-Carlo simulations for different noise sequences, to analyse the variability of the parameter estimates.

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Author contributions

Anca Maxim: Writing – original draft (lead); Writing – review and editing (lead); Robin De Keyser: Conceptualization (lead); Investigation (lead); Methodology (lead); Software (lead); Review and editing (supporting).

Conflict of interest

The authors declare no potential conflict of interests.

□

APPENDIX

A

This appendix is organized in three parts with the following structure:

- Subsection A.1 presents the theoretical proofs regarding subsection 2.1,
- Subsection A.2 depicts the theoretical proofs regarding subsection 2.3,
- Subsection A.3 provides a summary of the algorithms and the Matlab® software corresponding to the procedure detailed in subsection 2.4.

A.1 Theoretical proofs - regarding the formulas introduced in Section 2.1

This section presents the theoretical proofs regarding the relations (6) and (7) by means of two theorems.

Theorem 1. The areas A_1 and A_2 are equal; their value is $A = K(\tau_1 + \tau_2)$.

Proof of Theorem 1. The step response of a SO process model is given as:

$$\begin{aligned} Y(s) &= \frac{K}{(1+\tau_1 s)(1+\tau_2 s)}\frac{1}{s} \\ \Rightarrow\ y(t) &= K\left(1 + \frac{\tau_1}{\tau_2-\tau_1}e^{-t/\tau_1} - \frac{\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}\right) \end{aligned} \tag{A1}$$

Then, the area A_1 is obtained as:

$$\begin{aligned} A_1 &= \int_0^\infty [K - y(t)]dt \\ &= -K\frac{\tau_1}{\tau_2-\tau_1}\int_0^\infty e^{-t/\tau_1}dt + K\frac{\tau_2}{\tau_2-\tau_1}\int_0^\infty e^{-t/\tau_2}dt = K(\tau_1+\tau_2) \end{aligned} \tag{A2}$$

The step response of a FO process model is given as:

$$X(s) = \frac{K}{1+(\tau_1+\tau_2)s}\frac{1}{s} \ \Rightarrow\ x(t) = K(1 - e^{-t/(\tau_1+\tau_2)}) \tag{A3}$$

Then, the area A_2 is obtained as:

$$A_2 = \int_0^\infty [K - x(t)]dt = K\int_0^\infty e^{-t/(\tau_1+\tau_2)}dt = K(\tau_1+\tau_2) \tag{A4}$$

□

Remark 1. As demonstrated, the following relations are obtained:

$$\begin{aligned}\mathcal{A}(t\to\infty) &= \int_0^\infty [x(t)-y(t)]dt \\ &= \int_0^\infty [K-y(t)]dt - \int_0^\infty [K-x(t)]dt \\ &= A_1 - A_2 = 0 \;\Rightarrow\; S_1 + S_2 = 0\end{aligned} \tag{A5}$$

Remark 2. After calculating the area A from a *measured* step response, $(\tau_1+\tau_2)$ can be estimated as:

$$\tau_1+\tau_2 = \frac{A}{K} \tag{A6}$$

Theorem 2. The ratio of the areas $\frac{S}{A}$ depends only on the ratio Z of the time constants $\left(Z=\frac{\tau_1}{\tau_2}\right)$.

Proof of Theorem 2.

$$\begin{aligned}S &\triangleq \mathcal{A}(T) = \int_0^T [x(t)-y(t)]dt \\ &= \int_0^T \left[K(1-e^{-t/(\tau_1+\tau_2)}) - K\left(1+\frac{\tau_1}{\tau_2-\tau_1}e^{-t/\tau_1} - \frac{\tau_2}{\tau_2-\tau_1}e^{-t/\tau_2}\right)\right]dt \\ &= K(\tau_1+\tau_2)\left[e^{-T/(\tau_1+\tau_2)} + \frac{\tau_1^2}{\tau_2^2-\tau_1^2}e^{-T/\tau_1} - \frac{\tau_2^2}{\tau_2^2-\tau_1^2}e^{-T/\tau_2}\right]\end{aligned} \tag{A7}$$

Defining

$$\alpha \triangleq \frac{T}{\tau_1+\tau_2}\left(=\frac{KT}{A}\right) \tag{A8}$$

and using (A6), (A7) becomes:

$$S = A\left[e^{-\alpha} - \frac{1}{1-Z^{-2}}e^{-\alpha(1+Z^{-1})} - \frac{1}{1-Z^2}e^{-\alpha(1+Z)}\right] \tag{A9}$$

The following relation for overdamped systems results:

$$\frac{S}{A} = e^{-\alpha}\left[1 - \frac{e^{-\alpha Z}}{1-Z^2} - \frac{e^{-\alpha Z}}{1-Z^{-2}}\right] \tag{A10}$$

Regarding the underdamped systems, $\tau_1 = Me^{-j\varphi}$ and $\tau_2 = Me^{j\varphi}$ are complex conjugated numbers with $0<\varphi<\pi/2$ for stability. The ratio of the time constants then takes the following form:

$$Z \triangleq \frac{\tau_1}{\tau_2} = e^{-j2\varphi}, \text{ so } Z^{-1} = Z^* \text{ (complex conjugated)} \tag{A11}$$

Thus

$$\frac{e^{-\alpha Z^{-1}}}{1-Z^{-2}} = \frac{e^{-\alpha Z^*}}{1-Z^{*2}} = \left(\frac{e^{-\alpha Z}}{1-Z^2}\right)^*, \text{ because } \alpha \text{ is real.} \tag{A12}$$

For underdamped systems, (A10) becomes:

$$\frac{S}{A} = e^{-\alpha}\left[1 - 2Re\frac{e^{-\alpha Z}}{1-Z^2}\right] \tag{A13}$$

□

NOTE: In theory, T can be any value in the range $0<T<T_m$, but the theory is based on a nominal 2^{nd} order system without disturbances. In practice, we select T where the area S is maximum, in order to decrease the effect of all kind of errors on the calculated S (e.g. higher order dynamics, integration errors due to discrete-time, error on the gain K, disturbances on the measured step response).

The final result is a SO model of the form:

$$\frac{K}{(1+\tau_1 s)(1+\tau_2 s)} = \frac{K}{1+\frac{2\zeta}{\omega_n}s+\frac{1}{\omega_n^2}s^2} \tag{A14}$$

It results that:

$$\tau_1+\tau_2 = \frac{2\zeta}{\omega_n} \text{ and } \tau_1\tau_2 = \frac{1}{\omega_n^2} \tag{A15}$$

Consider (A15) to calculate ζ:

$$\zeta = \frac{\tau_1+\tau_2}{2\sqrt{\tau_1\tau_2}} = \frac{Z+1}{2\sqrt{Z}} \tag{A16}$$

It follows that the solution of the quadratic equation (A16) is:

$$Z = (\zeta - \sqrt{\zeta^2-1})^2 \tag{A17}$$

Therefore, there are 2 cases:

1. if $\zeta > 1$, then $Z = (\zeta - \sqrt{\zeta^2-1})^2$ is in the range $0 < Z < 1$. Then, referring to (A10), the solution for ζ is given by:

$$F(\zeta) \triangleq \frac{S}{A} - e^{-\alpha}\left[1 - \frac{e^{-\alpha Z}}{1-Z^2} - \frac{e^{-\alpha Z^{-1}}}{1-Z^{-2}}\right] = 0 \tag{A18}$$

2. if $\zeta < 1$, then $Z = (\zeta - j\sqrt{1-\zeta^2})^2$ is of the form $e^{-j2\varphi}$, with φ in the range $0 < \varphi < \frac{\pi}{2}$, $\varphi = \arccos\zeta$. Then, referring to (A13), the solution for ζ is given by:

$$F(\zeta) \triangleq \frac{S}{A} - e^{-\alpha}\left[1 - 2\mathrm{Re}\left\{\frac{e^{-\alpha Z}}{1-Z^2}\right\}\right] = 0 \tag{A19}$$

A.2 Theoretical proofs - regarding the alternative area introduced in Section 2.3

This section presents the theoretical proofs regarding the computation of the alternative area (17) using a two parts exposition. First the calculation of T_k values is provided, followed by the computation of A_k and $\underline{A}$ areas (ref. Fig. 3).

Calculation of T_k values

The relationship between the standard SO model for oscillatory systems (2) and its complex conjugated poles is:

$$\frac{K\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{Kp\overline{p}}{(s-p)(s-\overline{p})} \tag{A20}$$

where $p = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$ is the complex pole, with the relationships $p+\overline{p} = -2\zeta\omega_n$ and $p\overline{p} = \omega_n^2$.
The general step response for the SO model (A20) is:

$$Y(s) = \frac{Kp\overline{p}}{(s-p)(s-\overline{p})}\frac{1}{s} = K\left(\frac{1}{s} + \frac{\overline{p}}{p-\overline{p}}\frac{1}{s-p} + \frac{p}{\overline{p}-p}\frac{1}{s-\overline{p}}\right) \tag{A21}$$

with the time-domain equivalent:

$$y(t) = K\left(1 + \frac{\overline{p}}{p-\overline{p}}e^{pt} + \frac{p}{\overline{p}-p}e^{\overline{p}t}\right) = K(1+C+\overline{C}) = K(1+2\mathrm{Re}\{C\}) \tag{A22}$$

where $C = \frac{\overline{p}}{p-\overline{p}}e^{pt}$.
From (A22) results that

$$\begin{aligned} &y(t) = K \quad \text{if} \quad \mathrm{Re}\{C\} = 0; \\ &\Rightarrow \mathrm{Re}\left\{\frac{\overline{p}}{p-\overline{p}}e^{pt}\right\} = \mathrm{Re}\left\{\frac{-\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}}{2j\omega_n\sqrt{1-\zeta^2}}e^{-\zeta\omega_n T_k}e^{j\omega_n\sqrt{1-\zeta^2}T_k}\right\} = 0 \end{aligned} \tag{A23}$$

Notice that $\sqrt{1-\zeta^2}$ is real because $\zeta < 1$ for underdamped systems and ω_n is real because is the natural frequency. Hence, after simplifications, (A23) becomes:

$$\mathrm{Re}\left\{\frac{\zeta + j\sqrt{1-\zeta^2}}{j}e^{j\omega_n\sqrt{1-\zeta^2}T_k}\right\} = 0 \Rightarrow \mathrm{Re}\left\{\frac{e^{j\left(\frac{\pi}{2}-\arcsin(\zeta)\right)}}{e^{j\frac{\pi}{2}}}e^{j\omega_n\sqrt{1-\zeta^2}T_k}\right\} = 0 \tag{A24}$$

Further on, using the Euler's formula in (A24) we obtain:

$$\begin{aligned}
&-\frac{\pi}{2} + \left(\frac{\pi}{2} - \arcsin(\zeta)\right) + \omega_n\sqrt{1-\zeta^2}T_k = -\frac{\pi}{2} + k\pi \\
&\Rightarrow T_k = \frac{\arcsin(\zeta) + k\pi - \frac{\pi}{2}}{\omega_n\sqrt{1-\zeta^2}}
\end{aligned} \tag{A25}$$

Using (A25), one can compute the difference between successive T_k values, with $k = 2, 3, \dots$ as follows:

$$T_k - T_{k-1} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{A26}$$

In particular, for $k = 1$ and $T_0 = 0$ we obtain:

$$T_1 - T_0 = \frac{\arcsin(\zeta) + \frac{\pi}{2}}{\omega_n\sqrt{1-\zeta^2}} \tag{A27}$$

Notice that the oscillation period can easily be computed from (A26) as:

$$T_p = 2(T_k - T_{k-1}) = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} \tag{A28}$$

Calculation of A_k and $\underline{A}$ areas

Starting from (17) and using (A22) results that each A_k area is defined as:

$$\begin{aligned}
A_k &= \int_{T_{k-1}}^{T_k} |K - y(t)|dt \\
&= -K\int_{T_{k-1}}^{T_k}\left[\frac{\overline{p}}{p-\overline{p}}e^{pt} + \frac{p}{\overline{p}-p}e^{\overline{p}t}\right]dt \\
&= -2K * \mathrm{Re}\left\{\frac{\overline{p}/p}{p-\overline{p}}\left[e^{pT_k} - e^{pT_{k-1}}\right]\right\}
\end{aligned} \tag{A29}$$

The alternative area $\underline{A} = A_1 - A_2 + A_3 - A_4 + \cdots$ can be computed using (A29) as:

$$\begin{aligned}
\underline{A} &= 2K * \mathrm{Re}\left\{\frac{\overline{p}/p}{\overline{p}-p}[(e^{pT_1} - e^{pT_0}) - (e^{pT_2} - e^{pT_1})\right. \\
&\quad \left. + (e^{pT_3} - e^{pT_2}) - (e^{pT_4} - e^{pT_3}) + \cdots]\right\} \\
&= 2K * \mathrm{Re}\left\{\frac{\overline{p}/p}{\overline{p}-p}[-1 + 2(e^{pT_1} - e^{pT_2} + e^{pT_3} - \cdots)]\right\} \\
&= 2K * \mathrm{Re}\left\{\frac{\overline{p}/p}{\overline{p}-p}[-1 + 2(e^{pT_1} - e^{p(T_1+0.5T_p)}\right. \\
&\quad \left. + e^{p(T_1+T_p)} - e^{p(T_1+1.5T_p)} + \cdots)]\right\} \\
&= 2K * \mathrm{Re}\left\{\frac{\overline{p}/p}{\overline{p}-p}[-1 + 2e^{pT_1}(1 - e^{0.5pT_p}\right. \\
&\quad \left. + e^{pT_p} - e^{1.5pT_p} + \cdots)]\right\}
\end{aligned} \tag{A30}$$

Finally, (A30) can be written in a compact form (using the geometric series formula) as:

$$\underline{A} = 2K * \mathrm{Re}\left\{\frac{\overline{p}/p}{\overline{p}-p}\left(-1 + 2\frac{e^{pT_1}}{1 + e^{0.5pT_p}}\right)\right\} \tag{A31}$$

This result is also given in (18), and is used in the proposed methodology to compute the optimal value for the parameter ζ.

A.3 Summary of the algorithms and Matlab® code

In this section, details regarding how to apply the proposed algorithms, and the Matlab® software corresponding to the procedure detailed in subsection 2.4 are provided.

A.3.1 Summary of the algorithms

Given the measured step response $y(t)$, $0 \leq t \leq T_m$, in discrete time $y(k)$, $k = 0, 1, \ldots, N_s$ and $T_m = N_s T_s$, with T_s the sampling period;
Estimate the static gain K (e.g. by averaging the samples of the steady-state part of the step response).
Apply one of the two algorithms below within the generic procedure of Section 2.4.

1) Overdamped and reasonably underdamped step response

* Calculate the area $A = T_s \sum_{k=0}^{N_s} [K - y(k)]$.
* Calculate the step response $x(k)$, $0 \leq k \leq N_s$ of the FO system $\frac{K}{1+\tau s}$, with $\tau = \frac{A}{K}$.
* Calculate the area signal $\mathcal{A}(k)$ between $x(k)$ and $y(k)$ as:

$$\mathcal{A}(k) = \mathcal{A}(k-1) + T_s[x(k) - y(k)],\ k = 1...N_s, \text{with } \mathcal{A}(0) = 0.$$

* Find $k^* = \arg\max_k |\mathcal{A}(k)|$ and define the time $T = T_s k^*$ and the corresponding area $S = \mathcal{A}(k^*)$.
* Define the function $F(\zeta)$ for $0.5 \leq \zeta \leq 3$ as follows:

$$F(\zeta) = \frac{S}{A} - e^{-\alpha}\left[1 - 2\mathrm{Re}\left\{\frac{e^{-\alpha Z}}{1 - Z^2}\right\}\right], \text{if } 0.5 < \zeta < 1,$$

$$F(\zeta) = \frac{S}{A} - e^{-\alpha}\left[1 - \frac{e^{-\alpha Z}}{1 - Z^2} - \frac{e^{-\alpha Z^{-1}}}{1 - Z^{-2}}\right], \text{if } 1 < \zeta < 3,$$

with $\alpha \stackrel{\Delta}{=} \frac{T}{\tau}$ and $Z \stackrel{\Delta}{=} (\zeta - \sqrt{\zeta^2 - 1})^2$.
* Solution: $\zeta^* = \arg\min_\zeta |F(\zeta)|$ and $\omega_n^* = \frac{2\zeta^*}{\tau}$.

The algorithm involves the calculations of 2 areas and the minimisation of a 1-dimensional function of a single variable, (which is quite simple and which does not face any convergence problems).

2) Highly oscillatory step response

* Estimate the oscillation period T_p (e.g. by detecting the maxima of the integral signal $\mathcal{I}(k) = \mathcal{I}(k-1) + T_s[K - y(k)]$ for $k = 1, \ldots, N_s$, with $\mathcal{I}(0) = 0$).
* Calculate the area $\underline{A} = T_s \sum_{k=0}^{N_s} |K - y(k)|$.
* Define the function $F(\zeta)$ for $0.01 \leq \zeta \leq 0.5$ as follows:

$$F(\zeta) = \underline{A} - 2K\mathrm{Re}\left\{\frac{\bar{p}/p}{\bar{p} - p}\left(-1 + 2\frac{e^{pT_1}}{1 + e^{0.5pT_p}}\right)\right\},$$

with $p = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$ (and $\bar{p}$ the complex conjugated), $\omega_n = \frac{2\pi}{T_p\sqrt{1-\zeta^2}}$ and $T_1 = \frac{\arcsin(\zeta) + \frac{\pi}{2}}{\omega_n\sqrt{1-\zeta^2}}$.
* Solution: $\zeta^* = \arg\min_\zeta |F(\zeta)|$ and $\omega_n^* = \frac{2\pi}{T_p\sqrt{1-\zeta^{*2}}}$.

A.3.2 Matlab® code

```
function SysEst=RDK_SOPDT_FOPDT(StepMeas,Ts,K,Tp,MinDelay,MaxDelay)
% Robust estimation of a SOPDT (or FOPDT) model from
% highly corrupted stepresponse data
% Robin De Keyser - Ghent University, Belgium - Robain.DeKeyser@UGent.be - 070720
% StepMeas: (column) vector containing the measured stepresponse
% Ts: sampling period
% K: static gain
% Tp: oscillation period (only for highly oscillatory stepresponses; otherwise put Tp=0)
% MinDelay: minimum limit for deadtime (put 0 if no better guess)
% MaxDelay: maximum limit for deadtime (put -1 if no better guess)
% SysEst: estimated SOPDT model
%(for FOPDT model: uncomment line 20 and drop the 2nd order term in SysEst)

s=tf('s'); Res1=[]; Res2=[]; NDTmin=round(MinDelay/Ts);
if MaxDelay==-1, NDTmax=length(StepMeas)-2; else NDTmax=round(MaxDelay/Ts); end

if Tp==0
  Zv1=[0.50:0.01:0.99]; Zv2=[1.01:0.01:3.00]; %allows real and complex-conj poles
  %Zv1=[0.50:0.01:0.50]; Zv2=[1.01:0.01:3.00]; %to force real poles
  %Zv1=[0.50:0.01:0.50]; Zv2=[3.00:0.01:3.00]; %to force 1st order (FOPDT)
  Xv1=(Zv1-sqrt(Zv1.^2-1)).^2; Xv2=(Zv2-sqrt(Zv2.^2-1)).^2;
else
  Z=[0.01:0.01:0.50];
  SR=sqrt(1-Z.^2); O=2*pi./(Tp.*SR); p=-Z.*O+j*O.*SR; pb=conj(p);
  T1=(asin(Z)+0.5*pi)./(O.*SR);
  F=((pb./p)./(pb-p)).*(-1+2*exp(p.*T1)./(1+exp(0.5*p*Tp)));
end %if

for NDT=NDTmin:NDTmax
  TauD=NDT*Ts; StepResp=StepMeas(NDT+1:end);
  if Tp==0
   A=Ts*sum(K-StepResp); Tau=A/K; SysRef=K/(1+Tau*s);
   StepRef=step(SysRef,Ts*[0:length(StepResp)-1]');
   Diff=StepRef-StepResp; Area=0;
   for k=2:length(StepResp)
    Area(k)=Area(k-1)+Ts*(Diff(k-1)+Diff(k))/2;
   end
   [Val,Ind]=max(abs(Area)); S=Area(Ind); Alfa=Ts*(Ind-1)/Tau;
   Fv1=S/A-exp(-Alfa).*(1-2*real(exp(-Alfa.*Xv1)./(1-Xv1.^2)));
   Fv2=S/A-exp(-Alfa).*(1-exp(-Alfa.*Xv2)./(1-Xv2.^+2)-exp(-Alfa.*Xv2.^-1)./(1-Xv2.^-2));
   Fv=[Fv1 Fv2];
   [Val,Ind]=min(abs(Fv)); Zv=[Zv1 Zv2]; Zeta=Zv(Ind); Omega=2*Zeta/Tau;
  else
   A=Ts*sum(abs(K-StepResp)); Fz=A-2*K*real(F);
   [Val,Ind]=min(abs(Fz)); Zeta=Z(Ind); Omega=2*pi/(Tp*sqrt(1-Zeta^2));
 end %if
 SysEst=K*exp(-TauD*s)/(1+(2*Zeta/Omega)*s+(1/Omega^2)*s^2);
 StepEst=step(SysEst,Ts*[0:length(StepMeas)-1]');
 Err=StepMeas-StepEst; Cost=Err'*Err;
 Res1=[Res1; Cost TauD]; Res2=[Res2; SysEst];
end %for

[Val,Ind]=min(Res1(:,1)); SysEst=Res2(Ind);
```

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