

ARTICLE COMMENTARY | APRIL 06 2023

## Comment on “Arbitrary amplitude ion-acoustic supersolitons in negative ion plasmas with two-temperature superthermal electrons” [Phys. Plasmas 29, 092101 (2022)]



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*Physics of Plasmas* 30, 044701 (2023)

<https://doi.org/10.1063/5.0138474>



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Cite as: Phys. Plasmas 30, 044701 (2023); doi: 10.1063/5.0138474

Submitted: 12 December 2022 · Accepted: 27 February 2023 ·

Published Online: 6 April 2023



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## AFFILIATIONS

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## ABSTRACT

In a recent paper, “Arbitrary amplitude ion-acoustic supersolitons in negative ion plasmas with two-temperature superthermal electrons” [Phys. Plasmas 29, 092101 (2022)], Kumar and Mishra deal with the study of ion-acoustic solitary and supersolitary waves in a four-component plasma consisting of positive ions and negative ions along with two temperature superthermal electrons. Unfortunately, the reported results are incorrect.

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The results reported in a recent paper, entitled “Arbitrary amplitude ion-acoustic supersolitons in negative ion plasmas with two-temperature superthermal electrons,” by authors Kumar and Mishra<sup>1</sup> are incorrect, as detailed in this comment.

The authors have considered a plasma comprising two cold ion species, one of which is assumed to be positively charged, while the other one can be either positive or negative. (Two negative ion species cannot fulfill charge neutrality in equilibrium.) To include the two possibilities in one description, a normalized charge ratio is introduced,  $\varepsilon = z_2/z_1$ , which is later explicitly stated to be  $\pm 1$ . The structure of the momentum equations (2) and (4) in Ref. 1 dictates the choice  $\varepsilon_z = +1$  for the case of positive and negative ions (respectively, for the bi-ion mixture) or  $\varepsilon_z = -1$  for two positive ion species, a notation that is counter-intuitive and leads to confusion, as shown in the second paragraph of the right column on page 4.

The authors then proceed to discuss the velocity threshold beyond which the positive ion species is no longer real and thus find condition (14) for the upper Mach number  $M$ . Even though Eq. (14) is technically correct, the investigation of the existence conditions in Sec. III is incomplete and the resulting analysis is incorrect. As a matter of

fact, when the other ion species is also positive, there is a second limit on the same side (i.e., restricting the velocity of positive potential pulses)—imposed to preserve the reality of the second species related variables, e.g., the density expression (9) for  $\varepsilon_z = -1$ ; the equivalent condition should have been provided and should also appear as additional curves in Fig. 1. Indeed, for two positive ion species, the mass ratio will determine which upper limit is reached first. Conversely, for a negative ion component, one obtains a limit on the other side (i.e., restricting the velocity of negative potential pulses)—imposed by the reality of, e.g., the density expression (9) for  $\varepsilon_z = +1$ —the expression of which has not been given either. The latter limit should clearly have been taken into account, since Figs. 2 and 6 suggest that negative solitary waves exist, whereas in Figs. 3–5, the obtained pulse-shaped solitary waves are of positive polarity. To sum up, two additional equations should have been given after the condition (14) as essential parts of the discussion, and the corresponding curves are lacking in Fig. 1.

In addition to the aforementioned analytical shortcomings, Figs. 2–7 in the paper<sup>1</sup> are completely incorrect in various ways. In part (a) of Figs. 2–7, the pseudopotential graphs are grievously wrong for the equilibrium conditions at  $V(\phi) = 0$  and  $\phi = 0$ . To obtain

solitons or double layers, the undisturbed conditions to be imposed on the pseudopotential function  $V(\phi, M)$  given by (11) require an unstable maximum at the origin ( $\phi = 0$ ), thus  $V(0, M) = V'(0, M) = 0$  and  $V''(0, M) < 0$  (for all  $M$ ). [Strangely enough, even though the authors have successfully written down the above analytical requirements, in the lines preceding their Eq. (12), and have thus derived the correct superacoustic condition (12) emanating from these requirements, they have failed to produce satisfactory graphs in account of this condition.] The curves in Sec. IV show otherwise, as they pass through zero with an oblique tangent (thus flagrantly violating the above analytical constraints). Therefore, localized solutions (solitons and double layers) would be plainly excluded in the depicted case(s) (if the graphs were correct). Consequently,  $V(\phi, M)$  has only single roots, both at the origin  $\phi = 0$  and at the “soliton amplitude.” This means that, assuming for a minute that these graphs were correct, in reality only *periodic* nonlinear structures would be possible; hence, localized forms, i.e., “solitons” or “supersolitons” cannot exist; furthermore, the “double layer” condition would give a weird kind of shock, zero at  $\phi = 0$  and tending to  $\phi_{DL}$  at infinity.

As a matter of fact, even if periodic structures were within the scope of the article (judging from the figures in Sec. IV), the zero boundary soliton conditions at infinity are not valid for periodic solutions—yet these have been used for the intermediate integrations needed to establish the expression (11) of  $V(\phi)$ . This was recently discussed by Olivier and Verheest<sup>2</sup> for results based on reductive perturbation analysis, but should be equally valid for pseudopotential calculations. In a nutshell, the graphs in Sec. IV fail to reproduce nonlinear solutions of the fluid model correctly, whether these might be of periodic or aperiodic (localized) type.

Furthermore, in part (b) of Figs. 2–7, it is clear that the different “soliton” forms depicted are only defined over bounded intervals of  $\xi$ , unlike soliton domains, which are known to be unbounded. The hodo-graphs in part (c) of these figures do not correspond at the origin of the coordinate axes to what is seen in parts (a) and (b) and thus are also not correct.

Another inconsistency is that in part (b) of these figures, the dotted curves refer to “supersoliton” solution(s), but in parts (a) and (c),

they clearly correspond to “double layers.” In reading Fig. 2(a), for instance, the blue curve is for a supersoliton, and the black dashed curve is for a double layer, quite the opposite of what the authors claim in the figure caption. Another facet of the ambiguity lies in Fig. 2(b), where the blue curve is inside the black one. [It is assumed that the curve coding in (b) is the same as in (a).] In general, in such problems, when there are double layers, one can only get supersolitons beyond the double layers, and the curves should be encountered in this succession (i.e., transiting from double layers to supersolitons) upon increasing Mach number—i.e., amplitude—values. Analogous remarks hold for Figs. 3–7. In Fig. 2(a), the blue curve clearly shows that  $V'(\phi)$  is nonzero at  $\phi = 0$ , and also at  $\phi = -1.21$ . Between those two roots, only periodic motion (waves) is possible, since the force is nonzero in the endpoints, again in contrast to the claim that the blue curve represents a supersoliton. Similar conclusions are drawn from part (a) of Figs. 3–7.

## ACKNOWLEDGMENTS

Authors S.S.V. and I.K. gratefully acknowledge financial support from Khalifa University’s Space and Planetary Science Center (Abu Dhabi, UAE) under Grant No. KU-SPSC-8474000336. I.K. acknowledges the support from KU via CIRA (Competitive Internal Research Award) CIRA-2021-064 (8474000412) and FSU (Faculty Start-Up award) FSU-2021-012 (8474000352) projects.

## AUTHOR DECLARATIONS

### Conflict of Interest

The authors have no conflicts to disclose.

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- <sup>1</sup>K. Kumar and M. Mishra, “Arbitrary amplitude ion-acoustic supersolitons in negative ion plasmas with two-temperature superthermal electrons,” *Phys. Plasmas* **29**, 092101 (2022).
- <sup>2</sup>C. P. Olivier and F. Verheest, “A new reductive perturbation formalism for ion acoustic cnoidal waves,” *J. Plasma Phys.* **88**, 905880601 (2022).